Extrapolate Dictionaries in Celestial CFT and Conformal Collider Physics

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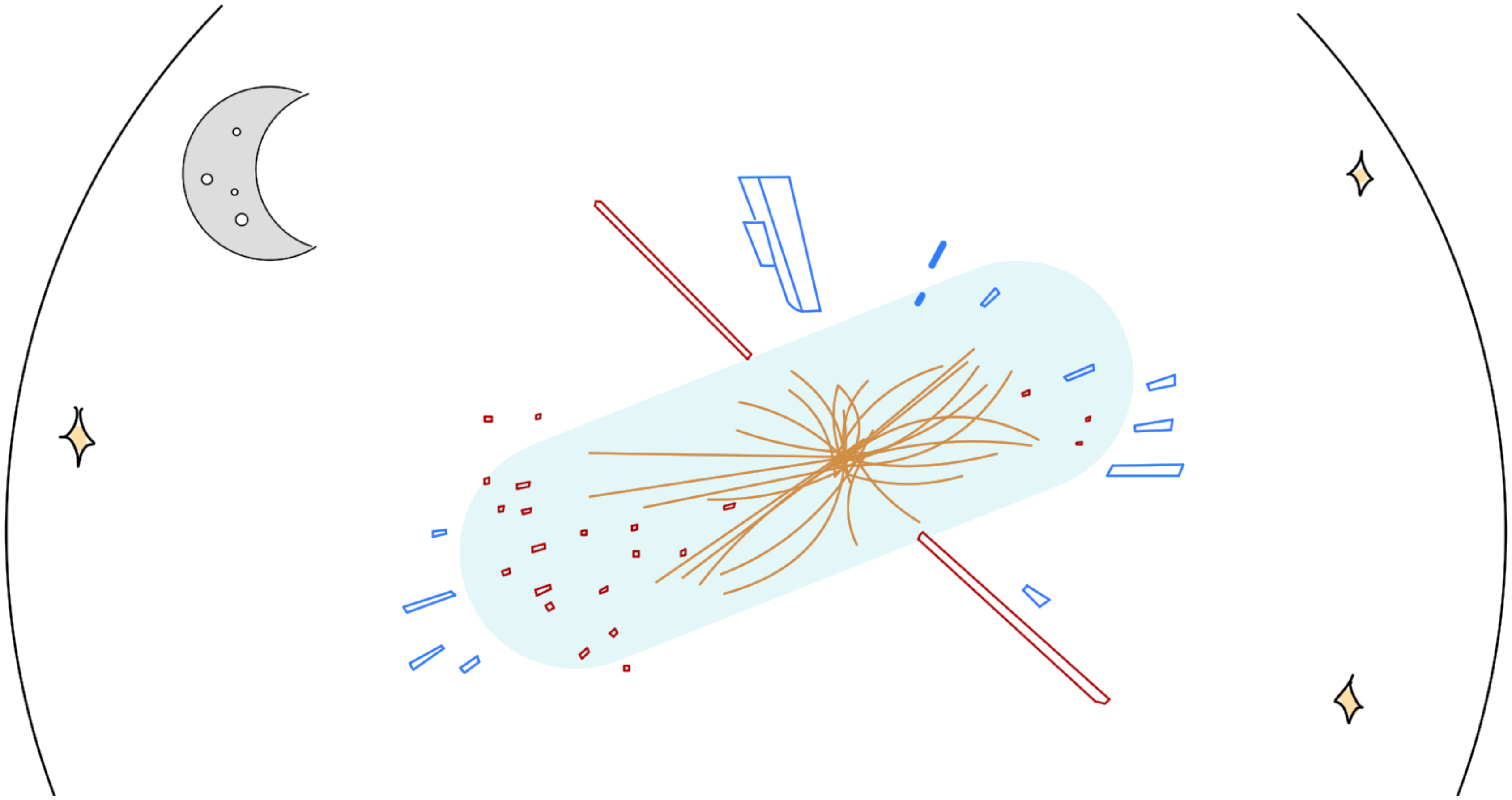
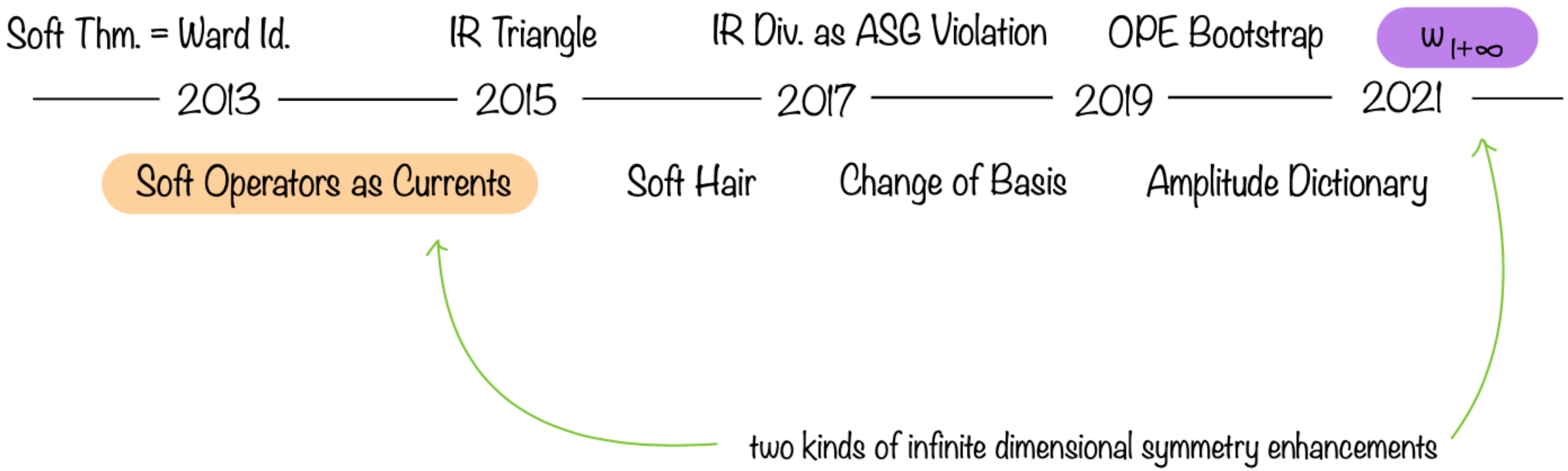
DESY 09/28/23

Celestial Holography proposes a duality between scattering in asymptotically flat spacetimes…



… and a CFT living on the celestial sphere.

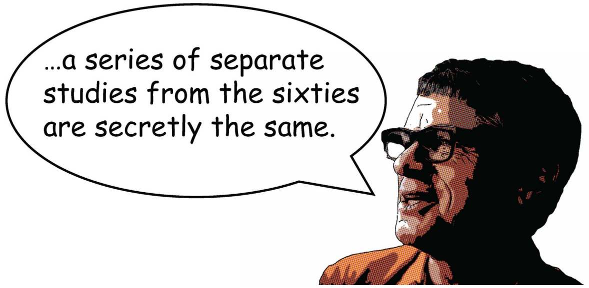
This program evolved from a **bottom-up** approach to flat holography…



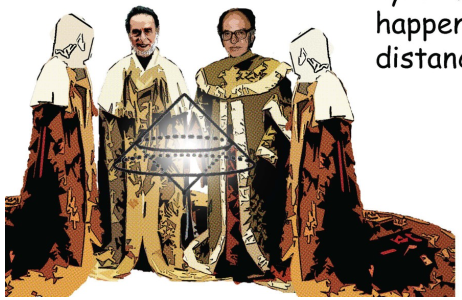
… recognizing **soft theorems as Ward Identities** for asymptotic symmetries and recasting the **soft operators as currents** in a codimension 2 CFT.

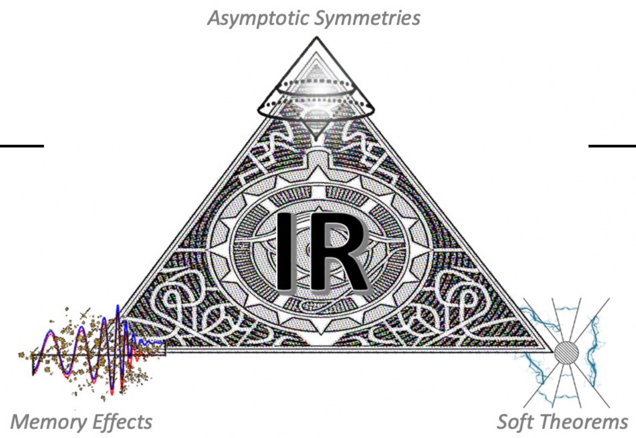
A Collision of Research Programs: Then

Our story starts with Strominger's suggestion that…

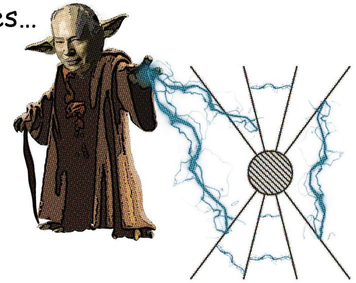


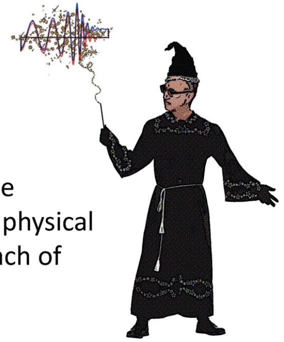
The relativists were systematizing what happens at long distances…





The quantum field theorists were worried about what was going on at low energies…





And, a little later, someone remembered there was a physical observable attached to each of these things….

A Collision of Research Programs: Then

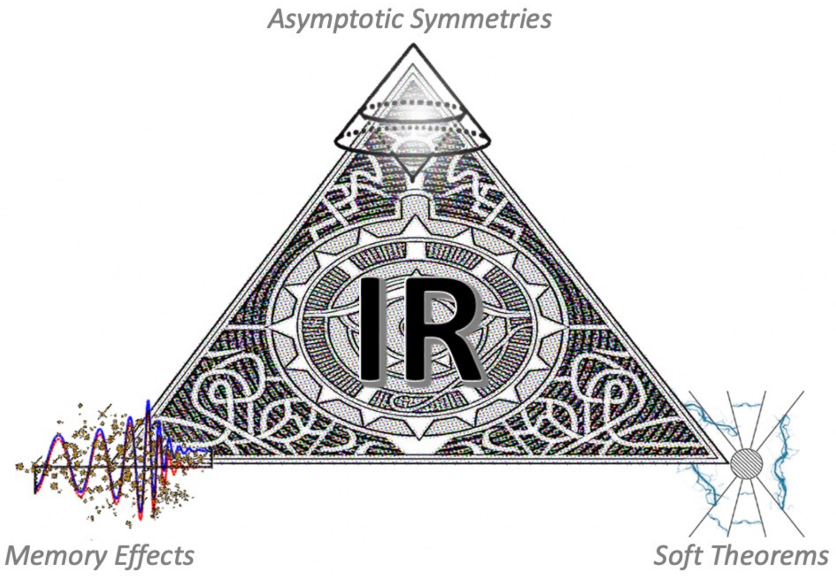
Boost Basis

2D Stress Tensor

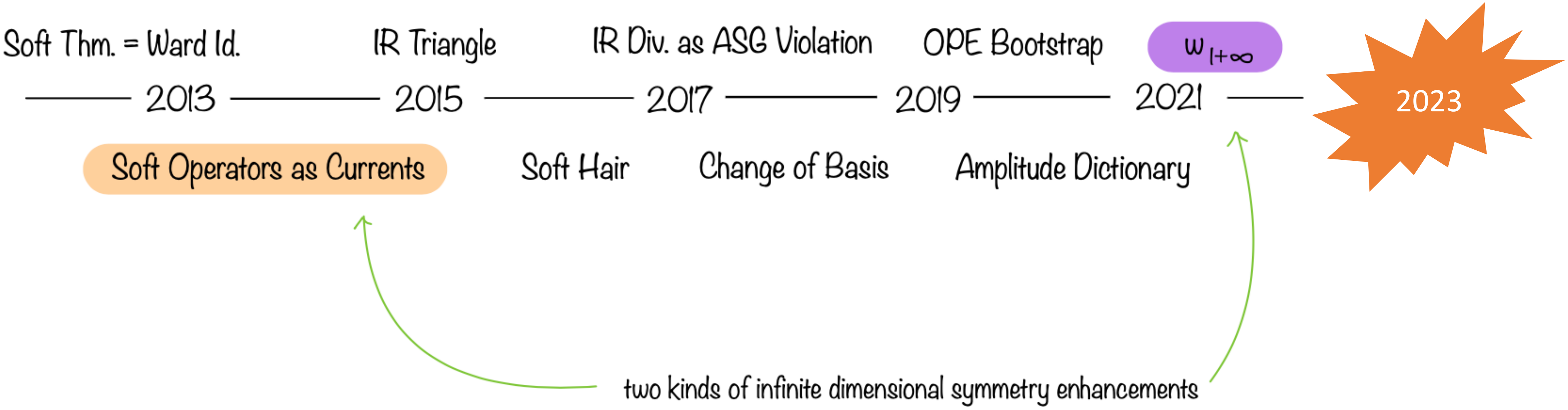
Superrotation
Symmetry

New Subleading
Soft Graviton Thm.

Spin Memory



A Collision of Research Programs: Now



A Collision of Research Programs: Now

conformally soft theorems

holomorphic collinear limits

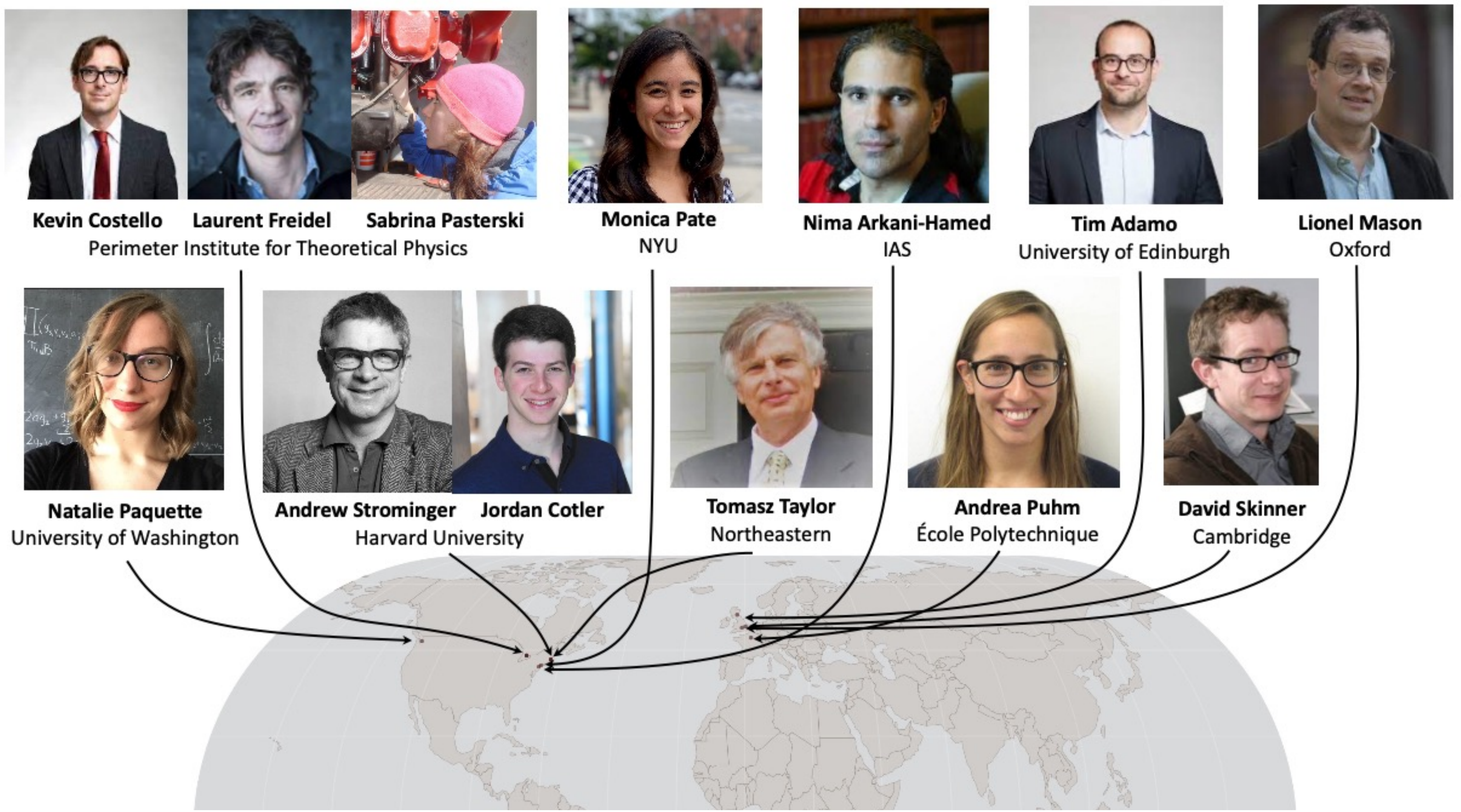
$Lw_{1+\infty}$ in gravity, YM analog, …

Holographic Symmetry Algebras

gravitational phase space

self-dual gravity

twisted holography


Kevin Costello
Laurent Freidel
Sabrina Pasterski
Perimeter Institute for Theoretical Physics
Monica Pate
NYU
Nima Arkani-Hamed
IAS
Tim Adamo
University of Edinburgh
Lionel Mason
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Jordan Cotler
Harvard University
Tomasz Taylor
Northeastern
Andrea Puhm
École Polytechnique
David Skinner
Cambridge


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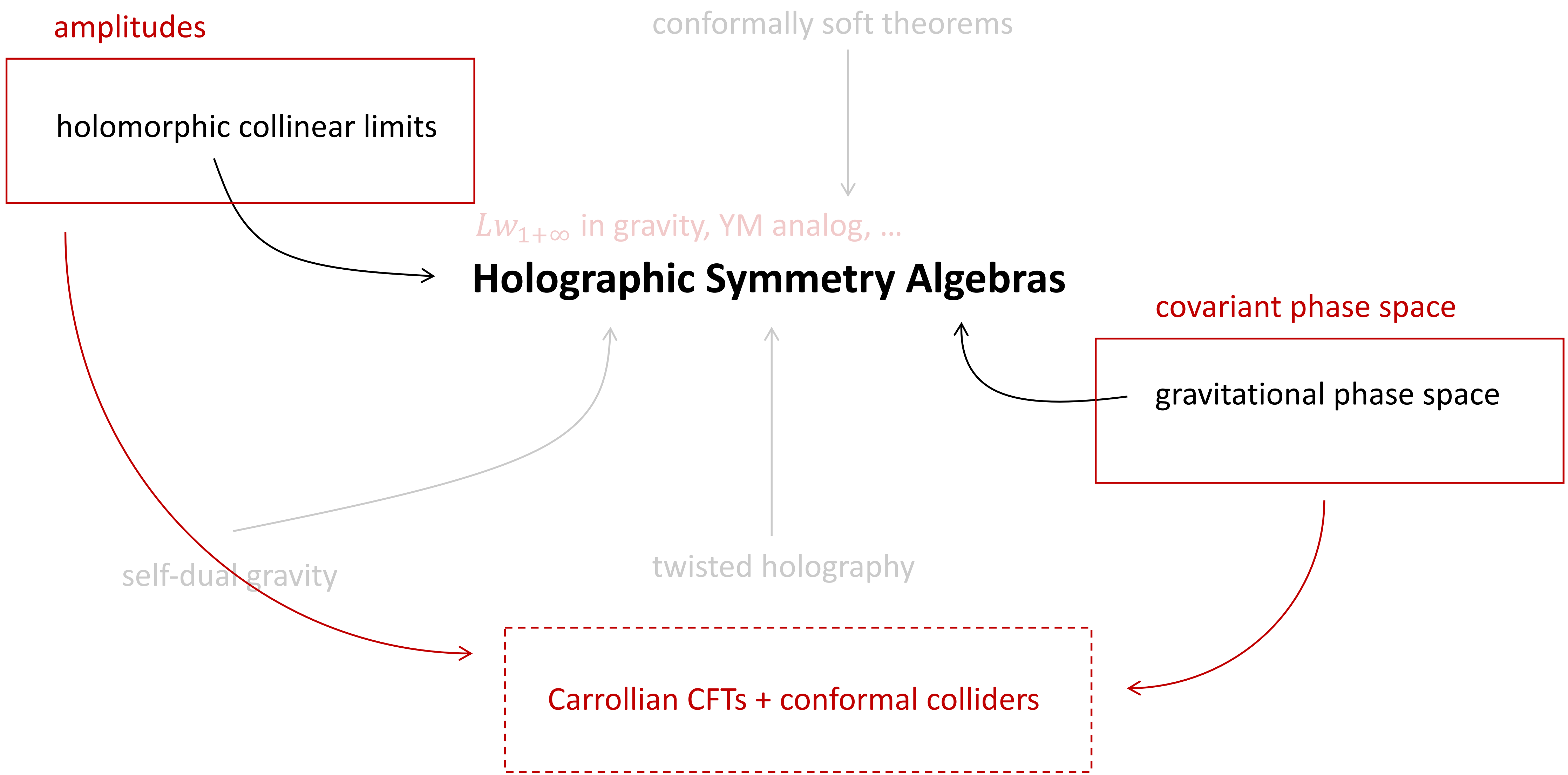
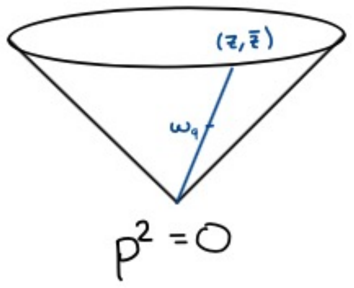
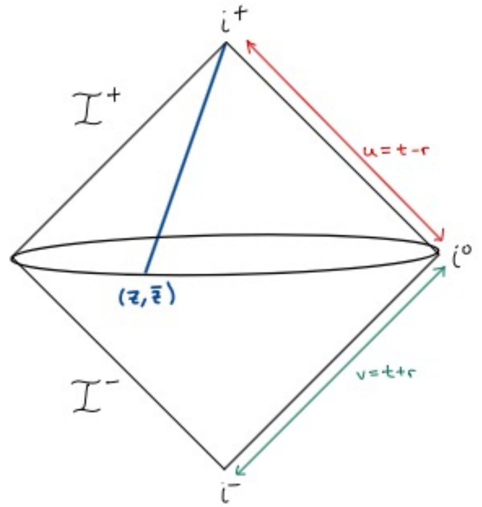

amplitudes
holomorphic collinear limits
conformally soft theorems
$Lw_{1+\infty}$ in gravity, YM analog, …
Holographic Symmetry Algebras
covariant phase space
gravitational phase space
self-dual gravity
twisted holography
Carrollian CFTs + conformal colliders

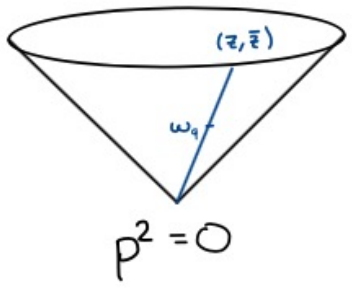


LSZ $\Leftrightarrow$ Extrapolate Dict.

$$\langle out|S|in\rangle_{boost} = \prod_i \lim_{r\to\infty} \int_{-\infty}^{\infty} \mathrm{d}\nu_i\, \nu_i^{-\Delta_i} \langle r\Phi(\nu_1, r, z_1, \bar{z}_1)...r\Phi(\nu_n, r, z_n, \bar{z}_n)\rangle$$

$$\nu = \{u, v\}$$





LSZ ⟺ Extrapolate Dict.

$$\langle out|S|in\rangle_{boost} = \prod_i \lim_{r\to\infty} \int_{-\infty}^{\infty} \mathrm{d}\nu_i \, \nu_i^{-\Delta_i} \, \langle r\Phi(\nu_1, r, z_1, \bar{z}_1) \ldots r\Phi(\nu_n, r, z_n, \bar{z}_n)\rangle$$

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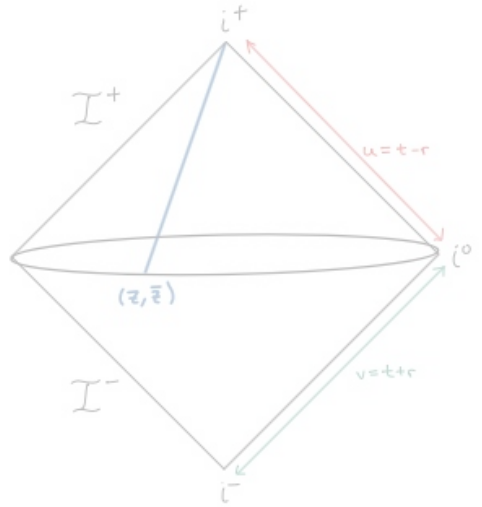
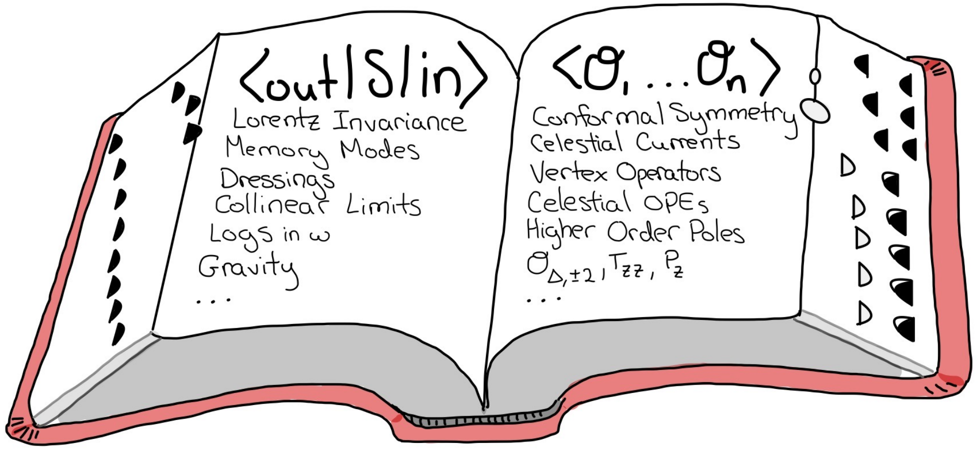


The Celestial Dictionary

4D Lorentz invariance = 2D global conformal symmetry

$$\langle \mathcal{O}^{\pm}_{\Delta_1}(z_1,\bar{z}_1)...\mathcal{O}^{\pm}_{\Delta_n}(z_n,\bar{z}_n)\rangle = \prod_{i=1}^{n}\int_0^\infty d\omega_i \omega_i^{\Delta_i-1}\langle out|\mathcal{S}|in\rangle$$

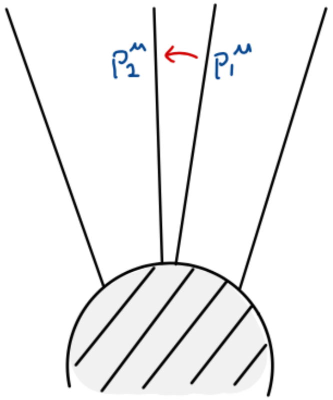
If we go to a boost basis, amplitudes transform as CFT correlators under the Lorentz group.

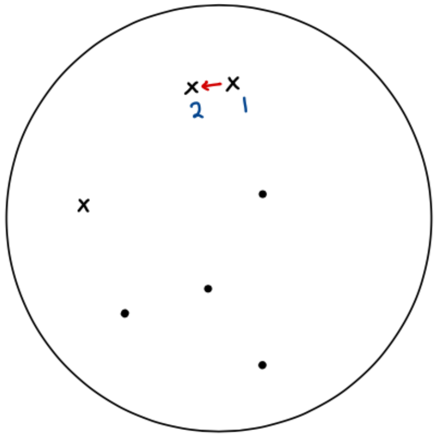


- Single particle operators have a continuous spectrum $\{u, \Delta, \omega\}$

   $\rightarrow$ Can analytically continue to integer Δ

- Collinear limits $\rightarrow$ Celestial OPE of single particle operators





Splitting Function → Celestial OPE → OPE of "Currents"

$$\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,+2}(z_2,\bar{z}_2) \sim -\frac{\kappa}{2}\frac{\bar{z}_{12}}{z_{12}}B(\Delta_1-1,\Delta_2-1)\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2)+\ldots\ ,$$

$$\begin{aligned}\mathcal{O}_{\Delta_1,+2}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2,-2}(z_2,\bar{z}_2) \sim& -\frac{\kappa}{2}\frac{\bar{z}_{12}}{z_{12}}B(\Delta_1-1,\Delta_2+3)\mathcal{O}_{\Delta_1+\Delta_2,-2}(z_2,\bar{z}_2)\\ &-\ \frac{\kappa}{2}\frac{z_{12}}{\bar{z}_{12}}B(\Delta_1+3,\Delta_2-1)\mathcal{O}_{\Delta_1+\Delta_2,+2}(z_2,\bar{z}_2)+\ldots\ ,\end{aligned}$$

For special weights, the SL(2,C) multiplets have primary descendants.

$$H^k(z,\bar{z}) \;:=\; \lim_{\epsilon\to 0} \epsilon\, \mathcal{O}_{k+\epsilon,2}(z,\bar{z}), \quad \Delta = k \;=\; 2,1,0,-1,\ldots$$

Assuming these multiplets shorten, we have

$$H^k(z,\bar{z}) \;=\; \sum_{m=\frac{k-2}{2}}^{\frac{2-k}{2}} \bar{z}^{-\frac{k-2}{2}-m}\, H^k_m(z)\ , \qquad\qquad w^p_n \;=\; \frac{1}{\kappa}\,(p-n-1)!(p+n-1)!\, H_n^{-2p+4}$$

Complexifying the celestial sphere variables and defining a holomorphic commutator

$$[A, B](z) = \frac{1}{2\pi i} \oint_z dw\, A(w) B(z)$$

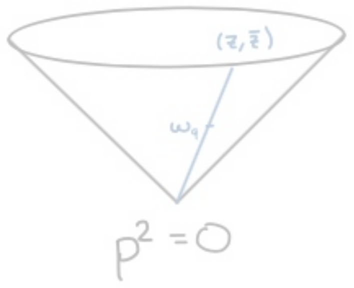
gives a $Lw_{1+\infty}$ symmetry algebra for appropriately rescaled modes

$$\left[w_n^p, w_m^q\right](z) = \left[n(q-1) - m(p-1)\right] w_{m+n}^{p+q-2}(z)$$

Do these symmetries beyond tree level, or the self-dual sector?

Can we realize them in the matter sector?

Can we really complexify the celestial sphere to define these currents?



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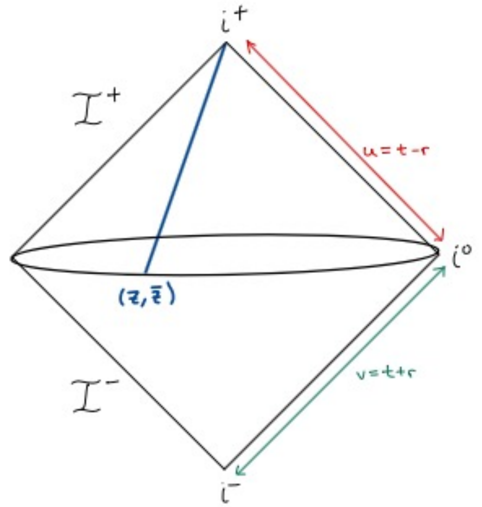


[Freidel, Pranzetti, Raclariu '21] we can try to realize the same holographic symmetry algebras using the standard phase space bracket

$$\big[A(z,\bar{z}),B(w,\bar{w})\big]_{\bar{w}}=\oint_{\bar{w}}\frac{d\bar{z}}{2\pi i}\,A(z,\bar{z})\,B(w,\bar{w})\quad\mapsto\quad i\hbar\,[A,B]=\{A,B\}_{P.B.}$$

From the extrapolate dictionary we have

$$
\begin{aligned}
C(u,z,\bar{z}) \;&=\; \frac{i\kappa}{8\pi^2}\int_0^\infty d\omega\left[a_-^\dagger(\omega,z,\bar{z})e^{i\omega u}-a_+(\omega,z,\bar{z})e^{-i\omega u}\right]\quad,\\
\bar{C}(u,z,\bar{z}) \;&=\; \frac{i\kappa}{8\pi^2}\int_0^\infty d\omega\left[a_+^\dagger(\omega,z,\bar{z})e^{i\omega u}-a_-(\omega,z,\bar{z})e^{-i\omega u}\right]\quad,
\end{aligned}
$$

with phase space bracket

$$
\{\partial_u\bar{C}(u,z,\bar{z}),\,C(u',w,\bar{w})\}\;=\;\frac{\kappa^2}{2}\,\delta(u-u')\,\delta^{(2)}(z-w)
$$

We can realize the w-infinity symmetry

$$\Big[W_s(z,\bar z), W_{s'}(w,\bar w)\Big] = \frac{i}{2}\Big[(s+1)\partial_w \delta^{(2)}(z-w)\, W_{s+s'-1}(z,\bar z) - (s'+1)\partial_z \delta^{(2)}(z-w)\, W_{s+s'-1}(w,\bar w)\Big] \; ,$$

using a tower of $(\Delta, J) = (3, s)$ operators

$$q_s = \partial_u^{-1} D_z\, q_{s-1} + \frac{(s+1)}{2} \partial_u^{-1}\Big(C\, q_{s-2}\Big) \; , \qquad W_s(z,\bar z) = \lim_{u\to-\infty} \hat q_s(u,z,\bar z) \; .$$

These involve a series of terms with more collinear gravitons

$$q_s = \sum_{k=1}^{\max(2,s+1)} q_s^k$$

With the linear and quadratic charges taking the form

$$W_s^1(z,\bar{z}) \;=\; \frac{1}{2}\frac{(-1)^{s+1}}{s!}\int_{-\infty}^{+\infty} du\, u^s\, D_z^{s+2}\,\partial_u\,\bar{C}(u,z,\bar{z})$$

$$W_{s,grav}^2(z,\bar{z}) \;=\; -\frac{1}{4}\sum_{l=0}^{s}(-1)^{s-l}\,\frac{(l+1)}{(s-l)!}\int_{-\infty}^{+\infty} du\, u^{s-l}\, D_z^{s-l}\left[C(u,z,\bar{z})\, D_z^l \partial_u^{2-l}\bar{C}(u,z,\bar{z})\right]$$

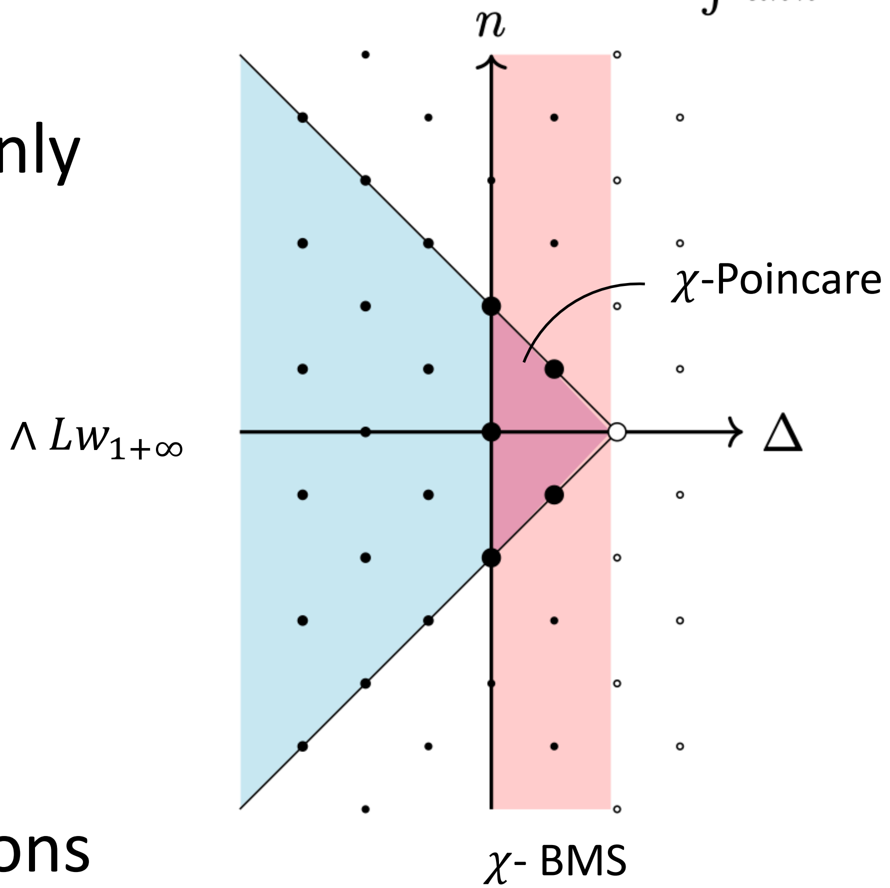
$\oint dz z^{n+\frac{\Delta}{2}-2}$

- We can realize the BMS subalgebra using only linear and quadratic modes.

$$Q^{\mathcal{I}^+} = Q_S^{\mathcal{I}^+} + Q_H^{\mathcal{I}^+}$$

- We can realize the wedge subalgebra using only quadratic operators.

- We need higher particle ($k \geq 3$) contributions to go outside the wedge.

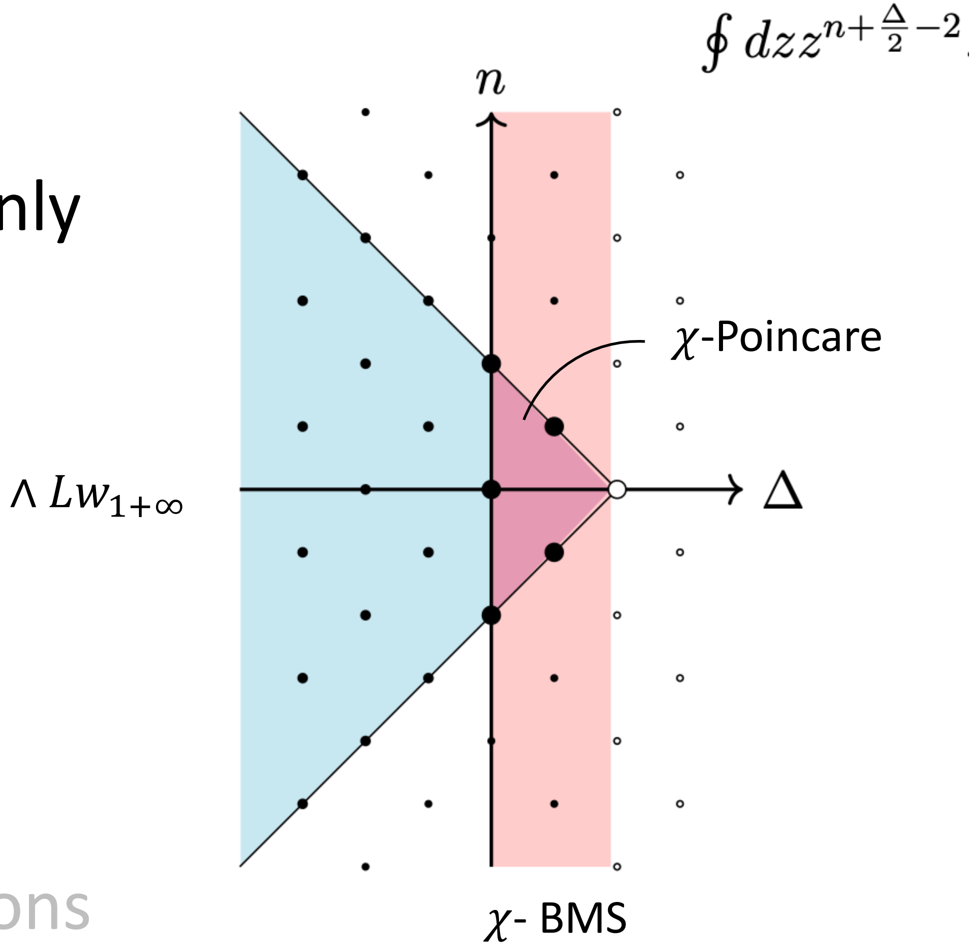


- We can realize the BMS subalgebra using only ~~linear and~~ quadratic modes.

$$Q_H^{\mathcal{I}^+} = Q_{H,grav}^{\mathcal{I}^+} + Q_{H,matter}^{\mathcal{I}^+}$$

- We can realize the wedge subalgebra using only quadratic operators.

- We need higher particle ($k \geq 3$) contributions to go outside the wedge.



The same is true for matter fields!

- BMS symmetry of light ray supported operators

$$\text{ex.} \quad Q^{\mathcal{I}^+}_{H,matter} \qquad \Leftrightarrow \qquad \mathcal{E}(z,\bar{z}) \;=\; \lim_{r\to\infty} r^2 \int du\, T_{uu}(u,r,z,\bar{z})$$

- $\wedge\, Lw_{1+\infty}$ realization for light sheet supported operators

  $\rightarrow$ Need mixed graviton/matter to realize $Lw_{1+\infty}$ with light ray operators

- These phase space representations amount to symmetries of the free m=0 theory

$$\langle out|Q^+[\xi]\mathcal{S} - \mathcal{S}Q^-[\xi]|in\rangle$$

- Free m=0 matter sector is a special case of the [Cordova Shao ’18] BMS result for general CFT_4.

Cordova and Shao '18 examined light-ray operators in unitary CFTs operators restricted to a light-sheet

$$\mathcal{T}(f) \equiv \int d^{d-2}x^{\perp} f(x^{\perp})\mathcal{E}(x^{\perp}) \ ,$$
$$\mathcal{R}(Y^A) \equiv \int d^{d-2}x^{\perp} Y^A(x^{\perp})\mathcal{N}_A(x^{\perp}) + \frac{1}{d-2}\int d^{d-2}x^{\perp}\partial_A Y^A(x^{\perp})\mathcal{K}(x^{\perp}) \ .$$

And found that one could generically construct a representation of the BMS algebra starting from the bulk matter stress tensor

$$[\mathcal{T}(f_1), \mathcal{T}(f_2)] = 0 \,,$$
$$[\mathcal{T}(f), \mathcal{R}(Y^A)] = i\mathcal{T}(g) \,, \qquad g = \frac{1}{d-2} f\partial_A Y^A - Y^A \partial_A f \,,$$
$$[\mathcal{R}(Y_1^A), \mathcal{R}(Y_2^A)] = i\mathcal{R}(Y_3^A) \,, \qquad Y_3^A = Y_1^B \partial_B Y_2^A - Y_2^B \partial_B Y_1^A \,.$$

All these light-ray supported operators are local on the celestial sphere. Seemingly by accident*

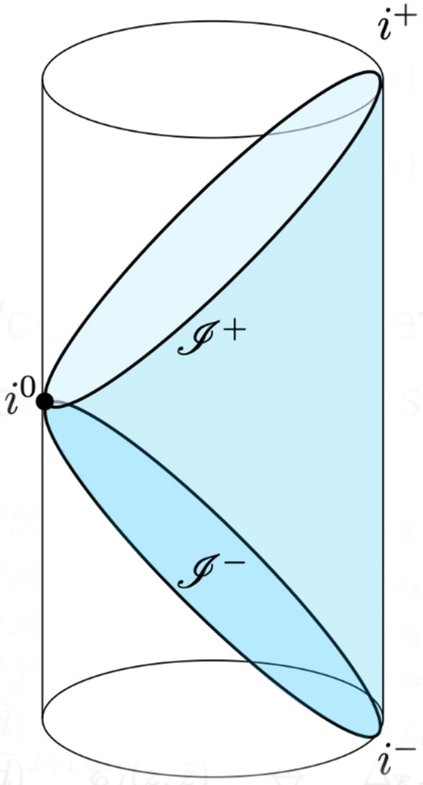
$$\mathscr{E}(z,\bar z) = \lim_{r\to\infty} r^2 \int du\, T_{uu}(u,r,z,\bar z)\ ,$$
$$\mathscr{K}(z,\bar z) = \lim_{r\to\infty} r^2 \int du\, u\, T_{uu}(u,r,z,\bar z)\ ,$$
$$\mathscr{N}_A(z,\bar z) = \lim_{r\to\infty} r^2 \int du\, T_{uA}(u,r,z,\bar z)\ ,$$

$$\mathscr{J}^a(z,\bar z) = \lim_{r\to\infty} r^2 \int du\, j^a_u(u,r,z,\bar z)\ ,$$
$$\mathscr{L}_n(z,\bar z) = \lim_{r\to\infty} r^2 \int du\, u^{n+2}\, T_{uu}(u,r,z,\bar z)\ ,$$
$$\mathscr{E}_J(z,\bar z) = \lim_{r\to\infty} r^2 \int du\ :\partial_u\phi\, \partial_u^{J-1}\phi:(u,r,z,\bar z)\ ,$$

these matter operators, and similar 'detector' [Belin et al '11; Kravchuk Simmons-Duffin '18] generalizations are themselves celestial primaries.

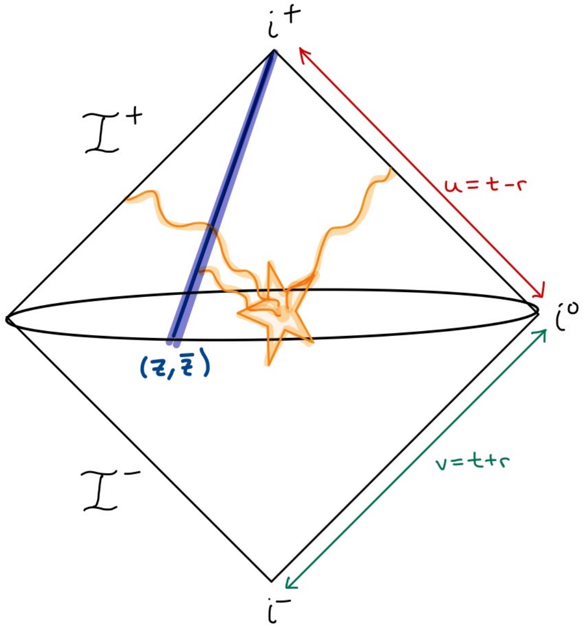
$$\begin{array}{rclclcl}
\mathscr{K}(z',\bar z') &=& (cz+d)^2\,(\bar c\bar z+\bar d)^2\,\mathscr{K}(z,\bar z) & \leftrightarrow & \Delta_{\mathscr{K}} = 2, & & J_{\mathscr{K}} = 0\ ,\\
\mathscr{N}_z(z',\bar z') &=& (cz+d)^4\,(\bar c\bar z+\bar d)^2\,\mathscr{N}_z(z,\bar z) & \leftrightarrow & \Delta_{\mathscr{N}_z} = 3, & & J_{\mathscr{N}_z} = 1\ ,\\
\mathscr{N}_{\bar z}(z',\bar z') &=& (cz+d)^2\,(\bar c\bar z+\bar d)^4\,\mathscr{N}_{\bar z}(z,\bar z) & \leftrightarrow & \Delta_{\mathscr{N}_{\bar z}} = 3, & & J_{\mathscr{N}_{\bar z}} = -1\ ,\\
\mathscr{J}^a(z',\bar z') &=& (cz+d)^2\,(\bar c\bar z+\bar d)^2\,\mathscr{J}^a(z,\bar z) & \leftrightarrow & \Delta_{\mathscr{J}^a} = 2, & & J_{\mathscr{J}^a} = 0\ ,\\
\mathscr{L}_n(z',\bar z') &=& (cz+d)^{1-n}\,(\bar c\bar z+\bar d)^{1-n}\,\mathscr{L}_n(z,\bar z) & \leftrightarrow & \Delta_{\mathscr{L}_n} = 1-n, & & J_{\mathscr{L}_n} = 0\ ,\\
\mathscr{E}_J(z',\bar z') &=& (cz+d)^{J+1}\,(\bar c\bar z+\bar d)^{J+1}\,\mathscr{E}_J(z,\bar z) & \leftrightarrow & \Delta_{\mathscr{E}_J} = 1+J, & & J_{\mathscr{E}_J} = 0\ .
\end{array}$$

All these light-ray supported operators are local on the celestial sphere. Seemingly by accident*



* or not: the subalgebra of CFT_4 preserving Scri is Poincare + Dilatations; primaries at i^0 – Including Light-transformed ones – are zero energy states, which form a closed sector

Conformal collider physics applies CFT techniques and insights from AdS/CFT to compute observables relevant to particle experiments.



The main objects – ANEC operators and their generalizations – appear in precision tests of QCD, in QI settings, to bound anomaly coefficients, and to identify good observables.

- The celestial and conformal collider literature both look at 4D correlation functions of operators that produce boost eigenstates

CCFT

$$\langle \mathcal{O}^{\pm}_{\Delta_1}(z_1, \bar{z}_1) \ldots \mathcal{O}^{\pm}_{\Delta_n}(z_n, \bar{z}_n) \rangle$$

Single Particle

Exclusive

vs

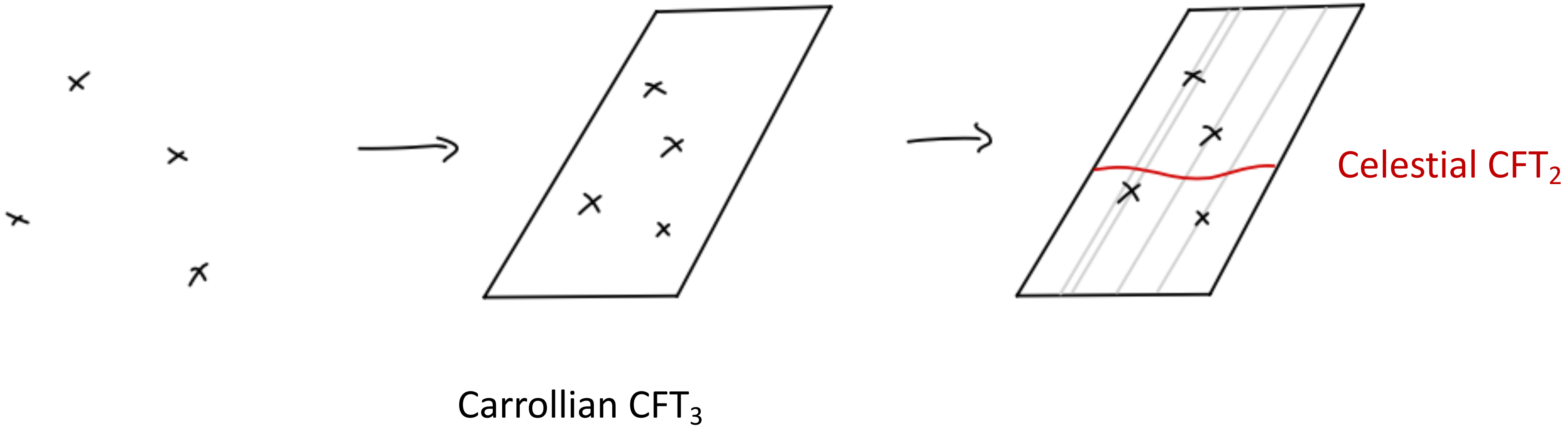
Conformal Colliders

$$\frac{\langle 0|\mathcal{O}^\dagger \mathcal{E}(\theta_1) \cdots \mathcal{E}(\theta_n) \mathcal{O}|0\rangle}{\langle 0|\mathcal{O}^\dagger \mathcal{O}|0\rangle}$$

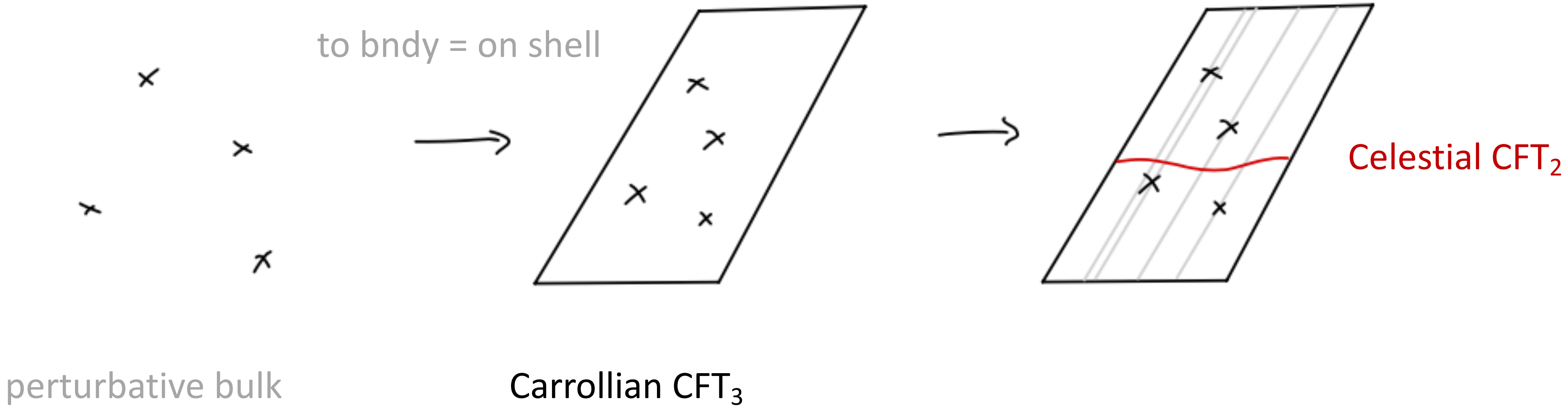
2-Particle

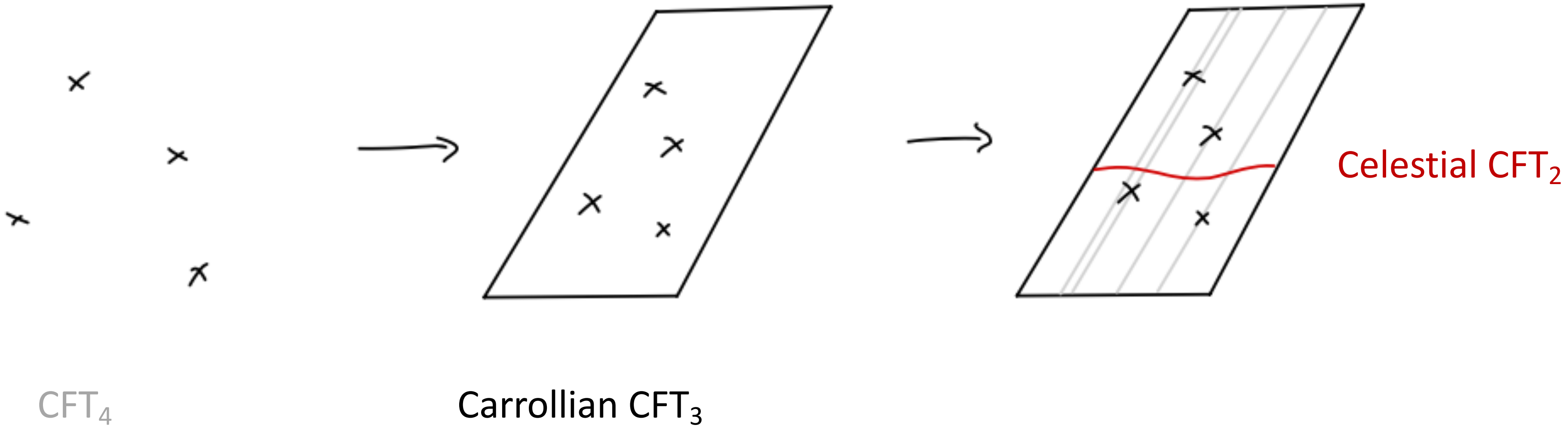
Inclusive

- Both are interested in the Celestial OPE.



Carrollian CFT_3
Celestial CFT_2


to bndy = on shell
perturbative bulk
Carrollian CFT$_3$
Celestial CFT$_2$



CFT_4
Carrollian CFT_3
Celestial CFT_2

Let’s close with three applications where this perspective is useful

- CCFT Spectrum (w/ J. Kulp)
- CCFT OPE (w/ Y. Hu + A. Ball)
- Relating CCFT Extrapolate Dictionaries (w/ E. Jørstad + A. Sharma)

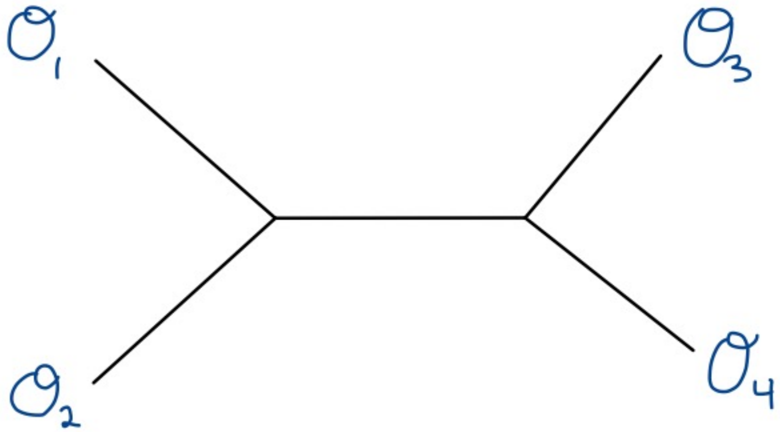
Fock Space ↔ 4D Hilbert Space ↔ 2D States ↔ 2D Operators

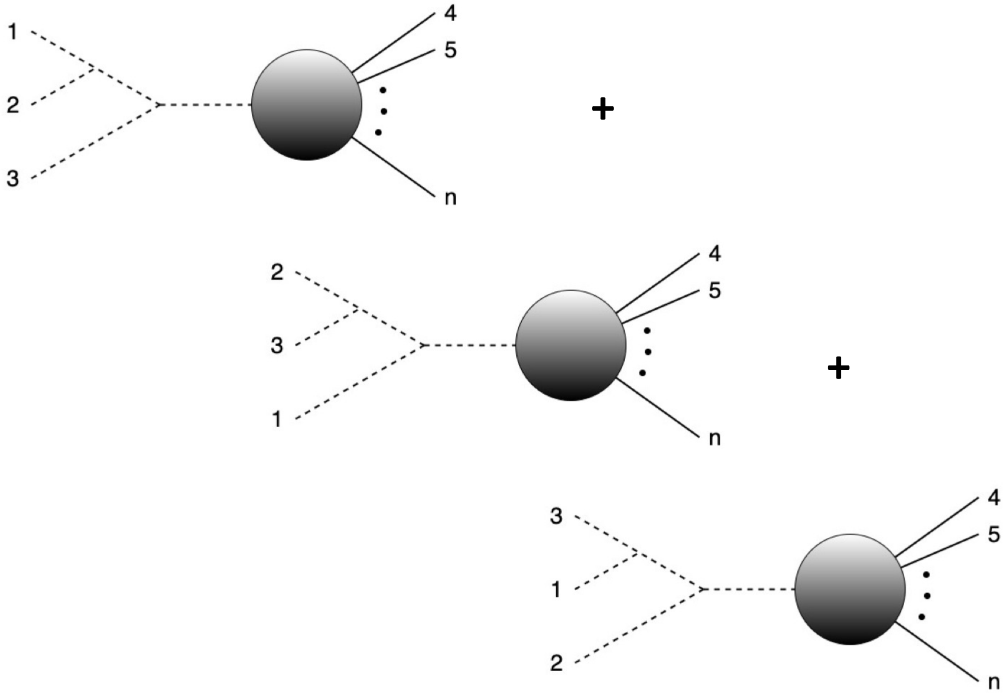
$$\mathcal{O}_\Delta(z,\bar{z}) \equiv \int_{-\infty}^{\infty} \mathrm{d}u\, u^{-\Delta} \lim_{r\to\infty} \left[r^\delta \Phi(u,r,z,\bar{z}) \right]$$

$$:\mathcal{O}^{(\rho)}\mathcal{O}:_\Delta(z,\bar{z}) \;\equiv\; \int_0^\infty d\omega\, \omega^{\Delta-\rho-1} \int_0^\omega d\omega_1\, \omega_1^{\rho-1}\, a^\dagger(\omega_1,z,\bar{z}) a^\dagger(\omega-\omega_1,z,\bar{z})$$

...

Celestial OPE vs Feynman Diagrams





Celestial OPE vs Feynman Diagrams

$$\mathcal{O}_{\Delta_1}(z_1,\bar{z}_1)\mathcal{O}_{\Delta_2}(z_2,\bar{z}_2)\mathcal{O}_{\Delta_3}(z_3,\bar{z}_3) \;\sim\; \left(\frac{1}{z_{13}z_{23}\bar{z}_{13}\bar{z}_{23}}\mathcal{C}_1 + \frac{(z_{23}\bar{z}_{23})^{\Delta_1-2}}{(z_{13}\bar{z}_{13})^{\Delta_1}}\mathcal{C}_2\right)\mathcal{O}_{\Delta_1+\Delta_2+\Delta_3-4}(z_3,\bar{z}_3)$$

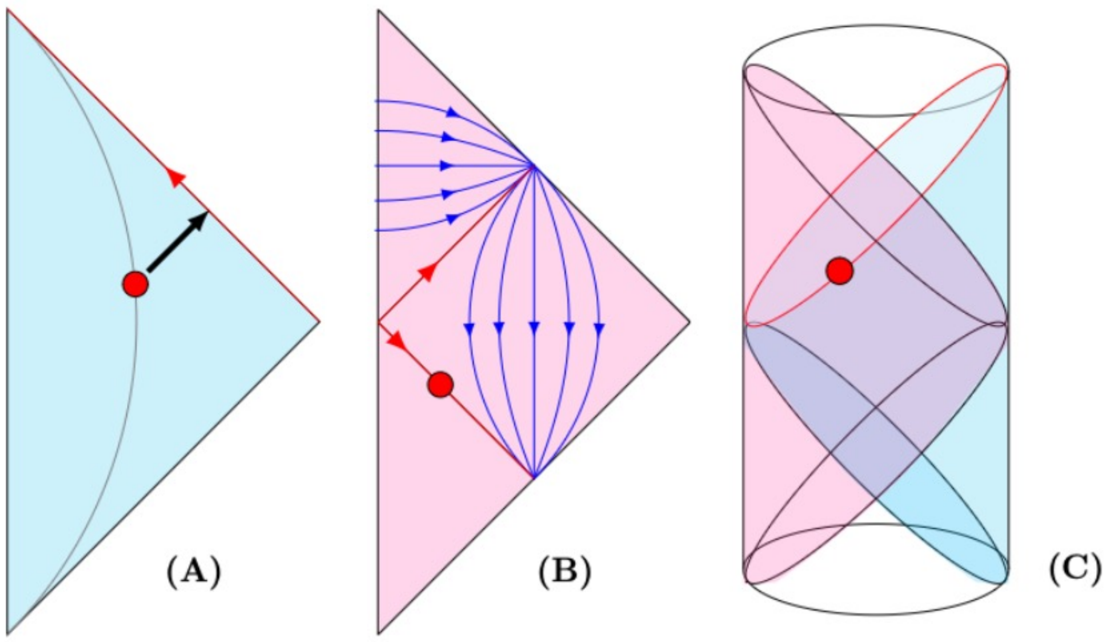
$$\mathcal{O}_{\Delta_2}(z_2,\bar{z}_2)\mathcal{O}_{\Delta_3}(z_3,\bar{z}_3) \;\sim\; \int d\Delta\, \frac{{C_{\Delta_2,\Delta_3}}^{\Delta}}{(z_{23}\bar{z}_{23})^{\frac{1}{2}(\Delta_2+\Delta_3-\Delta)}}\,\mathcal{O}_{\Delta}(z_3,\bar{z}_3) + \int d\Delta\, d\sigma\, \frac{{C_{\Delta_2,\Delta_3}}^{(\Delta,\sigma)}}{(z_{23}\bar{z}_{23})^{\frac{1}{2}(\Delta_2+\Delta_3-\Delta)}}\,\mathcal{R}^{\sigma}_{\Delta}(z_3,\bar{z}_3)$$

In those examples the 4D uplift help us understand the local behavior when two operators approach each other on the celestial sphere.

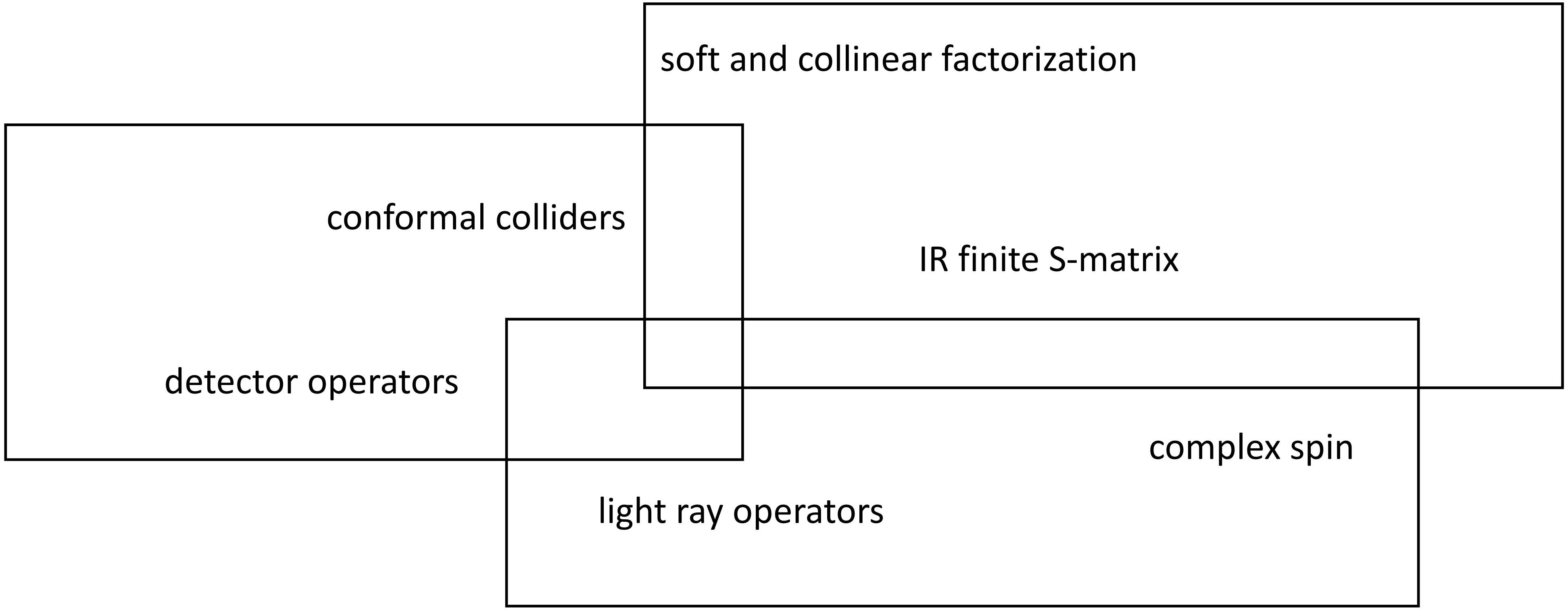
Uplifting to the Einstein cylinder vs the Poincare patch further relates:

→ 2D Shadows & 4D Inversions

→ different CCFT Dictionaries

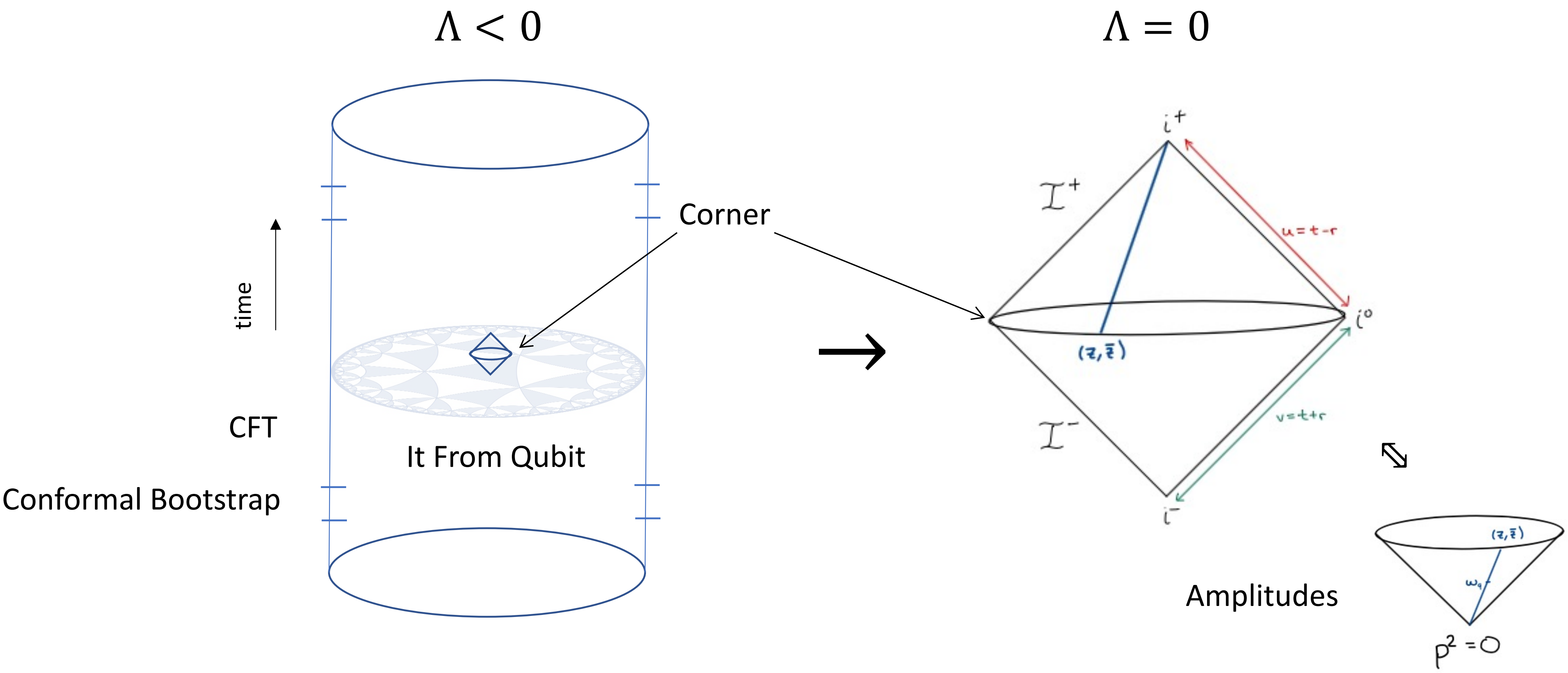


Conformal colliders are just one of many other ventures we can make direct contact with…



… just starting from the kinematics!

A Collision of Research Programs: Soon



A Collision of Research Programs: Soon

Amplitudes: mathematical structures → consistency → <u>S-matrix</u>

Conformal Bootstrap: <u>conformal symmetry</u> + <u>OPE</u> → space of CFTs

Quantum Gravity:

(It from Qubit) entanglement → spacetime

(Corner) <u>symmetries</u> → Hilbert space

Amplitudes

soft and collinear factorization

It From Qubit

conformal colliders

IR finite S-matrix

detector operators

complex spin

light ray operators

Bootstrap


EFFECTIVE FIELD THEORY
CHERN-SIMONS THEORY
ADS/CFT CORRESPONDENCE
COSMOLOGY
QED
RELATIVITY
AMPLITUDES
BLACK HOLES
SOFT THEOREMS
MEMORY
STRING THEORY
CONFORMAL FIELD THEORY
TWISTOR THEORY
QED
QUANTUM FIELDS
GENERAL RELATIVITY
GRAVITATION