Tensor models for chaotic CFT

Julian Sonner

DESY theory workshop
29 September 2023

Introduction and background

The plan

I Matrix models for quantum chaos and gravity

II An ETH matrix model (e.g. JT + matter)

III Statistics of crossing and averaging CFTs

IV A tensor model for chaotic CFT

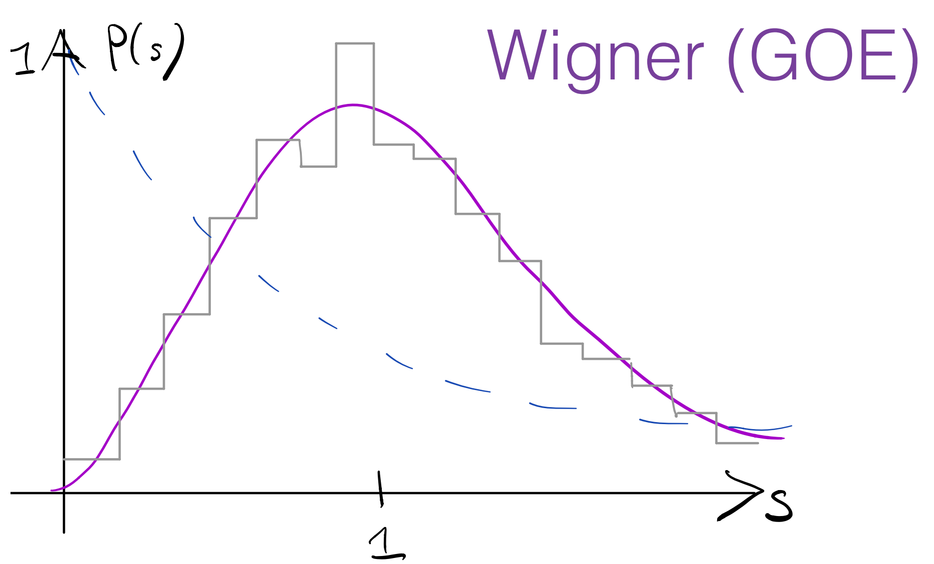
2209.02130 & 2209.02131 (PRX '23) with [**Jafferis, Kolchmeyer, Mukhametzhanov**]
and 2308.03829 with [**Belin, de Boer, Jafferis & Nayak**]

Ia) Matrix Models and Quantum Chaos

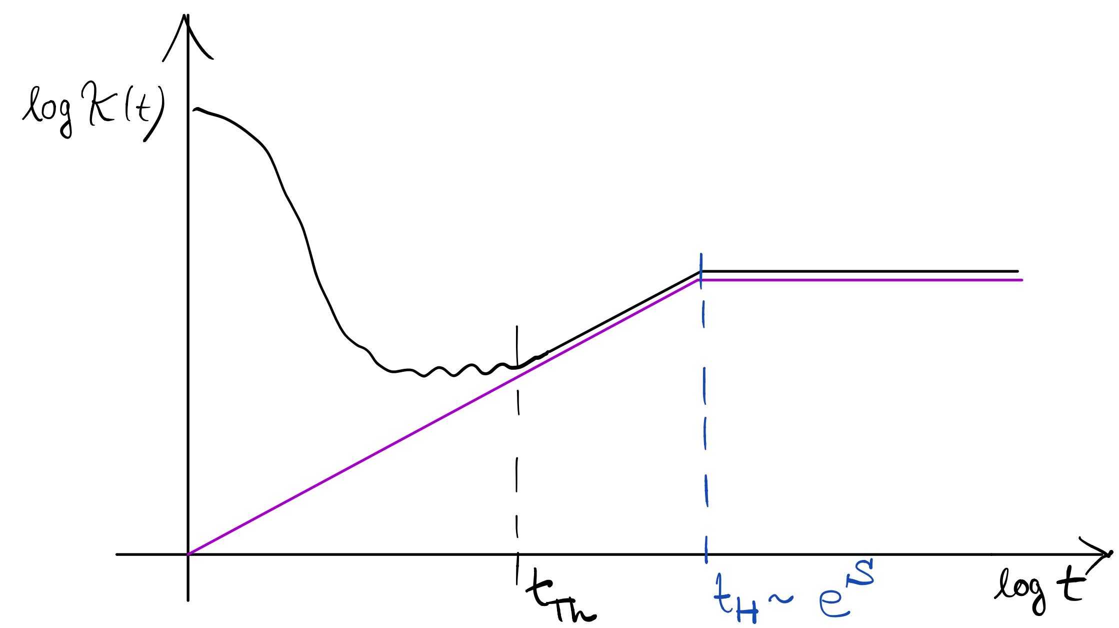
Matrix models = random matrix theory (RMT) = matrix integrals
[Wigner, Dyson,…]

Quantum chaotic H(amiltonian) $\cong$ RMT

Exemplified, e.g., by spectral probes:



2-level correlation



Spectral from factor

Chaotic Hamiltonians show RMT statistics (Gaussian, in absence of further inputs / information)

Ib) operators: Eigenstate Thermalization (ETH)

Attempts to answer: How do unitary quantum systems thermalize?

$$\langle i|\mathcal{O}|j\rangle = \overline{\mathcal{O}}(\overline{E})\delta_{ij} + e^{-S/2} f(E_i, E_j) R_{ij}$$

[Deutsch; Srednicki,...]

smooth function
theory dept.

random matrix
(Gaussian, for now)

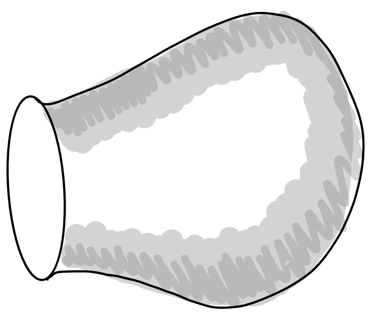
Smooth function of average energy

Goal: fit ETH and RMT into a joint framework, the "ETH matrix model"

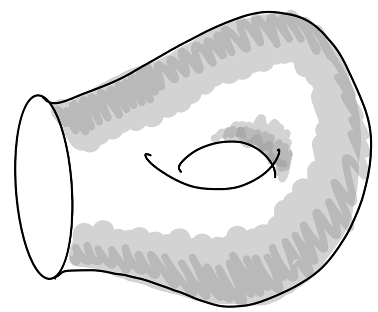
Remark: (universal) emergence of RMT and operator statistics can be understood in terms of a Goldstone EFT: [Wegner; Efetov; Altland, JS]

II) Random matrices and quantum gravity

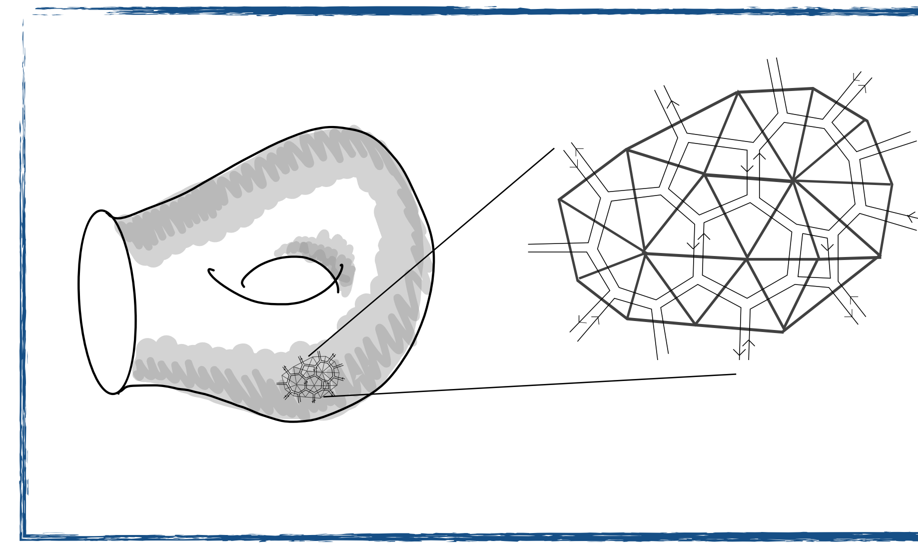
2D gravity can be defined from random triangulations of surfaces:



$+ \; e^{-S_0}$



$+ \cdots$



[Kazakov; Gross, Migdal; Moore, Shenker;… SSS]

$$\sum_{\text{top}} \int \mathcal{D}h e^{-S[h]} \quad \leftrightarrow \quad \int d\mu[H] e^{-\mathrm{tr} V(H;\lambda,N)}$$

RMT = discrete triangulation of 2D universes, continuum gravity path integral emerges from double-scaling limit

Recent developments suggest unity of points I & II

[SSS; Altland, Bagrets, JS, Nayak; Johnson,….]

A few motivating questions:

I What is the Wigner ensemble for a (holographic) CFT?

II Can we establish (in controlled examples) a connection to gravity?

III More insight about gravity as an ensemble of quantum theories?

An ETH matrix model

[Jafferis, Kolchmeyer, Mukhametzhanov, JS, PRX ’23]

De-Gaussing ETH

It is usually said that R_{ij} is a Gaussian random matrix, but it cannot be so

- Would lead to trivial OTO 2n-point functions, i.e. fail to capture Lyapunov-type chaos
- More generally, fails to produce crossing-invariant 4-pt function

$$\vdots$$

For example, to capture 4pt OTO (Lyapunov exponent), need

$$\overline{R_{ij}R_{jk}R_{kl}R_{li}}\Big|_{\text{conn}} = e^{-3S(\overline{E})} g^{(4)}(E_1, E_2, E_3, E_4)$$

Contributes to OTOC at O(1) due to Hilbert-space sums, determines Lyapunov exponent

[JS, Vielma; Foini, Kurchan, Murthy, Srednicki]

Packaging as a (two) matrix model

We have two inputs:

- Energy-level statistics: leads to random matrix H_{ij}
- Operator matrix-element statistics: leads to random matrix R_{ij}

$$\mathcal{Z} = \int d\mu[H] d\mu[\mathcal{O}] e^{-\mathrm{tr} V(H,O)}$$

In energy eigenbasis we have

$$V = -\sum_i \underbrace{V(E_i)}_{\text{Determined by matching } \boldsymbol{\rho}(E)} - e^{S_0} \sum_{i,j} \tfrac{1}{2} \underbrace{F_{ij}}_{\text{“Gaussian ETH”}} \mathcal{O}_{ij} \mathcal{O}_{ji} + e^{S_0} \sum_{i_1, \dots i_n} \underbrace{G^{(n)}_{i_1 \dots i_n}}_{\text{Determined by matching hierarchy of } g^{(n)}} \mathcal{O}_{i_1 i_2} \cdots \mathcal{O}_{i_n i_1}$$

The ETH matrix model

Comments:

- V(H) determines $\boldsymbol{\rho}(E)$, $\boldsymbol{\rho}^{(2)}(E, E')$,… gives connected energy correlations

- F_{ij}, $G^{(n)}{}_{ij}$…. non-Gaussianities of operator statistics

Like in standard ETH, these are free functions: how to determine them?

Input more knowledge by imposing constraints:

- EFT of quantum chaos (quantum-chaos universality)

- Determine non-Gaussianities by imposing additional constraints (e.g. modular crossing, s/t crossing,…). First example: JT + matter

[Jafferis, Kolchmeyer, Mukhametzhanov, JS, PRX '23]

⋮

Approximate CFTs

[de Boer, Belin, Jafferis, Nayak, JS;]

Random ensembles for chaotic CFT

[Cotler, Jensen; de Boer, Belin, Nayak, JS;
Chandra, Collier, Maloney, Hartman]

Let us think about the idea of "ETH matrix models" for (chaotic) CFT:

Do not expect straight-forward Wigner ("Altland-Zirnbauer")-type RMTs
BUT: RMT universality can hold 'spin-by-spin'

[very recent: Ubaldi, Perlmutter;
Haehl, Reeves, Rozali,…]

Most natural approach, average over CFT data

$$\{\Delta_i \, , \, J_i \, , \, C_{ijk}\}$$

These data define a CFT only if they satisfy consistency conditions

→ so what are the elements of such ensembles?

Definition of approximate CFT

[de Boer, Belin, Jafferis, Nayak, JS;]

An approximate CFT is the set of data $\{\Delta_i\ ,\ J_i\ ,\ C_{ijk}\}$, which *approximately satisfy* CFT constraints, in the sense that

- CFT constraints are only imposed for a subset $\{\mathbb{O}_{\mathrm{restr}}\}$ of "light operators"
- These constraints are only imposed up to a tolerance parameter $\mathbb{T}$ where $\mathbb{T} \sim e^{-S}$
- This can equivalently be seen as imposing restrictions $(n_{\mathrm{max}}, \Delta_{\mathrm{max}}, z^L_{\mathrm{min}} \ldots)$ on number of insertions, maximal dimension, minimal cross ratio,…
- Allows in principle large violations of CFT constraints (!), but in a way that is carefully correlated across the spectrum

The approximate bootstrap constraints open up islands to average over

Examples of approximate CFT

Take a true CFT and to shift the dimension of a single operator $\mathcal{O}_0$

$$\Delta_0 \to \Delta_0 + \epsilon$$

If $\mathcal{O}_0 \in \{\mathbb{O}_{\text{rest}}\}$ the correlation functions of $\mathcal{O}_0$ violate crossing arbitrarily strongly, in other words $\mathcal{O}_0 \notin \{\mathbb{O}_{\text{rest}}\}$

However, even light correlation functions will violate crossing, due to heavy operators in intermediate channels

In fact, we show that these kinds of violations can be counterbalanced by shifting other heavy operators in correlated fashion

Crucially this produces violations of crossing of operators $\in \{\mathbb{O}_{\text{rest}}\}$ uniformly bounded by ϵ in cross-ratio space

A tensor model for random CFT

Ensembles for random (2D)CFT

We will now construct a CFT ensemble for 2D CFT concretely

This will be a joint statistical model of the data for 2D CFT:

$$\{h_i, \bar{h}_i, C_{ijk}\}$$

random matrix
$h_i = \mathrm{eig}(L_{ij})$

random tensor

We can construct the resulting tensor model by imposing approximate bootstrap constraints (→ notion of approximate CFT)

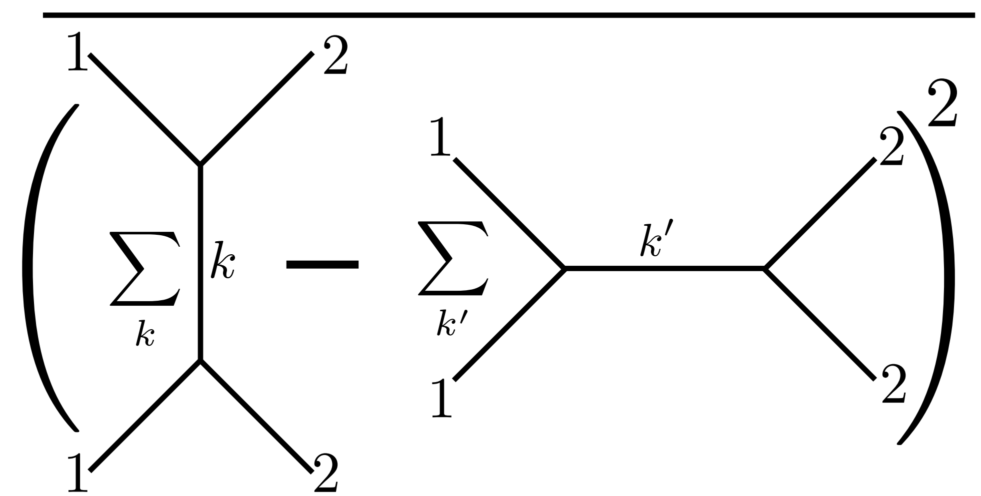
In order to define the measure can a) consider moments or b) use CFT constraints more directly

a) moments: crossing implies non-Gaussianity

Suppose crossing in the ensemble is satisfied on average, then

imposing the variance of crossing on average to be ≈ 0

$$\overline{\left(\langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_2\mathcal{O}_1\rangle_s - \langle \mathcal{O}_1\mathcal{O}_2\mathcal{O}_2\mathcal{O}_1\rangle_t\right)^2} =$$



$$\approx 0$$

This implies the existence of large non-Gaussianities among OPE coefficients

Virasoro 6j (crossing kernel)

$$\overline{C_{ijk}C_{iml}C_{njl}C_{nmk}}\Big|_c = \begin{Bmatrix} \mathcal{O}_k & \mathcal{O}_j & \mathcal{O}_i \\ \mathcal{O}_l & \mathcal{O}_m & \mathcal{O}_n \end{Bmatrix}$$

This fixes the second moment. What about higher moments?

Fixing all moments by statistics of crossing

In fact, by approximately enforcing crossing, we can construct the full potential. For example:

$$\sum_q \left(C_{i_1 i_2 q} C_{i_3 i_4 q} \delta^{(2)} \left(P_s - P_q\right) - C_{i_1 i_4 q} C_{i_2 i_3 q} \left| \mathbb{F}_{P_q P_s} \begin{bmatrix} P_3 & P_4 \\ P_2 & P_1 \end{bmatrix} \right|^2 \right) = 0$$

The 4-point crossing on the sphere thus is the constraint equation

$$M^{i_1 i_2}_{i_3 i_4}(P_s, \bar{P}_s) = 0$$

Modular crossing of torus 1-point function gives a similar (simpler) condition

The total potential is then defined as the maximum ignorance ensemble of CFT data that imposes (on average) these constraints

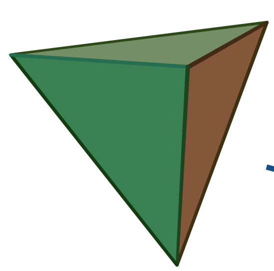
A tensor model for random CFT

Builds again a type of "ETH tensor model", with potential given by the square of the crossing equation

$$\mathcal{Z} = \int d\mu[H] d\mu[C] \exp\left(-V(H,C)\right)$$

with quartic non-linearity implementing the square of crossing

$$V_4 = 2 \sum_{i_1 \cdots i_4}{}' \sum_{p,q} \left(\frac{C_{i_1 i_2 p} C_{i_3 i_4 p} C_{i_1 i_2 q} C_{i_3 i_4 q}}{|\rho_0(p) C_0(12p) C_0(34p)|^2} \delta^{(2)}\left(P_p - P_q\right) - \right.$$



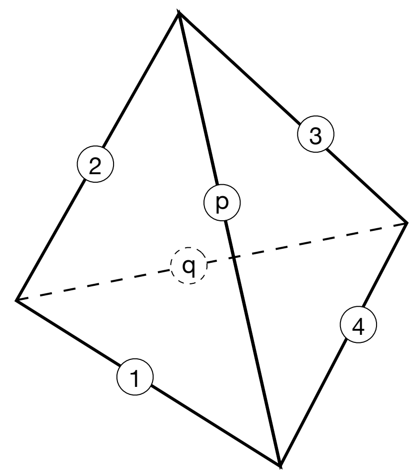
$$\left. \frac{C_{i_1 i_2 p} C_{i_3 i_4 p} C_{i_1 i_4 q} C_{i_2 i_3 q}}{|C_0(12p) C_0(34p) C_0(23q) C_0(14q)|^2} \begin{Bmatrix} \mathcal{O}_q & \mathcal{O}_4 & \mathcal{O}_1 \\ \mathcal{O}_p & \mathcal{O}_2 & \mathcal{O}_3 \end{Bmatrix} \right)$$

More terms in the potential come from modular crossing to fix $\rho(\Delta)$

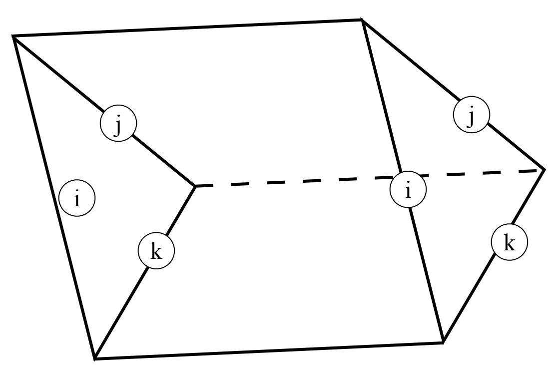
Propagator of the model comes from identity running in the sum

2D random CFT = 3d gravity?

Tensor diagrams are simplicial decompositions weighted by:



$$= \begin{Bmatrix} \mathcal{O}_q & \mathcal{O}_4 & \mathcal{O}_1 \\ \mathcal{O}_p & \mathcal{O}_2 & \mathcal{O}_3 \end{Bmatrix} |C_0(12p)C_0(34p)C_0(23q)C_0(14q)|^{-2}$$



$$= \quad C_0\left(P_i, P_j, P_k\right) C_0\left(\bar{P}_i, \bar{P}_j, \bar{P}_k\right)$$

Non-perturbative Schwinger-Dyson equations generate moments of approximate CFTs studied earlier

A new role for “simplicial-gravity” in 3D? [Boulatov; M. Gross, Ambjørn,…]

Discussion and outlook

Summary

Chaotic nature of QG-Hilbert space: re-interpret matrix-type models as approximate ensembles of generalised Wigner type

Appropriate structure: “ETH matrix models”. Closely related to recent developments in statistical physics

Chaotic or ‘random’ CFT are described by random matrix / tensor models with CFT constraints baked in

Apply to gravity through double-scaled ETH-matrix models (did not go into detail in this talk)

Low-energy supergravity as a moment-generating gadget refining/ updating statistical information $\Longrightarrow$ gravity is maximum entropy ensemble

Outlook

Establish JT+matter matrix model @ all genera through some generalisation of topological recursion to 2-matrix models?

Revisit tensor-models of 3D gravity from Wigner perspective?

Statistical models of higher-D CFTs:“The random CFT bootstrap”. Same tensor model applies, but geometry interpretation less clear

Statistical physics applications of ETH matrix models?

Thanks for your attention!