The cusp anomalous dimension of ABJM from a Thermodynamic Bethe Ansatz approach

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based on 2304.01924 with D.H. Correa and V.I. Giraldo-Rivera

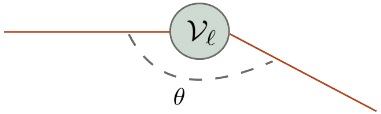
Outline

1. Integrability and Wilson lines in ABJM

$$|p\rangle_A = \sum_n e^{ipn} |\underbrace{\uparrow\uparrow\uparrow\uparrow}_{n}\downarrow\uparrow\uparrow\uparrow\rangle$$

2. An open spin chain for the ABJM's ½ BPS Wilson line

3. TBA equations for the cusp anomalous dimension



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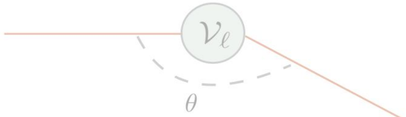
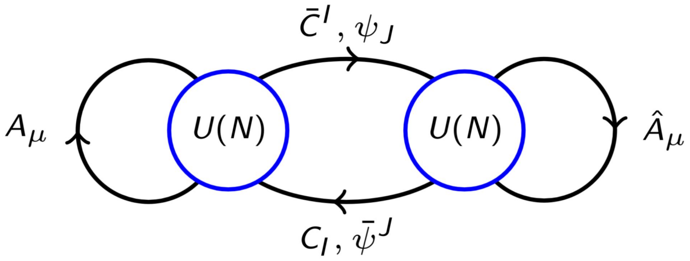


ABJM theory



[Aharony, Bergman, Jafferis, Maldacena (2008)]

Matrix of anomalous dimensions of single trace operators

$\equiv$

Hamiltonian of **integrable** and **periodic** spin chain

[Minahan, Zarembo (2008)]

Periodic spin chains in ABJM

We have an **alternating** spin chain.

Vacuum state

$$\mathrm{tr}[C_1\bar{C}^2C_1\bar{C}^2C_1\bar{C}^2C_1\bar{C}^2] \longleftrightarrow |0\rangle = |\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\rangle$$

Excited states (magnons)

$$\sum_n e^{ipn}\,\mathrm{tr}[\underbrace{C_1\bar{C}^2C_1\bar{C}^2\dots\bar{C}^2}_{n}C_3\bar{C}^2C_1\bar{C}^2] \longleftrightarrow |p\rangle_A = \sum_n e^{ipn}|\underbrace{\uparrow\uparrow\uparrow\uparrow}_{n}\downarrow\uparrow\uparrow\uparrow\rangle \qquad \textbf{Type A}$$

$$\sum_n e^{ipn}\,\mathrm{tr}[\underbrace{C_1\bar{C}^2C_1\bar{C}^2\dots C_1}_{n}\bar{C}^3C_1\bar{C}^2] \longleftrightarrow |p\rangle_B = \sum_n e^{ipn}|\uparrow\underbrace{\uparrow\uparrow\uparrow\uparrow}_{n}\downarrow\uparrow\uparrow\rangle \qquad \textbf{Type B}$$

Dispersion relation

Using the SU(2|2) symmetry of the spin chain one gets, for a Q-magnon,

$$E(\lambda) = \sqrt{Q^2 + 16 h^2(\lambda) \sin^2\left(\frac{p}{2}\right)}$$

[Beisert (2005)]

The $h(\lambda)$ function can not be fixed by symmetry, and it percolates in all the results obtained from integrability.

How can one compute $h(\lambda)$ non-perturbatively?

Motivation

How can one compute $h(\lambda)$ non-perturbatively?

By computing the same observable to all loops both from integrability and from another method.

Done for N=4 sYM in
[Correa, Maldacena, Sever (2012)]
[Drukker (2012)]

Cusp anomalous dimension from integrability

We focused on this step

Done for N=4 sYM in
[Gromov, Sever (2012)]

Bremsstrahlung function from integrability

Bremsstrahlung function from supersymmetric localization

[Correa, Henn, Maldacena, Sever (2012)] (N=4 sYM)
[Bianchi, Preti, Vescovi (2018)] (ABJM)

$h(\lambda)$

Current proposal: $\lambda = \dfrac{\sinh(2\pi h)}{2\pi}\, {}_3F_2\left(\tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}; 1; \tfrac{3}{2}; -\sinh^2(2\pi h)\right)$ [Gromov, Sizov (2014)]

Wilson loops and Γ_{cusp} in ABJM

One can use a U(N|N) superconnection to construct ½ BPS Wilson loops in ABJM.

$$W = \frac{1}{2N}\mathrm{tr}\left[\mathcal{P}\exp\left(i\int_{\infty}^{\infty}\mathcal{L}(\tau)\,d\tau\right)\right]$$

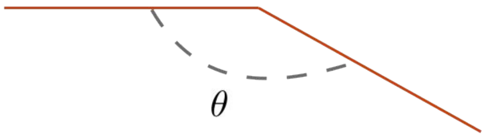
[Drukker, Trancanelli (2009)]

$$\mathcal{L}(\tau) = \begin{pmatrix} A_t - \frac{2\pi i}{k}\mathcal{M}_I^J C_I \bar{C}^J & -i\sqrt{\frac{2\pi}{k}}\eta\bar{\psi}_+^1 \\ -i\sqrt{\frac{2\pi}{k}}\bar{\eta}\psi_1^+ & \hat{A}_t - \frac{2\pi i}{k}\mathcal{M}_I^J \bar{C}^J C_I \end{pmatrix}$$

$$\mathcal{M} = \mathrm{diag}(-1,1,1,1)$$

$$\eta\bar{\eta} = -2i$$

When considering WL with a **geometrical cusp**, the renormalization introduces a **cusp anomalous dimension**.



$$\langle W^{\text{ren}}(\theta)\rangle = Z_{\text{cusp}}(\theta)\,\langle W(\theta)\rangle$$

[Polyakov (1980)]

$$\Gamma_{\text{cusp}}(\theta) := \frac{\partial \log Z_{\text{cusp}}(\theta)}{\partial \log \mu}$$

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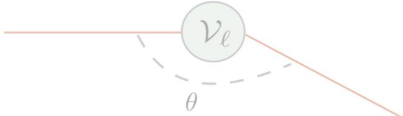

Anomalous dimensions of insertions along Wilson Lines

Using integrability, one can also study the anomalous dimensions of operators inserted along Wilson lines.

$$\mathcal{O}_W(\tau) = \frac{1}{2N}\mathrm{tr}\left[\mathcal{P}W(-\infty,\tau)\,\mathcal{O}(\tau)W(\tau,\infty)\right]$$

Matrix of anomalous dimensions of insertions along Wilson Lines

$$\equiv$$

Hamiltonian of **integrable** **open** spin chain

Studied for N=4 sYM in

[Drukker, Kawamoto (2006)]

Wilson line spin chain in ABJM

Vacuum state

$$\mathcal{V}_\ell = \begin{pmatrix} 0 & (C_1\bar{C}^2)^\ell C_1 \\ 0 & 0 \end{pmatrix}$$

It has **SU(1|2) symmetry**, in accordance with the string theory prediction.

Excited states (magnons)

Type A and **B** impurities propagating in the upper off-diagonal block.

$\Rightarrow$ Same scattering matrix as for the periodic spin chain.

Reflection matrix

We can use the **SU(1|2) symmetry** and **weak coupling expansions** to constrain the reflection matrix.

$$\mathbf{R} = \begin{pmatrix} R_A & 0 \\ 0 & R_B \end{pmatrix}$$

$$R_A = R_0^A \, \mathrm{diag}\left(1, 1, e^{-ip/2}, -e^{ip/2}\right)$$

$$R_B = R_0^B \, \mathrm{diag}\left(1, 1, e^{-ip/2}, -e^{ip/2}\right)$$

This result satisfies the **Boundary Yang-Baxter Equation** (indication of integrability).

How can we compute the two dressing phases R_0^A and R_0^B ?

Crossing equation

Consistency allows to derive a **crossing equation** for the dressing phases.

$$\begin{cases} x^+ + \dfrac{1}{x^+} - x^- - \dfrac{1}{x^-} = \dfrac{i}{h(\lambda)} \\ \dfrac{x^+}{x^-} = e^{ip} \end{cases}$$

Zhukowski variables

$$R_A^0(p)R_B^0(\bar{p}) = -\frac{\frac{1}{x^+} + x^+}{\frac{1}{x^-} + x^-}\frac{1}{\sigma(p, -\bar{p})}$$

$p \to -p \quad \text{and} \quad E \to -E$

BES phase

Dressing phases

We propose the following **all-loop solutions** to the crossing equation:

$$R_A^0(p) = -\frac{1}{R_0(p)} \underbrace{\left(\frac{\frac{1}{x^+}+x^+}{\frac{1}{x^-}+x^-}\right)}_{\text{Boundary bound states only for type A magnons}} \left(\frac{x^-}{x^+}\right)$$

$$R_B^0(p) = \frac{1}{R_0(p)} \left(\frac{x^-}{x^+}\right)$$

$$R_0(p) = \Bigg[\underbrace{\frac{1}{\sigma_B(p)\sigma(p,-p)} \left(\frac{1+\frac{1}{(x^-)^2}}{1+\frac{1}{(x^+)^2}}\right)}_{\text{Dressing phase for Wilson lines in N=4 super Yang-Mills}}\Bigg]^{\frac{1}{2}}$$

[Correa, Maldacena, Sever (2012)]
[Drukker (2012)]

These proposals passed tests both at **weak** and **strong coupling**.

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n

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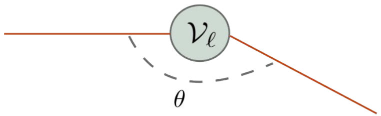
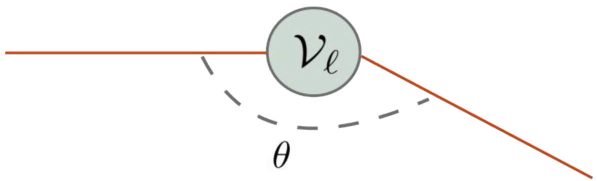


Γ_{cusp} from integrability

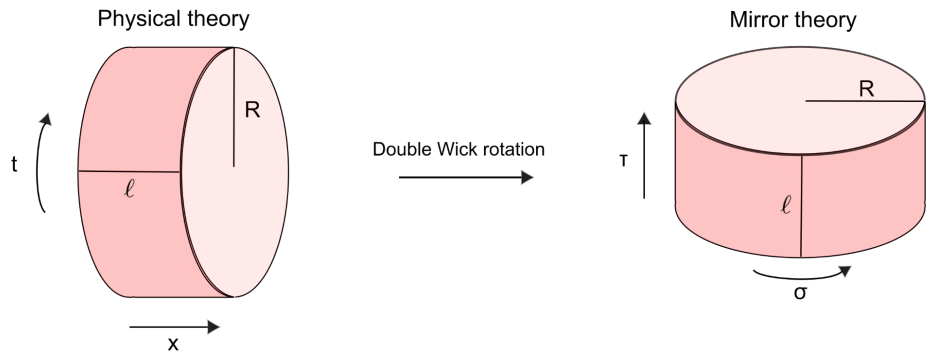
Let's consider a cusped WL with a $\mathcal{V}_\ell$ insertion at the position of the cusp.



The anomalous dimension $\Gamma(\ell)$ will be a function of ℓ . In the limit with no insertions $(\ell \to -\frac{1}{2})$ we recover Γ_{cusp} .

We need to compute the vacuum energy $E_0(\ell)$ at finite ℓ $\Rightarrow$ Thermodynamic Bethe Ansatz (TBA)

Thermodynamic Bethe Ansatz (TBA)



[Zamolodchikov (1990)]

[LeClair, Mussardo, Saleur, Skorik (1995)]

$$Z(\ell, R) \sim e^{-RE_0(\ell)} \longrightarrow \langle B_{\text{left}} | e^{-\ell H_{\text{mirror}}} | B_{\text{right}} \rangle$$

Y-system

From the double Wick rotation we get

$$E(\ell) = -\frac{1}{4\pi}\sum_{a=1}^{\infty}\int_0^{\infty} dq\ \log[1+Y^{I}_{a,0}(q)] - \frac{1}{4\pi}\sum_{a=1}^{\infty}\int_0^{\infty} dq\ \log[1+Y^{II}_{a,0}(q)]$$

The Y-functions are the solutions to a system of functional equations, the **Y-system**.

Proposal

Same Y-system as for the periodic chain.

Computed in
[Gromov, Levkovich-Maslyuk (2009)]
[Bombardelli, Fioravanti, Tateo (2009)]

Different asymptotic and analytic boundary conditions.

Alternatively, the Y-system can be rewritten as a set of Hirota equations, the **T-system**.

Test of TBA: leading finite-size correction

Energy of magnons in mirror theory

Charge conjugation

$$Y^{I}_{a,0}(q) \sim e^{-2\ell\,\epsilon_a(q)}\,\mathrm{Tr}\left[R_{A,a}(q)\,C\,R^{\theta}_{A,a}(-\bar{q})\,C^{-1}\right]$$
$$Y^{II}_{a,0}(q) \sim e^{-2\ell\,\epsilon_a(q)}\,\mathrm{Tr}\left[R_{B,a}(q)\,C\,R^{\theta}_{B,a}(-\bar{q})\,C^{-1}\right]$$

Momentum
in mirror theory

Reflection matrix of a-magnons

Rotated reflection matrix

Leading Lüscher corrections

[Goshal, Zamolodchikov (1993)]

Test of TBA: leading finite-size correction

For the fundamental magnons (a=1),

$$Y^{I}_{1,0} = Y^{II}_{1,0} = -e^{-2L\,\epsilon_1} \frac{(z^+ + z^-)^2}{2\,z^+ z^- \left(1 + \frac{z^+ + \frac{1}{z^+}}{z^- + \frac{1}{z^-}}\right)} R^0_A\left(z^+, z^-\right) R^0_A\left(-\tfrac{1}{z^-}, -\tfrac{1}{z^+}\right) T_{1,1}$$

Zhukowski variables in mirror theory

Then, for arbitrary bound-state magnons,

[Bajnok, Nepomechie, Palla, Suzuki (2012)]

Obtained from $\overbrace{T^{SU(1|2)}_{a,1} \equiv T^{SU(2|1)}_{1,a}}$

Computed in [Bajnok et al (2013)]

$$Y^{I}_{a,0} = Y^{II}_{a,0} = -e^{-2L\,\epsilon_a} \frac{(z^+ + z^-)^2}{2\,z^+ z^- \left(1 + \frac{z^+ + \frac{1}{z^+}}{z^- + \frac{1}{z^-}}\right)} \underbrace{R^0_{I,a}\left(z^+, z^-\right) R^0_{I,a}\left(-\tfrac{1}{z^-}, -\tfrac{1}{z^+}\right)}_{\text{Obtained from fusion rules}} T_{a,1}$$

Test of TBA: leading finite-size correction

Therefore, from the TBA formula for the vacuum energy we get

Jacobi Polynomials

$$E_0(\ell) = -2h^{2\ell+2}\sin^2\frac{\theta}{2}\sum_{k=0}^{\infty}\frac{P_k^{(0,1)}(-\cos\theta)}{(k+1)^{2\ell+1}} + \mathcal{O}(h^{2\ell+3})$$

Finally, in limit with no insertions in the cusp $(\ell \to -\frac{1}{2})$ we recover

We use $h(\lambda) = \lambda + \mathcal{O}(\lambda^2)$

[Minahan et al (2010)]

[Leoni et al (2010)]

$$\Gamma_{\text{cusp}}(\theta) = E_0(-1/2) = -\lambda\underbrace{\left(\frac{1}{\cos\frac{\theta}{2}} - 1\right)}_{\text{Previously computed from Feynman diagrams in [Griguolo et al (2012)]}} + \mathcal{O}(\lambda^2)$$

Previously computed from Feynman diagrams in [Griguolo et al (2012)]

Conclusions

1. We studied the **integrability of the ½ BPS Wilson lines** in ABJM, computing the **all-loop reflection matrices.**

2. We proposed a set of **TBA equations for the cusped Wilson loop**.
   First step towards a direct test of the current proposal for $h(\lambda)$ **.**

3. We successfully reproduced de **one-loop cusp anomalous dimension from the TBA equations**.

Future directions

1. To study the **small angle limit** (*Bremsstrahlung limit*) of the TBA equation.

2. To **test** the dressing phases at **higher perturbative orders**.

3. To iterate the TBA equations to reproduce the **two-loop cusp anomalous dimension**.

4. Derivation of the **Quantum Spectral Curve** (QSC) for the cusped Wilson loop.

Thank you!