Soft Scattering & Holography: BMS Symmetries in Higher Dimensions

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New Perspectives in Conformal Field Theoriy and Gravity
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Based on 2304.09330 with Prahar Mitra, Aaron Poole, Bilyana Tomova

Starting from the general asymptotic analysis of higher dimensional gravity with null asymptotic boundaries 2108.01203[hep-th] FC, we study the asymptotic symmetries of 6-dimensional Asymptotically locally flat (AlF) spacetimes

in order to pinpoint **universal features** of

- gravitational S-matrix
- flat holography

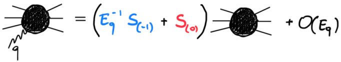
(the results extend to all even dimensions)

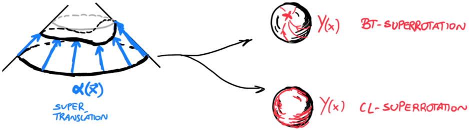
Motivations and aims

A look from the bulk

Strominger '14: Soft theorems are Ward identities of the asymptotic symmetries of **4-dimensional** asymptotically flat spacetimes at null infinity*

soft theorems $\leftrightarrow$ asymptotic symmetries





A look from the boundary

- **Take home lesson:** bulk QFT results can be derived from boundary relations.

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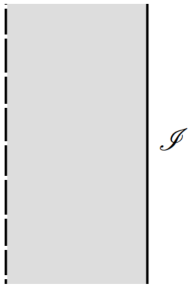
Inspires

$$\text{Flat Holography} \sim \begin{array}{l} \text{Celestial holography,} \\ \text{Carrollian holography,} \\ \text{Cauchy holography,} \\ \text{Wedge holography,} \\ \vdots \end{array}$$

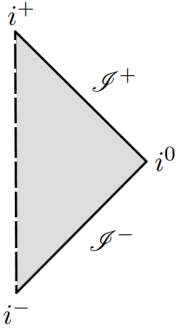
A look from the boundary

AdS Holography



$Z_{CFT} = Z_{grav}$

Flat Holography



Structure X $\rightarrow$ S-matrix
(Celestial CFT
Carrollian CFT, ...)

A look from the boundary

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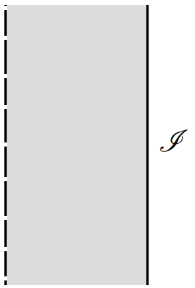

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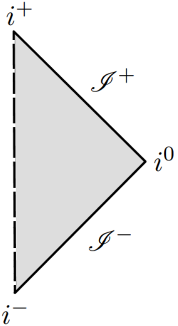

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Point of view:

soft theorems $\leftrightarrow$ asymptotic symmetries

Universal feature (or check) for flat holography models!

Our goal

- Soft theorems exist in any dimension greater or equal to 4

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Is the relationship

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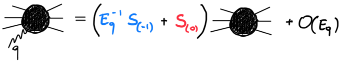
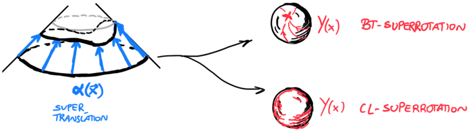
- **Bonus...** Stay tuned until the end!

Cartoon



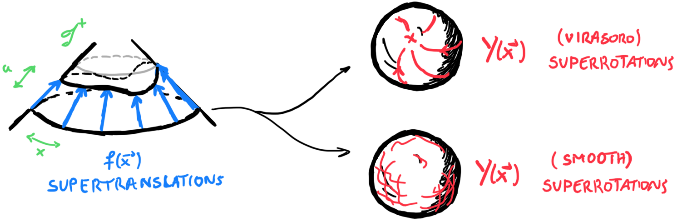
q
$= \left(E_q^{-1} S_{(-1)} + S_{(0)}\right)$
$+ \mathcal{O}(E_q)$



α(x̂)
SUPER_
TRANSLATION
Y(x)
BT-SUPERROTATION
Y(x)
CL-SUPERROTATION

From BMS...

- $BMS = Lorentz \ltimes Supertranslations \subset Superrotations \ltimes Supertranslations$



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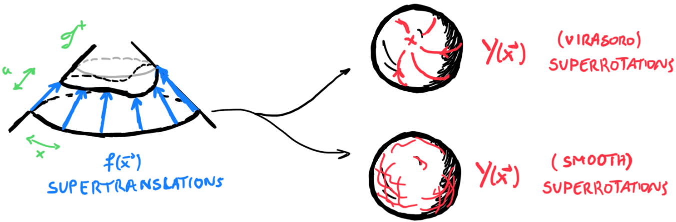

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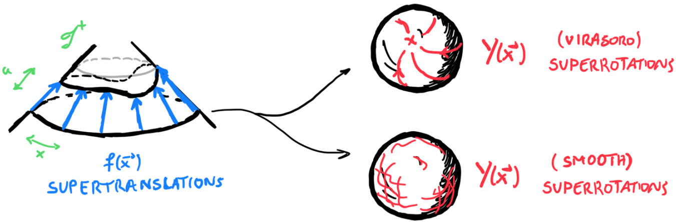

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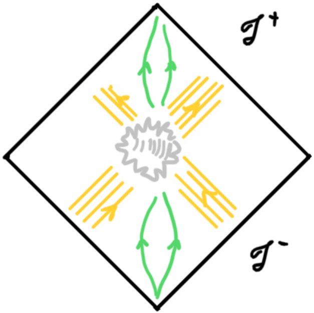
- Such symmetries are generated on the phase space by a set of charges $Q_{as} = \{Q_{st}, Q_{sr}\}$

...to the S-matrix and Soft Theorems

- BMS act on $|in\rangle$ and $|out\rangle$ states of a scattering process

$$|out\rangle = \mathcal{S}\,|in\rangle \text{ up to the action of } Q_{as}$$



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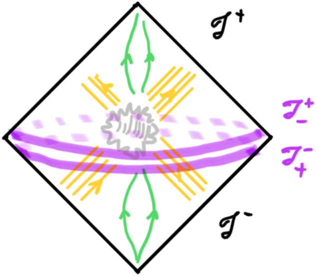

Antipodal matching
$Q_{as}|_{\mathscr{I}^+_-} = Q_{as}|_{\mathscr{I}^-_+}$
(conditions on fields near i^0)

Strominger's **conjecture**:
$[Q_{as}, \mathcal{S}] = 0$

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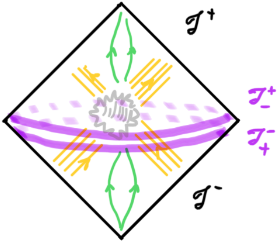

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$$\langle out| \, [Q_{st}, \mathcal{S}] \, |in\rangle = 0 \Leftrightarrow \lim_{E_q \to 0} E_q \mathcal{M}_{n+1} = S_{(-1)} \mathcal{M}_n ,$$

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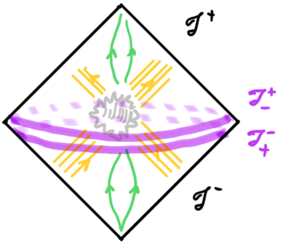

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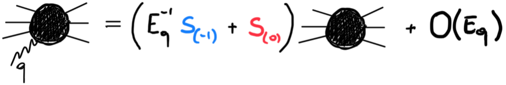
- namely,

$$\mathscr{M}_{n+1}(p_1, \dots p_n, q) = \left(E_q^{-1} S_{(-1)} + S_{(0)}\right) \mathscr{M}_n(p_1, \dots p_n) + O(E_q)$$



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Shopping list

1. Solution space for Einstein gravity with null asymptotic boundaries

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4. Derivation of soft theorems from the well-defined charges

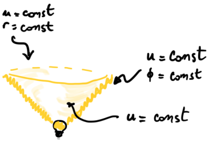
Details

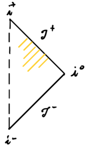
Higher even dimensions

1. Solution space for gravity with null asymptotic boundaries

- Bondi-Sachs gauge

$$ds^2 = -e^{2\beta}U\,du^2 - 2e^{2\beta}\,du\,dr + r^2 h_{AB}(dx^A - W^A\,du)(dx^B - W^B\,du)$$





- with **asymptotically locally Minkowskian** boundary condition

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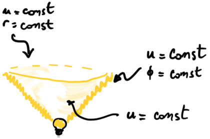

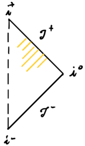

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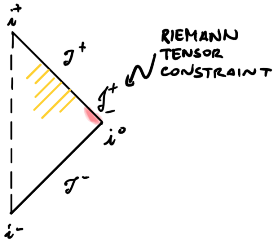
- Large-r solution (of $R_{\mu\nu} = 0$) has analytic terms in r^{-n} and logarithmic terms $r^{-n}\log^m r$ [2108.01203 FC]

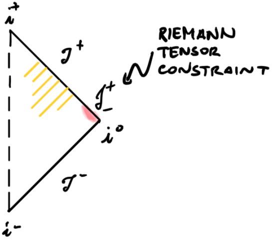
2. Imposition of boundary conditions near i^0



$D = d + 2$ spacetime dimensions, d even

$$r^d e^{\mu}_{\hat{\mu}} e^{\nu}_{\hat{\nu}} e^{\rho}_{\hat{\rho}} e^{\sigma}_{\hat{\sigma}} R_{\mu\nu\rho\sigma}[G]|_{\mathscr{I}^+_-} = 0$$

- **No flux** of radiation near i^0
- **Remove** all logarithmic terms from solution space



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RADIATION ORDER

$$\Rightarrow \begin{cases} \text{Analytic metric expansions, with} \quad h_{AB} = q_{AB} + \dfrac{h_{(1)AB}}{r} + \dots + \dfrac{h_{(\frac{d}{2})AB}}{r^{\frac{d}{2}}} + \dots \\ \text{and conformally flat } q_{AB}(x) = e^{2\Phi(x)} \hat{q}_{AB}(x), \quad \hat{q}_{AB}(x) = \partial_A \chi^C(x) \partial_B \chi^D(x) \delta_{CD} \end{cases}$$

! Supertranslation and Superrotation large gauge modes are **overleading w.r.t. radiative order**

3. Construction of asymptotic charges

Covariant phase space procedure

$$
\begin{aligned}
&1)\,\delta L = EoM\,\delta g + d\theta,\\
&2)\,\text{Presymplectic current}\,\theta \sim p\delta q\\
&3)\,\text{Symplectic form}\quad \Omega(\delta,\delta') = \delta\theta = \delta p \wedge \delta q\\
&4)\,\text{Hamiltonian}\quad \delta H_c = \Omega(\delta,\delta_c)
\end{aligned}
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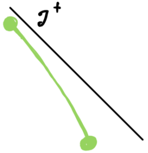
$$\theta \to \theta + dY \Rightarrow \Omega \to \Omega + \delta dY$$



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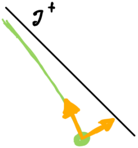

We build the counterterm Y in a **local** and **covariant** way in terms of the induced metric on the celestial sphere and the extrinsic curvature

3. Construction of asymptotic charges

- The resulting symplectic form reads

$$\Omega(\delta, \delta_{BMS}) = \frac{1}{4\pi G}\int d^4x\sqrt{q}\, f\,\delta M + \frac{1}{16\pi G}\int d^4x\sqrt{q}\, Y^A \delta N_A + \frac{1}{16\pi G}\int d^4x\sqrt{q}\,\delta\chi^A(\delta_\xi - \mathscr{L}_Y + 4\omega)P_A$$

N.B.: The Weyl mode is fixed in terms of the other dynamical variables to make Ω invertible

$$\boxed{\text{The last term is not } \delta\text{-exact} \Rightarrow \nexists \quad H_{BMS} \quad s.t. \quad \delta H_{BMS} = \Omega(\delta, \delta_{BMS})}$$

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- Way out: define the BMS charges from the integrable part of Ω

$$T_f \equiv \frac{1}{4\pi G}\int d^4x\sqrt{q}\, f M, \qquad J_Y \equiv \frac{1}{16\pi G}\int d^4x\sqrt{q}\, Y^A N_A.$$

They represent the BMS algebra faithfully

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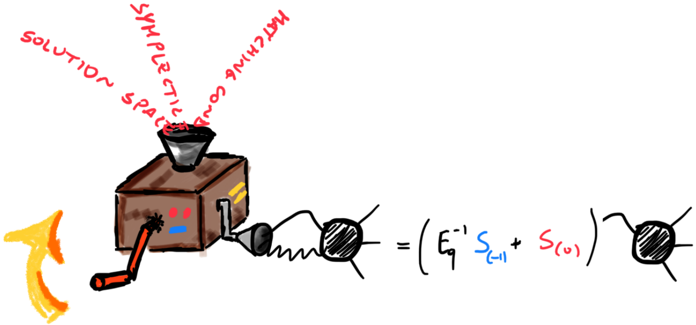

We derive them for amplitudes involving a single soft graviton and hard particles of any spin.

Closing Remarks

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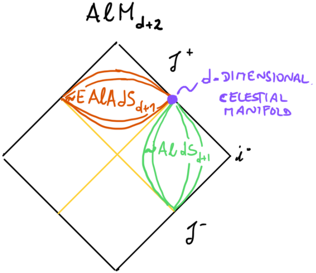

$$h = q + \frac{h_{(1)}}{r} + \cdots + \frac{1}{r^{\frac{d}{2}}}\left(h_{(\frac{d}{2})} + h_{(\frac{d}{2},1)} \log r\right) + \ldots$$

match the logarithmic coefficient of $AlAdS_{d+1}$ spacetimes which is related to the Weyl anomaly [2108.01203 FC]

$AlM_{d+2} \leftrightarrow (E)Al(A)dS_{d+1}$

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Thank you!