Vacua of gauged $N = 2$ supergravities arising as special subloci of the complex structure moduli space of String Compactifications

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Motivation

- $d = 4$, $N = 2$ Supergravity theories appear as low energy EFTs of Calabi-Yau Threefold compactifications in type II string theory
- The target space of the scalar fields can be identified with the Moduli space of the underlying family of Calabi-Yau Threefolds

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- Conifold Singularities are understood in the context of String theory
  - Appearance of additional states which become massless on the singular sublocus [Strominger, 1995]
  - Geometrical: Topology changing transition to a different Moduli space [Greene, Morrison, Strominger, 1995]

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- Locally in $N = 2$ Gauge Theories: Interpretation as a super-Higgs mechanism between vector- and hypermultiplets [Katz, Morrison, Plesser, 1996] [Klemm, Mayr, 1996]
- Here: Find a global description in terms of $N = 2$ Supergravity

➢ Consider Minkowski Vacua of $N = 2$ gauged Supergravity

Gauged $N = 2$ Supergravity

- Spectrum
  - Gravity Multiplet $(g_{\mu\nu}, \psi_{\mu A}, A_\mu)$
  - n_v Vectormultiplets $(A^i_\mu, \lambda^{iA}, t^i)$
  - n_h Hypermultiplets (ζ_α, q^u)

[Andrianopoli, Bertolini, Ceresole, D'Auria, Ferrara, Fré, Magri, 1996]
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- The target space of the scalars is locally $\mathcal{M}_V \times \mathcal{M}_H$ with
  - $\mathcal{M}_V$ local special Kähler manifold parametrized by n_v complex scalars t^i
  - $\mathcal{M}_H$ quaternionic Kähler manifold parametrized by $4n_h$ real scalars q^u

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- Bosonic part of the Action

$$S_{sc} = \int \frac{1}{2} R \star 1 + g_{i\bar{j}} \mathcal{D}t^i \wedge \star \mathcal{D}\bar{t}^{\bar{j}} + h_{uv} \mathcal{D}q^u \wedge \star \mathcal{D}q^v$$
$$+ \frac{1}{2} Im(\mathcal{N})_{ij} F^i \wedge \star F^j + \frac{1}{2} Re(\mathcal{N})_{ij} F^i \wedge F^j - V$$

- Gauge coupling via covariant derivatives

$$\mathcal{D}t^i = \partial t^i - \hat{k}^i_\lambda \hat{\Theta}^\lambda_\Lambda A^\Lambda \qquad \mathcal{D}q^u = \partial q^u - k^u_\lambda \Theta^\lambda_\Lambda A^\Lambda$$

  and scalar potential $V(t^i, q^u)$
  - $\hat{k}^i_\lambda$ and k^u_λ are killing vectors on $\mathcal{M}_V$ and $\mathcal{M}_H$ respectively

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The Scalar Potential

- Scalar Potential

$$V(t^i, q^u) = -6|S|^2 + \frac{1}{2}|W|^2 + |N|^2$$

- $N = 2$ supersymmetric Minkowski Vacuum iff

$$|S|^2 = 0 \qquad |W|^2 = 0 \qquad |N|^2 = 0$$

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- Resulting constraints for gauging isometries of $\mathcal{M}_H$

$$\begin{aligned} 0 &= X^\Lambda \Theta_\Lambda^\lambda \mathcal{P}_\lambda^a \\ 0 &= g^{i\bar{j}} (\nabla_{\bar{j}} \bar{X}^\Lambda) \Theta_\Lambda^\lambda \mathcal{P}_\lambda^a \\ 0 &= \bar{X}^\Lambda \Theta_\Lambda^\lambda k_\lambda^u \end{aligned}$$

  - X^Λ: $2(n_v + 1)$-dimensional period vector of $\mathcal{M}_V$
  - $\mathcal{P}_\lambda^a$: killing prepotential of the isometries k_λ^u on $\mathcal{M}_H$
  - Θ_Λ^λ: embedding tensors representing the gauge charges

[Louis, Smyth, Triendl, 2012]

Minkowski vacua in $N = 2$ Supergravity

- $(X^\Lambda, \nabla_i X^\Lambda)$ is full ranked matrix and $g^{i\bar{j}}$ invertible
  - $0 = X^\Lambda \Theta_\Lambda^\lambda \mathcal{P}_\lambda^a$
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  - ➢ $\Theta_\Lambda^\lambda \mathcal{P}_\lambda^a = 0$ for $a = 1, 2, 3$
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  - ➢ Non-trivial vev for k_λ^u creates holomorphic conditions among the X^Λ
  - ➢ Super-Higgs mechanism

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- Massive Modes:
  - $3n$ (real) hypermultiplet scalars
  - n (complex) vectormultiplet scalars
  - n gauge bosons eating n additional (real) hypermultiplet scalars
- These combine to n long massive vectormultiplets

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Conifold Singularities

- Effective Supergravity description breaks down on conifold singularities
  - Topology change as a non-trivial 3-cycle shrinks to a point
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- Model:
  - $\mathcal{M}_V$: Local special Kähler manifold with a conifold transition of codimension 1
  - $\mathcal{M}_H$: Universal hypermultiplet $(\phi, \sigma, C, \bar{C})$
  - ➢ Transition Locus reproduced as Minkowski vacuum by stabilizing C and $\bar{C}$

Relation to String Compactifications

- $\mathcal{M}_V$ corresponds to the complex structure Moduli space of type IIB Calabi-Yau compactifications
  - Local special Kähler structure is realized by the variation of Hodge structure

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- Spotable by
  - a factorization of the intermediate Jacobian
  - a reduction of the Picard-Fuchs ideal
  - a vanishing locus of a period and its dual

Summary and Outlook

- Transition Loci on Moduli Spaces of Calabi-Yau Manifolds correspond to Minkowski Vacua of the corresponding gauged $N = 2$ supergravity

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  - Systematic search for transition subloci on two-dimensional local special Kähler manifolds
    - Use arithmetic techniques to spot factorizations of the Hodge structure

[Candelas, de la Ossa, Elmi, van Straten, 2019]
[Candelas, de la Ossa, Kuusela, McGovern, 2023]