Comments on Non-invertible Symmetries in Argyres-Douglas Theories

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based on arXiv:2303.16216 [hep-th]
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Non-invertible Symmetries in $d \geq 3$

The Paradigma of Non-invertible Symmetries

1. Consider a 4d theory with a 0-form symmetry and a $\mathbb{Z}_N^{[1]}$ 1-form symmetry with the following mixed 't Hooft anomaly
$$\mathcal{A} \sim \frac{1}{2}\int_{\mathcal{M}_5} A_1 \wedge \mathcal{P}(B_2)\,,$$
with $\mathcal{P}(B_2)$ the Pontryagin square of B_2.
2. The defect $D(\mathbf{M}_3, B_2)$ is **anomalous** under $\mathbb{Z}_N^{[1]}$ transformations. However, a gauge invariant operator is
$$D(\mathbf{M}_3, B_2) e^{\frac{i}{2}\int_{\mathbf{M}_4}\mathcal{P}(B_2)} \text{ with } \partial\mathbf{M}_4 = \mathbf{M}_3\,.$$
3. Gauging the $\mathbb{Z}_N^{[1]}$ 1-form symmetry, $D(\mathcal{M}_3, b_2)$ is **not** well-defined, but it must be **coupled** to a 3d **TQFT** that cancel the anomaly.
4. Computing the fusion rules for the resulting defect, it is **non-invertible**.

Recent reviews: S. Schafer-Nameki, arXiv: 2305.18296 (hep-th); L. Bhardwaj, L. E. Bottini, L. Fraser-Taliente, L. Gladden, D. S. W. Gould, A. Platschorre, H. Tillim, arXiv: 2307.07547 (hep-th); S.-H. Shao, arXiv: 2308.00747 (hep-th)

Global Structure for $\mathcal{N}=4$ $\mathfrak{su}(2)$ SYM

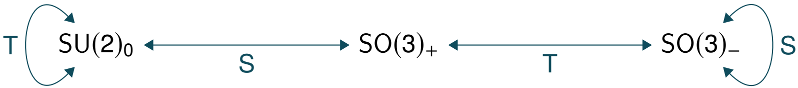
Global Structure of the Gauge Group

$\mathfrak{su}(2)$ admits three choices for the **global structure** of the gauge group, i.e. $\mathrm{SU}(2)_0$, $\mathrm{SO}(3)_+$ and $\mathrm{SO}(3)_-$, differing by their spectra of line operators.[1]

Montonen-Olive duality group $\mathrm{SL}(2,\mathbb{Z})$

The different global structures are related by $\mathrm{SL}(2,\mathbb{Z})$ **duality**, by acting with S and T, such as

$$\mathrm{S}\,:\,\tau\to-\frac{1}{\tau}\,,\ \mathrm{T}\,:\,\tau\to\tau+1\,.$$

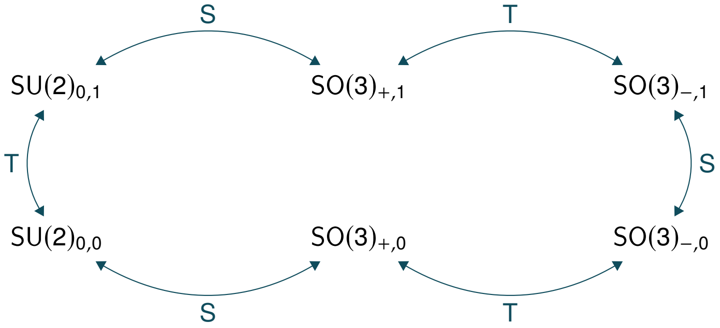


[1] O. Aharony, N. Seiberg, Y. Tachikawa, *JHEP* **08**, 115, arXiv: 1305.0318 (hep-th)

1-form Symmetry for $\mathcal{N} = 4$ $\mathfrak{su}(2)$ SYM

1-form Symmetry

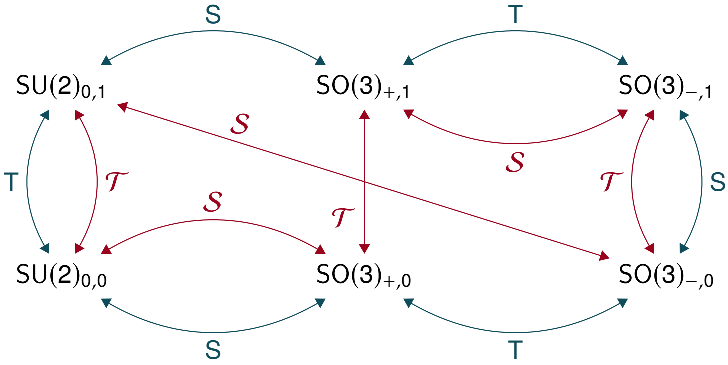
1. $\mathfrak{su}(2)$ has a $\mathbb{Z}_2^{[1]}$ 1-form symmetry, and we call B_2 the background gauge field associated to it.
2. The **counterterm** $\frac{1}{2}\int \mathcal{P}(B_2)$ can be added to the Lagrangian without modifying the theory.
3. Such a term is generated by acting with S and T on the theory with B_2 turned.[2]



[2] J. Kaidi, G. Zafrir, Y. Zheng, *JHEP* **08**, 053, arXiv: 2205.01104 (hep-th)

Gauging $\mathbb{Z}_2^{[1]}$ 1-form symmetry in $\mathcal{N} = 4$ $\mathfrak{su}(2)$ SYM

1. $\mathcal{T}$: stack with counterterm $\frac{1}{2}\int \mathcal{P}(B_2)$.
2. $\mathcal{S}$: gauge $\mathbb{Z}_2^{[1]}$ 1-form symmetry.[3]



[3] J. Kaidi, G. Zafrir, Y. Zheng, *JHEP* **08**, 053, arXiv: 2205.01104 (hep-th)

Non-invertible Symmetry for $\mathcal{N} = 4$ $\mathfrak{su}(2)$ SYM

0-form Symmetry

$\mathfrak{su}(2)$ at $\tau = i$ has also a $\mathbb{Z}_2^{[0]}$ 0-form symmetry generated by S, with **mixed anomaly** with B_2 as

$$\mathcal{A} \sim \frac{1}{2} \int_{\mathcal{M}_5} A_1 \wedge \mathcal{P}(B_2)\,.$$

Example of Non-invertible Defect

$$SU(2)_{0,0}(\tau)\big|_{\tau=i} \xrightarrow{\mathcal{S}} SO(3)_{+,0}(\tau)\big|_{\tau=i} \xrightarrow{\mathrm{S}} SU(2)_{0,0}\left(-\frac{1}{\tau}\right)\bigg|_{\tau=i}$$

$$SU(2)_{0,0}(\tau)\big|_{\tau=i} \xrightarrow{\qquad\qquad \mathcal{S}\mathrm{S} \qquad\qquad} SU(2)_{0,0}\left(-\frac{1}{\tau}\right)\bigg|_{\tau=i}$$

The defect generated by $\mathcal{S}$S is not invertible.

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Argyres-Douglas Theories

1. Let us consider the following Seiberg-Witten (SW) curve for a rank-1 theory
$$\Sigma_u \; : \; y^2 = x^3 + f(u)x + g(u)\,.$$
2. We demand scale invariance of the curve imposing
$$u \longrightarrow \xi^r u\,, \;\; \xi \in \mathbb{C}^*\,.$$
3. We impose the homogeneity of the curve, and $r\,[\lambda_{\text{SW}}] = 1$, with
$$\frac{\partial\lambda}{\partial u} \sim \frac{dx}{y}\,.$$

Rank-1 AD theories

$$\Delta[u] = r[u] = \frac{6}{6-n} \text{ or } \frac{4}{4-n} \geq 1, \text{ with } n \in \mathbb{Z}_+\,.$$

8 theories are realized, and 3 of them have fractional scaling dimension for u: **Argyres-Douglas (AD) theories.**[4]

Recent reviews: M. Martone, presented at the Young Researchers Integrability School and Workshop 2020: A modern primer for superconformal field theories, arXiv: 2006.14038 (hep-th); M. Akhond, G. Arias-Tamargo, A. **Mininno**, H.-Y. Sun, Z. Sun, Y. Wang, F. Xu, *SciPost Phys. Lect. Notes*, arXiv: 2112.14764 (hep-th)

[4] P. C. Argyres, M. R. Douglas, *Nucl. Phys. B* **448**, 93–126, arXiv: hep-th/9505062; P. C. Argyres, M. Plesser, N. Seiberg, E. Witten, *Nucl. Phys. B* **461**, 71–84, arXiv: hep-th/9511154

Argyres-Douglas Theories Then: $\mathcal{N} = 2$ SU(3) SYM

The first AD theory was found starting from the 4d $\mathcal{N} = 2$ SU(3) SYM and extracting the SW curve on its Coulomb Branch (CB). At the AD point, one finds:

Minimal $\mathcal{N} = 2$ SCFT

1. Rank-1 theory with CB operator of dimension $\frac{6}{5}$.
2. Its central charges are[5]

$$(a, c) = \left(\frac{43}{120}, \frac{11}{30}\right)$$

3. **No** $\mathcal{N} = 2$ Lagrangian description.

Recent reviews: M. Martone, presented at the Young Researchers Integrability School and Workshop 2020: A modern primer for superconformal field theories, arXiv: 2006.14038 (hep-th); M. Akhond, G. Arias-Tamargo, A. **Mininno**, H.-Y. Sun, Z. Sun, Y. Wang, F. Xu, *SciPost Phys. Lect. Notes*, arXiv: 2112.14764 (hep-th)

[5] A. D. Shapere, Y. Tachikawa, *JHEP* **09**, 109, arXiv: 0804.1957 (hep-th)

Argyres-Douglas Theories Now

Argyres-Douglas Theories

Any 4d $\mathcal{N}=2$ SCFT with CB operators of **fractional** dimension is called Argyres-Douglas theory.

Not Only Rank-1 Theories

1. They can be realized in **class $\mathcal{S}$**, i.e. 6d $\mathcal{N}=(2,0)$ theory compactified on a Rienmann surfaces with (ir)regular punctures.[6]
2. They can be realized via **geometric engineering** in type IIB via compactification on singular Calabi-Yau 3-folds,[7] e.g. (G,G') theories with $G,G'=A,D,E$.
3. They can also be realized via twisted compactification of 6d $\mathcal{N}=(1,0)$ theories.[8]

[6] D. Gaiotto, G. W. Moore, A. Neitzke, arXiv: 0907.3987 (hep-th); D. Xie, *JHEP* **01**, 100, arXiv: 1204.2270 (hep-th); Y. Wang, D. Xie, *Phys. Rev. D* **94**, 065012, arXiv: 1509.00847 (hep-th); Y. Wang, D. Xie, *Phys. Rev. D* **100**, 025001, arXiv: 1805.08839 (hep-th)

[7] A. D. Shapere, C. Vafa, arXiv: hep-th/9910182; S. Cecotti, A. Neitzke, C. Vafa, arXiv: 1006.3435 (hep-th); D. Xie, S.-T. Yau, arXiv: 1510.01324 (hep-th)

[8] M. Del Zotto, C. Vafa, D. Xie, *JHEP* **11**, 123, arXiv: 1504.08348 (hep-th); K. Ohmori, H. Shimizu, Y. Tachikawa, K. Yonekura, *JHEP* **07**, 014, arXiv: 1503.06217 (hep-th); K. Ohmori, H. Shimizu, Y. Tachikawa, K. Yonekura, *JHEP* **12**, 131, arXiv: 1508.00915 (hep-th)

Geometric Engineering in Type IIB Compactification

Calabi-Yau with an Isolated Hypersuface Singularity

Consider type IIB compactification on a non-compact Calabi-Yau manifolds defined as[9]

$$W := \{F(x_1, x_2, x_3, x_4) = 0\} \subset \mathbb{C}^4 \, ,$$

where F is a polynomial, such that $F = dF = 0$ has a unique solution at the **isolated** point.

The polynomial must be

1. Quasi-homogeneous:

$$F(\xi^{q_i} x_i) = \xi F(x_i) \text{ with } q_i > 0 \, .$$

2. The weights q_i satisfy

$$\sum_i q_i > 1 \, .$$

Recent review M. Akhond, G. Arias-Tamargo, A. **Mininno**, H.-Y. Sun, Z. Sun, Y. Wang, F. Xu, *SciPost Phys. Lect. Notes*, arXiv: 2112.14764 (hep-th)

[9] A. D. Shapere, C. Vafa, arXiv: hep-th/9910182; S. Cecotti, A. Neitzke, C. Vafa, arXiv: 1006.3435 (hep-th)

Geometric Engineering in Type IIB Compactification

Mini-versal Deformations

$$\hat{F} = F + \sum_{\alpha} g^{\alpha}(x_i)\lambda_{\alpha} \,,$$

is a **generalized** SW geometry, where λ_{α} represent the CB data in the 4d SCFT.

The SW differential is replaced by the holomorphic 3-form

$$\Omega = \frac{\prod_i dx_i}{d\hat{F}} \,.$$

Imposing

1. The homogeneity of the geometry,
2. $[\Omega] = 1$,

the set of deformation parameters λ_{α} are of three kinds:

1. $\Delta(\lambda_{\alpha}) > 1$: **CB operators**.
2. $\Delta(\lambda_{\alpha}) < 1$: **couplings**. In particular, $\Delta(\lambda_{\alpha}) = 0$ are the exactly **marginal deformations**.
3. $\Delta(\lambda_{\alpha}) = 1$: **masses**.

(G, G') Singularities

Consider, e.g., the following ADE surface singularities:

$$t^2 = F_{A_k}(x, y) = x^2 + y^{k+1}\,, \quad t^2 = F_{D_k}(x, y) = x^2 y + y^{k-1}\,.$$

(G, G')

$$F = F_G(x_1, x_2) + F_{G'}(x_3, x_4) = 0\,.$$

Example: (A_1, A_3)

$$F = x_1^2 + x_2^2 + x_3^2 + x_4^4 \text{ such that } \hat{F} = x_1^2 + x_2^2 + x_3^2 + x_4^4 + \lambda_1 + \lambda_2 x_4 + \lambda_3 x_4^2\,.$$

From the conditions on the coordinates:

$$[x_1] = [x_2] = [x_3] = 2[x_4] = \frac{2}{3} \Longrightarrow [\lambda_1] = \frac{4}{3}\,, \; [\lambda_2] = 1\,, \; [\lambda_3] = \frac{2}{3}\,.$$

This is an AD theory of rank 1 and a mass, with $\mathfrak{su}(2)$ flavor symmetry (type III Kodaira singularity), also known as $D_3(\mathrm{SU}(2))$.

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(A_2, D_4)

$$F = x_1^2 + x_2^3 + x_3^2 x_4 + x_4^3\,,$$

such that

$$\hat{F} = x_1^2 + x_2^3 + x_3^2 x_4 + x_4^3 + \lambda_1 + \lambda_2 x_2 + \lambda_3 x_3 + \lambda_4 x_4 + \lambda_5 x_2 x_4 + \lambda_6 x_2 x_3 + \lambda_7 x_3^2 + \lambda_8 x_2 x_3^2\,.$$

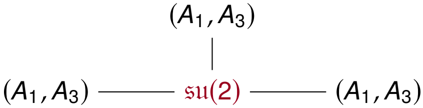
As before, we compute the dimensions of the coordinates

$$[x_1] = 1\,,\ \ [x_2] = [x_3] = [x_4] = \frac{2}{3}\,,$$

implying

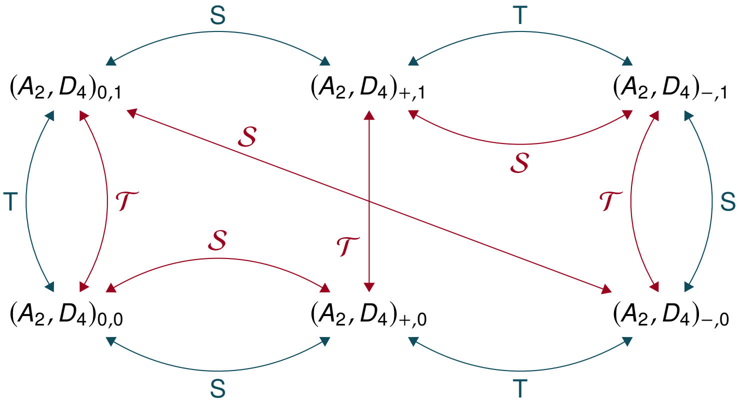
1. $\mathrm{CB}_{(A_2,D_4)} = \left\{\frac{4}{3}, \frac{4}{3}, \frac{4}{3}, 2\right\} = 3\,\mathrm{CB}_{(A_1,A_3)} + \mathfrak{su}(2)\,.$
2. $a_{(A_2,D_4)} = 3\,a_{(A_1,A_3)} + a_{\mathfrak{su}(2)}$ and $c_{(A_2,D_4)} = 3\,c_{(A_1,A_3)} + c_{\mathfrak{su}(2)}\,.$
3. One-dimensional conformal manifold.

$(A_2, D_4)\ :$ [10]



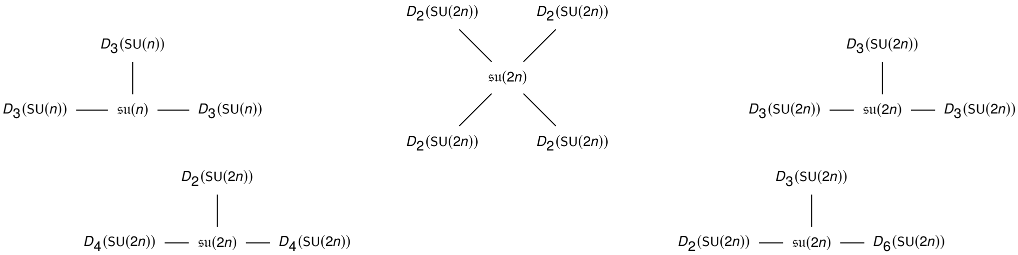
[10] C. Closset, S. Schafer-Nameki, Y.-N. Wang, *JHEP* **02**, 003, arXiv: 2007.15600 (hep-th); M. Buican, T. Nishinaka, *J. Phys. A* **49**, 465401, arXiv: 1603.00887 (hep-th); C. Closset, S. Giacomelli, S. Schafer-Nameki, Y.-N. Wang, arXiv: 2012.12827 (hep-th)

Non-invertible Symmetries in Argyres-Douglas Theories



Generalizations

1. One can consider 6d $\mathcal{N} = (1,0)$ theories, i.e. F-theory on $(\mathbb{C}^2 \times T^2)/\Gamma$, with $\Gamma \subset SU(3)$, on T^2.
2. Tuning the parameters, one obtains a 4d $\mathcal{N} = 2$ SCFT as gauging of $\mathfrak{su}(N)$ of $D_{p_i}(SU(N))$ theories.



3. The complex structure τ of the T^2 gives rise to a $SL(2,\mathbb{Z})$ group, and it is the gauge coupling of the $\mathfrak{su}(N)$ gauge group.
4. The 1-form symmetry comes from the compactification of the 2-form symmetry in 6d.
5. If $a = c$, the conformal manifold is one-dimensional: we obtain **generalization** of the example of (A_2, D_4).
6. If $a \neq c$, the conformal manifold is larger, but for certain theories, **restricting** to the one-dimensional submanifold, the conclusions are **similar**.

Thank you!