Superconformal algebras for (twisted) connected sums

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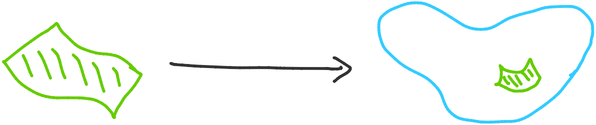
Introduction and motivation

The setup

String theory (typically type II) as a sigma-model.



$$\Sigma \longrightarrow \mathbb{M}_{10-d} \times \mathcal{M}_d \ ,$$

- Superconformal field theory in the worldsheet.
- Target compact manifold with holonomy group G.

What is the relation between them?

Howe–Papadopoulos W-symmetry

At a classical level in the sigma model, covariantly constant forms give rise to worldsheet symmetries and conserved currents.

[Howe, Papadopoulos 93]

Example: $\mathcal{N} = (1,1)$ sigma model with target $\mathcal{M}$.

$$S[X] = \int \mathrm{d}z^+ \mathrm{d}z^- \mathrm{d}\theta^+ \mathrm{d}\theta^- \, (g_{ij} + B_{ij}) \, D_+ X^i D_- X^j \, .$$

- Suppose $\mathcal{M}$ has a covariantly constant p-form ϕ, $\nabla\phi = 0$.
- Then $S[X]$ is invariant under a chiral symmetry whose associated current is

$$J^- = \frac{1}{p!} \, \phi_{i_1 \cdots i_p} \, DX_+^{i_1} \cdots D_+ X^{i_p} .$$

Quantum picture

After quantization, currents become chiral operators of the worldsheet $\mathcal{N} = 1$ SCFT.

Covariantly constant forms $\longleftrightarrow$ Operators

We **extend** the $\mathcal{N} = 1$ Virasoro algebra by adding these operators. We use the following dictionary:

Target space	Cov. const. forms	Operators	Algebra
$\mathbb{R}$ or $\mathbb{S}^1$	$\mathrm{d}t$	ψ_t	Fr^1
Calabi-Yau n-fold	(ω_n, Ω_n)	$(J_n, A_n + iB_n)$	Od_n

The algebras relevant for this talk are the Free fermion and the Odake algebra. [Odake 89]

The Schoen Calabi–Yau manifold

What is it?

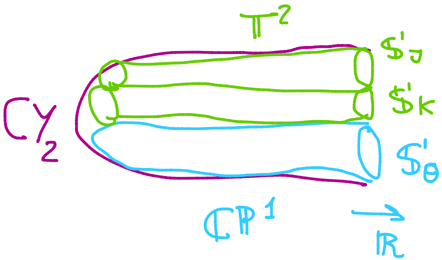
- Introduced in [Schoen 88], but we will mostly follow [Braun, Schäfer-Nameki 18].
- SU(3) holonomy.
- It is a CICY, the split bicubic (bi-elliptic fibration).

$$\left[\begin{array}{c|cc} \mathbb{CP}^1 & 1 & 1 \\ \mathbb{CP}^2 & 3 & 0 \\ \mathbb{CP}^2 & 0 & 3 \end{array}\right].$$

- It is a crepant resolution of $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$.
- We are interested in its realization as a connected sum.

Key ingredient

Asymptotically cylindrical (ACyl) Calabi–Yau 2-fold.

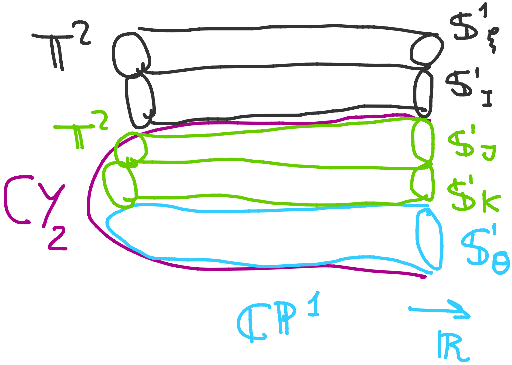


$$\text{ACyl } \mathrm{CY}_2 \longrightarrow \mathrm{CY}_1 \times \mathbb{S}^1_\theta \times \mathbb{R}^+ \,.$$

$$\omega_2 \longrightarrow \omega_{2,\infty} = \mathrm{d}\theta \wedge \mathrm{d}x_J + \mathrm{d}t \wedge \mathrm{d}x_K \,,$$
$$\Omega_2 \longrightarrow \Omega_{2,\infty} = (\mathrm{d}x_J - i\mathrm{d}\theta) \wedge (\mathrm{d}x_K - i\mathrm{d}t) \,.$$

Building blocks

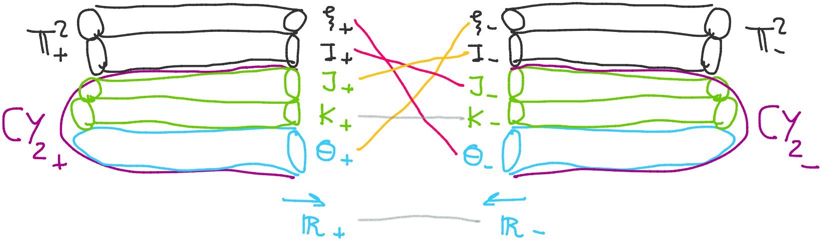
Trivial torus bundle over Calabi–Yau 2-fold.



$$\omega_3 = \mathrm{d}\xi \wedge \mathrm{d}x_I + \omega_2 \,, \qquad \Omega_3 = (\mathrm{d}x_I - i\mathrm{d}\xi) \wedge \Omega_2 \,,$$

Construction

Glue the open manifolds along their asymptotic ends



Well-defined $(\omega_3, \Omega_3) \implies$ SU(3) holonomy

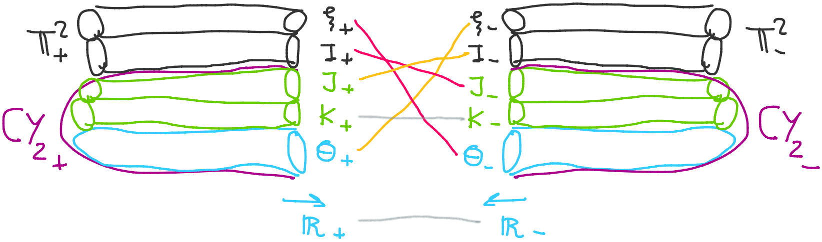
The Diamond and its consequences

Correspondence

How can we use the piecewise geometric data?

- Use the **dictionary** to associate an algebra to each geometry.
- Asymptotic relations give ansätze of algebra inclusions.
- Algebras arranged as a **unique** diamond of inclusions, mimicking the geometric construction.

The Diamond



$$
\begin{array}{ccc}
 & \mathrm{Fr}^6 & \\
\cup & & \cup \\
\left(\mathrm{Fr}^2 \oplus \mathrm{Od}_2\right)_+ & & \left(\mathrm{Fr}^2 \oplus \mathrm{Od}_2\right)_- \\
\cup & & \cup \\
 & \mathrm{Od}_3 &
\end{array}
$$

Application: mirror symmetry

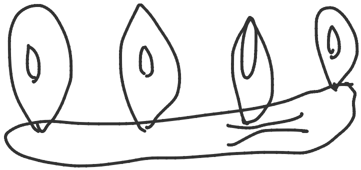
Two manifolds are mirror symmetric if they give rise to the same physical theory after compactification.

Superconformal algebras are invariant

$$\Downarrow$$

Mirror map = algebra automorphism

In geometry, SYZ approach: mirror symmetry arises from T-dualities on a torus fibration. [Strominger, Yau, Zaslow 96]



Mirror automorphisms for the Schoen manifold

Each algebra has automorphisms with a geometric interpretation.

$$\mathrm{Fr}^1 \longrightarrow \text{T-duality map } \mathbf{T}$$
$$\mathrm{Od}_n \longrightarrow \text{Mirror symmetry map } \mathbf{M}$$
$$\text{Phase rotation } \mathbf{Ph}^{\pi}$$

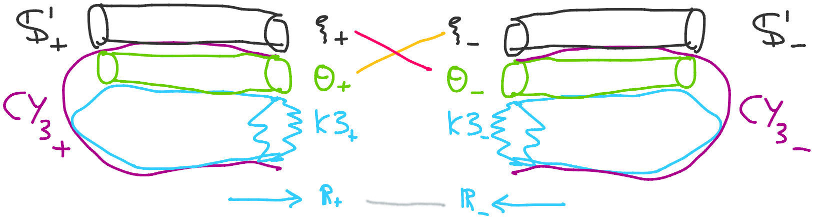
The Schoen manifold has an SYZ fibration. The $\mathbb{T}^3$ fibres are given by the coordinates (x_I, x_J, x_K) in our setting. This defines an automorphism **on the whole diamond**.

Fr^6	$\mathbf{T}_{I+} \circ \mathbf{T}_{J+} \circ \mathbf{T}_{K+}$
$\left(\mathrm{Fr}^2 \oplus \mathrm{Od}_2\right)_+$	$\mathbf{T}_{I+} \circ \mathbf{M}$
$\left(\mathrm{Fr}^2 \oplus \mathrm{Od}_2\right)_-$	$\mathbf{T}_{J+} \circ \mathbf{M}$
Od_3	$\mathbf{M} \circ \mathbf{Ph}^{\pi}$

A comment on other manifolds

ETCS & GCS

Diamonds exist for manifolds of holonomies G_2 and Spin(7) built as (generalized) connected sums.



In particular, the Diamond can be used to shed light on mirror symmetry for Spin(7) manifolds.

Conclusion and outlook

Conclusions

- Diamond reproducing geometry.

$$\begin{array}{ccc} & \mathrm{Fr}^6 & \\ \rotatebox{45}{$\subset$} & & \rotatebox{-45}{$\supset$} \\ \left(\mathrm{Fr}^2 \oplus \mathrm{Od}_2\right)_+ & & \left(\mathrm{Fr}^2 \oplus \mathrm{Od}_2\right)_- \\ \rotatebox{-45}{$\supset$} & & \rotatebox{45}{$\subset$} \\ & \mathrm{Od}_3 & \end{array}$$

- Connection between automorphisms and mirror symmetry.
- Holonomies $SU(3)$, G_2 and $Spin(7)$ (so far).

Future directions

- Diamonds for other manifolds
  (e.g. higher-dimensional Calabi–Yau).
- Understanding manifolds with the same Diamond.
- Generalization to more elaborated geometric constructions.
- Mirror symmetry analysis for G_2 Extra Twisted Connected Sum manifolds (ETCS).
- Connection with chiral de Rham complex.

Thank you! Questions?