Filling a (twist) gap in our knowledge of 2d CFT

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Motivation I: Misconceptions about 2d CFTs

- All 2d CFTs are solved. Reply: (otherwise I wouldn't be giving this talk)
- All compact 2d CFTs are rational. Reply: ($c = 1$ compact boson @ irrational R, CY NLσ-models)
- All compact irrational 2d CFTs are continuously connected to a nearby rational CFT. Reply: (today's construction)

Motivation II: Theorems about 'generic' 2d CFTs

- Modular invariance + $c > 1 \to \infty$-many Virasoro primaries [Cardy, 1986].
- Rationality (finitely many primaries) therefore implies enhanced chiral algebra (CA): extra currents. This means $\tau_{\text{gap}} = \Delta - J = 0$. There are irrational theories with enhanced CA, but $\tau_{\text{gap}} > 0$ implies irrationality (and just Virasoro CA).

- Modular bootstrap: unitary, $c > 1$, $\tau_{\text{gap}} > 0 \implies \exists\, \tau \to (c-1)/12$
- Virasoro light-cone bootstrap: above + compact $\implies$ Regge trajectories, asymptotic density of OPE coeffs.

Main premise

- To quote some experts: 'Either we are missing key constraints or we don't have any example of what a generic 2d CFT looks like' or 'What we need is a 3d Ising model in 2d (which is NOT the 2d Ising model!)'
- Our goal is to explicitly construct a generic 2d CFT, i.e. a compact, unitary 2d CFT with $c > 1$ and $\tau_{\text{gap}} > 0$

How to build a 2d CFT with $\tau_{\mathbf{gap}} > 0$ (I)

- A systematic way to produce new CFTs is taking N copies of a solved one and coupling them preserving some symmetry. Ideally, we use RG in the perturbative regime to find IR fixed points.
- The $4 - \epsilon$ expansion of N free scalars has produced a wealth of fixed points [Osborn and Stergiou, 2018], but properties at $\epsilon = 2$ are hard to study.
- Directly in 2d, we can couple the solved Ising model. Using a $q-$states deformation makes them weakly coupled but non-unitary [Dotsenko et al., 1999], with $\Delta_{\epsilon_i \epsilon_j} = 2 - \#(q-2) + \dots$

$$S = \sum_{i=1}^{N} S^i_{q-\mathrm{Potts}} + g \sum_{i<j} \epsilon^i \epsilon^j \tag{1}$$

How to build a 2d CFT with $\tau_{\mathbf{gap}} > 0$, (II)

- Take instead the full family of minimal models $\mathcal{M}_m$ with $c = 1 - 6/(m(m+1))$. [Zamolodchikov, 1987] showed that as $m \to \infty$ there is a weakly coupled RG flow:

$$\mathcal{M}_m + g \int \phi_{(1,3)} \to \mathcal{M}_{m-1} \tag{2}$$

- We propose an N copy version of this flow:

$$S_{\mathrm{CMM}} = \sum_{i=1}^{N} S_m^i + g_\epsilon \int \mathrm{d}^2 x\, N^{-\frac{1}{2}} \sum_{i=1}^{N} \phi^i_{(1,3)} \tag{3}$$
$$+ g_\sigma \int \mathrm{d}^2 x \binom{N}{4}^{-\frac{1}{2}} \sum_{i<j<k<l}^{N} \phi^i_{(1,2)} \phi^j_{(1,2)} \phi^k_{(1,2)} \phi^l_{(1,2)}\,.$$

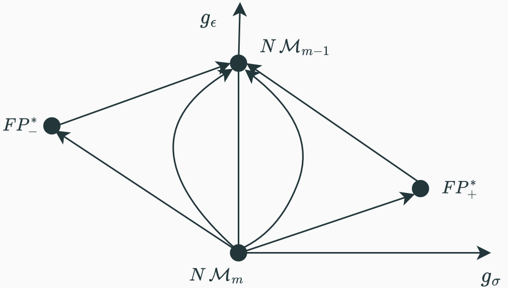
- The operators ϵ and σ have $\Delta = 2 - \frac{\#}{m}$, and close under OPE.

Perturbative RG analysis (I)

- The β-function equations read:

$$\begin{aligned}\beta_\sigma &= \frac{6}{m} g_\sigma - \frac{4\pi\sqrt{3}}{\sqrt{N}} g_\sigma g_\epsilon - 6\pi \binom{N-4}{2} \binom{N}{4}^{-\frac{1}{2}} g_\sigma^2\,, \\ \beta_\epsilon &= \frac{4}{m} g_\epsilon - \frac{4\pi}{\sqrt{3N}} g_\epsilon^2 - \frac{2\pi\sqrt{3}}{\sqrt{N}} g_\sigma^2\end{aligned} \tag{4}$$

- There are 4 real fixed points, focus on 2 non-trivial ones:



Perturbative RG analysis (II)

- Straightforward to extract anomalous dimensions, e.g.

$$\Delta = 2 \pm \frac{2\sqrt{6}}{m}\,(N=4)\,; \quad \Delta = 2 \pm \frac{\sqrt{6} \pm \sqrt{870}}{6m}\,(N=6) \tag{5}$$

- Similarly, we can extract the IR central charge using the integrated $c-$theorem:

$$\Delta c^*_\pm = -\frac{2N}{m^3}\frac{\sqrt{3}Q(N) \mp 3\sqrt{P(N)}}{\sqrt{P(N)}}\,, \tag{6}$$

which is also irrational for $N \geq 6$.

- Irrational CFT data hints at irrationality following Vafa's theorem [Vafa, 1988].

Lifting of currents and evidence of $\tau_{\mathbf{gap}} > 0$ (I)

- The decoupled UV theory has a huge set of conserved currents due to the enhanced chiral algebra $\mathcal{V}_{\mathrm{UV}} = \mathrm{Vir}^N$. Count by expanding Z^N in Vir characters.
- The simplest question is whether out of the T^i how many spin 2 operators remain conserved. We would hope $\mathrm{Vir}^N \to \hat{\mathrm{Vir}}$, with $\hat{T} = \sum T^i$.
- Use multiplet recombination

$$\bar{\partial} T_J = (b g_\sigma + O(g_\sigma^2)) V_{J-1} \tag{7}$$

for a divergence operator V_{J-1}. The coefficient b is proportional to C_{TVO}.

Lifting of currents and evidence of $\tau_{\mathbf{gap}} > 0$ (II)

- The anomalous dimension matrix γ^{ij} is then proportional to $C^2_{T^i V^j O}$. In this case the candidate V^i read

$$V^i = \sum_{(j<k<l)\neq i} (\partial\phi^i_{(1,2)})\phi^j_{(1,2)}\phi^k_{(1,2)}\phi^l_{(1,2)} - \frac{1}{4}\partial(\phi^i_{(1,2)}\phi^j_{(1,2)}\phi^k_{(1,2)}\phi^l_{(1,2)})\,. \tag{8}$$

- There is an $N-1$ fold degenerate anomalous dimension $3(g^*_\sigma\pi)^2/(N-1)$ associated to $T^i - T^{i+1}$, as well as a zero anomalous dimension associated to $\hat{T}$.
- We still need to go on until $J=\infty$. At $J=4$ the S_N singlet current:

$$T_4 = \Sigma L_{-4} - \frac{5}{3}\Sigma L^2_{-2} + \frac{9}{N-1}(\Sigma L_{-2})^2 \tag{9}$$

gets an anomalous dimension by 'eating' some V_3.

Lifting of currents and evidence of $\tau_{\mathbf{gap}} > 0$ (III)

- Proceeding we find:

ℓ \ N	4	5	6	7
4	(1, 1)	(1, 2)	(1, 2)	(1, 2)
6	(2, 2)	(2, 5)	(2, 6)	(2, 6)
8	(4, 7)	(4, 17)	(4, 22)	(4, 23)
10	(5, 18)	(7, 50)	(7, 69)	(7, 75)

Lifting of currents and evidence of $\tau_{\mathrm{gap}} > 0$ (IV)

- The counting is overwhelming for $N > 4$, all currents should lift, but we explicitly check γ_{IJ} has maximal rank in the singlet sector.
- This is 'enough' for the orbifolded theory, and indirectly suggests the same should happen for currents in non-trivial S_N irreps (but explicitly checked up to spin 5).
- Interestingly, for $N = 4$ a spin 4 current in the (2,2) of S_4 and a spin 6 singlet survive at one loop. Could the theory flow to a $W(2, 4, 4, 6)$ chiral algebra?

Summary

- In generic 2d CFTs all non-identity primaries should have $\Delta > J$.
- We can construct candidate theories by finding IR fixed point of weakly coupled sets of Virasoro minimal models
- The evidence that the gap lifts is overwhelming. Though not rigorous, the burden of proof is now on the other side.

Some open directions

- Make the problem finite by deriving an upper bound on the maximum spin gap for currents. (Numerical Bootstrap?)
- Study the models at large N
- Study the models non-perturbatively (Numerical Bootstrap, Monte Carlo, Hamiltonian truncation)

Thank you!

Thank you for your attention!

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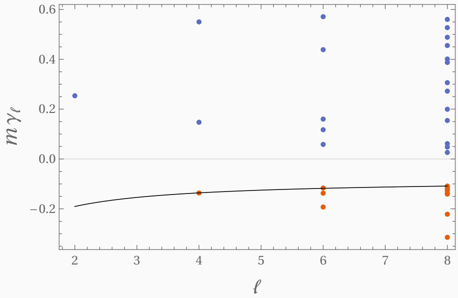
John Cardy

Alexander Zamolodchikov

A quantum Regge trajectory?

- Virasoro lightcone bootstrap predicts quantum Regge trajectories that asymptote to usual CFT Regge trajectories at large c (we can make N large).
- We explicitly computed one-loop anomalous dimension of double-trace operators schematically of the form $O_J = \sum_{i<j} \phi^i_{(1,2)} \mathcal{L}_{-J} \phi^j_{(1,2)}$ and found:



- At least one trajectory has the lightcone bootstrap large N behavior. We can track it down to finite N.

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