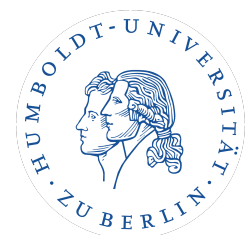


FERMIONS ACROSS DIMENSIONS

Julien Barrat

28.09.2023

DESY Theory Workshop



RTG 2575:

Rethinking
Quantum Field Theory

TO APPEAR

*Scalar-fermion correlators in
Yukawa CFTs across dimensions,*

JB, I. Burić, V. Schomerus, P. van Vliet.

1. **YUKAWA**
CFT

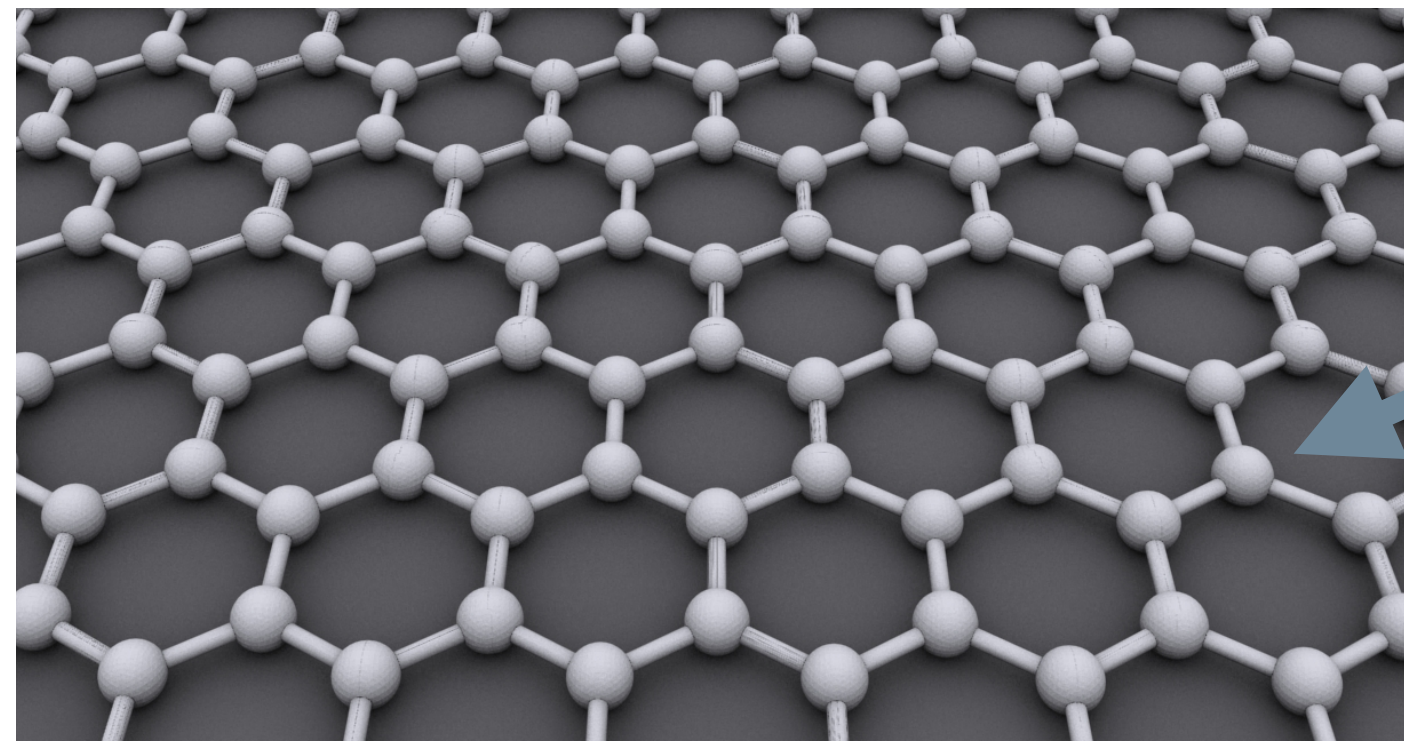
2. **SCALAR-FERMION**
CORRELATORS

1.

YUKAWA
CFT

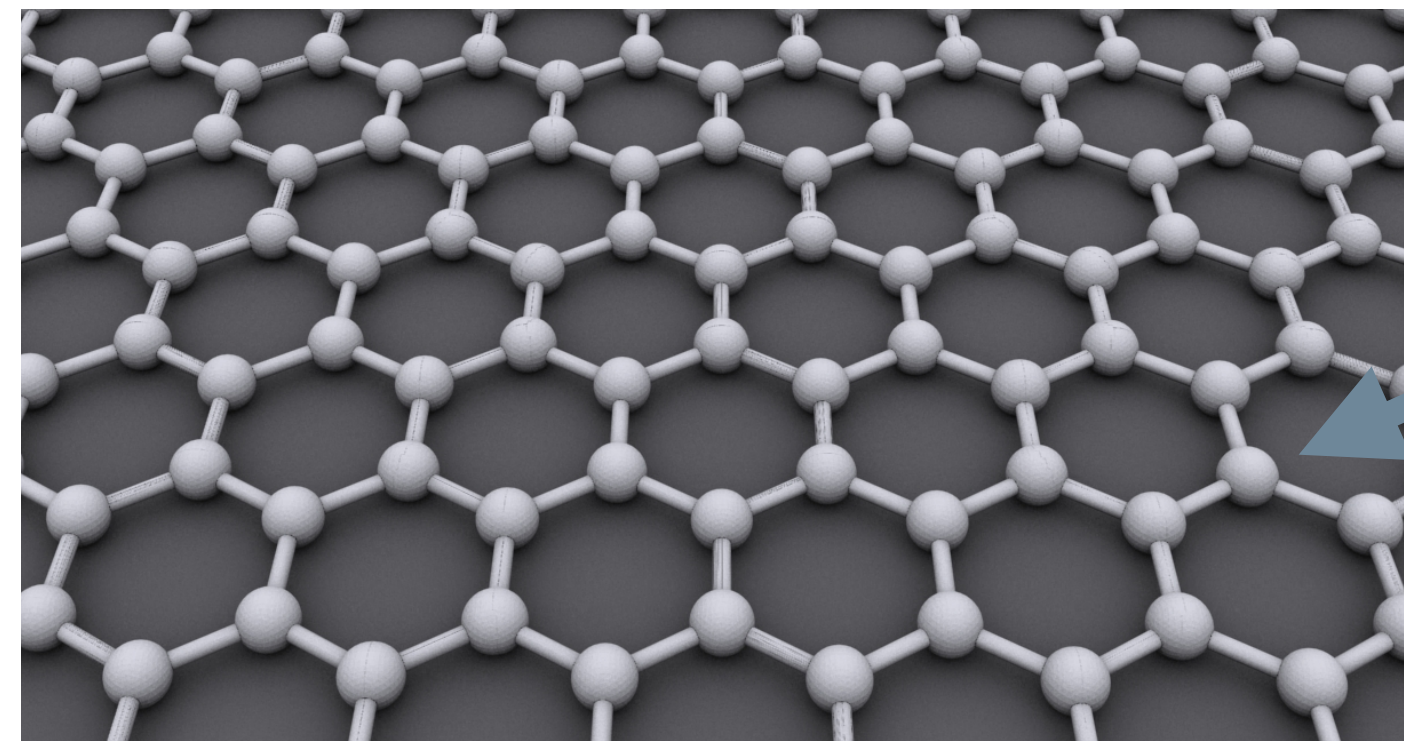


GRAPHENE



carbon atoms

GRAPHENE



carbon atoms

$2d$ material

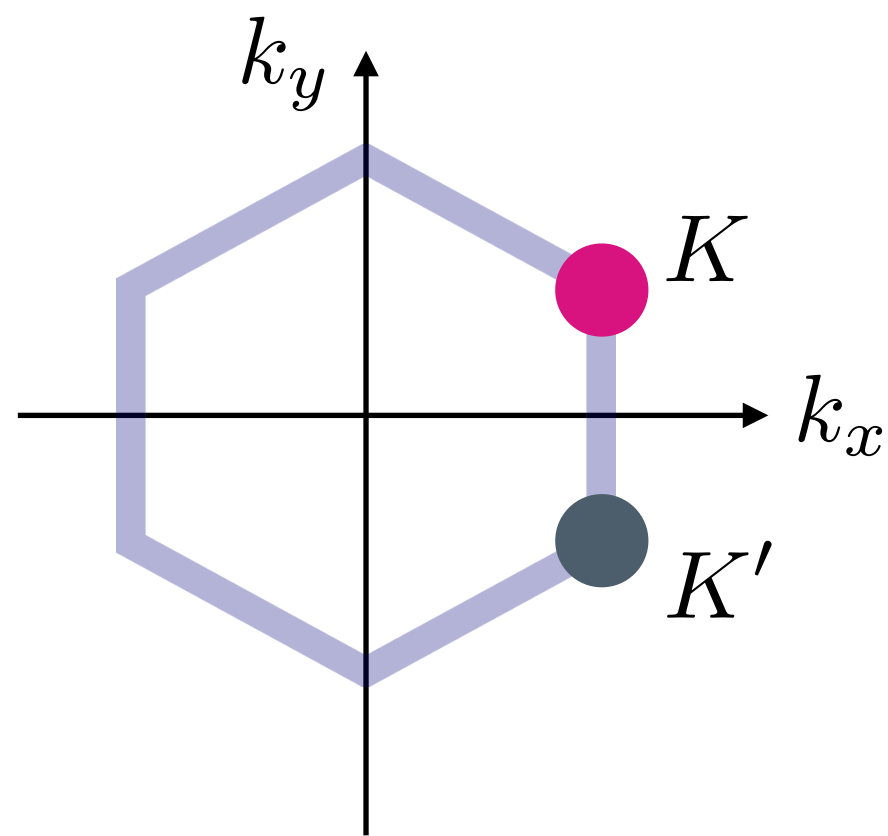
Excellent conductor

Robust, flexible, light

Lorentz symmetry

GRAPHENE $\rightarrow$ QFT

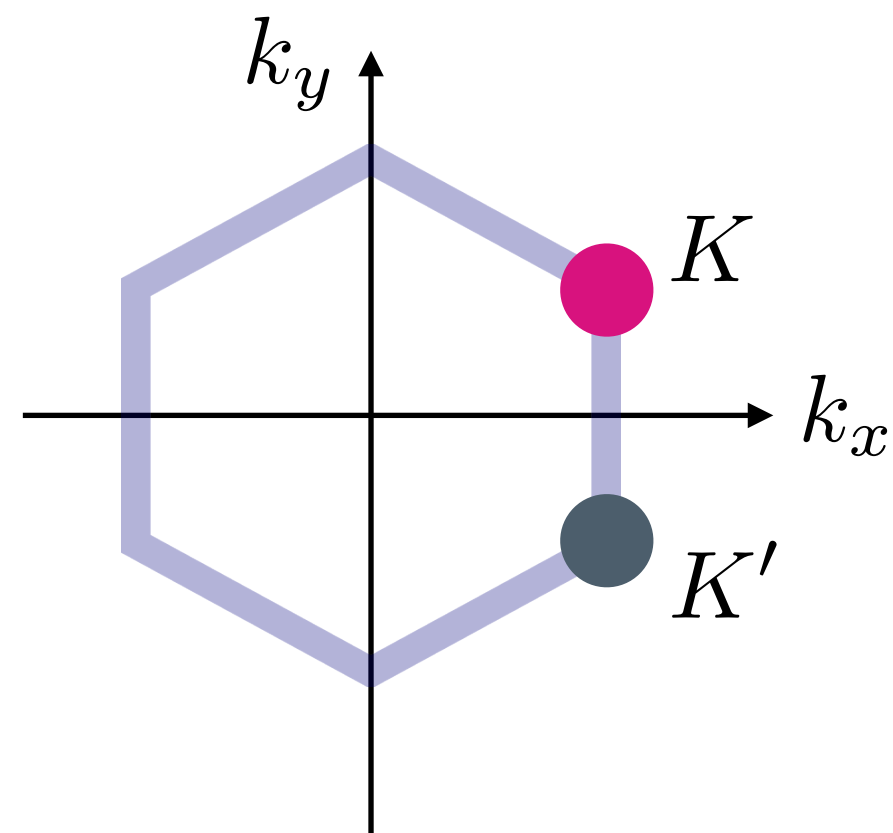
Reciprocal lattice



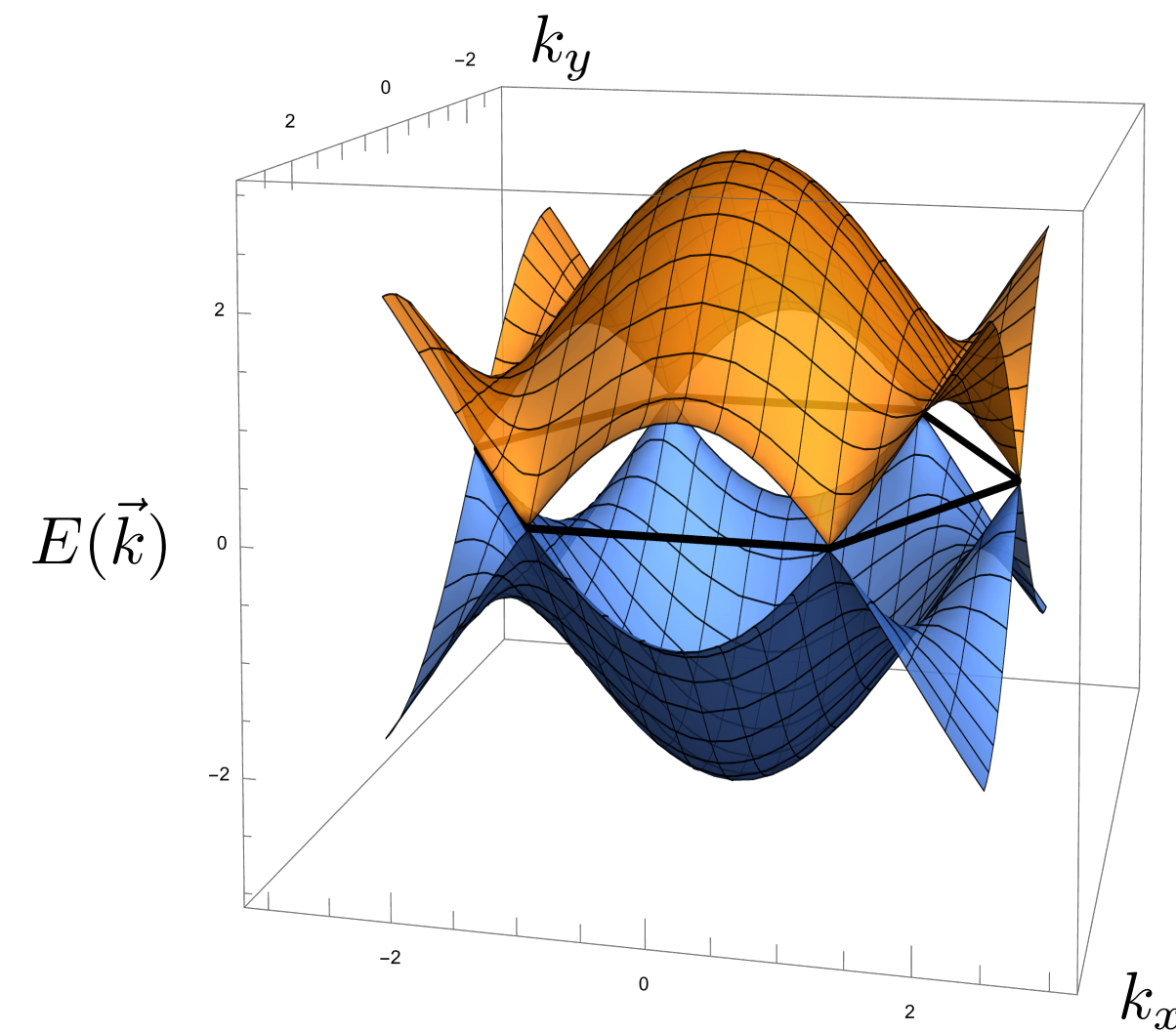
$$\mathcal{H} = -t \sum_{\text{sites, spin}} \left(a_{\sigma,i}^\dagger b_{\sigma,j} + \text{h.c.} \right)$$

GRAPHENE $\rightarrow$ QFT

Reciprocal lattice



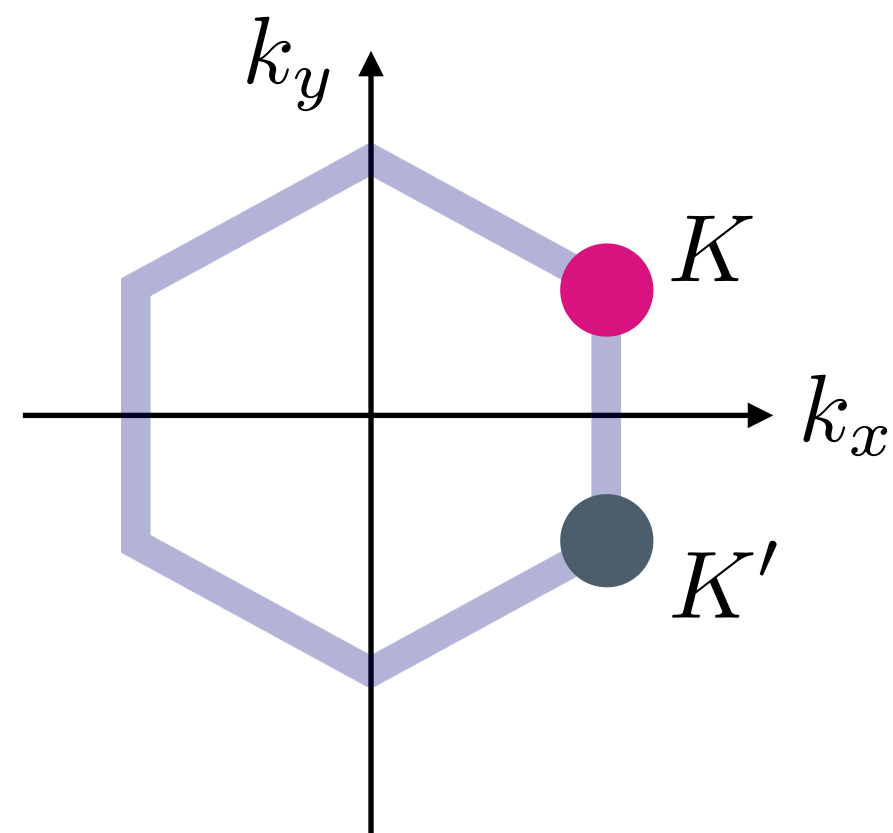
Energy bands



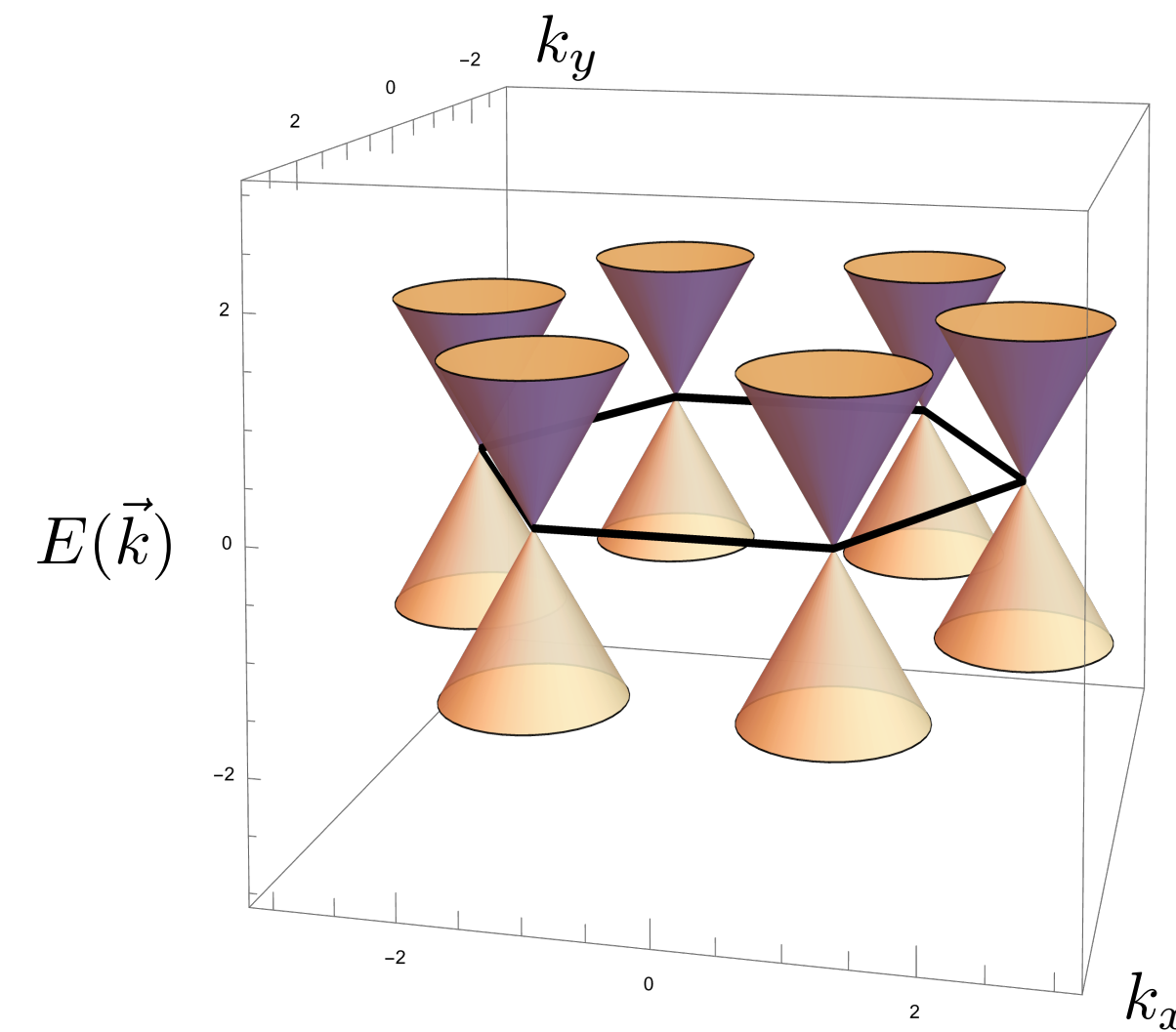
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GRAPHENE $\rightarrow$ QFT

Reciprocal lattice



Energy bands

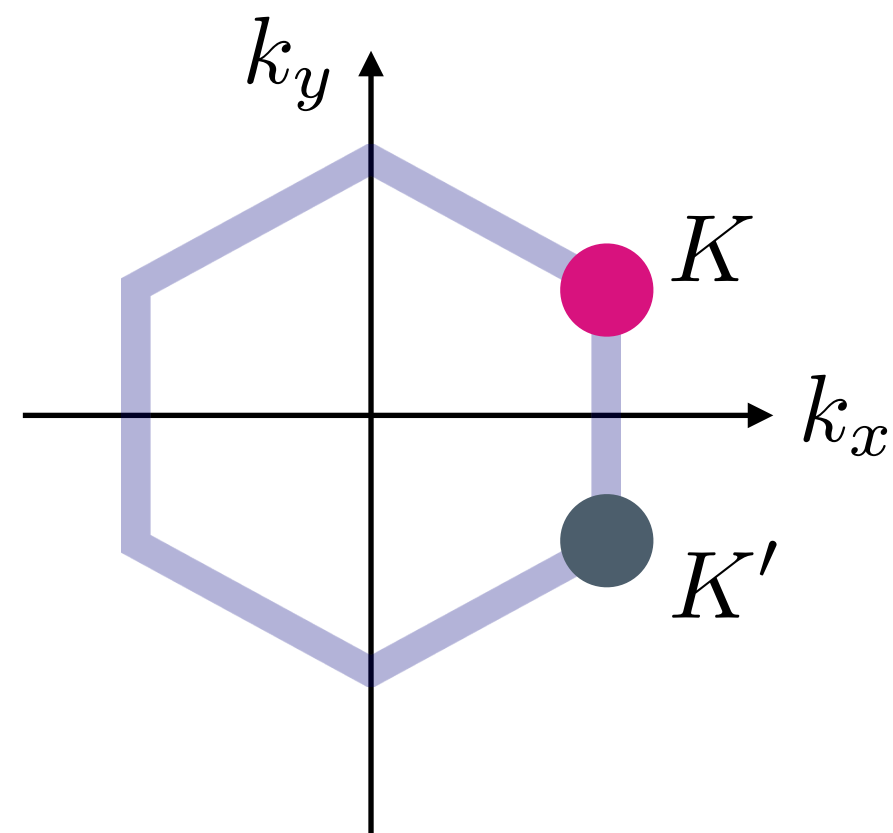


$$E_{\pm}(\vec{k}) = \pm v_F |\vec{k}|$$

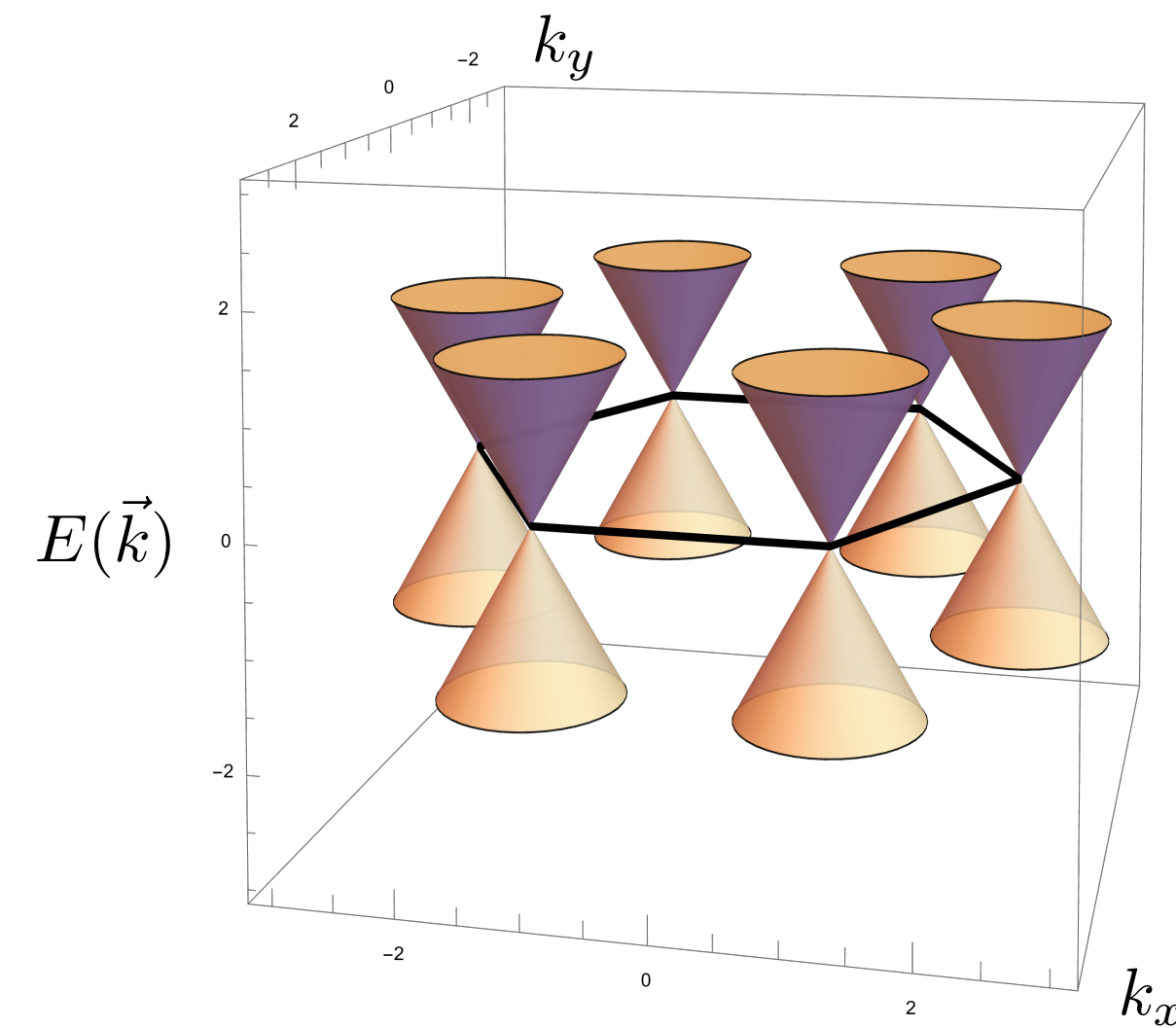
(relativistic!)

GRAPHENE $\rightarrow$ QFT

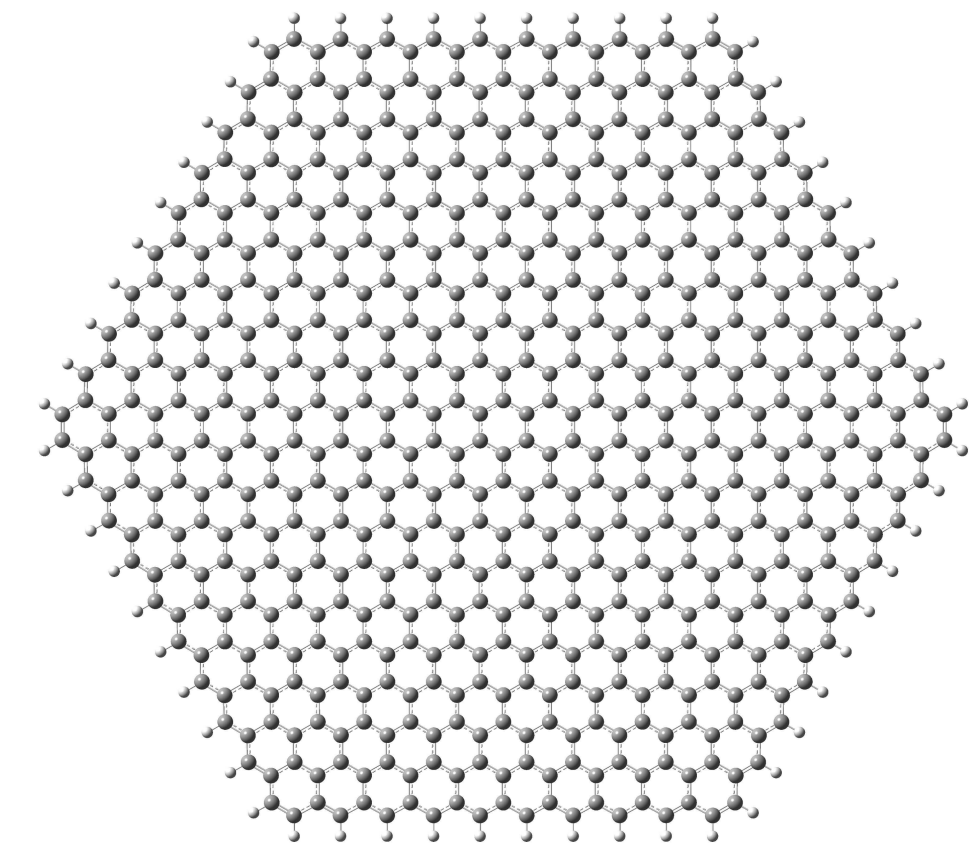
Reciprocal lattice



Energy bands



Continuum limit



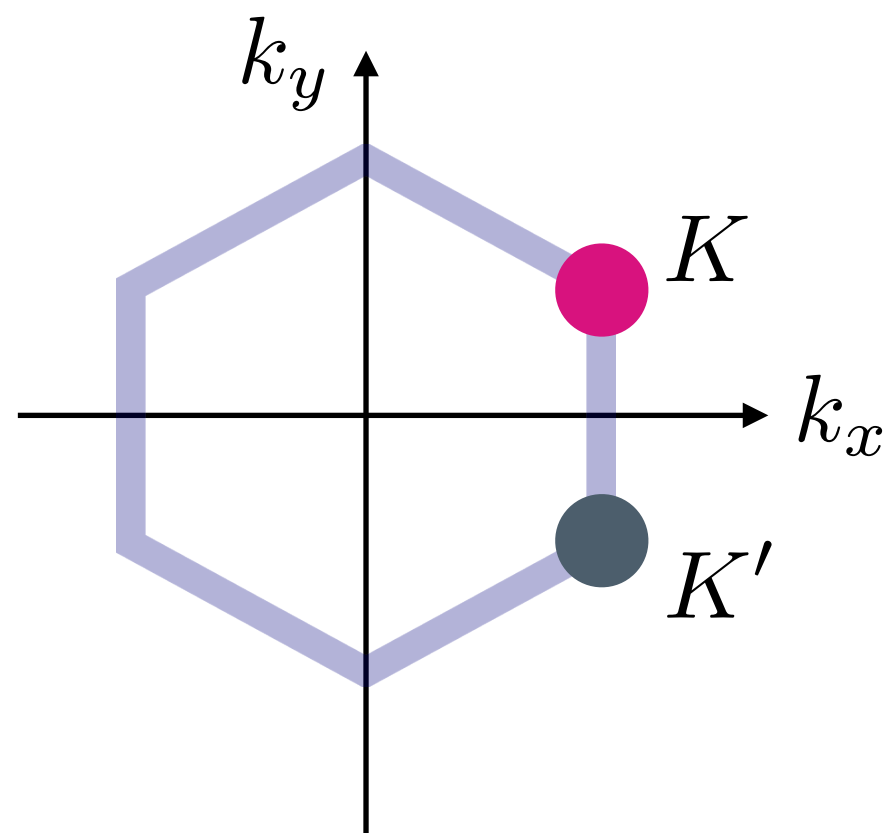
$$S_{\text{free}} = \int d^3x i \bar{\psi}^a \sigma \cdot \nabla \psi^a$$

[Semenoff, '84]

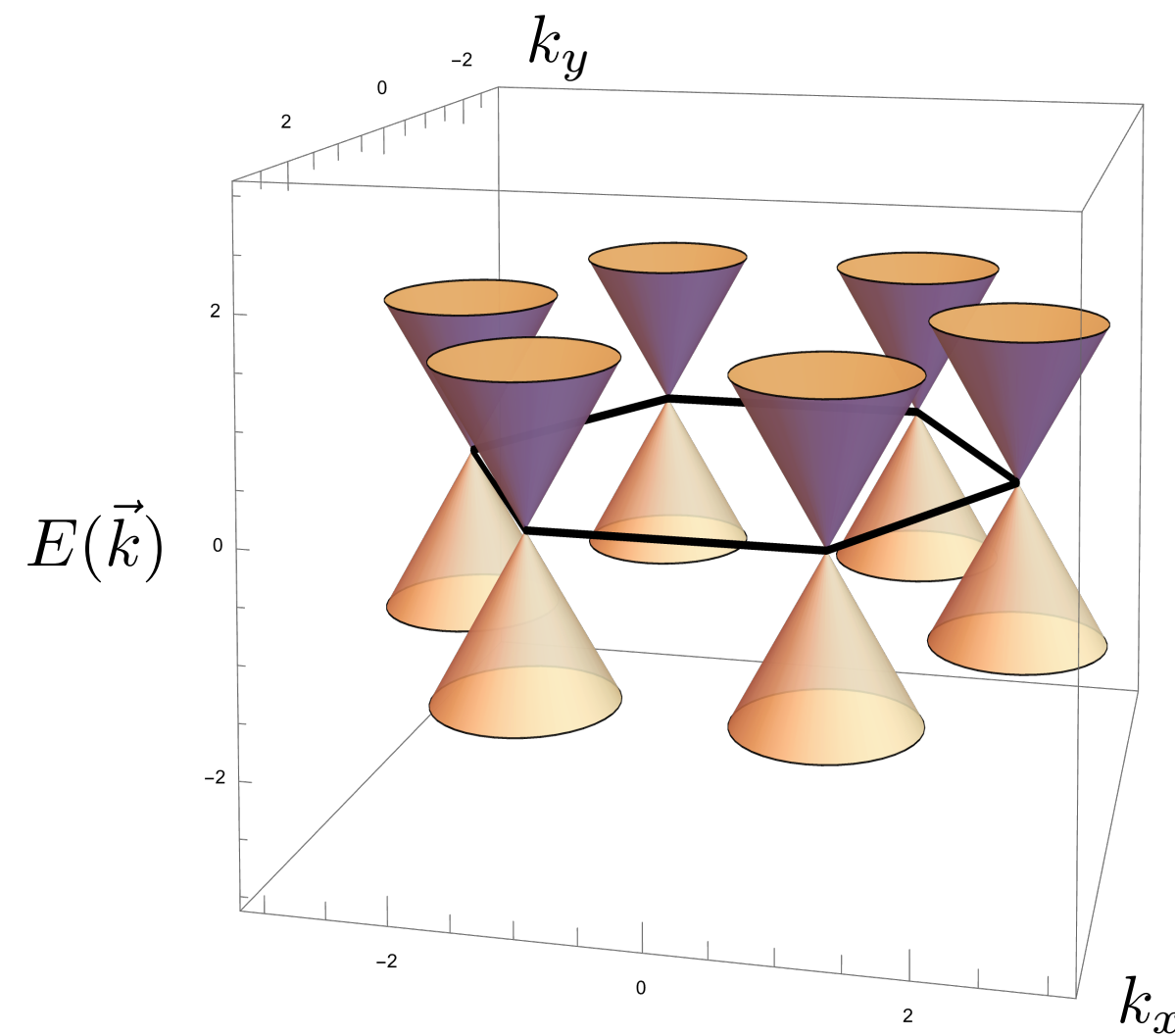
$$a = 1, 2$$

GRAPHENE $\rightarrow$ QFT

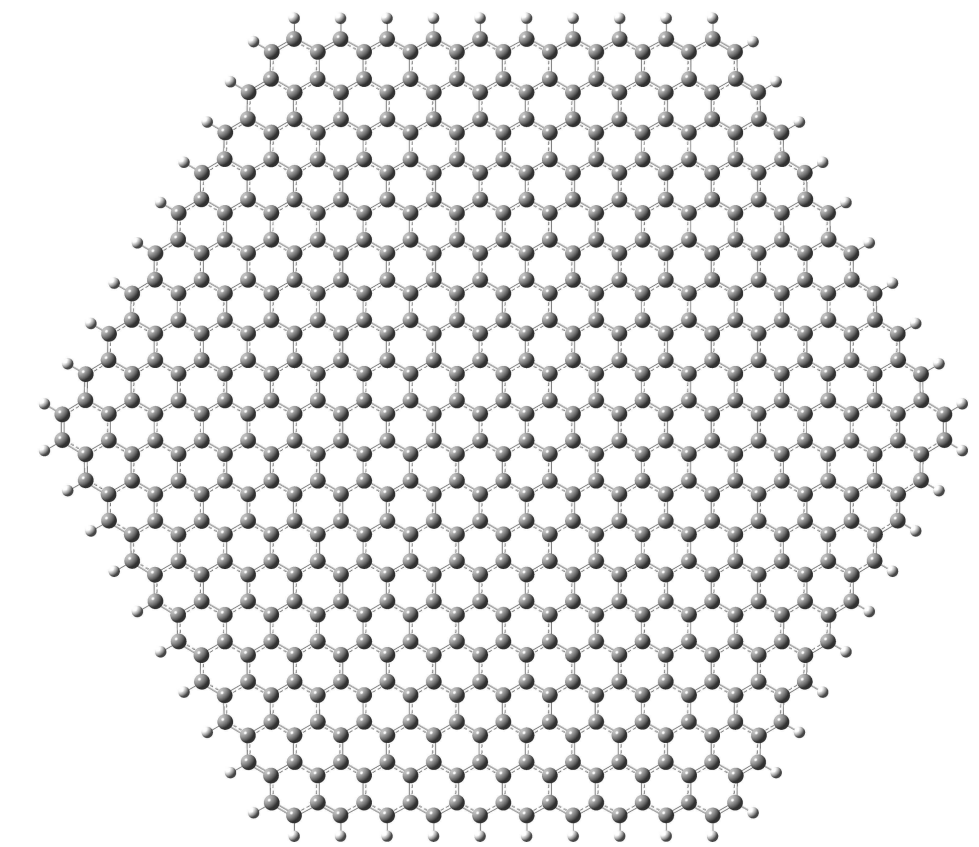
Reciprocal lattice



Energy bands



Continuum limit



$$S = \int d^3x \left(i\bar{\psi}^a \not{\partial} \psi^a + \frac{g^2}{2} (\bar{\psi}^a \psi^a) (\bar{\psi}^b \psi^b) \right) \quad a = 1, 2$$

[Herbut, '09]

YUKAWA QFT

UV completion

$$S = \int d^3x \left(i\bar{\psi}^a \not{\partial} \psi^a + \frac{1}{2} \partial_\mu \phi^I \partial_\mu \phi^I + g \bar{\psi}^a \Sigma^I \phi^I \psi^a + \frac{\lambda}{4!} \phi^I \phi^I \phi^J \phi^J \right)$$

[Hubbard, Stratonovich, '57]

[Gross-Neveu, '74]

YUKAWA QFT

UV completion

$$S = \int d^3x \left(i\bar{\psi}^a \not{\partial} \psi^a + \frac{1}{2} \partial_\mu \phi^I \partial_\mu \phi^I + g \bar{\psi}^a \Sigma^I \phi^I \psi^a + \frac{\lambda}{4!} \phi^I \phi^I \phi^J \phi^J \right)$$

$$(a = 1, \dots, N_f, \quad I = 1, \dots, N)$$

[Hubbard, Stratonovich, '57]

[Gross-Neveu, '74]

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Fixed points: $\beta_g(g_\star) = \beta_\lambda(\lambda_\star) = 0$

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[Gross-Neveu, '74]

Fixed points: $\beta_g(g_\star) = \beta_\lambda(\lambda_\star) = 0$

Conformal bootstrap

don't use action, assume fixed points exist $\longrightarrow$ many results!

[Alday, Bissi, Gimenez-Grau, Gromov, Henriksson, Kaviraj, Kravchuk, Liendo, Paulos, Poland, Rong, Rychkov, Schomerus, v. Vliet, Zhiboedov, ...]

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[Hubbard, Stratonovich, '57]

[Gross-Neveu, '74]

$$\text{Fixed points: } \beta_g(g_\star) = \beta_\lambda(\lambda_\star) = 0$$

Conformal bootstrap

don't use action, assume fixed points exist $\longrightarrow$ many results!

ε -expansion

$$d = 4 - \varepsilon$$

[Wilson, Fisher, '74]

$d = 4$ free theory

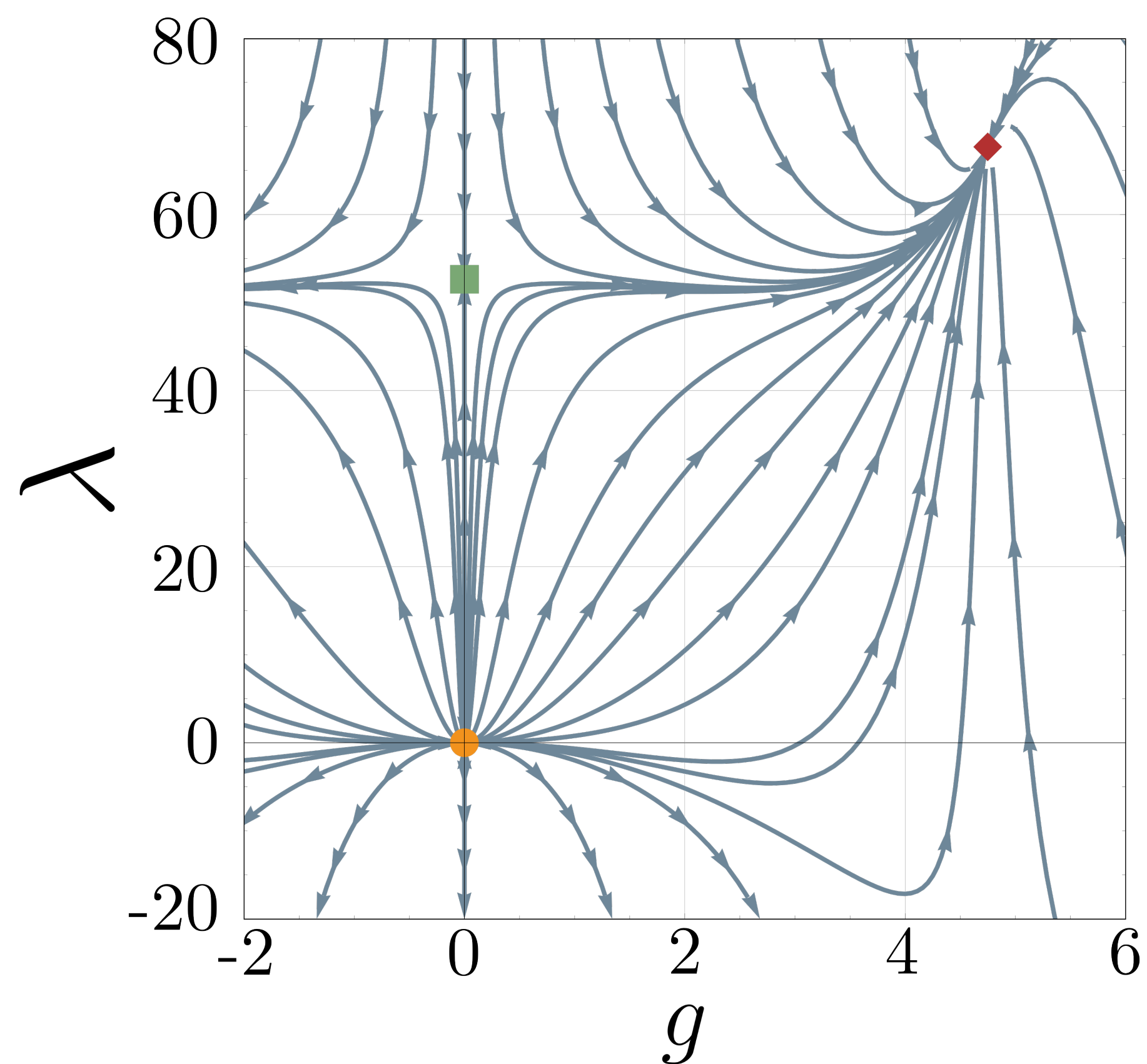
$$\downarrow \varepsilon \rightarrow 1$$

$d = 3$ interacting theory

[Alday, Bissi, Gimenez-Grau, Gromov, Henriksson, Kaviraj, Kravchuk, Liendo, Paulos, Poland, Rong, Rychkov, Schomerus, v. Vliet, Zhiboedov, ...]

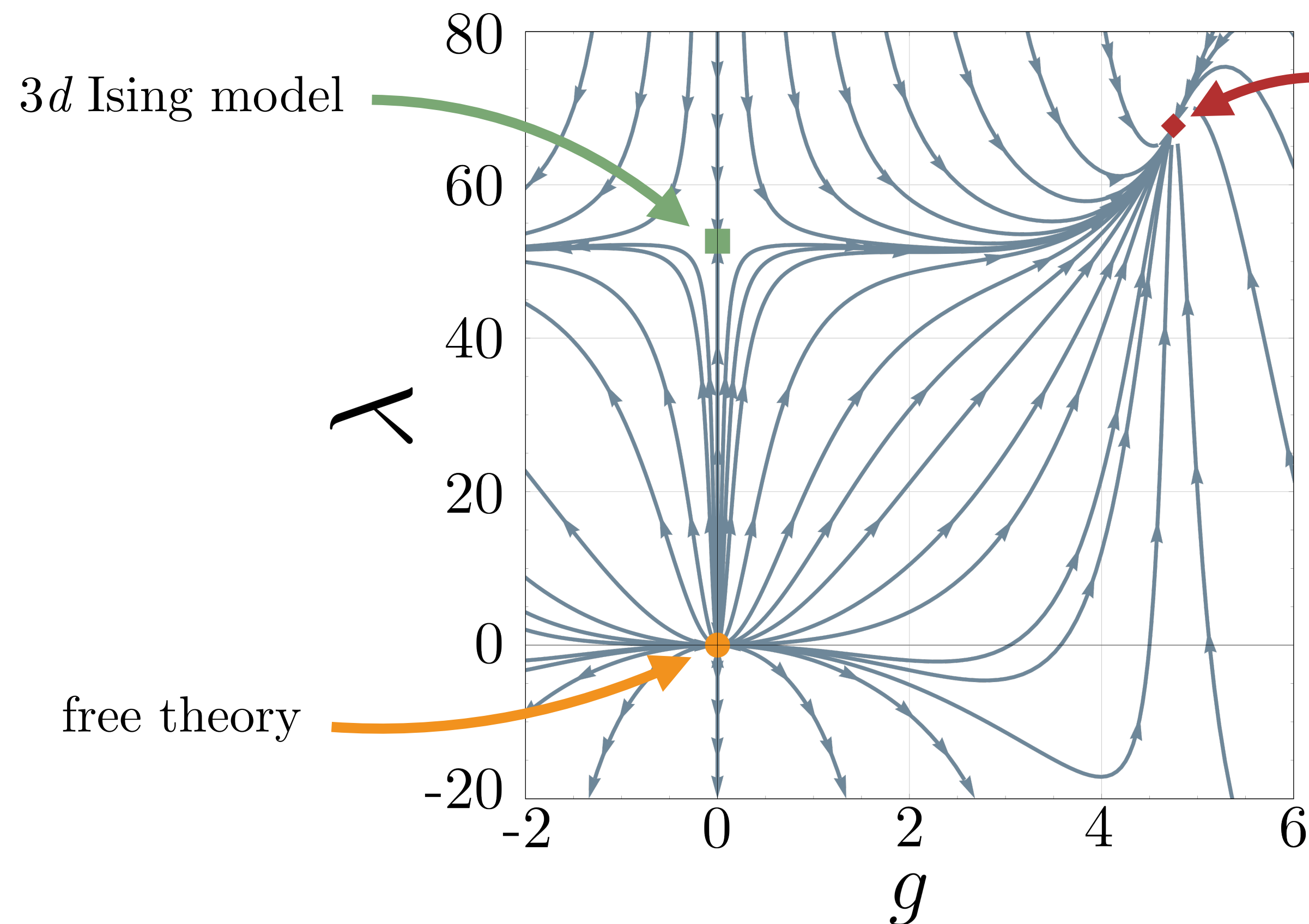
PHASE TRANSITIONS

RG flow ($N = 1, N_f = 1/4$)



PHASE TRANSITIONS

RG flow ($N = 1, N_f = 1/4$)



WFY fixed point
($\mathcal{N} = 1$ SUSY)

$$\beta_g(g_\star) = \beta_\lambda(\lambda_\star) = 0$$

$$g_\star^2 = \frac{64\pi^2}{7}\varepsilon + \dots$$

$$\lambda_\star = \frac{8\pi^2}{21}(5 + \sqrt{1705})\varepsilon + \dots$$

[Karkkainen, Lacaze, Lacock, Petersson, '94]

[Rosenstein, Yu, Kovner, '93]



2.

**SCALAR-FERMION
CORRELATORS**

2. SCALAR-FERMION CORRELATORS

AN INTERESTING CORRELATOR

$$\langle \phi\phi\bar{\psi}\psi \rangle$$

2. SCALAR-FERMION CORRELATORS

AN INTERESTING CORRELATOR

$$\langle \phi \phi \bar{\psi} \psi \rangle$$

OPE

$$\overline{\phi\phi} \sim \mathbb{1} + \phi^2 + \dots$$

$$\overline{\phi\psi} \sim \bar{\psi} + \dots$$

2. SCALAR-FERMION CORRELATORS

AN INTERESTING CORRELATOR

$$\langle \phi \phi \bar{\psi} \psi \rangle$$

OPE

$$\overline{\phi\phi} \sim \mathbb{1} + \phi^2 + \dots \qquad \overline{\phi\bar{\psi}} \sim \bar{\psi} + \dots$$

Tensor structure

$$d = 4 \qquad \langle \phi \phi \bar{\psi} \psi \rangle = \mathbb{T}_1 f_1(z, \bar{z}) + \mathbb{T}_2 f_2(z, \bar{z}) \qquad \mathbb{T}_1 := \frac{\bar{s} \not{x}_{34} s}{x_{12}^2 x_{34}^4} \quad [\text{Cuomo, Karateev, Kravchuk, '17}]$$

$$\mathbb{T}_2 := \frac{\bar{s} \not{x}_{31} \not{x}_{12} \not{x}_{24} s}{|x_{12}|^3 |x_{34}|^3 |x_{13}| |x_{24}|}$$

$$d = 3 \qquad \langle \phi \phi \bar{\psi} \psi \rangle = \mathbb{T}_1 f_1(z, \bar{z}) + \mathbb{T}_2 f_2(z, \bar{z}) + \mathbb{T}_3 f_3(z, \bar{z}) + \mathbb{T}_4 f_4(z, \bar{z})$$

2. SCALAR-FERMION CORRELATORS

AN INTERESTING CORRELATOR

$$\langle \phi \phi \bar{\psi} \psi \rangle$$

OPE

$$\overline{\phi \phi} \sim \mathbb{1} + \phi^2 + \dots$$

$$\overline{\phi \bar{\psi}} \sim \bar{\psi} + \dots$$

Tensor structure

$$d = 4$$

$$\langle \phi \phi \bar{\psi} \psi \rangle = \underbrace{\mathbb{T}_1 f_1(z, \bar{z}) + \mathbb{T}_2 f_2(z, \bar{z})}_{\text{parity-even}}$$

$$\mathbb{T}_1 := \frac{\bar{s} \not{x}_{34} s}{x_{12}^2 x_{34}^4} \quad [\text{Cuomo, Karateev, Kravchuk, '17}]$$

$$\mathbb{T}_2 := \frac{\bar{s} \not{x}_{31} \not{x}_{12} \not{x}_{24} s}{|x_{12}|^3 |x_{34}|^3 |x_{13}| |x_{24}|}$$

$$d = 3$$

$$\langle \phi \phi \bar{\psi} \psi \rangle = \underbrace{\mathbb{T}_1 f_1(z, \bar{z}) + \mathbb{T}_2 f_2(z, \bar{z})}_{\text{parity-even}} + \underbrace{\mathbb{T}_3 f_3(z, \bar{z}) + \mathbb{T}_4 f_4(z, \bar{z})}_{\text{parity-odd}}$$

2. SCALAR-FERMION CORRELATORS

AN INTERESTING CORRELATOR

$$\langle \phi \phi \bar{\psi} \psi \rangle$$

OPE

$$\overline{\phi\phi} \sim \mathbb{1} + \phi^2 + \dots$$

$$\overline{\phi\psi} \sim \bar{\psi} + \dots$$

Tensor structure

$$d = 4$$

$$\langle \phi \phi \bar{\psi} \psi \rangle = \mathbb{T}_1 f_1(z, \bar{z}) + \mathbb{T}_2 f_2(z, \bar{z})$$



$$d = 3$$

$$\langle \phi \phi \bar{\psi} \psi \rangle = \mathbb{T}_1 f_1(z, \bar{z}) + \mathbb{T}_2 f_2(z, \bar{z}) + \mathbb{T}_3 f_3(z, \bar{z}) + \mathbb{T}_4 f_4(z, \bar{z})$$

$$\mathbb{T}_1 := \frac{\bar{s} \not{x}_{34} s}{x_{12}^2 x_{34}^4} \quad [\text{Cuomo, Karateev, Kravchuk, '17}]$$

$$\mathbb{T}_2 := \frac{\bar{s} \not{x}_{31} \not{x}_{12} \not{x}_{24} s}{|x_{12}|^3 |x_{34}|^3 |x_{13}| |x_{24}|}$$

2. SCALAR-FERMION CORRELATORS

CONFORMAL BLOCKS

$$\langle \overline{\phi\phi} \overline{\psi\psi} \rangle$$

2. SCALAR-FERMION CORRELATORS

CONFORMAL BLOCKS

$$\langle \overline{\phi\phi} \overline{\psi\psi} \rangle$$

Calogero-Sutherland
Hamiltonian



$$d = 4$$

$$C_2^{d=4} = \begin{pmatrix} H_0^{(6)} + 8 & 0 \\ 0 & H_0^{(4)} \end{pmatrix}$$



$$d = 3$$

$$C_2^{d=3} = \begin{pmatrix} H_0^{(5)} + 6 & 0 \\ 0 & H_0^{(3)} \end{pmatrix}$$

2. SCALAR-FERMION CORRELATORS

CONFORMAL BLOCKS

$$\langle \overline{\phi\phi} \overline{\psi\psi} \rangle$$

Calogero-Sutherland
Hamiltonian

$$\begin{array}{ccc}
 d=4 & C_2^{d=4} = \begin{pmatrix} H_0^{(6)} + 8 & 0 \\ 0 & H_0^{(4)} \end{pmatrix} & \uparrow \\
 \downarrow & \xrightarrow{\hspace{10em}} & C_2^d = \begin{pmatrix} H_0^{(d+2)} + 2d & 0 \\ 0 & H_0^{(d)} \end{pmatrix} \\
 d=3 & C_2^{d=3} = \begin{pmatrix} H_0^{(5)} + 6 & 0 \\ 0 & H_0^{(3)} \end{pmatrix} &
 \end{array}$$

2. SCALAR-FERMION CORRELATORS

CONFORMAL BLOCKS

$$\langle \overline{\phi\phi} \overline{\psi\psi} \rangle$$

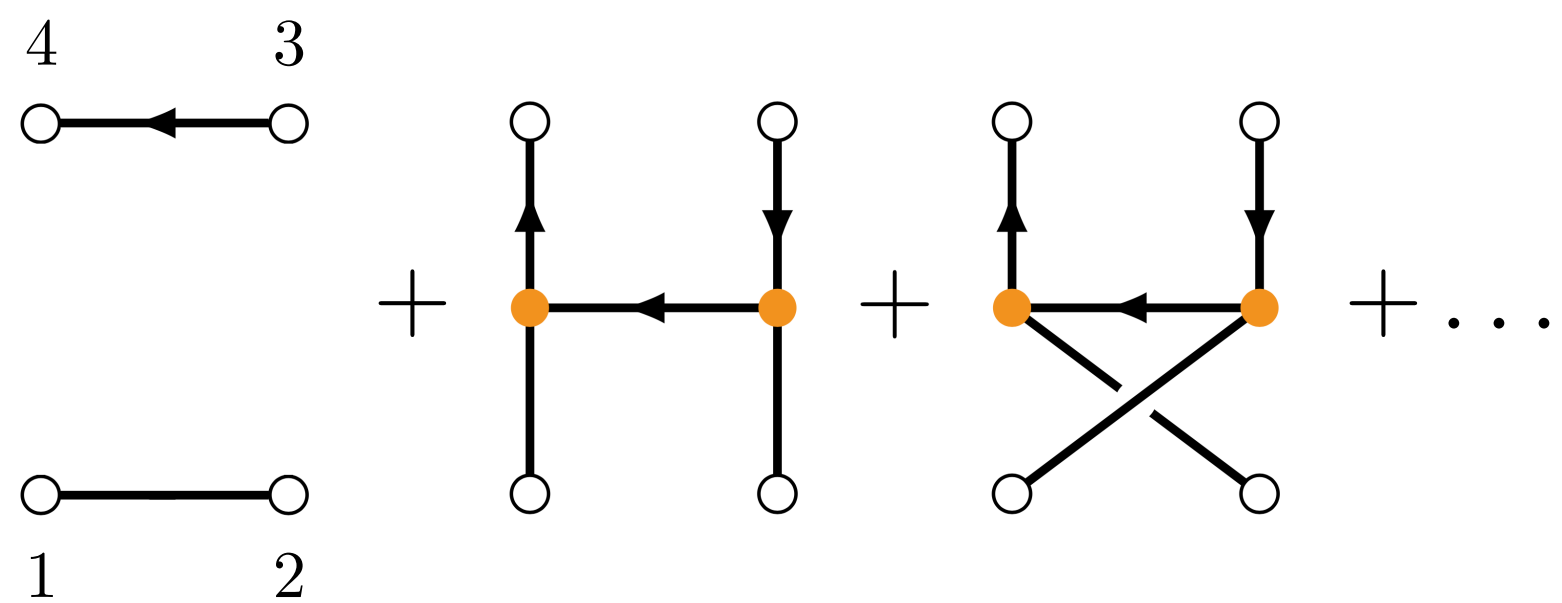
Calogero-Sutherland
Hamiltonian

$$\begin{array}{ccc}
 d=4 & C_2^{d=4} = \begin{pmatrix} H_0^{(6)} + 8 & 0 \\ 0 & H_0^{(4)} \end{pmatrix} & \\
 \downarrow & \xrightarrow{\hspace{10em}} & \\
 d=3 & C_2^{d=3} = \begin{pmatrix} H_0^{(5)} + 6 & 0 \\ 0 & H_0^{(3)} \end{pmatrix} & C_2^d = \begin{pmatrix} H_0^{(d+2)} + 2d & 0 \\ 0 & H_0^{(d)} \end{pmatrix} \\
 & & \text{scalar blocks}
 \end{array}$$

$$\begin{bmatrix} f_1(z, \bar{z}) \\ f_2(z, \bar{z}) \end{bmatrix} = \sum_{\Delta} \lambda_{\phi\phi\mathcal{O}_{\Delta}} \lambda_{\mathcal{O}_{\Delta}\bar{\psi}\psi} \begin{bmatrix} c & \frac{z+\bar{z}}{z\bar{z}} \\ 0 & 2 \end{bmatrix} \begin{bmatrix} g_{\Delta,\ell}^{(d)}(z, \bar{z}) \\ g_{\Delta+1,\ell-1}^{(d+2)}(z, \bar{z}) \end{bmatrix}$$

2. SCALAR-FERMION CORRELATORS

PERTURBATIVE CALCULATION

$$\langle \phi \phi \bar{\psi} \psi \rangle =$$


The equation shows the perturbative expansion of the scalar-fermion correlator $\langle \phi \phi \bar{\psi} \psi \rangle$. The expansion is represented by a series of Feynman diagrams:

- The first diagram is a tree-level exchange of a scalar particle between two fermion lines. The external legs are labeled 1, 2, 3, and 4. The internal line is a scalar propagator.
- The second diagram is a one-loop correction involving a fermion loop. The external legs are labeled 1, 2, 3, and 4. The internal lines are fermion and scalar propagators.
- The third diagram is a two-loop correction involving a fermion loop and a scalar loop. The external legs are labeled 1, 2, 3, and 4. The internal lines are fermion and scalar propagators.
- The expansion continues with higher-order terms, indicated by the ellipsis $\dots$.

2. SCALAR-FERMION CORRELATORS

PERTURBATIVE CALCULATION

$$\langle \phi \phi \bar{\psi} \psi \rangle =$$

Integral:

$$\sim \varepsilon \int d^4 x_5 \int d^4 x_6 I_{15} I_{26} \not{\partial}_3 I_{36} \not{\partial}_6 I_{56} \not{\partial}_5 I_{45} + \mathcal{O}(\varepsilon^2)$$

$$I_{ij} := \frac{1}{4\pi^2 x_{ij}^2}$$

2. SCALAR-FERMION CORRELATORS

PERTURBATIVE CALCULATION

$$\langle \phi \phi \bar{\psi} \psi \rangle =$$

Integral:

$$\sim 4\pi^2 \varepsilon \not{x}_{23} \gamma^\mu \gamma^\nu I_{23} \partial_4^\nu \int d^4 x_5 x_{25}^\mu I_{15} I_{25} I_{35} I_{45} + \mathcal{O}(\varepsilon^2)$$

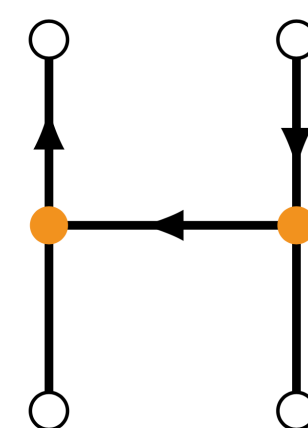
$$I_{ij} := \frac{1}{4\pi^2 x_{ij}^2}$$

1. Star-triangle relation: $\int d^4 x_6 I_{26} \not{\partial}_3 I_{36} \not{\partial}_6 I_{56} = -4\pi^2 \not{x}_{23} \not{x}_{25} I_{23} I_{25} I_{35}$

2. SCALAR-FERMION CORRELATORS

PERTURBATIVE CALCULATION

$$\langle \phi \phi \bar{\psi} \psi \rangle =$$

Integral: 

$$\sim 4\pi^2 \varepsilon \not{x}_{23} \gamma^\mu \gamma^\nu I_{23} \partial_4^\nu \int d^4 x_5 x_{25}^\mu I_{15} I_{25} I_{35} I_{45} + \mathcal{O}(\varepsilon^2)$$

$$I_{ij} := \frac{1}{4\pi^2 x_{ij}^2}$$

1. Star-triangle relation:
$$\int d^4 x_6 I_{26} \not{\partial}_3 I_{36} \not{\partial}_6 I_{56} = -4\pi^2 \not{x}_{23} \not{x}_{25} I_{23} I_{25} I_{35}$$

2. Tensor decomposition

2. SCALAR-FERMION CORRELATORS

PERTURBATIVE CALCULATION

$$\langle \phi \phi \bar{\psi} \psi \rangle = \mathbb{T}_1 f_1(z, \bar{z}) + \mathbb{T}_2 f_2(z, \bar{z})$$

2. SCALAR-FERMION CORRELATORS

PERTURBATIVE CALCULATION

$$\langle \phi \phi \bar{\psi} \psi \rangle = \text{Tr} f_1(z, \bar{z}) + \text{Tr} f_2(z, \bar{z})$$

$$\left. \frac{f_i(z, \bar{z})}{\kappa(z, \bar{z})} \right|_{\mathcal{O}(\varepsilon)} = a_i(z, \bar{z}) \frac{\log(z\bar{z})}{(1-z)(1-\bar{z})} + b_i(z, \bar{z}) \frac{\log((1-z)(1-\bar{z}))}{z\bar{z}} + i c_i(z, \bar{z}) \ell(z, \bar{z})$$

2. SCALAR-FERMION CORRELATORS

PERTURBATIVE CALCULATION

$$\langle \phi \phi \bar{\psi} \psi \rangle = \text{T} f_1(z, \bar{z}) + \text{T} f_2(z, \bar{z})$$

$$\left. \frac{f_i(z, \bar{z})}{\kappa(z, \bar{z})} \right|_{\mathcal{O}(\varepsilon)} = a_i(z, \bar{z}) \frac{\log(z\bar{z})}{(1-z)(1-\bar{z})} + b_i(z, \bar{z}) \frac{\log((1-z)(1-\bar{z}))}{z\bar{z}} + i c_i(z, \bar{z}) \ell(z, \bar{z})$$

$$\kappa(z, \bar{z}) := \frac{(z\bar{z})^2}{512\pi^6 |z - \bar{z}|^2}$$

$$i|z - \bar{z}| \ell(z, \bar{z}) := \frac{\pi^2}{3} + \log(-x_1) \log(-x_2) + 2(\text{Li}_2(x_1) + \text{Li}_2(x_2)) \\ + \log((1-z)(1-\bar{z})) \log\left(-\frac{z + \bar{z} - i|z - \bar{z}| - 2z\bar{z}}{z + \bar{z} + i|z - \bar{z}|}\right)$$

$$x_1 := \frac{2}{2 - z - \bar{z} - i|z - \bar{z}|}, \quad x_2 := (1-z)(1-\bar{z}) x_1$$

2. SCALAR-FERMION CORRELATORS

PERTURBATIVE CALCULATION

$$\langle \phi \phi \bar{\psi} \psi \rangle = \mathbb{T}_1 f_1(z, \bar{z}) + \mathbb{T}_2 f_2(z, \bar{z})$$

$$\left. \frac{f_i(z, \bar{z})}{\kappa(z, \bar{z})} \right|_{\mathcal{O}(\varepsilon)} = \boxed{a_i(z, \bar{z})} \frac{\log(z\bar{z})}{(1-z)(1-\bar{z})} + \boxed{b_i(z, \bar{z})} \frac{\log((1-z)(1-\bar{z}))}{z\bar{z}} + \boxed{i c_i(z, \bar{z})} \ell(z, \bar{z})$$

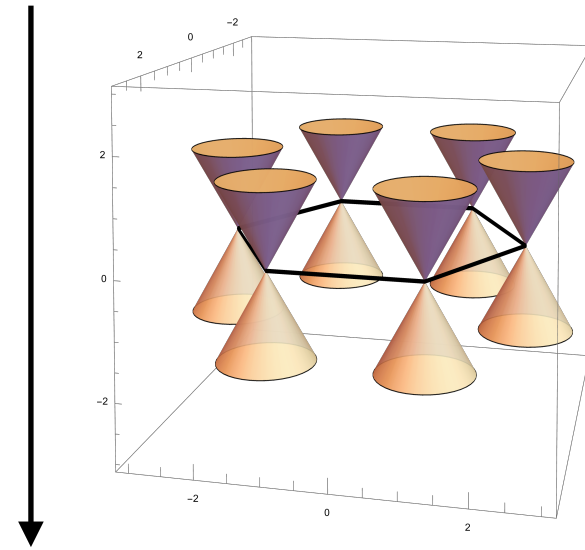
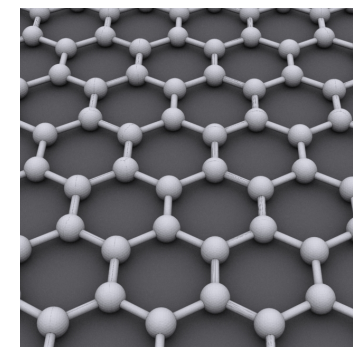
$$a_1(z, \bar{z}) = p(z, \bar{z}) - q(z, \bar{z}), \quad b_1(z, \bar{z}) = q(z, \bar{z}), \quad c_1(z, \bar{z}) = p(z, \bar{z})$$

$$a_2(z, \bar{z}) = \frac{1}{2} q(z, \bar{z}) (p(z, \bar{z}) + 2), \quad b_2(z, \bar{z}) = z\bar{z} p(z, \bar{z}), \quad c_2(z, \bar{z}) = q(z, \bar{z})$$

$$p(z, \bar{z}) := z + \bar{z} - 4$$

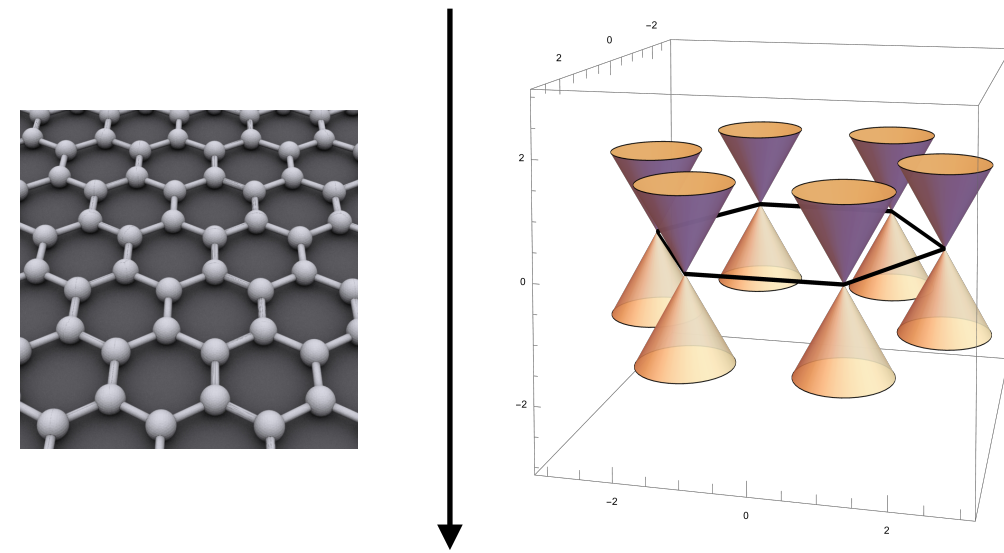
$$q(z, \bar{z}) := 2(z\bar{z} - z - \bar{z})$$

Phase transitions in graphene



Yukawa CFT

Phase transitions in graphene



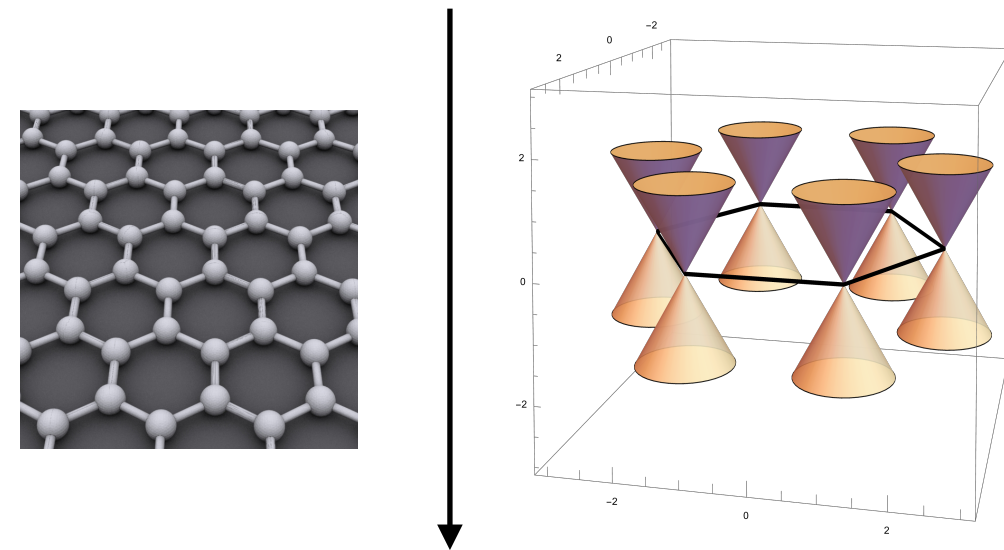
Yukawa CFT

$$\langle \phi \phi \bar{\psi} \psi \rangle$$

$$\begin{bmatrix} f_1(z, \bar{z}) \\ f_2(z, \bar{z}) \end{bmatrix} = \sum_{\Delta} \lambda_{\phi \phi \mathcal{O}_{\Delta}} \lambda_{\mathcal{O}_{\Delta} \bar{\psi} \psi} \begin{bmatrix} c & \frac{z+\bar{z}}{2} \\ 0 & \frac{z-\bar{z}}{2} \end{bmatrix} \begin{bmatrix} g_{\Delta, \ell}^{(d)}(z, \bar{z}) \\ g_{\Delta+1, \ell-1}^{(d+2)}(z, \bar{z}) \end{bmatrix}$$

$$\langle \phi \phi \bar{\psi} \psi \rangle = \begin{array}{c} \circ \text{---} \circ \\ \circ \text{---} \circ \end{array} + \begin{array}{c} \circ \quad \circ \\ | \quad | \\ \bullet \text{---} \bullet \\ | \quad | \\ \circ \quad \circ \end{array} + \begin{array}{c} \circ \quad \circ \\ | \quad | \\ \bullet \quad \bullet \\ \diagup \quad \diagdown \\ \bullet \quad \bullet \\ | \quad | \\ \circ \quad \circ \end{array} + \dots$$

Phase transitions in graphene



Yukawa CFT

$$\langle \phi \phi \bar{\psi} \psi \rangle$$

$$\begin{bmatrix} f_1(z, \bar{z}) \\ f_2(z, \bar{z}) \end{bmatrix} = \sum_{\Delta} \lambda_{\phi \phi \mathcal{O}_{\Delta}} \lambda_{\mathcal{O}_{\Delta} \bar{\psi} \psi} \begin{bmatrix} c & \frac{z+\bar{z}}{2} \\ 0 & \frac{z-\bar{z}}{2} \end{bmatrix} \begin{bmatrix} g_{\Delta, \ell}^{(d)}(z, \bar{z}) \\ g_{\Delta+1, \ell-1}^{(d+2)}(z, \bar{z}) \end{bmatrix}$$

$$\langle \phi \phi \bar{\psi} \psi \rangle = \begin{array}{c} \text{---} \text{---} \\ \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \\ | \quad | \\ \bullet \quad \bullet \\ | \quad | \\ \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \\ | \quad | \\ \bullet \quad \bullet \\ | \quad | \\ \text{---} \end{array} + \dots$$

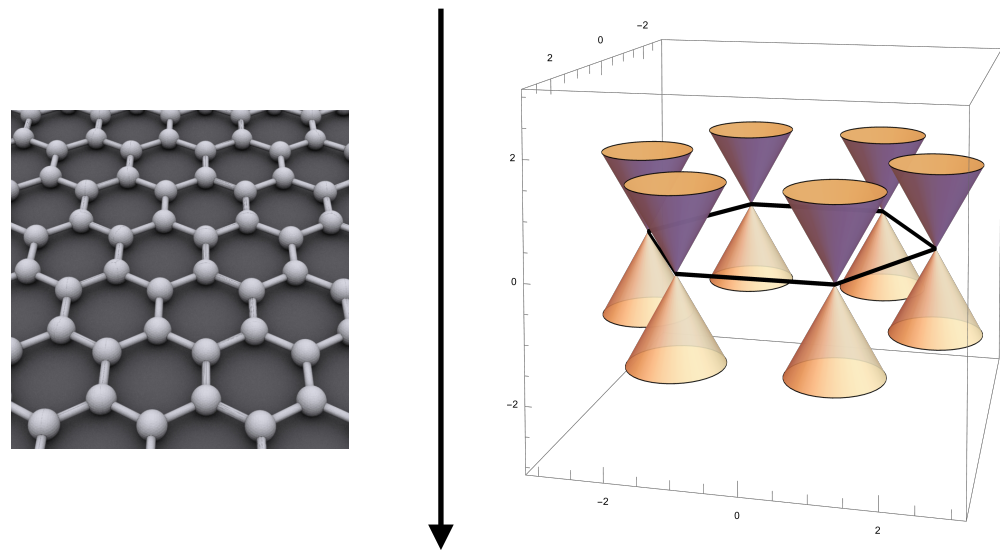
Magnetic line

$$\bullet \quad \bullet \quad \bullet \quad S \longrightarrow S + h \int d\tau \phi(\tau)$$

$$\bullet \quad \bullet \quad \bullet \quad \text{[Giombi, Helfenberger, Khanchandani, '23]}$$

$$\bullet \quad \bullet \quad \bullet \quad \text{[JB, Liendo, van Vliet, '23]}$$

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$$\langle \phi \phi \bar{\psi} \psi \rangle = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \dots$$

Magnetic line

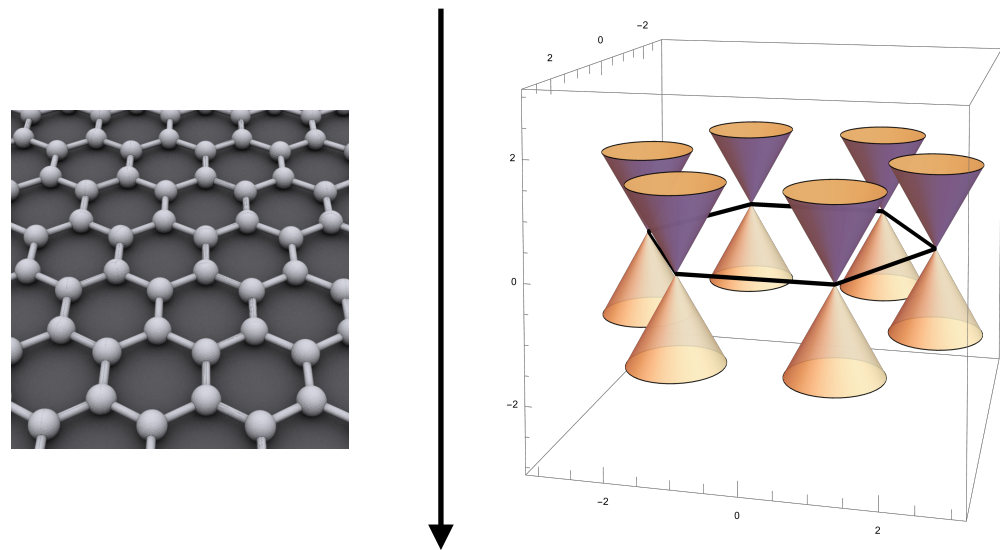
● ● ● $S \longrightarrow S + h \int d\tau \phi(\tau)$

● ● *h* ● [Giombi, Helfenberger, Khanchandani, '23]

● ● ● [JB, Liendo, van Vliet, '23]

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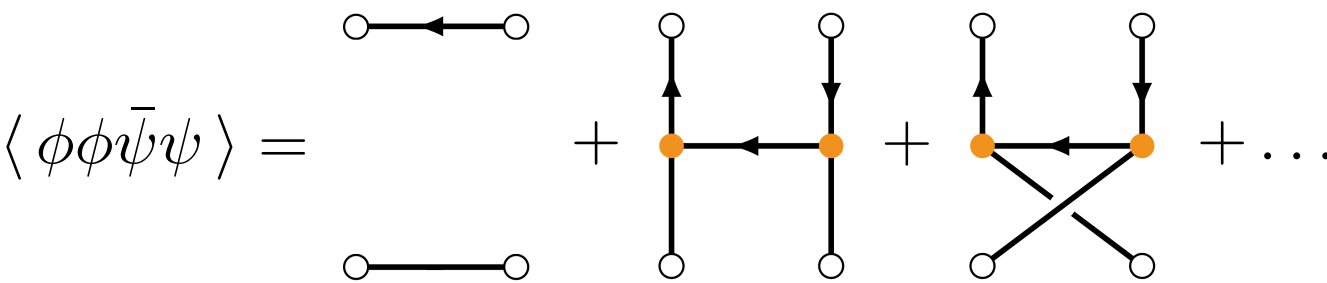
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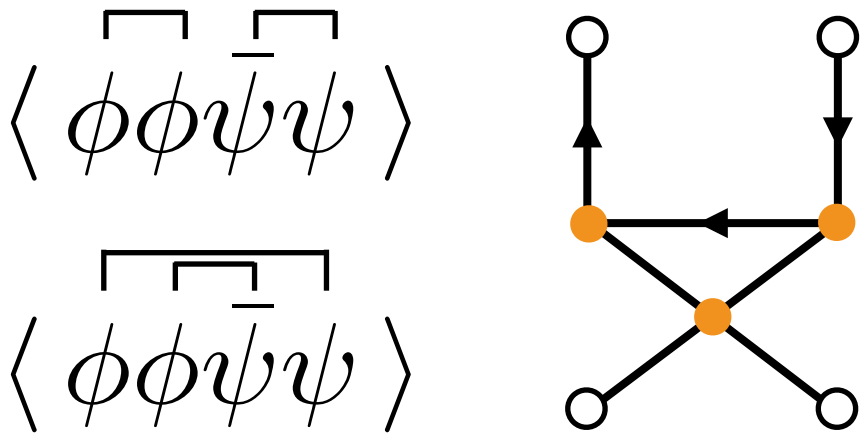
Magnetic line

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● ● h ● [Giombi, Helfenberger, Khanchandani, '23]

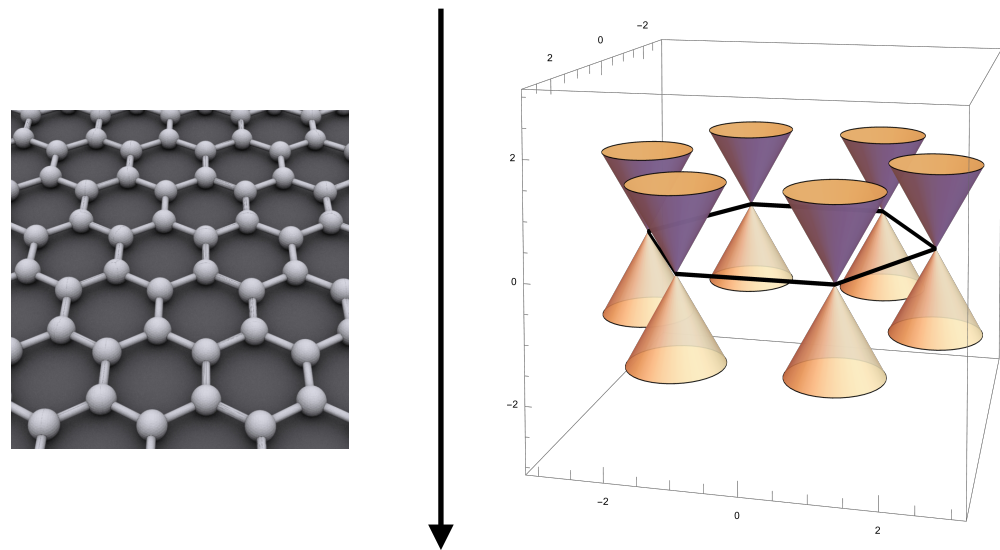
● ● ● [JB, Liendo, van Vliet, '23]

CFT data



SEE PHILINE'S TALK!

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$$\langle \phi \phi \bar{\psi} \psi \rangle = \text{diagram 1} + \text{diagram 2} + \text{diagram 3} + \dots$$

Magnetic line

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$S \longrightarrow S + h \int d\tau \phi(\tau)$

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● *h*

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[Giombi, Helfenberger, Khanchandani, '23]

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[JB, Liendo, van Vliet, '23]

CFT data

$$\langle \overline{\phi \phi} \bar{\psi} \psi \rangle \quad \text{diagram 1}$$
$$\langle \overline{\phi \phi} \bar{\psi} \psi \rangle \quad \text{diagram 2}$$

THANK YOU

SEE PHILINE'S TALK!