Line defects in $O(N)$ and Yukawa CFTs

Philine van Vliet

DESY Theory Workshop 2023

September 28, 2023

Based on:
ArXiv: 2304.13588 Barrat, Liendo, PvV, 2023
ArXiv: 2208.11715 Gimenez-Grau, Lauria, Liendo, PvV, 2022

Motivation

- See Davide's and Julien's talk!

Magnetic line defect in Yukawa CFTs

- Localized magnetic field line defect defined across dimensions in Yukawa or fermionic CFTs [Giombi et al., 2023]:

Gross-Neveu-Yukawa — $d = 4 - \epsilon$

$d = 3$

Gross-Neveu — $d = 2 + \tilde{\epsilon}$

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Gross-Neveu-Yukawa — $d = 4 - \epsilon$

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Gross-Neveu — $d = 2 + \tilde{\epsilon}$

- Magnetic line defect is the same in $O(N)$ and Yukawa CFTs:

$$S = \int d^d x \left(\frac{1}{2}(\partial_\mu \phi_a)^2 + i\bar{\psi}^i \partial\!\!\!/ \psi^i + g\bar{\psi}^i(\Sigma_a \cdot \phi_a)\psi^i + \frac{\lambda}{4!}(\phi_a^2)^2 \right) + h \int_D d\tau \phi_1 \,.$$

$$a = 1, \cdots, N \,, \quad i = 1, \cdots, N_f \,.$$

Bootstrapping conformal defects

- 2-pt bulk correlators depend on cross ratios r, w:

$$\langle\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle\rangle = \frac{\mathcal{F}(r,w)}{|x_1^\perp|^\Delta |x_2^\perp|^\Delta}\,,$$

$$r\left(\tau_{12}, x_1^\perp, x_2^\perp\right)\,,\ w\left(x_1^\perp, x_2^\perp\right)\,.$$

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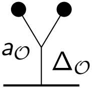
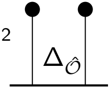
- Solve defect crossing equation

$\sum_{\mathcal{O}} \lambda_{\mathcal{O}} a_{\mathcal{O}}$  $= \sum_{\hat{\mathcal{O}}} (\hat{b}_{\mathcal{O}})^2$ 

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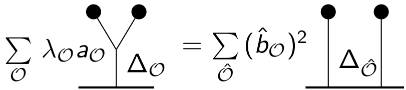

- 4-pt defect correlators depend on cross ratio χ:

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$$\chi \equiv \frac{\tau_{12}\tau_{34}}{\tau_{13}\tau_{24}}\,, \qquad 1-\chi \equiv \frac{\tau_{23}\tau_{14}}{\tau_{13}\tau_{24}}\,.$$

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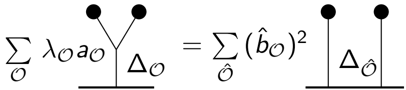

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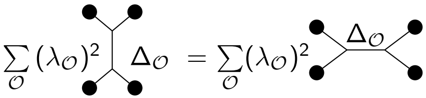

Yukawa CFT in the ε-expansion

- Recap of Julien's talk.
- Start from action:

$$S = \int d^d x \left(\frac{1}{2}(\partial_\mu \phi_a)^2 + i\bar{\psi}^i \partial\!\!\!/ \psi^i + g\bar{\psi}^i(\Sigma_a \cdot \phi_a)\psi^i + \frac{\lambda}{4!}(\phi_A^2)^2 \right) .$$

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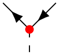
- Feynman vertices:

 $:= \; -\lambda_0 \int d^d x_5 \, I_{15} I_{25} I_{35} I_{45} \, ,$  $:= \; -g_0 \int d^d x_4 \, \partial\!\!\!/_1 I_{14} \Sigma^a \partial\!\!\!/_4 I_{34} I_{24} \, .$

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- Fixed points at $O(\varepsilon)$:

$$\frac{\lambda_\star}{(4\pi)^2} = \frac{3\kappa_2 \varepsilon}{2\kappa_1 (N+8)} \,, \quad \frac{g_\star^2}{(4\pi)^2} = \frac{\varepsilon}{2\kappa_1} \,.$$

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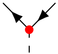
 $:= -\lambda_0 \int d^d x_5 \, I_{15} I_{25} I_{35} I_{45} \,,$  $:= -g_0 \int d^d x_4 \, \partial\!\!\!/_1 I_{14} \Sigma^a \partial\!\!\!/_4 I_{34} I_{24} \,.$

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$$\kappa_1 := 2N_f - N + 4 \,,$$
$$\kappa_2 := 2(4-N) - \kappa_1 + \sqrt{12\,(N^2+16) + \kappa_1(\kappa_1 + 12(N+4))} \,.$$

Line defect in Yukawa CFT in the ε-expansion

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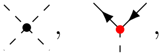

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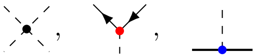

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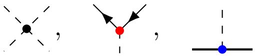

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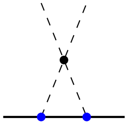
Bulk correlators

- Scalar correlator (see Davide's talk)

$$\langle\langle \Phi^a(x_1)\Phi^b(x_2)\rangle\rangle = \frac{1}{|x_1^\perp|^{\Delta_\phi}|x_2^\perp|^{\Delta_\phi}} F^{a,b}(z,\bar{z})$$

- Main contributing diagram is:



$$\sim \int \frac{d\tau_3 d\tau_4 d^d x_5}{(\hat{x}_{35}^2)^{\frac{d-2}{2}}(\hat{x}_{45}^2)^{\frac{d-2}{2}}(x_{15}^2)^{\frac{d-2}{2}}(x_{25}^2)^{\frac{d-2}{2}}} \sim \frac{\lambda_\star h_\star^2 H(z,\bar{z})}{|x_1^\perp||x_2^\perp|}\,.$$

$+$ self-energy corrections at $O(\varepsilon)$.

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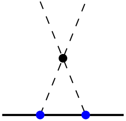

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- We know H in terms of conformal block of ϕ^2

[Gimenez-Grau, 2022][Bianchi, Bonomi, de Sabatta, 2022]

$$H(z,\bar{z}) = |1-z|^2|z|^{-2}(\partial_\Delta - 1 - \log 2)\, f_{2,0}(z,\bar{z})$$

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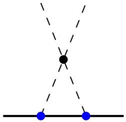

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$$H(z,\bar{z}) = |1-z|^2|z|^{-2}(\partial_\Delta - 1 - \log 2)\, f_{2,0}(z,\bar{z})$$

- Leads to new CFT data depending on N_f which reduces to $O(N)$ data for $N_f = 0$.

New correlator $\langle\langle\bar{\Psi}\Psi\rangle\rangle$

- Presence of fermions leads to new type of correlator:

$$\langle\langle\bar{\Psi}(s_1, x_1)\Psi(s_2, x_2)\rangle\rangle = \frac{\bar{s}_1 s_2}{|x_1^\perp|^{\Delta_\psi}|x_2^\perp|^{\Delta_\psi}} G(z, \bar{z})$$

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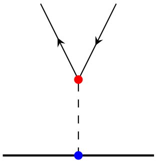
- First nontrivial order starts at $O(\sqrt{\varepsilon})$:

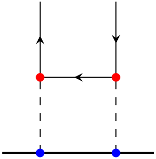


$$\sim \bar{s}_1^A s_2^B g_\star h_\star \delta^{ij} \int d\tau_3 \int d^4x_4\, \not{\partial}_1 I_{14}\, \Sigma^1\, \not{\partial}_2 I_{24} I_{34}$$

$$= -\frac{\pi\, g_0 h_0\, \delta^{ij}}{8(|x_1^{\perp}|+|x_2^{\perp}|)} \bar{s}_1 \left(\frac{\not{x}_1 \not{x}_2}{|x_1^{\perp}||x_2^{\perp}|} + 1 \right) s_2\,,$$

New correlator $\langle\langle\bar{\Psi}\Psi\rangle\rangle$ - outlook

- Second contribution at $O(\varepsilon)$:



$$\sim -g_\star^2 h_\star^2\, \bar{s}_1\, \bar{\partial}\!\!\!/_1 \int d\tau_3 \int d\tau_4\, I_{24} \left(\int d^4 x_5\, \not{x}_{54}\, I_{15} I_{25} I_{35} I_{45} \right) \not{x}_{24}\, s_2\,.$$

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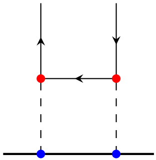

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- Tackle integral in brackets by tensor decomposition:

$$\int d^4x_5 \, x\!\!\!/_{54} \, I_{15} I_{25} I_{35} I_{45} = \frac{2}{\phi_K} j_{123;4} \, ,$$

with ϕ_K a Kibble function, and $j_{123;4}$ in terms of known integrals.

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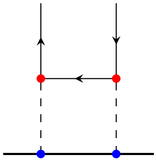

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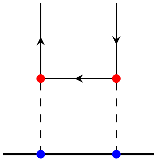

$$\sim -g_\star^2 h_\star^2\, \bar{s}_1\, \overleftarrow{\not\partial}_1 \int d\tau_3 \int d\tau_4\, I_{24} \left(\int d^4x_5\, \not{x}_{54}\, I_{15} I_{25} I_{35} I_{45} \right) \not{x}_{24}\, s_2\,.$$

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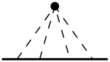
- Expression in terms of conformal blocks? Can be solved using analytic bootstrap?
$\rightarrow$ need fermionic blocks across dimensions! (see Julien's talk).

Defect correlators

- Now consider correlators of defect operators $\hat{\phi}, \hat{\psi}$ on the line.

$$\langle\, \hat{\phi}^a(\tau_1)\, \hat{\phi}^b(\tau_2)\, \hat{\phi}^c(\tau_3)\, \hat{\phi}^d(\tau_4)\, \rangle \sim f^{\hat{a}\hat{b}\hat{c}\hat{d}}(\chi)$$

$$= \text{discon.} + $$  $$+ \mathcal{O}(\varepsilon^2)\,.$$

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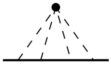
$$= \text{discon.} +$$  $$+\, \mathcal{O}(\varepsilon^2)\,.$$

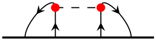
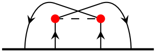
- E.g. correlator of $\hat{\phi}^1$ and $t^{\hat{a}}$ at $O(\varepsilon)$:

$$f^{1\hat{a}1\hat{b}}(\chi) = \delta^{\hat{a}\hat{b}}\chi^{\Delta_{\hat{\phi}^1}+\Delta_t} + \varepsilon\delta^{\hat{a}\hat{b}}\frac{\kappa_2}{\kappa_1(N+8)}\left(\chi\log(1-\chi) + \frac{\chi^2}{1-\chi}\log\chi\right)\,.$$

Defect correlators

- Also here: new type of correlators.

$$\langle \hat{\bar{\psi}}^i(\tau_1)\hat{\psi}^j(\tau_2)\hat{\bar{\psi}}^k(\tau_3)\hat{\psi}^l(\tau_4)\rangle \sim f^{ijkl}_{12,34}(\chi) - \frac{\chi^3}{(1-\chi)^3} f^{ilkj}_{14,32}(1-\chi)$$

$= \text{discon.} +$  $+$  $+ \mathcal{O}(\varepsilon^{\frac{3}{2}})\,.$

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- Also here: new type of correlators.

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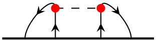
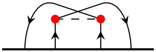
$$= \text{discon.} +$$  $$+$$  $$+\, \mathcal{O}(\varepsilon^{\frac{3}{2}})\,.$$

- with

$$f^{ijkl}_{12,34}(\chi) = \delta^{ij}\delta^{kl}(\bar{s}_1\gamma^0 s_2)(\bar{s}_3\gamma^0 s_4) + \frac{\varepsilon}{64\kappa_1}\delta^{ij}\delta^{kl}(\bar{s}_1\Sigma^a\gamma^0 s_2)(\bar{s}_3\Sigma^a\gamma^0 s_4)$$
$$\times \frac{\chi}{(1-\chi)^2}\left((1-\chi)(2-\chi) + \chi^2(2-\chi)\log\chi + \chi(1-\chi)^2\log(1-\chi)\right)\,.$$

Defect correlators

- Defect conformal data [Barrat, Liendo, PvV 2023] :

$$\hat{\Delta}_{\hat{t}_{\hat{a}}} = 1\,, \quad \hat{\Delta}_{\hat{\phi}} = 1 - \frac{(N-4)\varepsilon}{\kappa_1} + \mathcal{O}(\varepsilon^2)\,, \quad \hat{\Delta}_{\hat{\psi}} = \frac{3}{2} - \frac{\left(2 - \frac{N}{\kappa_1}\right)\varepsilon}{4} + \mathcal{O}(\varepsilon^2)\,,$$

$$\hat{\lambda}_{\hat{\phi}\hat{\phi}\hat{\phi}} = \frac{3\pi\varepsilon}{8} \frac{(4\kappa_1 - N_f)\sqrt{2(4-N)\kappa_2}}{\kappa_1{}^2\sqrt{N+8}} + \mathcal{O}(\varepsilon^2)\,, \quad \hat{\lambda}_{\hat{t}\hat{t}\hat{\phi}} = \frac{\hat{\lambda}_{\hat{\phi}\hat{\phi}\hat{\phi}}}{3} + \mathcal{O}(\varepsilon^2)\,.$$

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$N = 3$	$N_f = 0$	$N_f = 1$	$N_f = 2$
Δ_t	1	1	1
$\Delta_{\hat{\phi}_1}$	$1 + \epsilon$	$1 + \frac{2\epsilon}{3}$	$1 + \frac{\epsilon}{5}$
Δ_{s_-}	$2 + 0.35502$	$2 + 0.78832$	$2 + 0.75055$
Δ_T	$2 + 0.\bar{18}\epsilon$	$2 + 0.433\epsilon$	$2 + 0.29\bar{09}\epsilon$
Δ_A	3	3	3
Δ_V	$2 + 1.\bar{18}\epsilon$	$2 + 1.100\epsilon$	$2 + 0.49\bar{09}\epsilon$
$\lambda_{tt\hat{\phi}}$	0.947226ϵ	2.1893ϵ	1.6075ϵ
$\lambda_{\hat{\phi}\hat{\phi}\hat{\phi}}$	2.84168ϵ	6.56789ϵ	4.82249ϵ

Numerical bootstrap for $O(3)$ CFTs

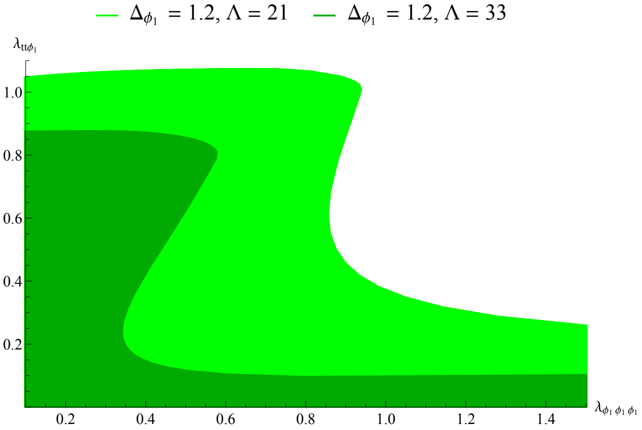

Figure: Upper bounds on the OPE coefficient $\lambda_{\phi_1\phi_1\phi_1}$ as a function of $\lambda_{tt\phi_1}$.

[Gimenez-Grau, Lauria, Liendo, PvV 2022]

- We found a series of cusps.

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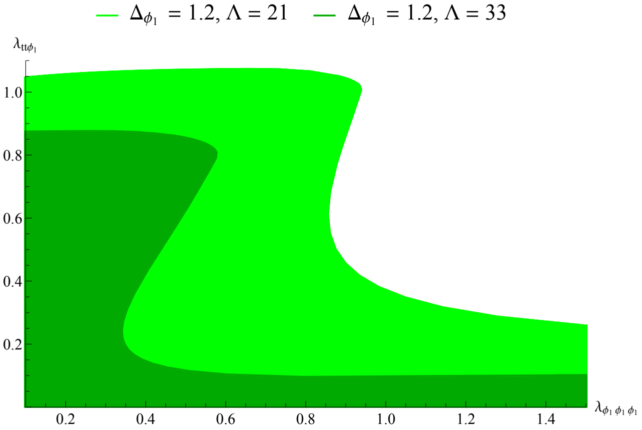

Figure: Upper bounds on the OPE coefficient $\lambda_{\phi_1\phi_1\phi_1}$ as a function of $\lambda_{tt\phi_1}$.

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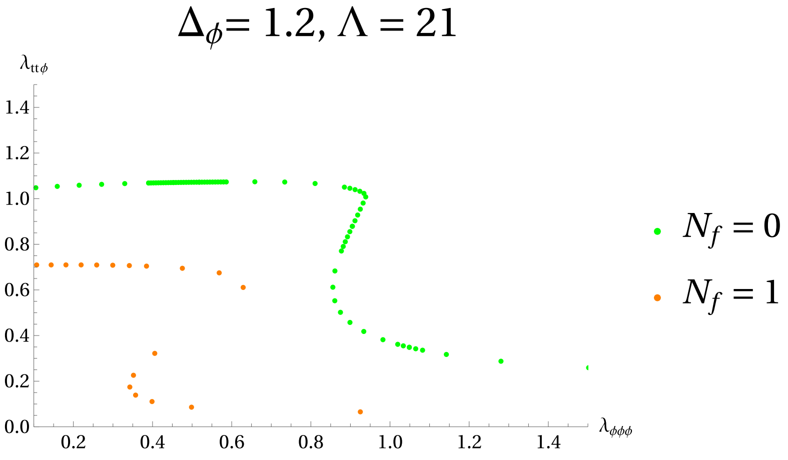
- We found a series of cusps. Wrong point?

Numerical bootstrap for chiral Heisenberg model?

- Yukawa CFT?



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- Yukawa theories have emergent SUSY: supersymmetric version of same setup?
- Study superconformal defect in $3d\,\mathcal{N}=2$ SCFT:

$$OSP(2|4) \rightarrow SU(1,1|1) \times U(1)\ \text{(monodromy)}\ ,$$ [Gimenez-Grau, Liendo 2022]

Thank you!

Backup

Extended objects

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- There is no conserved stress-energy tensor on the defect. Instead, there exist a displacement operator D

$$\partial_\mu T^{\mu d}(x) = -D(x^a)\delta(x^d),$$

CFT correlation functions

- Fixed 2- and 3-pt correlators:

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle = \frac{1}{x_{12}^{2\Delta}}\,,$$

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- Conformal data: $\{\Delta_i, \lambda_{ijk}, \ell_i\}$
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Determine conformal data by relying on very little input:

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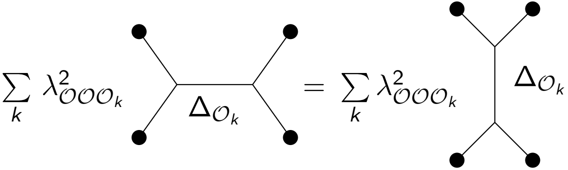
Can solve the central crossing equations either numerically or analytically.



Results

CFTs in $d = 4 - \varepsilon$ dimensions

- Start from the action for $O(N)$ CFTs defined in $d = 4 - \varepsilon$ dimensions:

$$S = \int d^d x \left(\frac{1}{2}(\partial_\mu \phi_a)^2 + \frac{\lambda_0}{4!}(\phi_a^2)^2 \right) , a = 1, \cdots , N .$$

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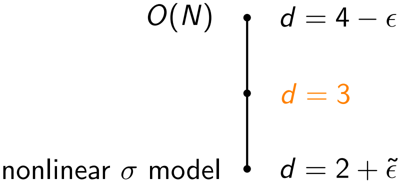
- Define conformal data perturbatively:

$$\Delta = \Delta^{(0)} + \varepsilon\Delta^{(1)} + \mathcal{O}(\varepsilon^2) ,$$
$$\lambda_{ijk} = \lambda_{ijk}^{(0)} + \varepsilon\lambda_{ijk}^{(1)} + \mathcal{O}(\varepsilon^2) .$$

Conformal line defect in $O(N)$ CFTs

- Connections between different models in various dimensions:



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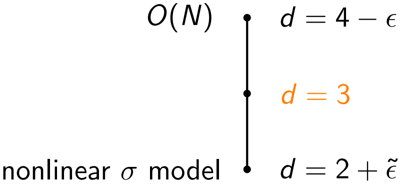

- Add a line defect to the action of $O(N)$ CFT:

$$S = \int d^d x \left(\frac{1}{2} (\partial_\mu \phi_a)^2 + \frac{\lambda_0}{4!} (\phi_a^2)^2 \right) + h_0 \int_{-\infty}^{\infty} d\tau \, \phi_1(x(\tau)) \, .$$

Magnetic line defect

- Defect breaks bulk $O(N)$ symmetry to defect $O(N-1)$ symmetry:

$$SO(d,1) \times O(N)_F \to SO(2,1) \times SO(d-1) \times O(N-1)_F$$

- Breaking of $O(N)$ introduces a new operator: the tilt

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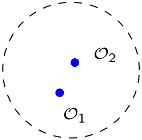
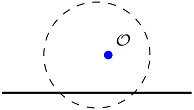
Defect OPE and defect blocks

Besides the usual bulk OPE:



$$\mathcal{O}_i(x_1)\mathcal{O}_j(x_2) \sim \sum_k \lambda_{ijk} C(x_{12}, \partial_2)\mathcal{O}_k(x_2)$$

we gain access to the bulk-to-defect OPE



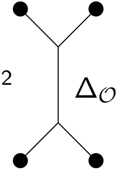
$$\mathcal{O}(x) \sim \sum_k \frac{b_{\mathcal{O}\hat{\mathcal{O}}}}{|x^\perp|^{\Delta-\hat{\Delta}}} C\left(|x^\perp|^2 \hat{\partial}^2\right) \hat{\mathcal{O}}_k(\hat{x})$$

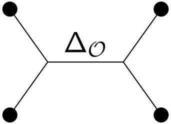
to expand bulk operators in defect operators. With the two OPEs, the 2-point function can be expanded in defect channel or bulk channel blocks:

$$\mathcal{F}(r,w) = \sum_{\hat{\mathcal{O}}_k} \hat{b}_{1k}\hat{b}_{2k}\hat{f}_{\hat{\Delta}_k,\hat{j}_k,s_k}(r,w)\,, \quad \mathcal{F}(r,w) = \sum_{\mathcal{O}_k} \lambda_{12k}\, a_k f^{\Delta_{12}}_{\Delta_k,\ell_k}(r,w)\,.$$

One-dimensional CFTs

- Operators ordered on the line: $\tau_1 < \tau_2 < \tau_3 < \tau_4$.
- Crossing symmetry follows from cyclicity.

$\sum_{\mathcal{O}} (\lambda_{\mathcal{O}})^2$ 

$= \sum_{\mathcal{O}} (\lambda_{\mathcal{O}})^2$ 

$$\sum_{\mathcal{O}} \lambda_{ij\mathcal{O}} \lambda_{kl\mathcal{O}} (1-\xi)^{\Delta_k+\Delta_j} g_{\Delta}^{\Delta_{ij},\Delta_{kl}}(\xi) = \sum_{\mathcal{O}} \lambda_{kj\mathcal{O}} \lambda_{il\mathcal{O}} \xi^{\Delta_i+\Delta_j} g_{\Delta}^{\Delta_{kj},\Delta_{il}}(1-\xi)\,.$$

- No parallel spin ℓ. Instead, there is parity (and transverse spin s)

$$\mathcal{S}: \qquad \tau \to -\tau\,, \quad \mathcal{S}(\psi(\tau)) = (-1)^{S_\psi}\psi(-\tau)\,, \quad S_\psi = 0, 1\,.$$

The numerical bootstrap

- Functionals to rule out possible solutions

$$\lambda^2_{\mathcal{O}'}\alpha\Big(F_{\Delta_{\mathcal{O}'}}\Big) = -\alpha\Big(F_0\Big) - \sum_{\mathcal{O}} \lambda^2_{\mathcal{O}}\alpha\Big(F_{\Delta_{\mathcal{O}}}\Big)\,.$$

- Upper bound if you can find α s.t.

$$\alpha\Big(F_{\Delta_{\mathcal{O}'}}\Big) = 1\,, \quad \alpha\Big(F_{\Delta_{\mathcal{O}}}\Big) \geq 0\,.$$

- Allowed and disallowed solutions for conformal dimensions.
- Upper and lower bounds for OPE coefficients.

Magnetic line bootstrap

- Feature for $\Delta_{\phi_1} = 1.55$ disappears.

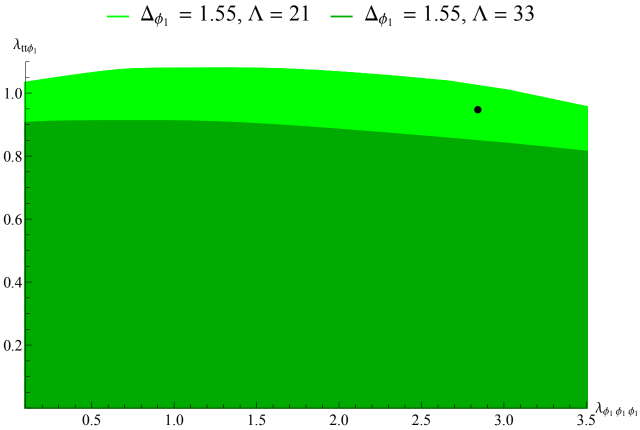
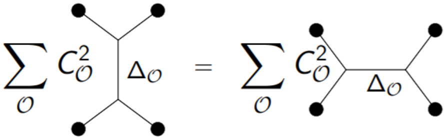


Figure: Upper bounds on the OPE coefficient $\lambda_{\phi_1\phi_1\phi_1}$ as a function of $\lambda_{tt\phi_1}$.

Crossing equations



- We obtain the following crossing equations:

$$\sum_{\mathcal{O}} \lambda_{ij\mathcal{O}} \lambda_{kl\mathcal{O}} (1-\xi)^{\Delta_k+\Delta_j} g_{\Delta}^{\Delta_{ij},\Delta_{kl}}(\xi) = \sum_{\mathcal{O}} \lambda_{kj\mathcal{O}} \lambda_{il\mathcal{O}} \xi^{\Delta_i+\Delta_j} g_{\Delta}^{\Delta_{kj},\Delta_{il}}(1-\xi)\,.$$

- where the conformal blocks are:

$$g_{\Delta}^{\Delta_{ij},\Delta_{kl}}(\xi) = \xi^{\Delta}\,{}_2F_1(\Delta-\Delta_{ij},\Delta+\Delta_{kl};2\Delta;\xi)\,.$$

- Straightforward solutions: generalized free fields.

$O(N)$ line defect: more results

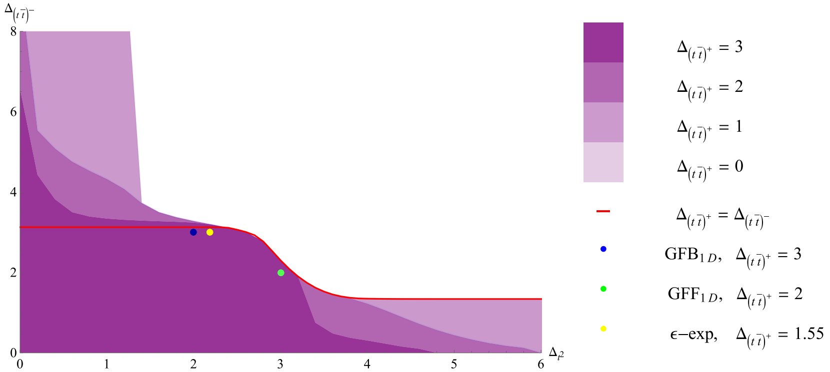


Figure: Bounds on the dimension of the $\mathcal{S}$-parity odd scalar $(t\bar{t})^-$ vs. the $\mathcal{S}$-parity even scalar $(t\bar{t})^+$ gap vs. the gap on t^2. $\Lambda = 33$.

Magnetic line bootstrap

- We bootstrap the tilt and the fundamental scalar:

$$\langle t(\tau_1)\bar{t}(\tau_2)t(\tau_3)\bar{t}(\tau_4)\rangle\,, \quad \langle \phi_1(\tau_1)\phi_1(\tau_2)\phi_1(\tau_3)\phi_1(\tau_4)\rangle\,,$$
$$\langle t(\tau_1)\bar{t}(\tau_2)\phi_1(\tau_3)\phi_1(\tau_4)\rangle\,.$$

- Special feature: externals appear in OPE

$$\phi_1 \times \phi_1 \sim \mathbf{1} + \phi_1 + s_- + \cdots\,, \quad (t\times\bar{t})^+ \sim \mathbf{1} + \phi_1 + s_- + \cdots\,,$$
$$(t\times\bar{t})^- \sim A + \cdots\,, \quad t\times t \sim T + \ldots\,, \quad t\times\phi_1 \sim t + V + \cdots\,,.$$

- Reminiscent of Ising model island!
- Gap assumptions from ε-expansion results at $O(\varepsilon)$:

$$\Delta_{s_-} = 2.36\,, \qquad \Delta_A = 3\,, \qquad \Delta_T = 2.18\,, \qquad \Delta_V = 3.18\,,$$

Magnetic line bootstrap

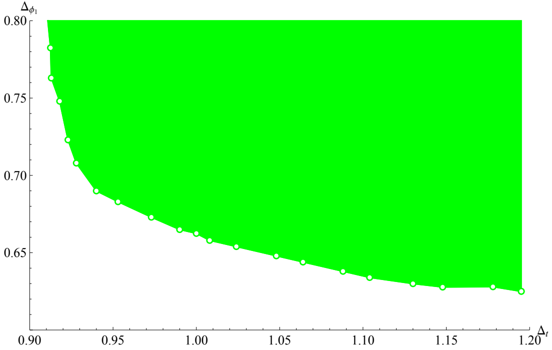


Figure: Bounds on the scaling dimensions Δ_{ϕ_1} and Δ_t for $\Lambda = 21$.

- Close to rediscovering the tilt!
- Set $\Delta_t = 1$ and bootstrap

$$(\lambda_{\phi_1\phi_1\phi_1})^2 + (\lambda_{tt\phi_1})^2 \,, \quad \tan\theta = \frac{\lambda_{\phi_1\phi_1\phi_1}}{\lambda_{tt\phi_1}} \,.$$

Monodromy line defect

- Start from $N = 2$ real scalars combined in a complex scalar Φ that satisfies

$$\Phi(r, \theta + 2\pi, \vec{x}) = e^{2\pi i v}\Phi(r, \theta, \vec{x}) , \quad v \sim v + 1 , \quad v \in [0, 1) .$$

- The defect modes Ψ of Φ will have fractional transverse spin $s \in \mathbb{Z} + v$ and dimensions [Söderberg 2017][Giombi et al. 2021]

$$\Delta_{\Psi_s} = 1 + |s| - \frac{\varepsilon}{2} + \frac{1}{5}\frac{v(v-1)}{|s|}\varepsilon + O(\varepsilon^2) .$$

- Generalization of Z_2 Ising twist defect [Gaiotto et al. 2013] [Billó et al. 2013]

Monodromy bootstrap

- D appears in the OPE of defect modes:

$$\Psi_v \times \bar{\Psi}_{v-1} \sim \text{D} + \cdots \ .$$

- We bootstrap the defect modes of the fundamental scalar

$$\langle \Psi_v(\tau_1)\Psi_v(\tau_2)\bar{\Psi}_v(\tau_3)\bar{\Psi}_v(\tau_4)\rangle \ , \quad \langle \Psi_{v-1}(\tau_1)\Psi_{v-1}(\tau_2)\bar{\Psi}_{v-1}(\tau_3)\bar{\Psi}_{v-1}(\tau_4)\rangle \ ,$$
$$\langle \Psi_v(\tau_1)\Psi_{v-1}(\tau_2)\bar{\Psi}_{v-1}(\tau_3)\bar{\Psi}_v(\tau_4)\rangle \ .$$

- Only information we give is appearance of D and the external dimensions of $\Delta_{\Psi_v}, \Delta_{\Psi_{v-1}}$.

Monodromy bootstrap

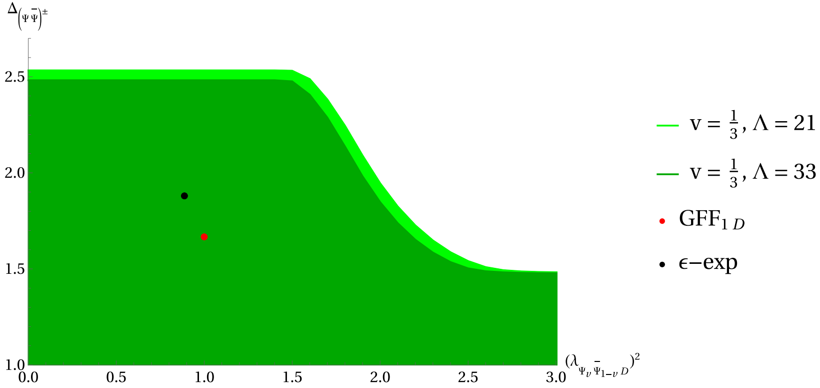


Figure: Bounds on the dimension of the first singlet in $\Psi_s \times \bar{\Psi}_s$ $\Delta_{(\Psi\bar{\Psi})^{\pm}}$ versus the OPE coefficient $(\lambda_{\Psi_v\bar{\Psi}_{1-v}D})^2$.