Understanding the tower weak gravity conjecture

Cesar Fierro Cota

Based on: 2212.09758 and 23XX.XXXXX

In collaboration with: Alessandro Mininno, Timo Weigand, and Max Wiesner.

27.09.2023

Outline

- **Part I:** The weak gravity conjecture — an overview
- **Part II** $\begin{cases} A : \text{The asymptotic weak gravity conjecture} \\ B : \text{The weak gravity conjecture with no infinite towers} \end{cases}$

Part I

–In this section statements are independent of string theory.

The weak gravity conjecture

Consider a U(1) gauge theory coupled to gravity.

The weak gravity conjecture:
There must exist a **super-extremal state** of charge q and mass m, such that

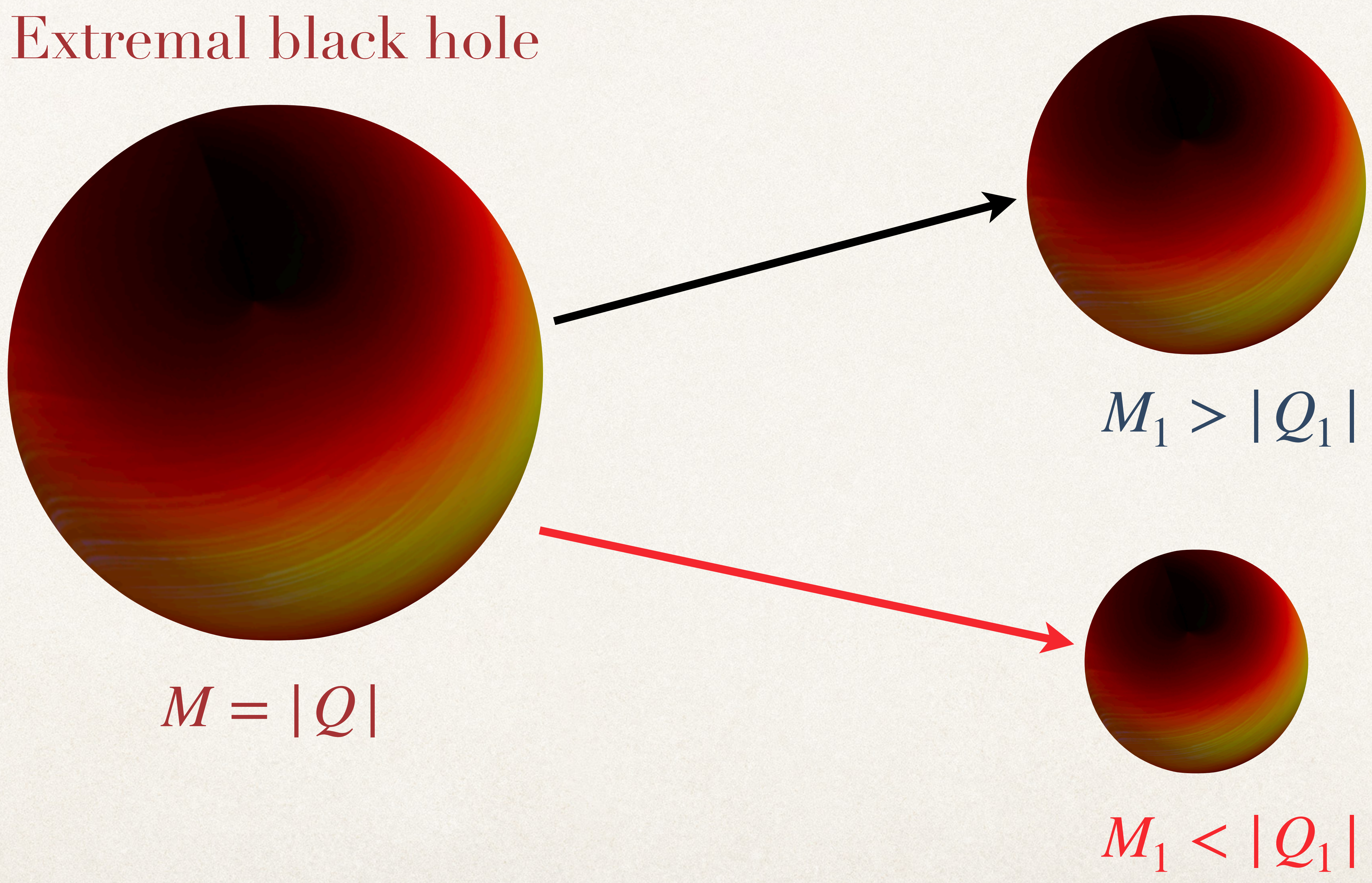
$$\frac{|q|}{m} \geq \frac{|Q|}{M}\bigg|_{\text{Ext}} . \tag{1}$$

Q: Black hole charge
M: Black hole mass

[Arkani-Hamed,Motl,Nicolis,Vafa'06]

The weak gravity conjecture — motivation

Extremal black holes per se are unstable objects and, thus, they should decay.



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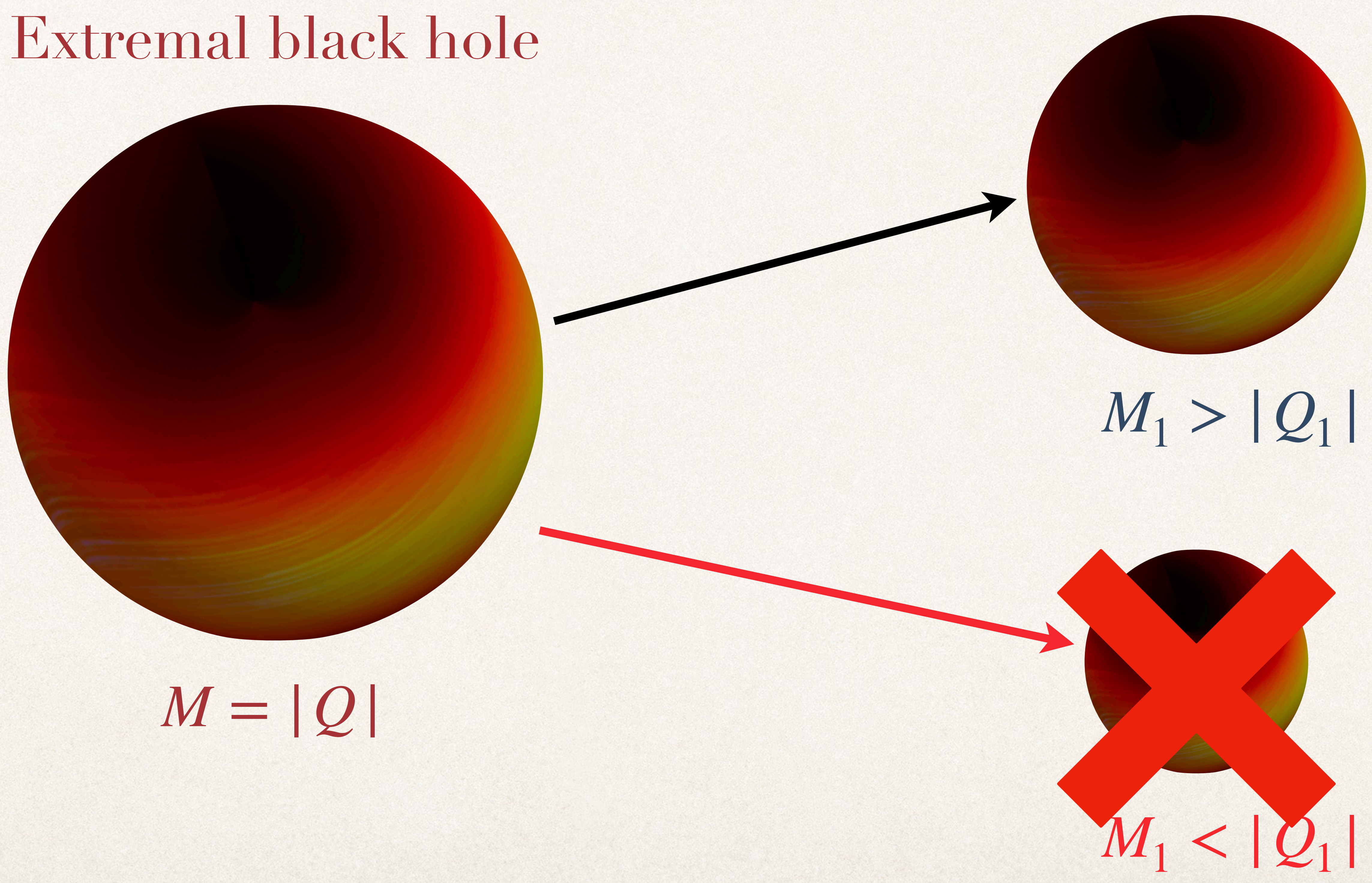

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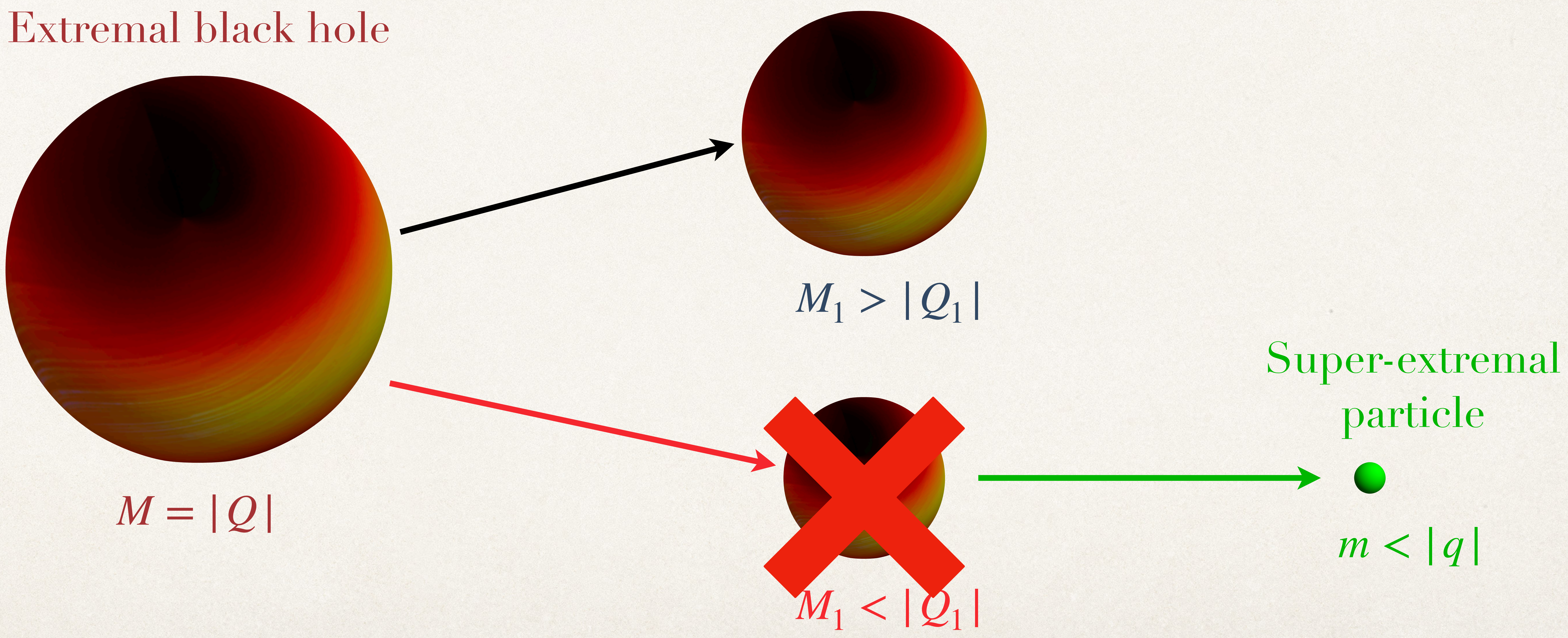

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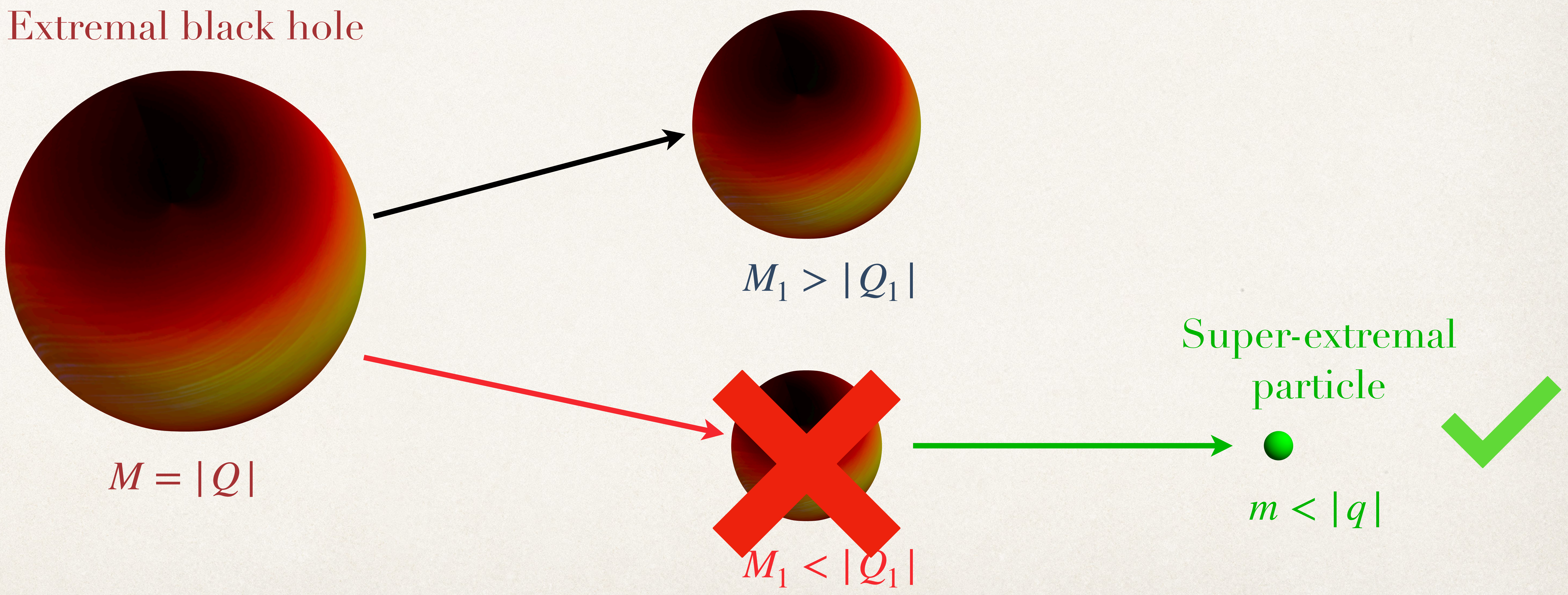

The weak gravity conjecture — extensions

The weak gravity conjecture inequality can be generalized for a theory in D-dimensions with several abelian gauge factors, as well as several scalar fields.

However, one has to be careful about the multiple $U(1)$ gauge group factors.

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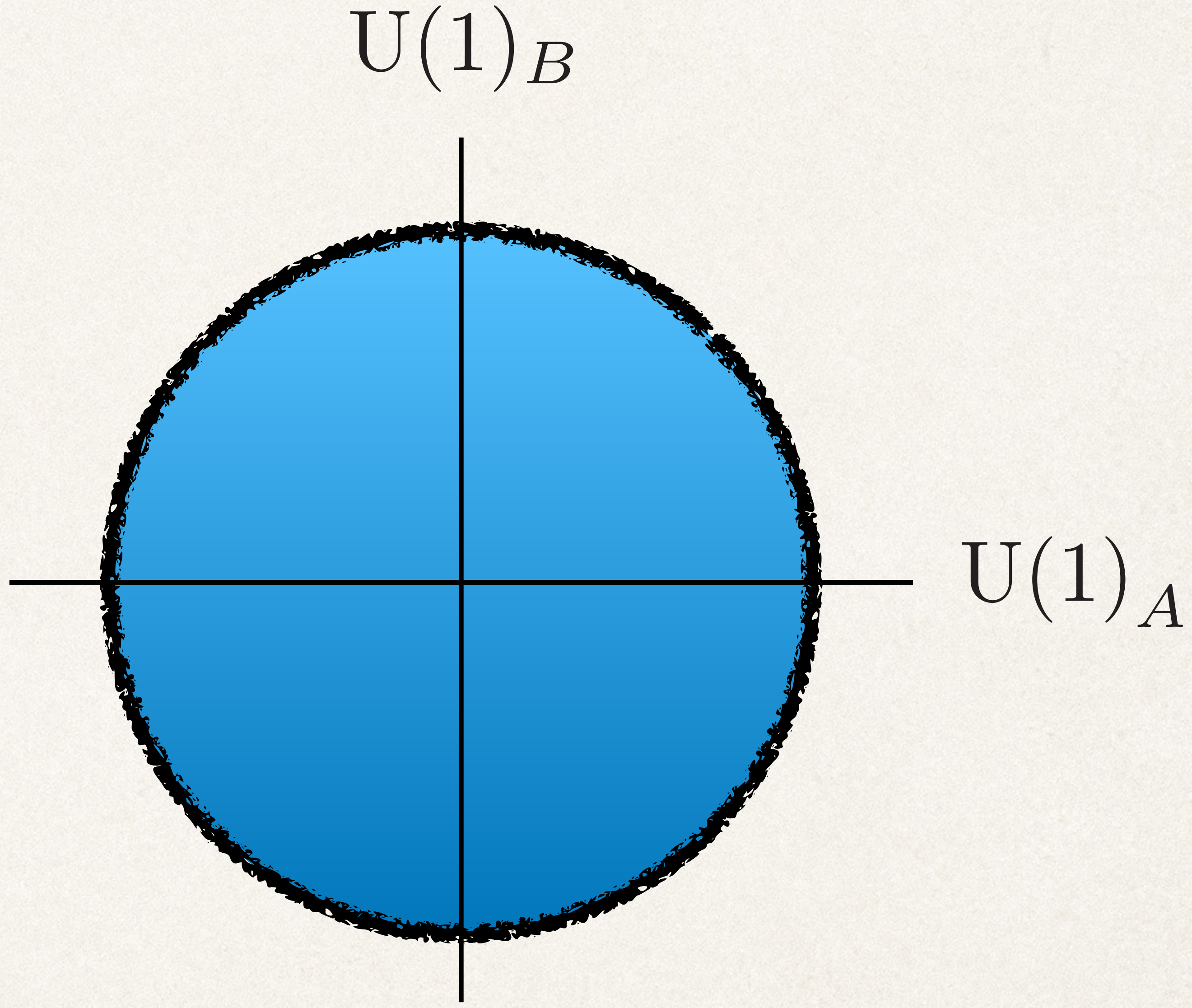

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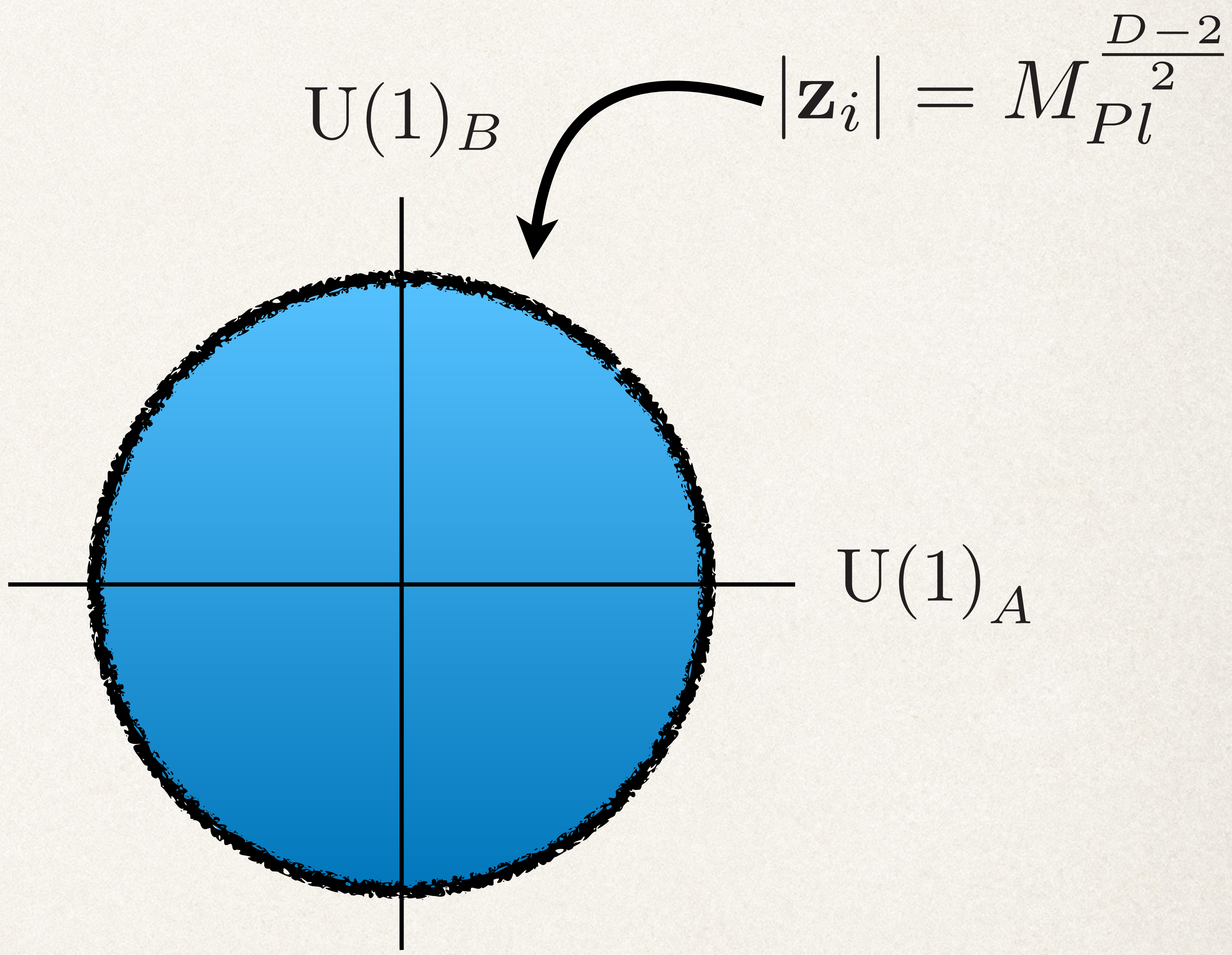

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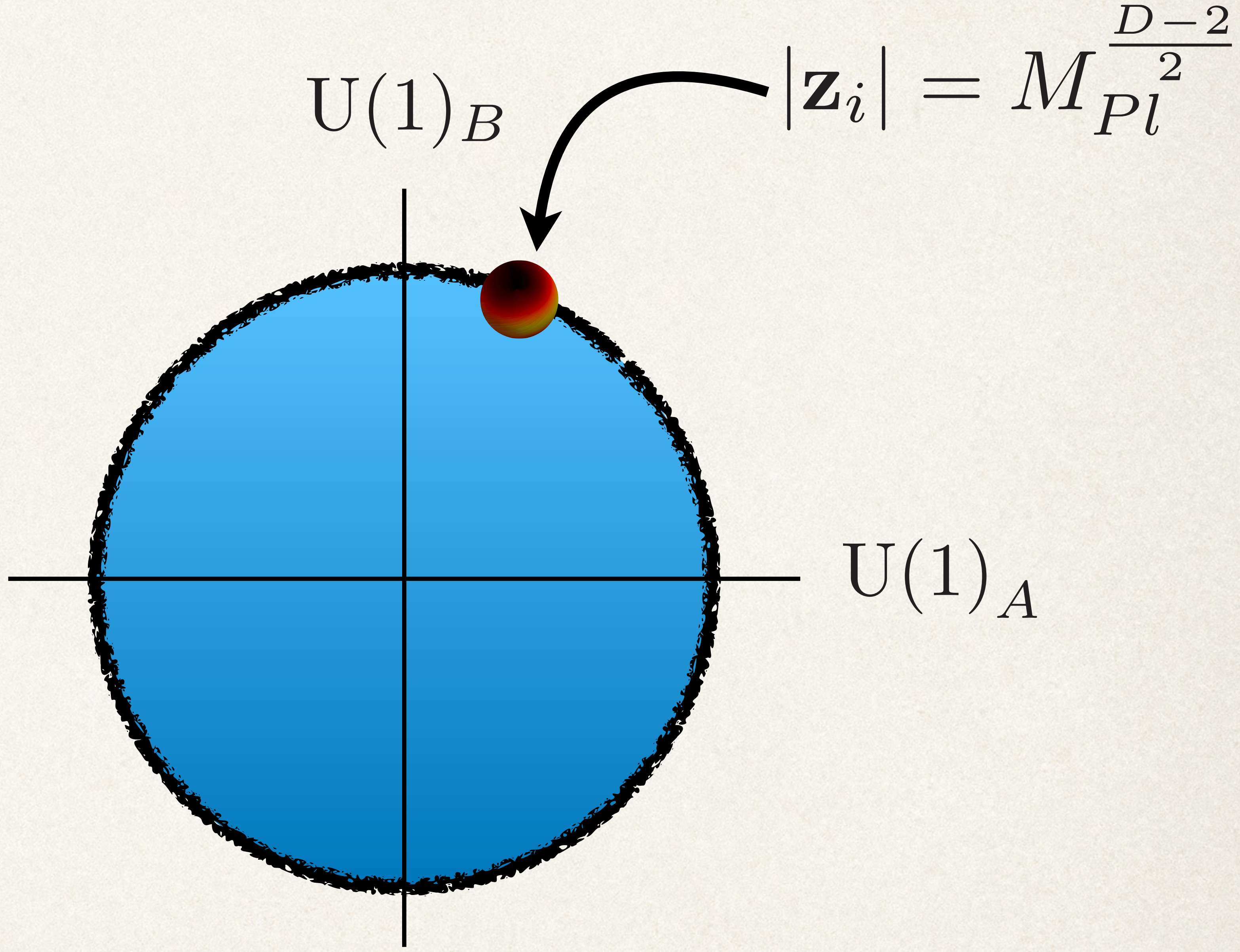

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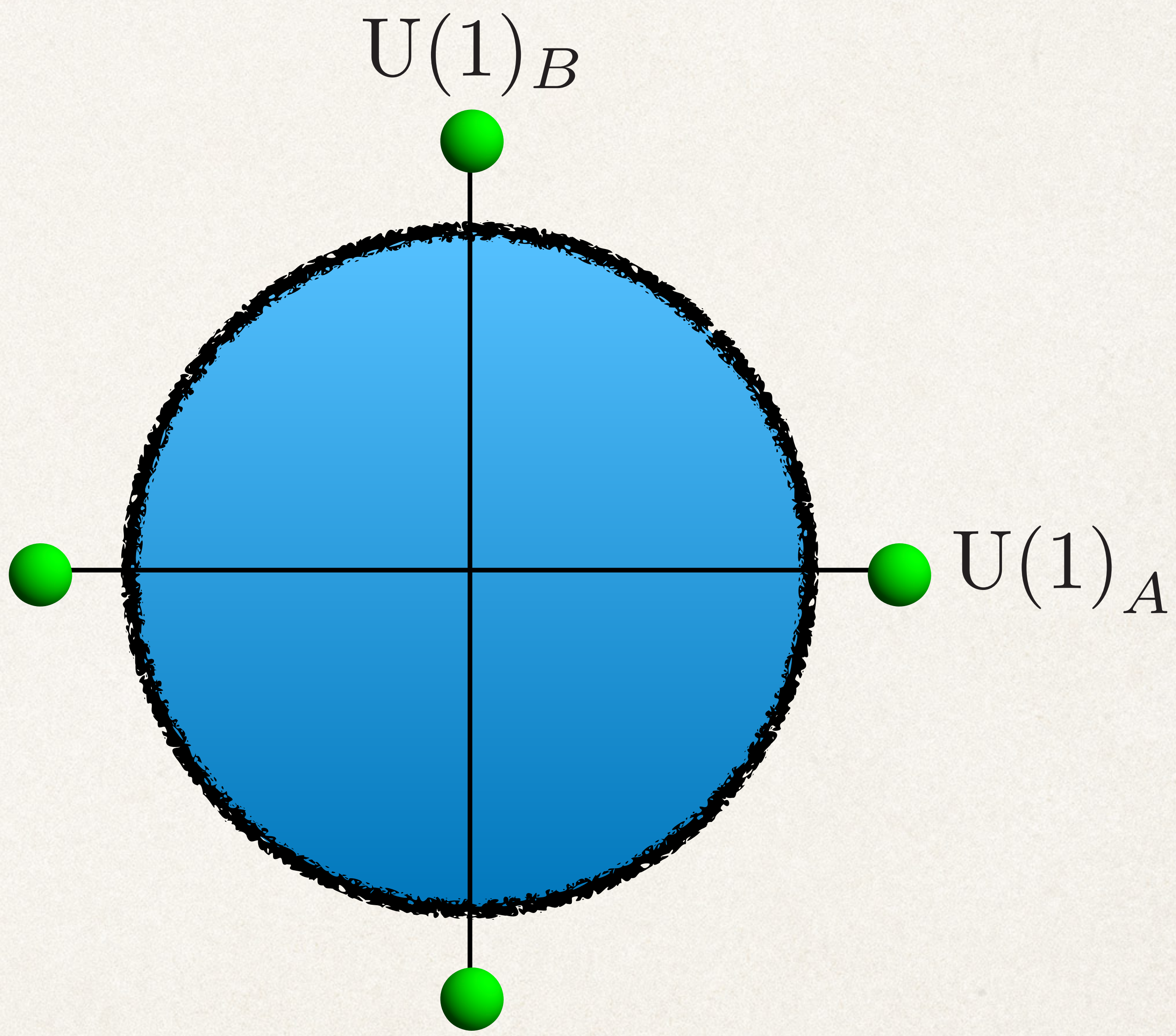

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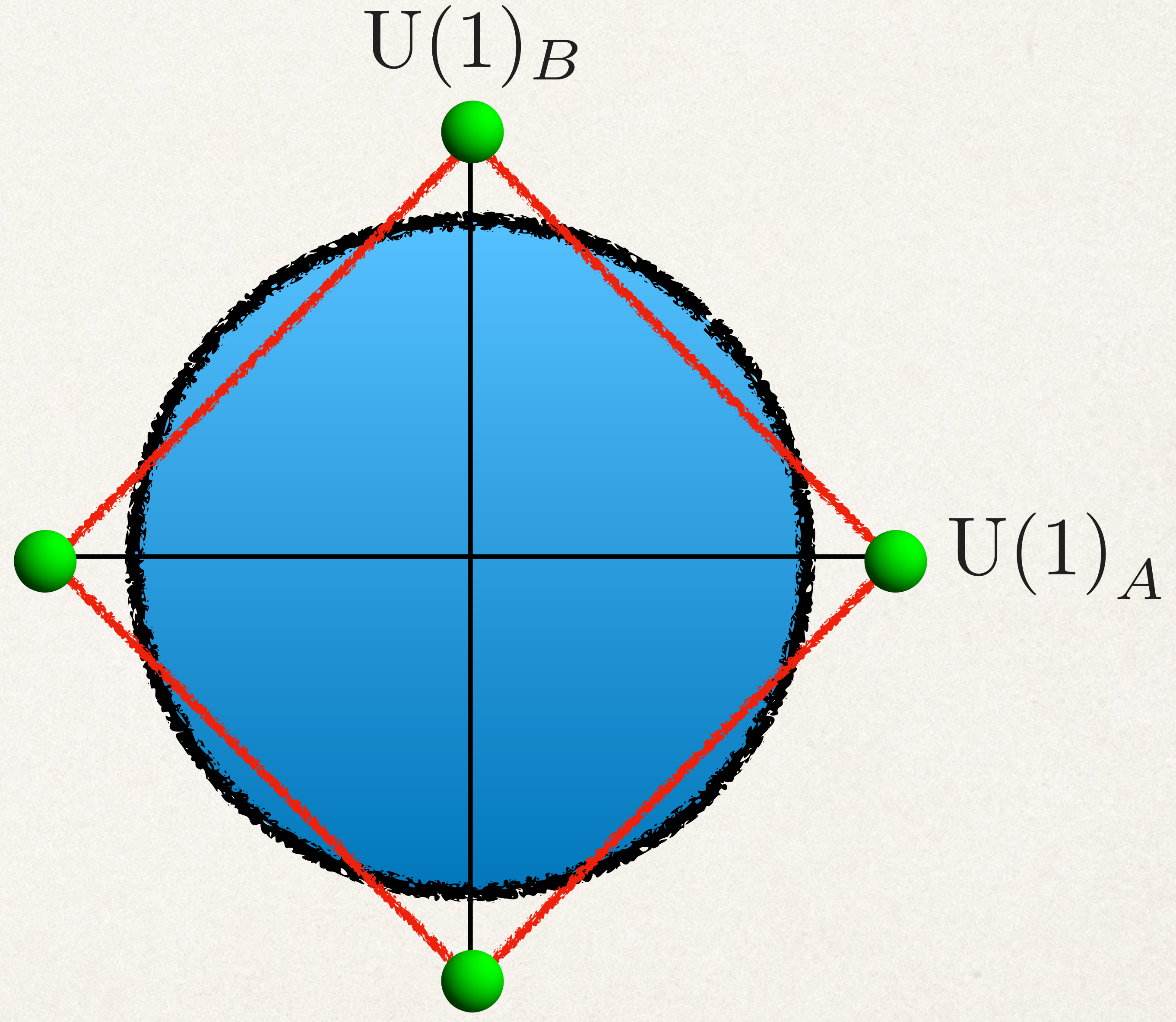

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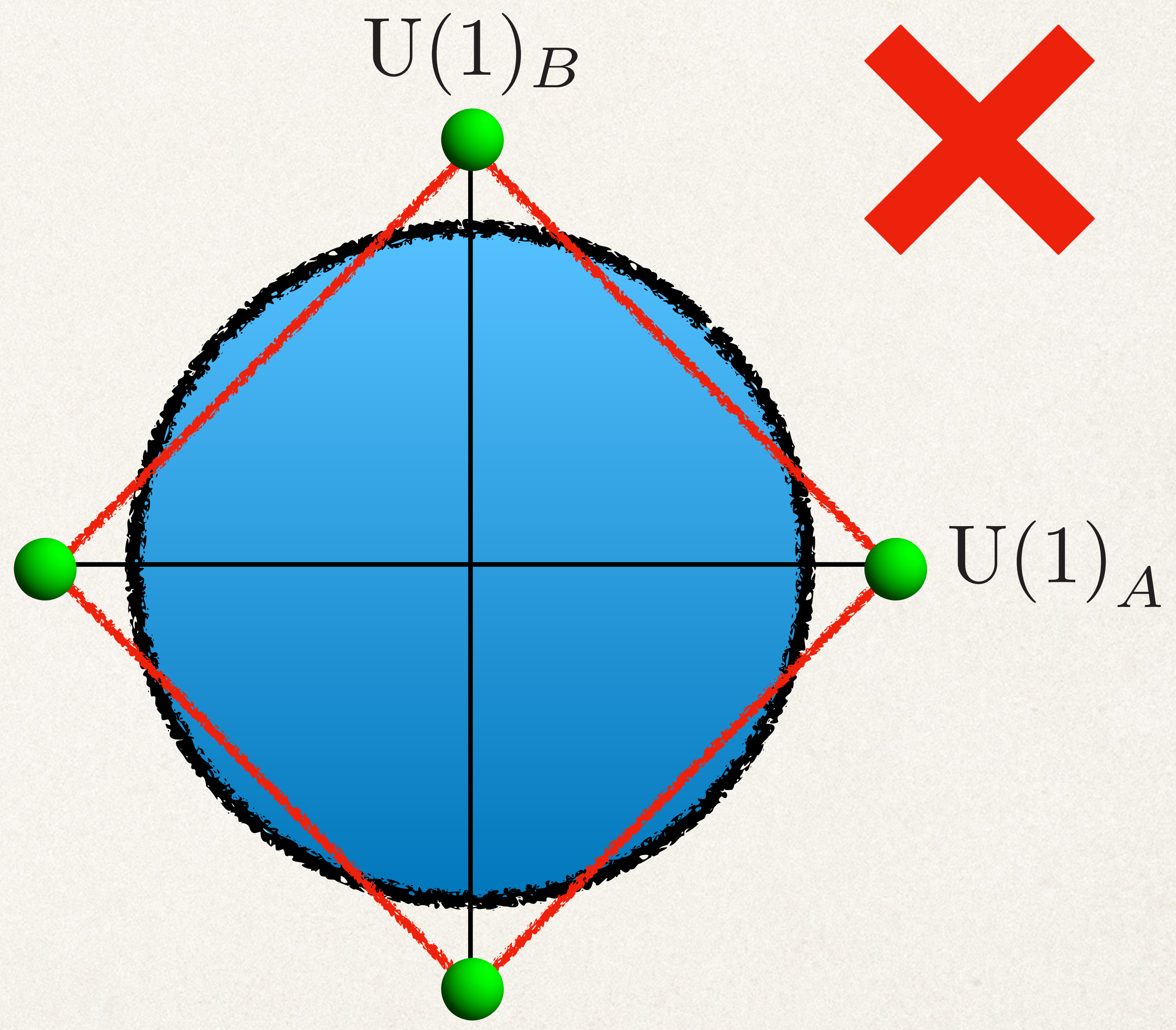

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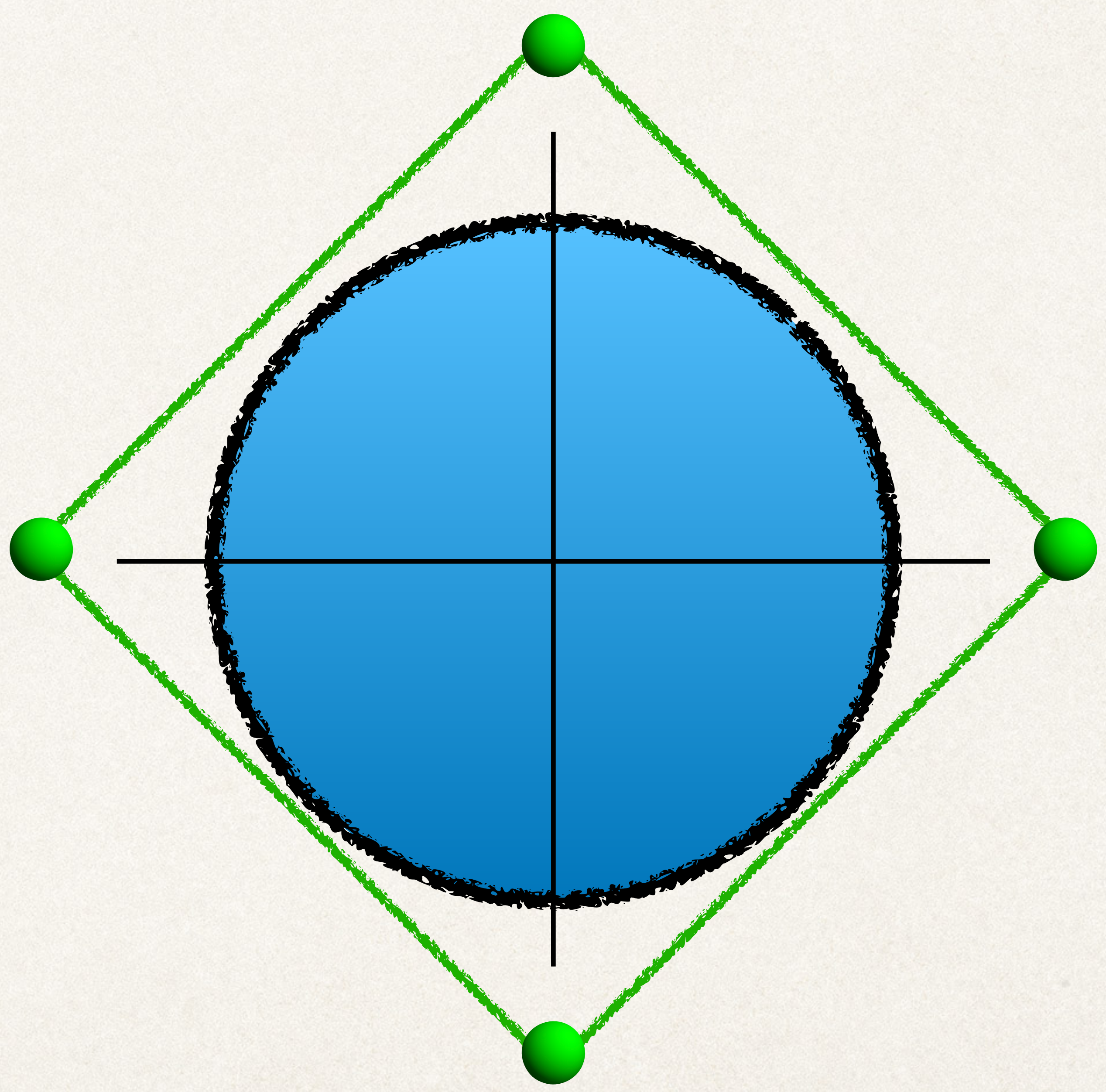

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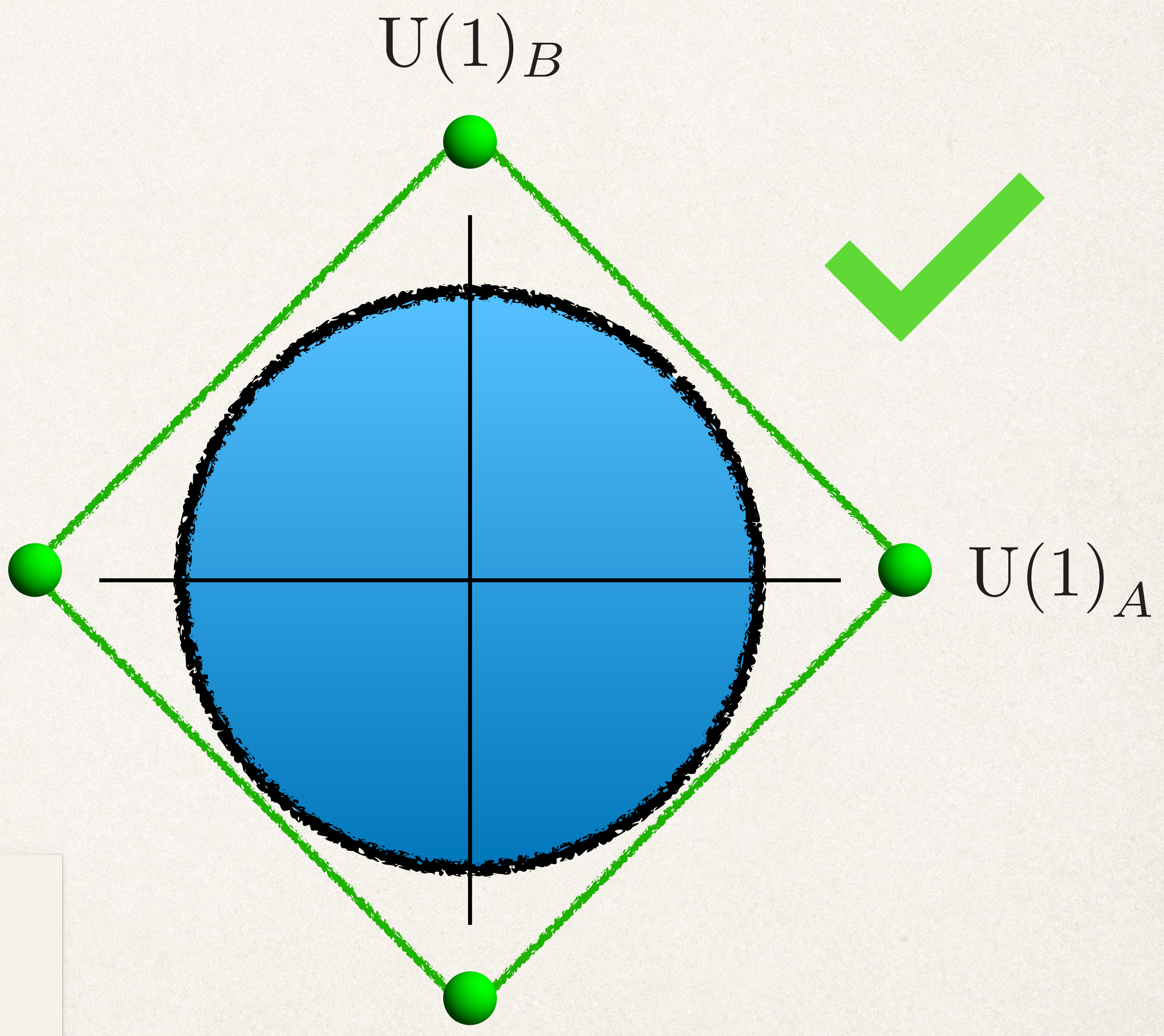

Convex Hull Condition: [Cheung, Remmen'14]
There is a set $\{\mathbf{z}_i\}_{i\in\mathcal{J}}$ whose convex hull contains the unit ball.

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Convex hull condition satisfied whenever: [Heidenreich, Reece, Rudelius'15]

$$(m_D r_{S^1})^2 \geq \frac{1}{4z_D^2(z_D^2 - 1)}$$

The weak gravity conjecture — extensions

Convex hull condition under Kaluza-Klein reduction:

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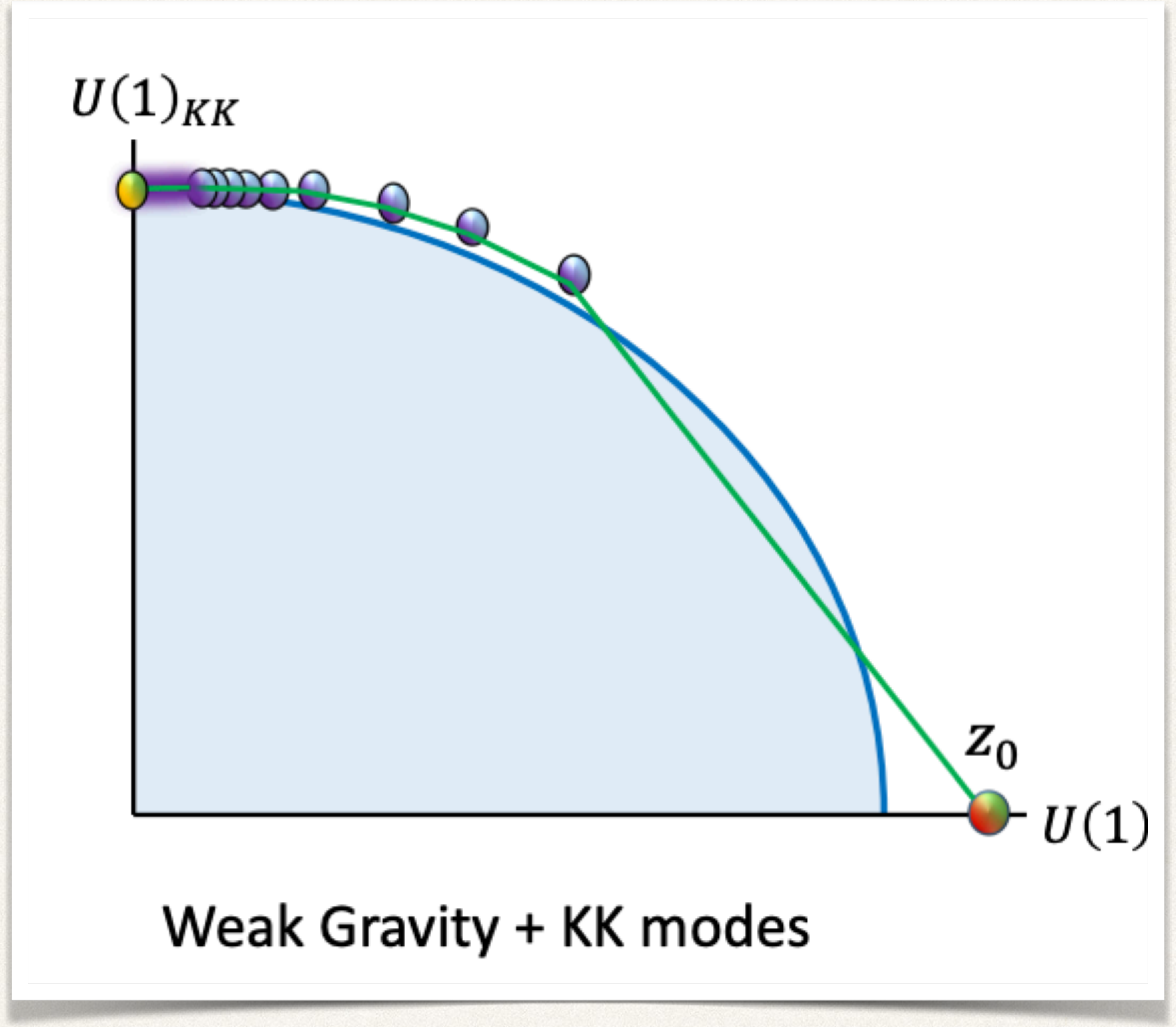

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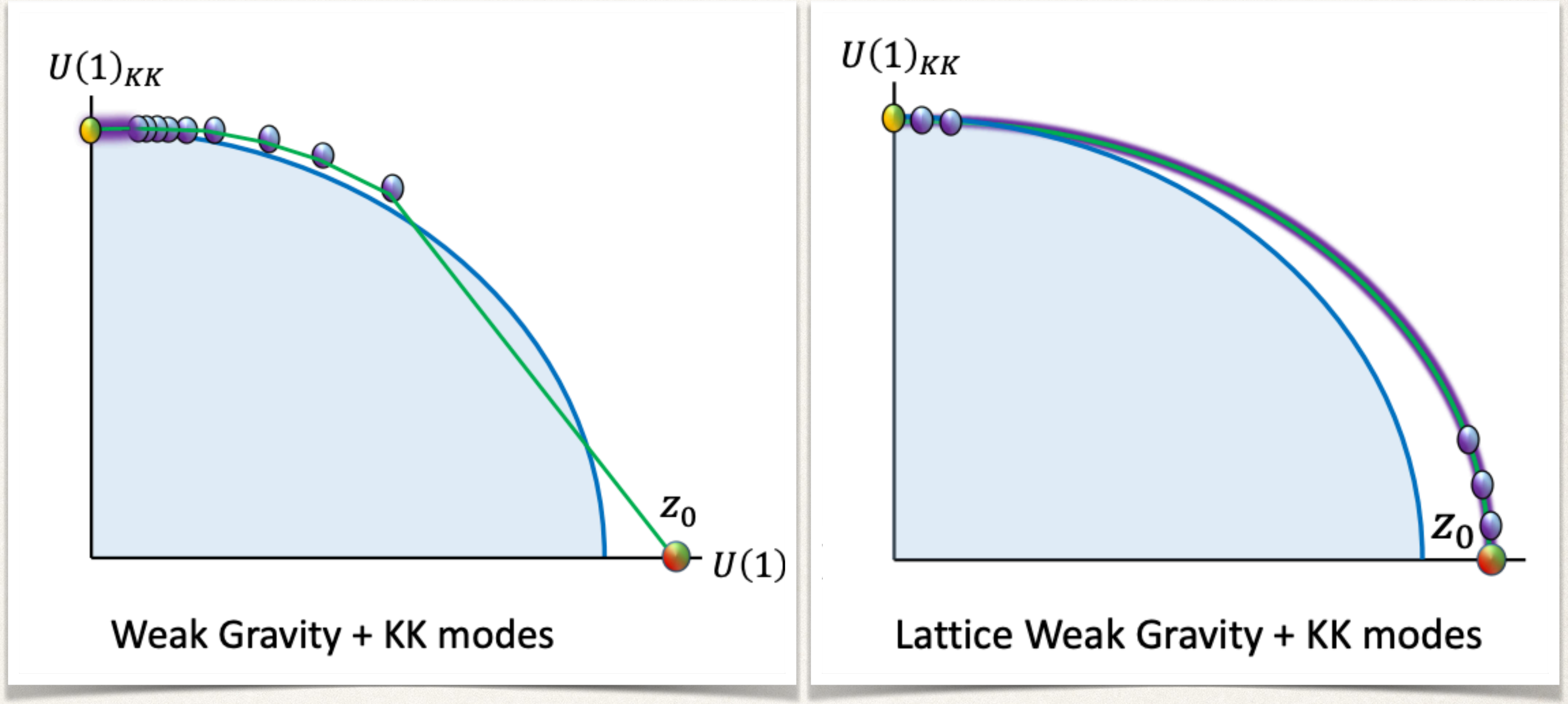


[Pictures: Palti'19]

Resolution: [Heidenreich, Reece, Rudelius'15]

1. The effective field theory breaks down at $r_{S^1} = r_{\text{min.}} \sim \Lambda_D^{-1}$.
2. There exists an infinite tower of super-extremal states such that $z_D \geq 1$.

The tower weak gravity conjecture

Evidence for the tWGC in string theory:

[Heidenreich, Reece, Rudelius'15'16'17]

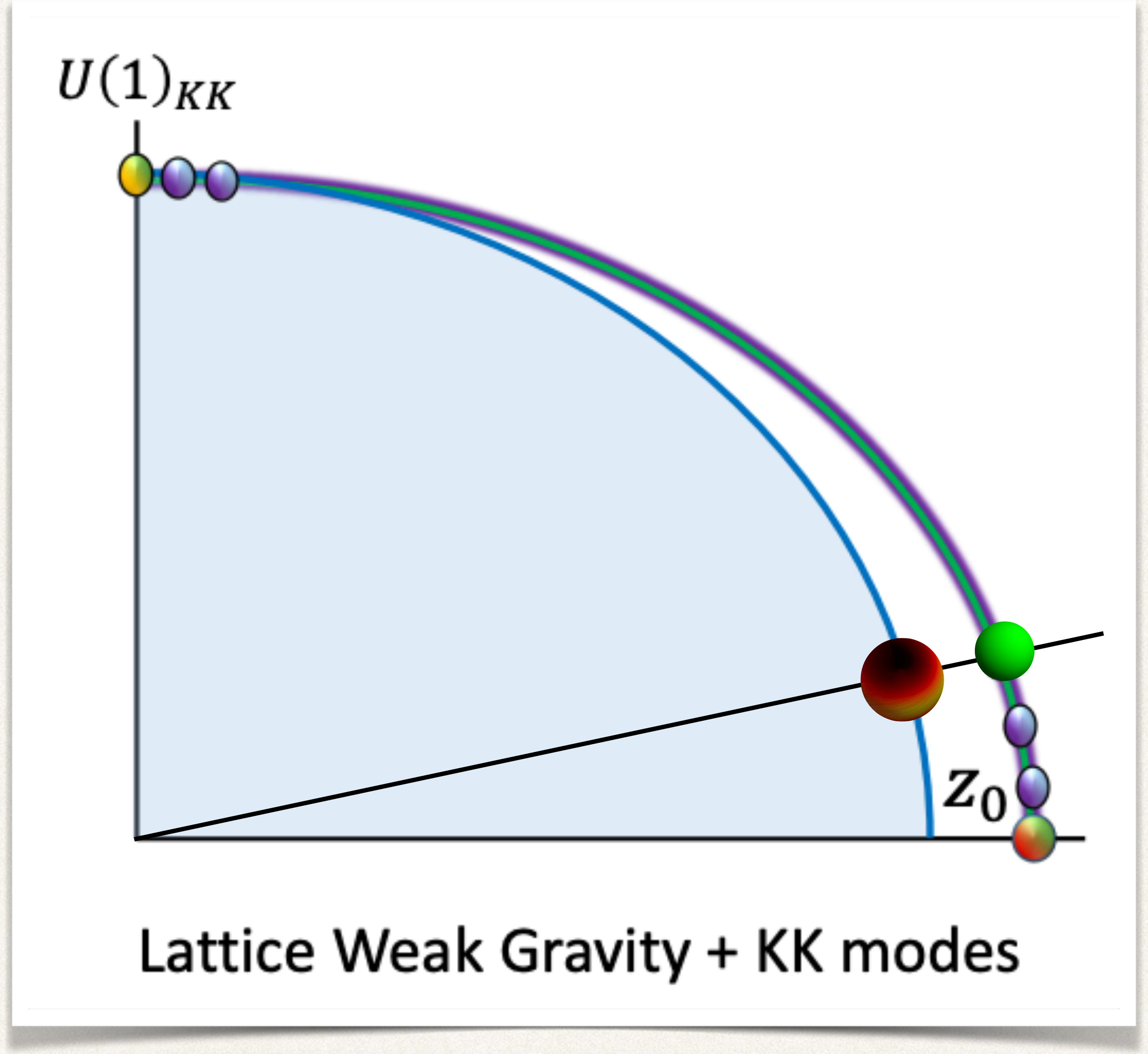
[Lee, Lerche, Weigand'18]

[Klaewer, Lee, Weigand, Wiesner'18]

[CFC, Klemm, Schimannek'20]

[CFC, Mininno, Weigand, Wiesner'22]

Most examples are related to the heterotic string.



Also:

[Alim, Heidenreich, Rudelius'21]

[Gendler, Heidenreich, McAllister, Rudelius'22]

Super-extremal states here are black holes.

The tower weak gravity conjecture:[Heidenreich, Reece, Rudelius'15'16[Andriolo, Junghans, Noumi, Shiu'16]

For every rational direction $\hat{Q}$ in charge space, a super-extremal state of charge and mass must exist, such that $\vec{q}/m \propto \hat{Q}$.

Part II

–From now on: F-theory/M-theory compactified on a Calabi-Yau threefold.

Part II — A: The asymptotic weak gravity conjecture

–Q: When can we relate super-extremal states with particle-like states in the EFT?

The asymptotic weak gravity conjecture

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We call *asymptotic weak gravity conjecture* those limits in which the weak gravity conjecture can be described such that

$$\frac{\Lambda_{\text{WGC}}}{\Lambda_{\text{QG}}} \to 0\,.$$

The asymptotic weak gravity conjecture

In M-theory on a Calabi-Yau threefold X_3, an asymptotic limit $\Lambda_{\rm WGC}/\Lambda_{\rm QG} \to 0$ is realized in the Kähler moduli space of a fibered X_3 by shrinking its fiber while blowing its base.
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Gauge group: $\mathrm{U}(1) = c_\alpha \mathrm{U}(1)^\alpha$ $\longleftrightarrow$ $C = c_\alpha C^\alpha$; $\{C^\alpha\}$ is basis of curves in X_3 and $c_\alpha \in \mathbb{Z}_{\geq 0}$.

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Tower weak gravity conjecture in the asymptotic limit holds for [CFC, Mininno, Weigand, Wiesner'22]

1. A T^2-fibered Calabi-Yau threefold $\pi : X_3 \to B_2$.
2. A K3-fibered Calabi-Yau threefold $\rho : X_3 \to \mathbb{P}^1$.

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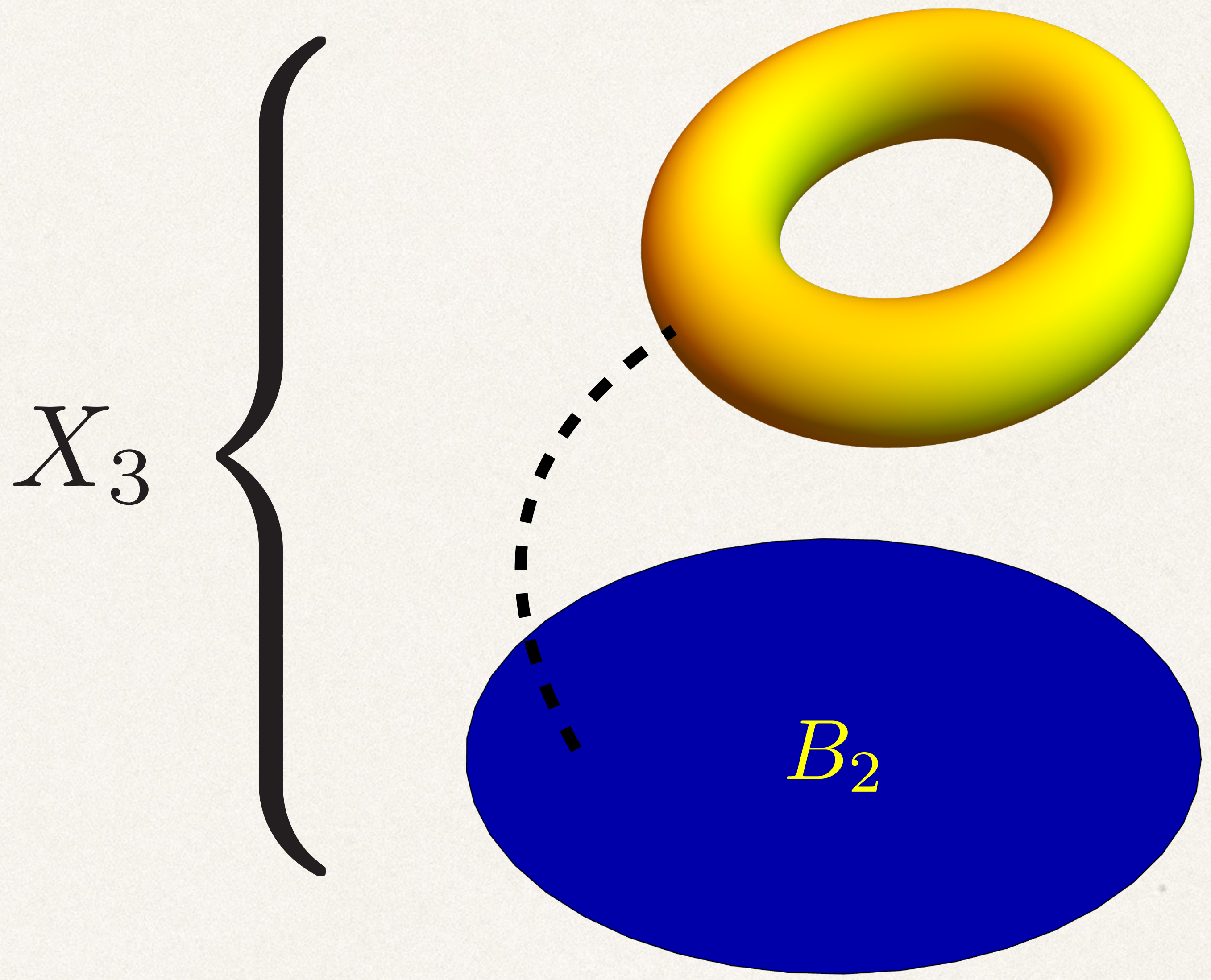

Super-extremal states counted by Gopakumar-Vafa invariants
$n^0_{nT^2} = -\chi(X_3),\ \ n \in \mathbb{N}.$

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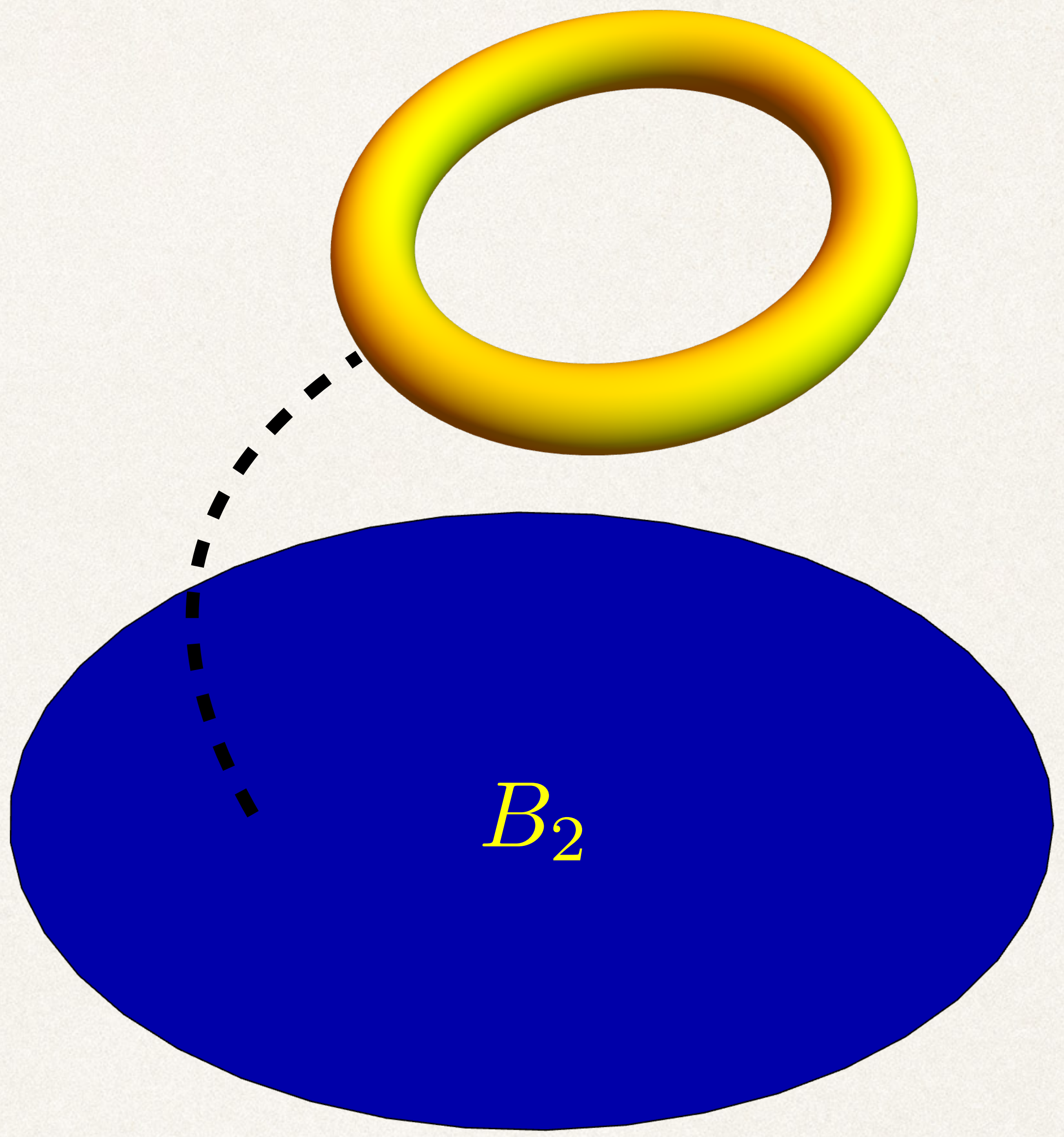

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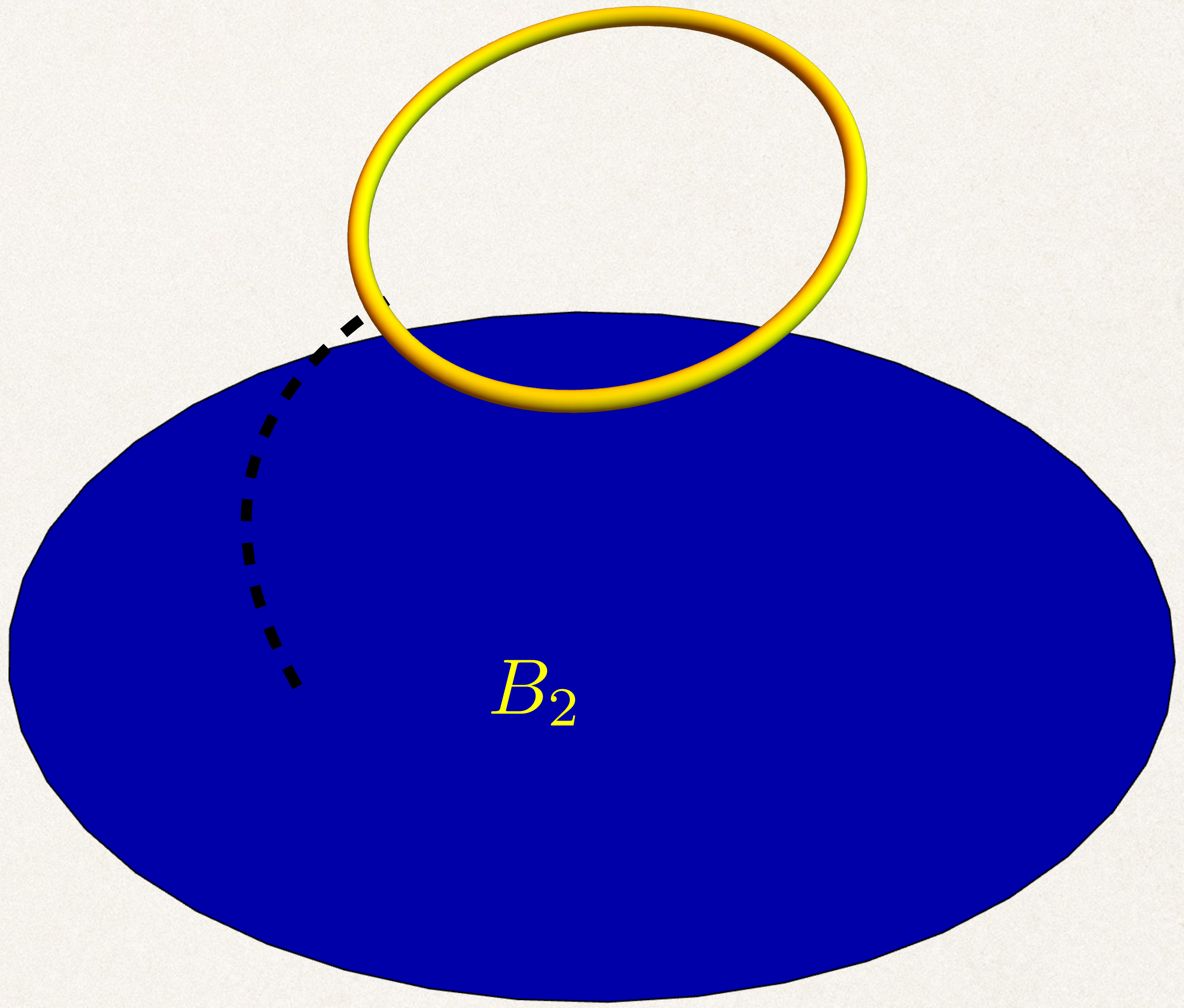

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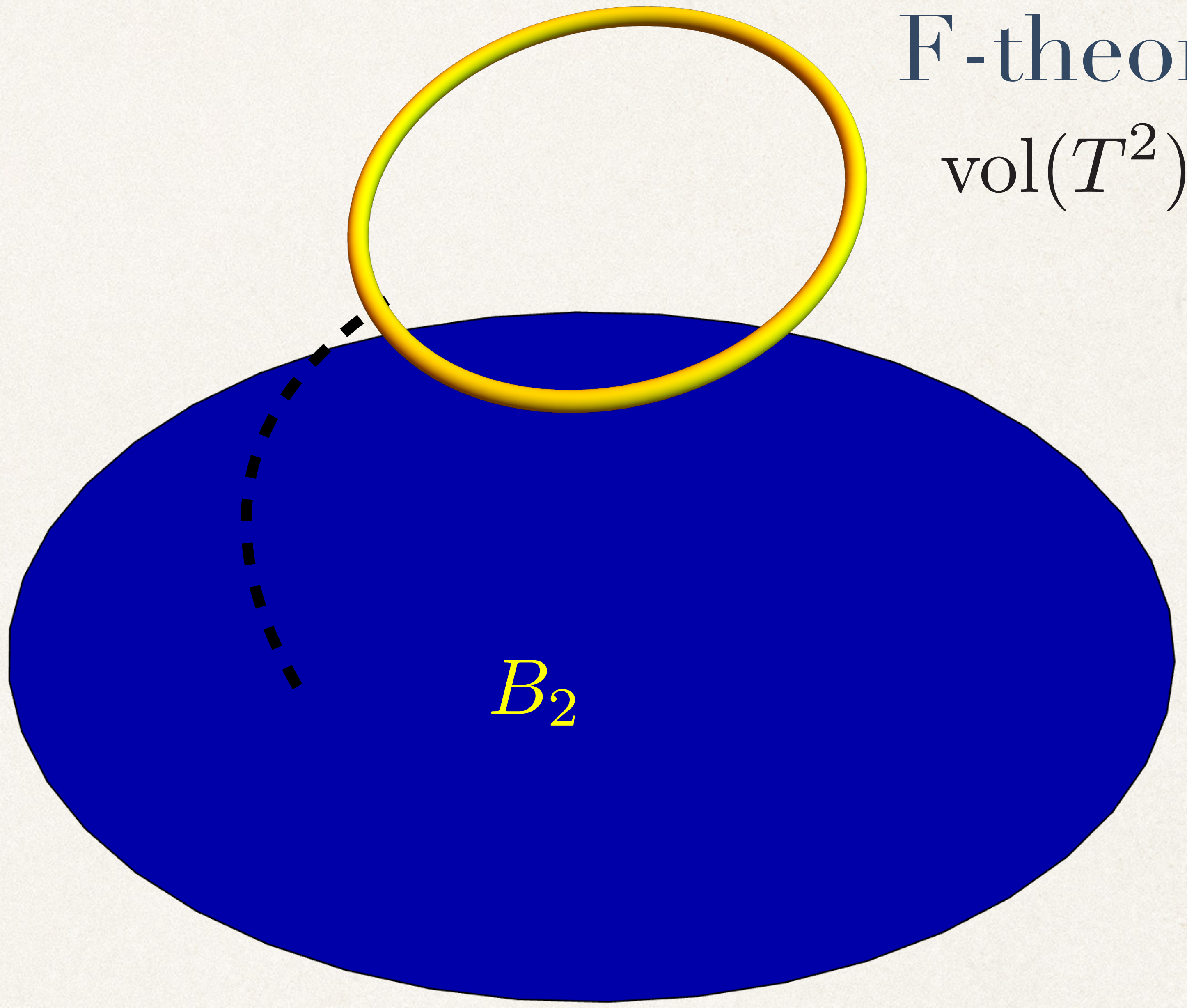

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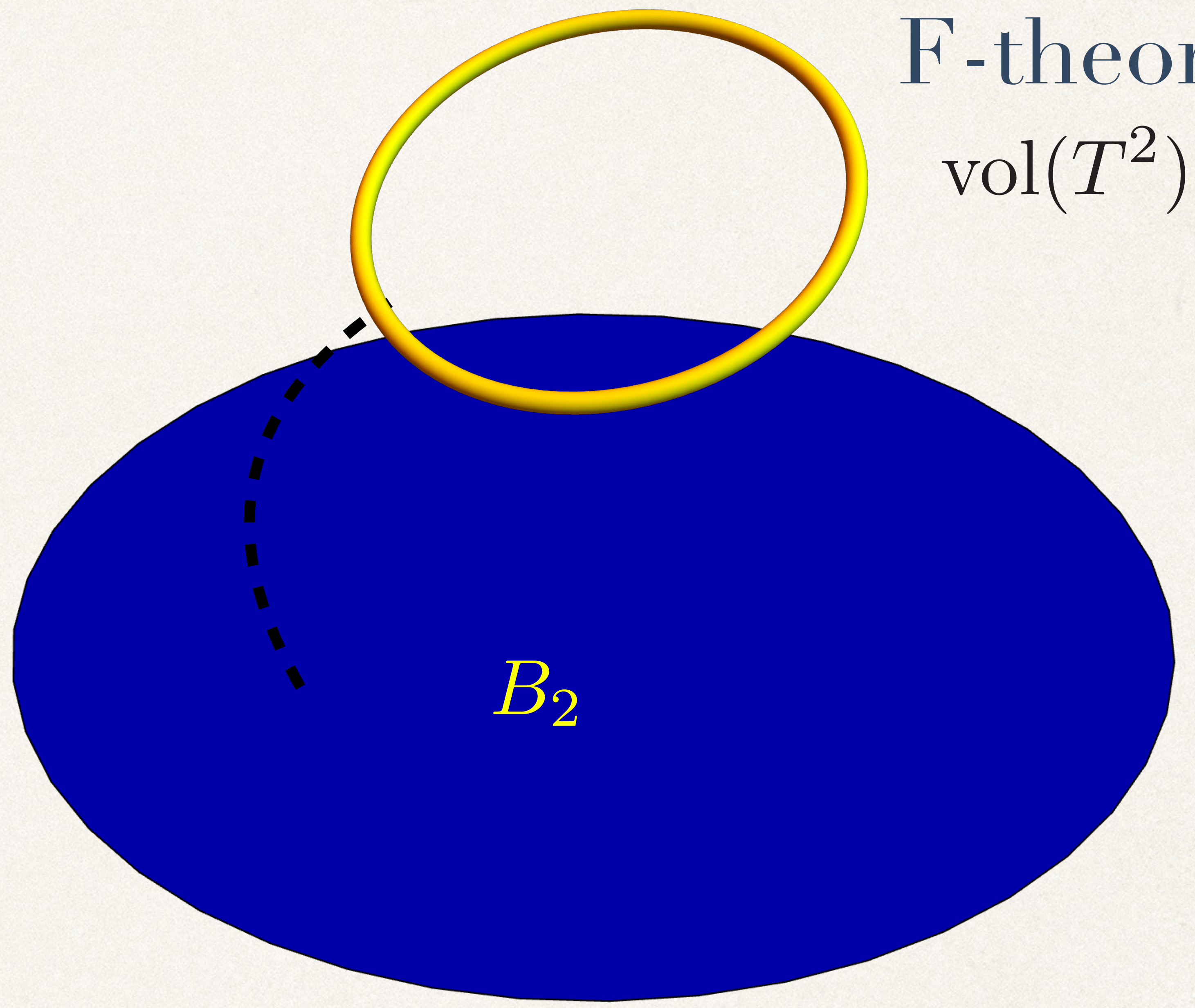

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Super-extremal states are Kaluza-Klein modes of a six-dimensional theory realized by F-theory.

[Lee, Lerche, Weigand'19]

The asymptotic weak gravity conjecture

2. K3-fibered Calabi-Yau threefold $\rho : X_3 \to \mathbb{P}^1$.
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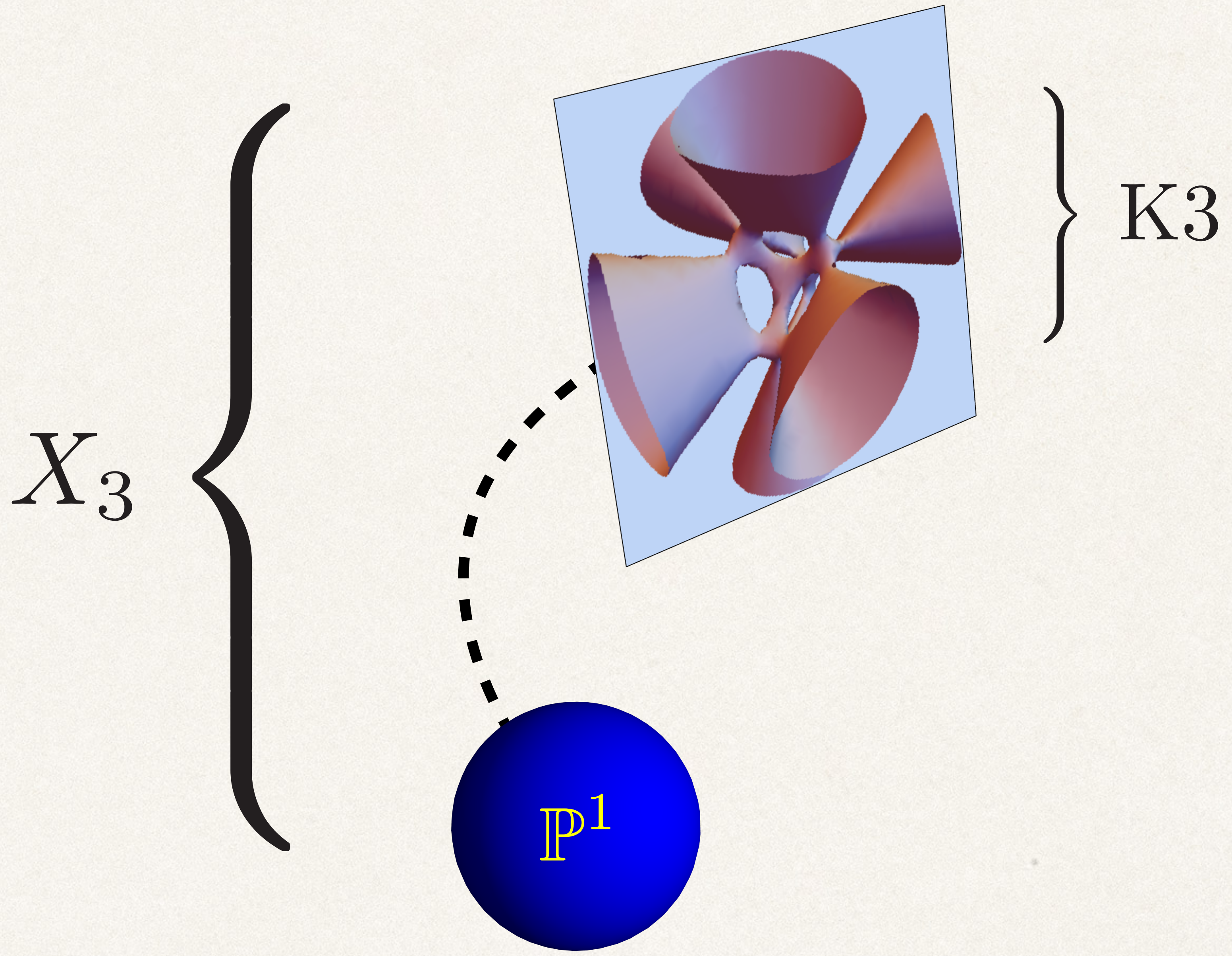

Super-extremal states counted by
Donaldson-Thomas invariants related to K3-fiber.

[Bouchard, Creutzig, Diaconescu, Doran, Quigley, Sheshmani'16]
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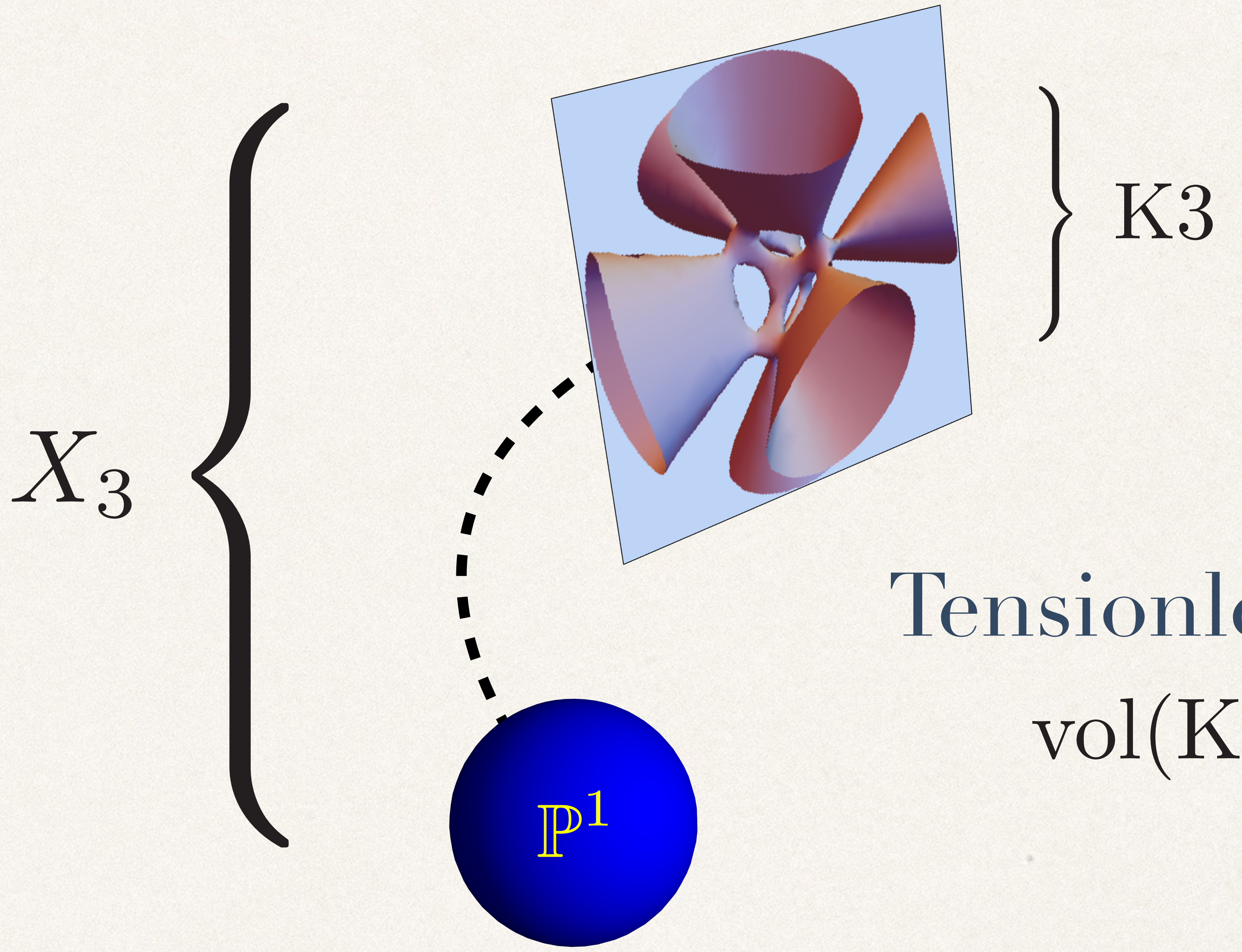

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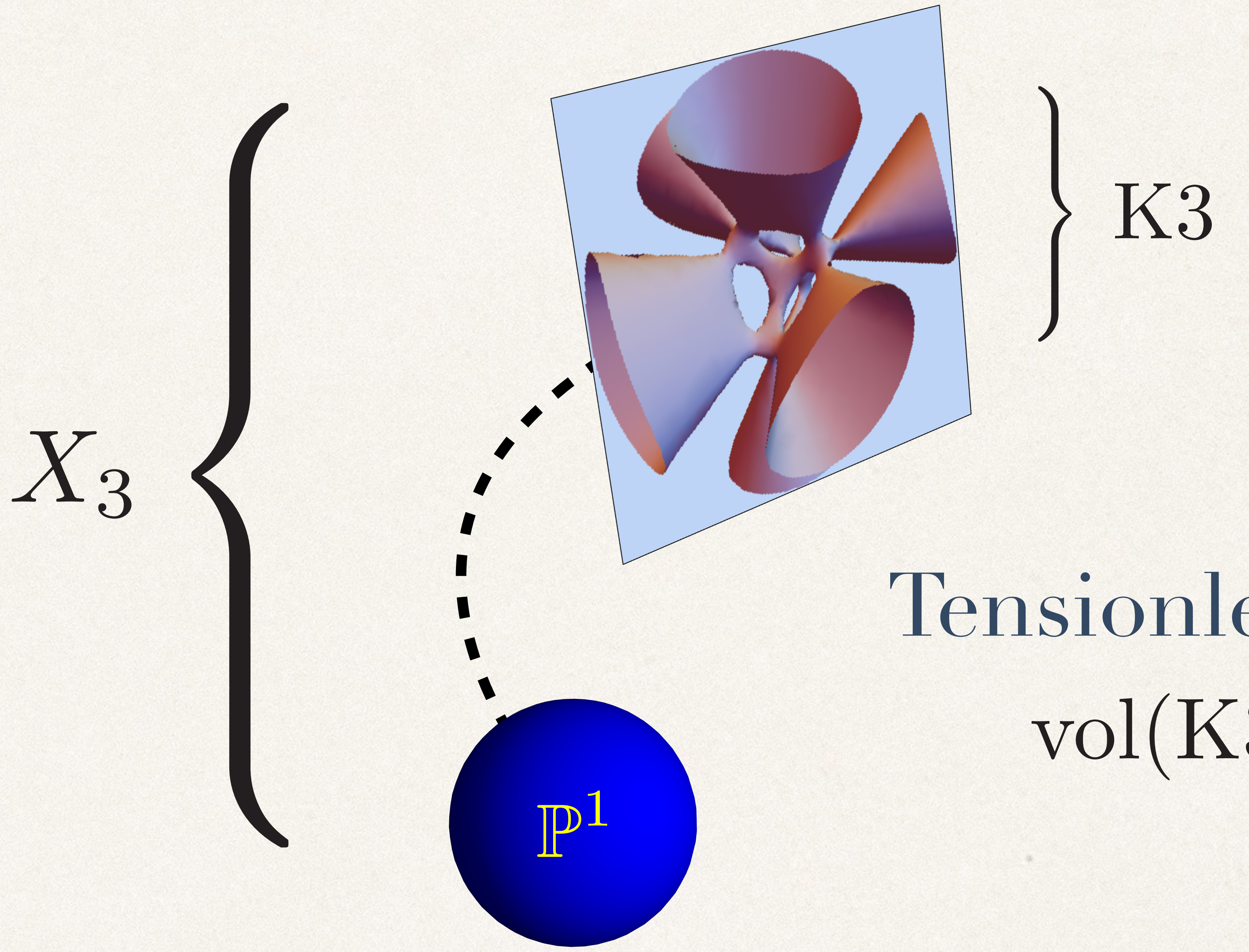

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Super-extremal states are excitations of heterotic strings in five dimensions.

[Lee, Lerche, Weigand'19]

Part II — B: The weak gravity conjecture with no infinte tower

–*Q: What about* U(1) *strongly coupled super-extremal states?*

The weak gravity conjecture with no ∞-tower

Philosophy: Not every U(1) gauge theory requires a tower of super-extremal states in the spectrum of the theory.

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Super-extremal states can be either particle-like states or black holes.

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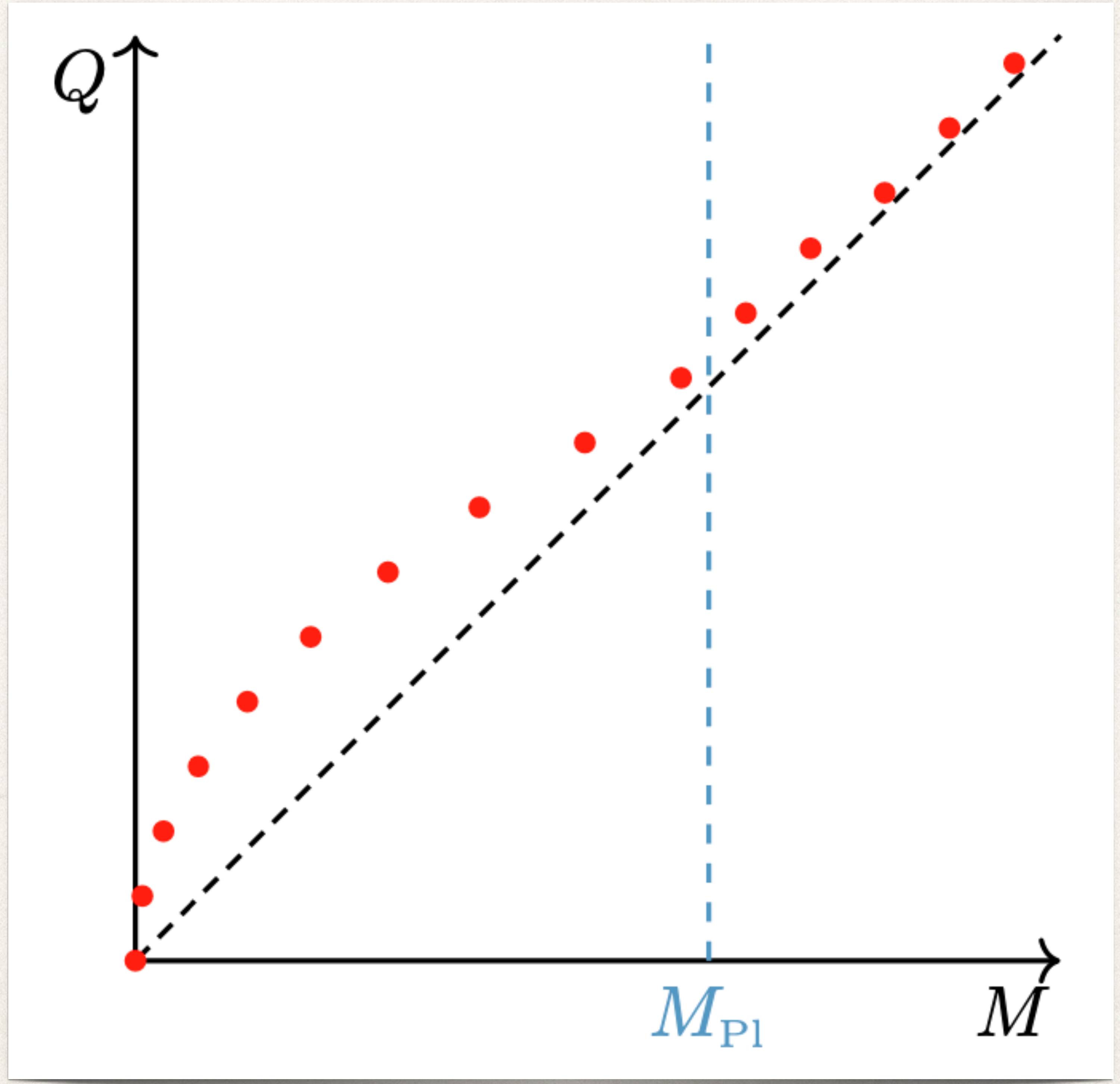

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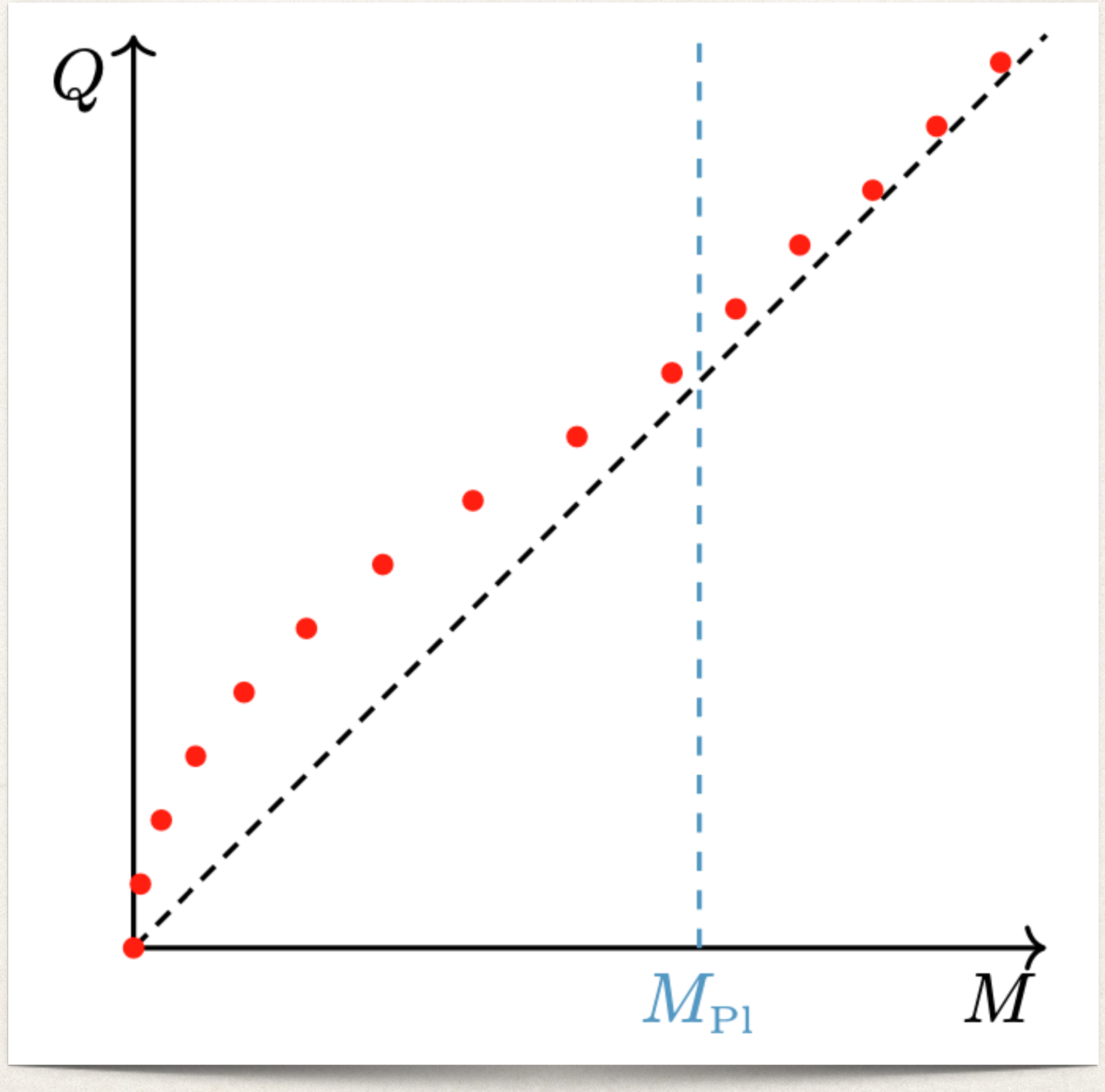

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Extremal black boles: $g^2_{\rm U(1)} q^2 M_{\rm Pl}^{D-2} = \gamma m^2$

Black holes can transition into particles when:

1. $g^2_{\rm U(1)} M_{\rm Pl}^{D-2} \to 0$.
2. $\gamma \to \infty$.

The weak gravity conjecture with no ∞-tower

Conjecture: [CFC, Mininno, Weigand, Wiesner' to appear]

In a D-dimensional U(1) gauge theory coupled to gravity, there **can** exist a tower of super-extremal states **if**:

i) An infinite distance limit exists in the field moduli space, such that $g^2_{U(1)} M_{\rm Pl}^{D-4} \to 0$.

– *Emergent string conjecture limits.*

ii) A finite distance limit exists in the field moduli space, such that $g^2_{U(1)} M_{\rm Pl}^{D-4} \to \infty$.

– *Strongly coupled dynamic limits.*

The weak gravity conjecture with no ∞-tower

Examples when the tower of super-extremal states is **NOT** required.

The weak gravity conjecture with no ∞-tower

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The convex hull condition holds for all $r_{S^1} \geq r_{\mathrm{min.}}$ via non-trivial charged massless matter.

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Examples when the tower of super-extremal states is **NOT** required.

b) F-theory on a T^2-fibered Calabi-Yau threefold
$\pi : X_3 \to B_2$ with $B_2 = dP_9$.

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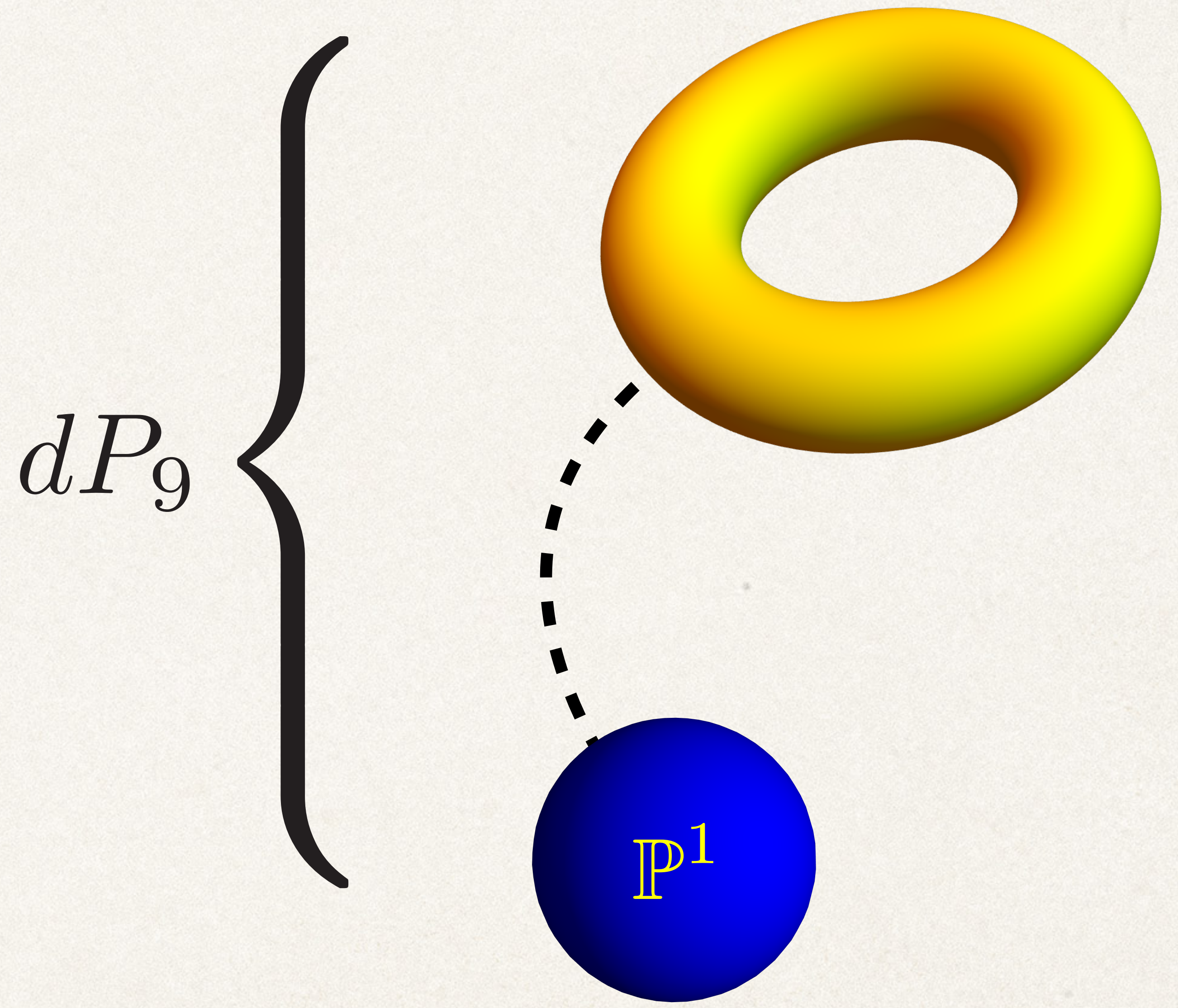
Rational elliptic surface:



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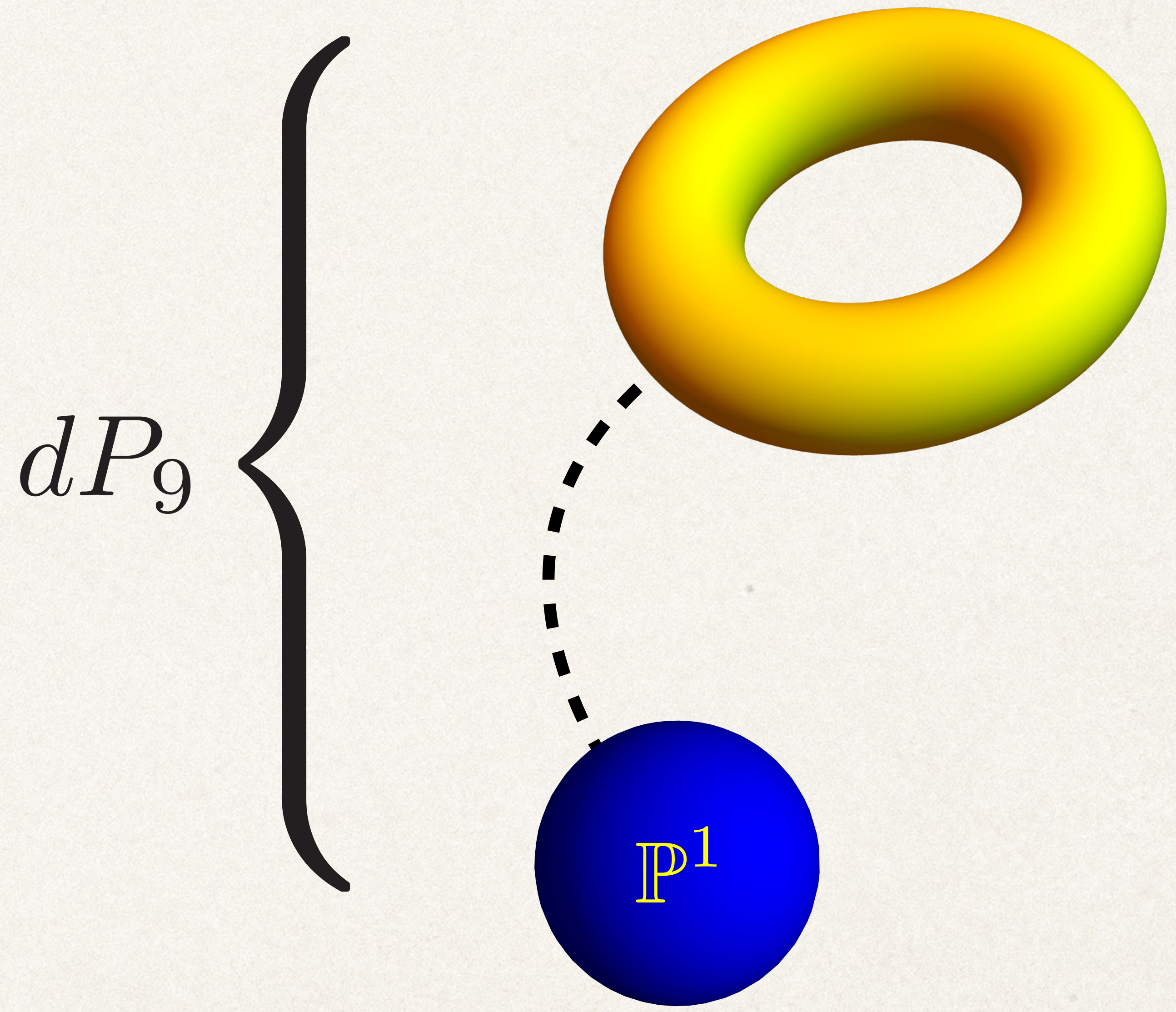

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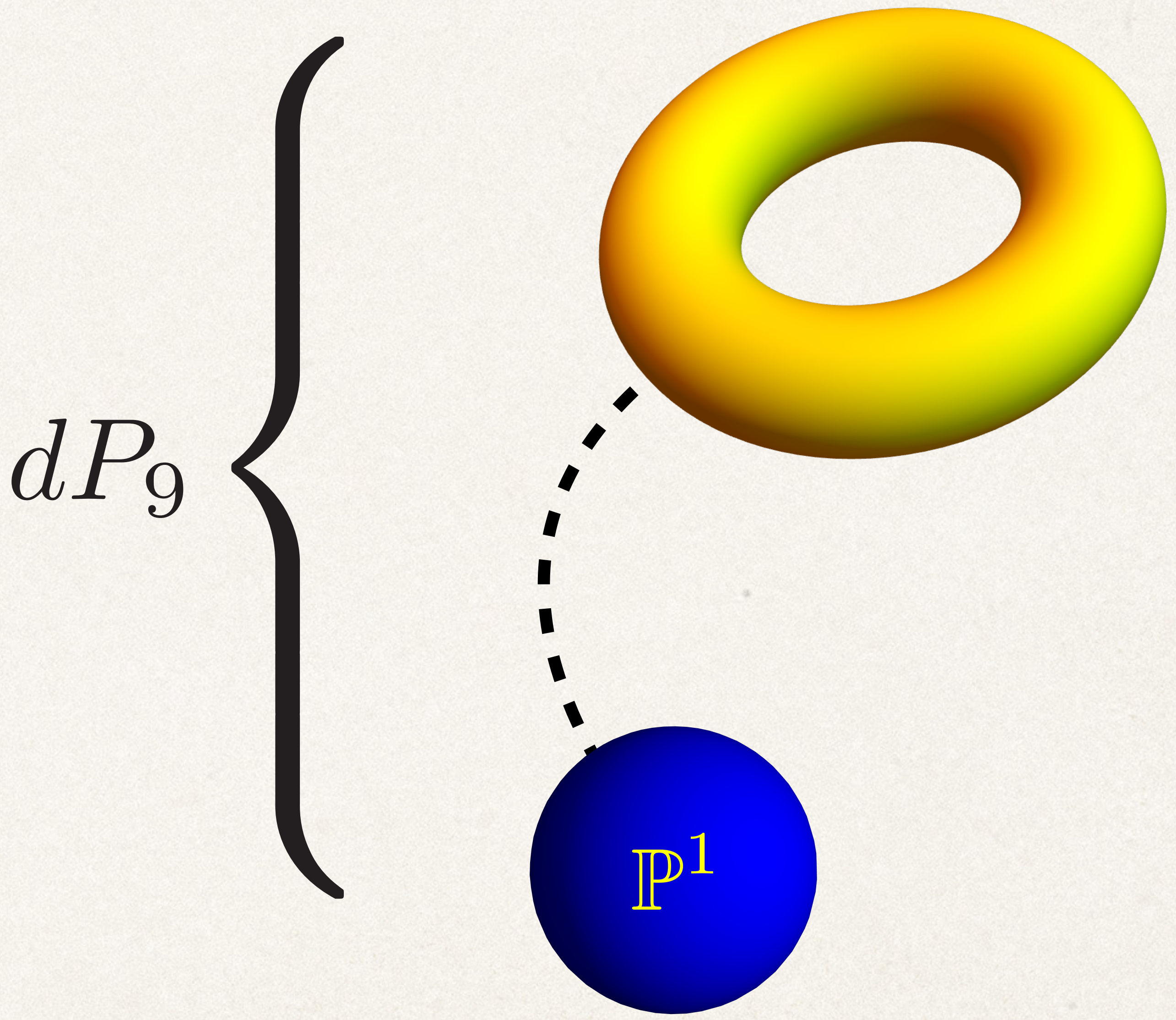

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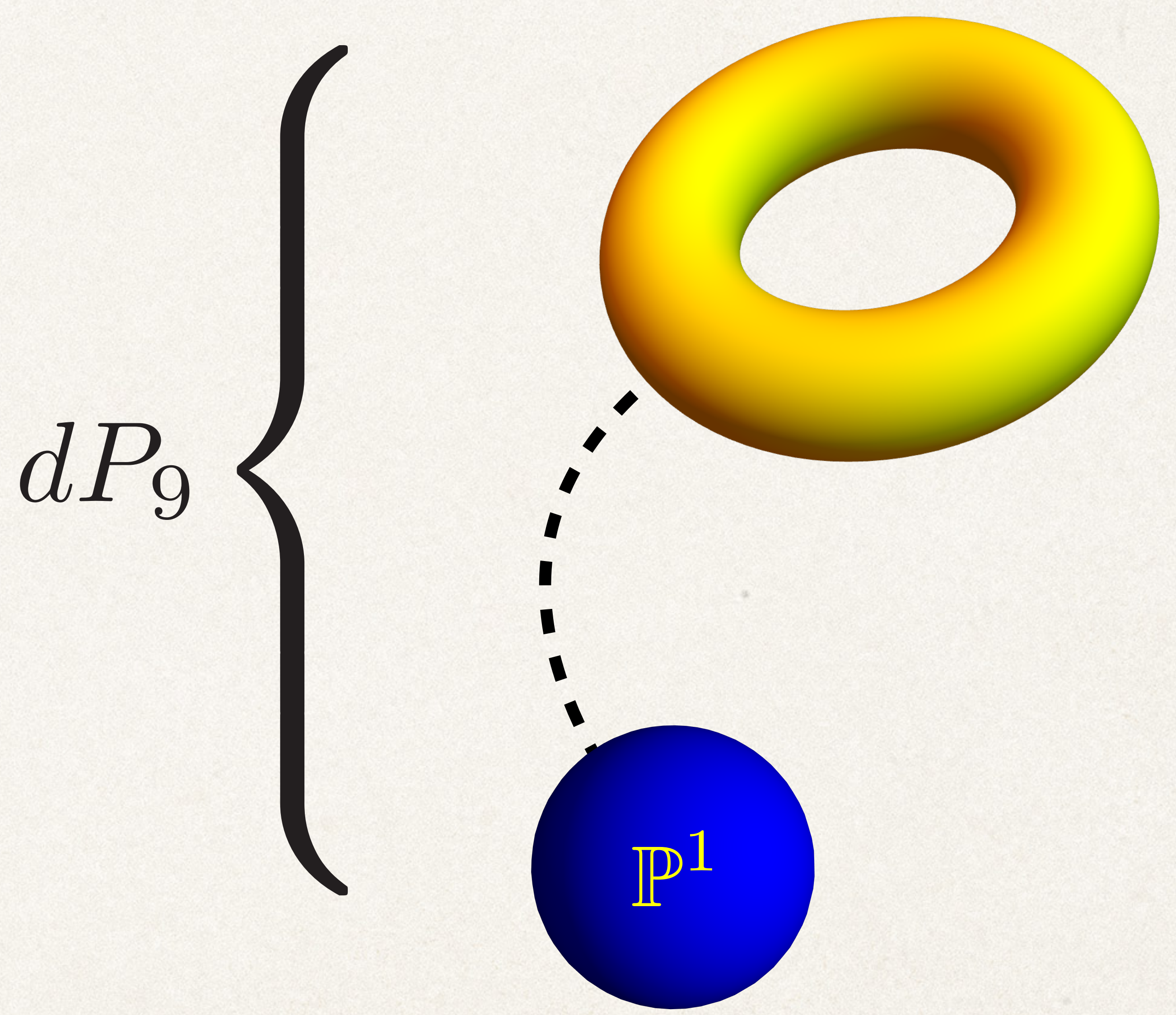

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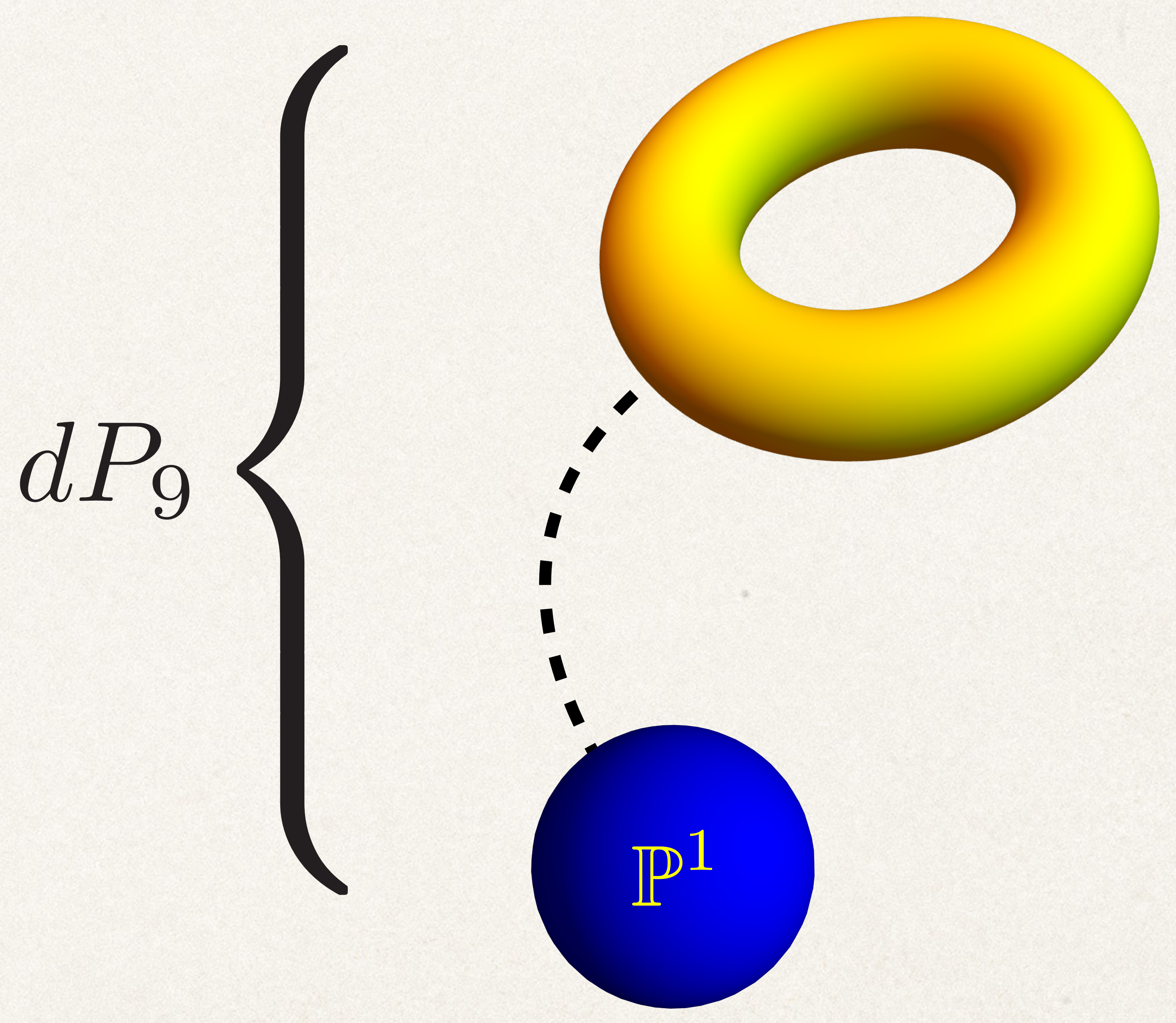

Upshot: Only a finite set of massive super-extremal states from $[p, q]$-string or E-strings.

Outlook

- **Part I:** We introduced the tower weak gravity conjecture as a constraint for the consistency of theories under Kaluza-Klein reduction.

- **Part II — A:** In five-dimensional theories, we ask ourselves when is the tower of super-extremal states relevant for the EFT. For similar results for four-dimensional $\mathcal{N} = 1$ theories, see [CFC, Mininno, Weigand, Wiesner '22x2] .

- **Part II — B:** Based on black hole arguments, we formulated a criterion to find when is the tower of super-extremal states necessary.

[CFC, Mininno, Weigand, Wiesner ' to appear]