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On the Boundary Conformal Field Theory Approach to Symmetry-Resolved Entanglement

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Symmetry Resolution-Main Idea

- Symmetries in quantum theory induce superselection rules
- The theory decomposes into sectors labeled by a fixed charge
- Conservation law constrains entanglement between two subregion
- Only sum of subregion charges is conserved
- Symmetry-resolved entanglement is entanglement in sector with fixed subregion charge

Motivation

- Contains more information than entanglement entropy
- Describes many body localization [Lukin et al.: 1805.09819]
- Understanding entanglement in AdS/CFT
- Applications to AQFT: Symmetry resolution of modular flow and modular operator [Di Giulio, Erdmenger: 2305.02343]

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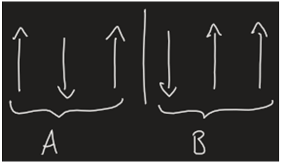
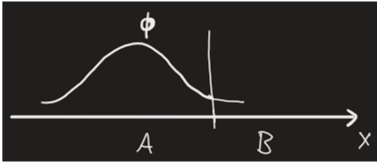
Regularization

- Definition of entanglement requires bipartition of Hilbert space

$$\mathscr{H}_{\text{tot}} = \mathscr{H}_A \otimes \mathscr{H}_B \tag{1}$$

- Evident for lattice theories
- Problematic in quantum field theory [Cardi, Tonni: 1608.01283]
- Regularization in field theory spirit requires boundary conditions



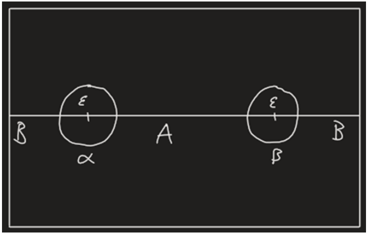


The Cutting Operation

- Cutting out small circles around the entangling points provides regularization
- Boundary conditions α, β specify the cutting operation [Ohmori, Tachikawa: 1406.4167]

$$\mathscr{H} \to \mathscr{H}_{\alpha,A,\beta} \otimes \mathscr{H}_{\beta,B,\alpha} \tag{2}$$

- The factorized Hilbert space is different from other regularizations-only the lowest lying states coincide



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Definitions

- Consider CFT with additional Abelian symmetry group $\mathscr{A}$
- Choose conformal boundary conditions that preserve at least parts of the symmetry
- Reduced density matrix decomposes into **charge blocks**

$$\rho_A = \bigoplus_{Q_A} P(Q_A)\rho_A(Q_A) \tag{3}$$

- **Symmetry-resolved entanglement entropy (SREE)**

$$S_1(Q_A) = -\mathrm{tr}\left(\rho_A(Q_A)\log\rho_A(Q_A)\right) \tag{4}$$

Definition of SRRE

- Define **charged partition functions** [Goldstein, Sela: 1711.09418]

$$Z_n(Q_A) = \mathrm{tr}\left(\rho_A^n \Pi_{Q_A}\right) \tag{5}$$

- Replica partition function of all states with fixed subregion charge
- **Symmetry-resolved Rényi entropy (SRRE)**

$$S_n(Q_A) = \frac{1}{1-n} \log\left[\frac{Z_n(Q_A)}{Z_1(Q_A)^n}\right] \tag{6}$$

- SREE can be recovered by taking $n \to 1$ limit
- **The key objects to compute are the charged partition functions**

Representation Theory

- The subregion Hilbert space will decompose into irreducible representations $\mathscr{H}_Q$ of the symmetry group $\mathscr{A}$
- The charged partition functions can be rewritten as

$$Z_n(Q) = \mathrm{tr}_{\mathscr{H}_Q}\left(q^{n\left(L_0 - \frac{c}{24}\right)}\right) \equiv \chi_Q(q^n) \tag{7}$$

- $\chi_Q(q)$ are known in the literature as **characters**
- Boundary conditions determine which charges enter the spectrum and their multiplicities

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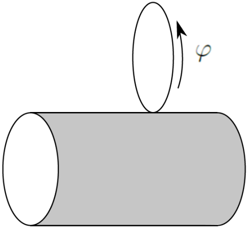
Example: Compact Free Boson

- Bosonic field φ, action

$$S = \frac{g}{2} \int_{\Sigma} \mathrm{d}^2 x \partial_\mu \varphi \partial^\mu \varphi, \qquad \varphi = \varphi + 2\pi R \tag{8}$$

- Translation $U(1) \otimes U(1)$ symmetry $\varphi \to \varphi + a$
- Conservation of momentum and winding quantum numbers m and w
- $\mathbb{Z}_2$ symmetry $\varphi \to -\varphi$



Boundary Conditions

- Separate boundary conditions need to be chosen at the two entangling points for the cutting operation
- Two possible boundary conditions that preserve conformal symmetry: Dirichlet (D) and Neumann (N)
- DD and NN boundary conditions preserve one conservation law of either w or m
- ND and DN break both conservation laws, only $\mathbb{Z}_2$ symmetry remaining

Results

DD and NN boundary conditions

- A single $U(1)$ of the original two copies remains
- Symmetry-resolved Rényi entropy

$$S_n(Q) = \frac{1}{1-n} \log \left(\frac{\eta(q)^n}{\eta(q^n)} \right) \tag{9}$$

- **Exact** equipartition to all orders in the cutoff
- Equipartition due to the exact form of the characters
- To leading order in the cutoff, familiar terms appear

$$S_1(Q) = \frac{1}{3} \log \frac{L}{\epsilon} - \frac{1}{2} \log \left(\frac{2}{\pi} \log \frac{L}{\epsilon} \right) - \frac{1}{2} + \mathcal{O}(\epsilon) \tag{10}$$

Results

DN and ND boundary conditions:

- Both copies of $U(1)$ are broken, resolve w.r.t. remaining $\mathbb{Z}_2$
- SRRE agrees to results in the literature to lowest order in the cutoff

$$S_n(\pm) = \frac{1}{6}\frac{1+n}{n}\log\frac{L}{\epsilon} - \log 2 + \dots \tag{11}$$

- Equipartition broken at higher orders

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Conclusions

- Symmetry-resolved entanglement entropy calculates entanglement in a sector with fixed subregion charge
- SREE can be calculated directly by calculating characters
- Gives correct leading term, also correction to arbitrary order
- Equipartition exact to all orders for $U(1)$ symmetry
- Mixed boundary conditions break $U(1)$ symmetry, resolution w.r.t. $\mathbb{Z}_2$ still possible

Outlook

- Formalism resolves spacetime symmetries
  [Northe: 2303.07724]

Open questions:

- non-Abelian symmetries
- Holographic interpretation of higher order terms
- Holographic implementation of boundary conditions
- Connection to Wilson line prescription