Broken global symmetries and defect conformal manifolds

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DESY workshop

Based on arXiv: 2003.17157 with N. Drukker and G. Sakkas
and arXiv: 2212.03886 with N. Drukker

Content

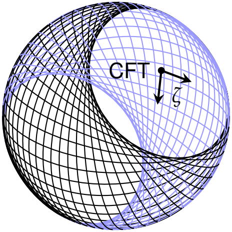
- Defect Conformal Manifold $\left\{\begin{array}{l}\text{Defect}\\ \text{Conformal Manifold}\end{array}\right.$
- Riemannian Structure $\left\{\begin{array}{l}\text{Zamolodchikov Metric}\\ \text{Curvature Tensor}\end{array}\right.$
- Examples: 1/2 and 1/3 BPS Wilson Loops in ABJM

Conformal Manifold

A conformal manifold $\mathcal{M}_{\mathrm{CFT}}$ is a family of CFT's parametrized by couplings $\{\zeta^i\}$. For each value of the couplings a different point on the manifold is a different CFT.

$$S \to S + \sum_i \zeta^i \int d^D x \mathcal{O}_i$$

where $\mathcal{O}_i$ are exactly marginal operators.



This manifold admits a Riemannian structure:

- Zamolodchikov Metric
- Curvature tensor

Geometric Structure

- Zamolodchikov Metric [Zamolodchikov]

$$g_{ij} = \langle \mathcal{O}_i(\infty)\mathcal{O}_j(0)\rangle = C_{\mathcal{O}}\delta_{ij}, \quad \mathcal{O}_i(\infty) \equiv \lim_{x\to\infty} |x|^{2D}\mathcal{O}_i(x)$$

- Curvature tensor [Kutasov]

$$R_{ijkl} = \frac{1}{2}(\partial_j\partial_k g_{il} - \partial_i\partial_k g_{jl} - \partial_j\partial_l g_{ik} + \partial_i\partial_l g_{jk})$$

  - Take the first term for instance

$$\begin{aligned}\partial_j\partial_k g_{il} &= \left(\partial_j\partial_k\langle e^{-\int \zeta^i \mathcal{O}_i d^D x}\mathcal{O}_i(\infty)\mathcal{O}_l(0)\rangle\right)\Big|_{\zeta^i=0} \\ &= \iint d^D x_1 d^D x_2 \langle \mathcal{O}_j(x_1)\mathcal{O}_k(x_2)\mathcal{O}_i(\infty)\mathcal{O}_l(0)\rangle\end{aligned}$$

The curvature tensor is a sum of double integrals of 4-pt functions ⇒ integrals over cross-ratios. [Friedan, Konechny]

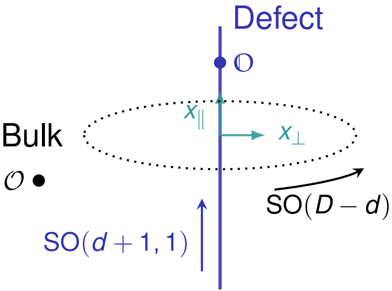
Defect

A conformal defect of dimension d in a D dimensional CFT breaking

- conformal symmetry:

$$SO(D+1,1) \to SO(d+1,1) \times SO(D-d)$$

- **global symmetry**: $G \to G'$



Defect Conformal Manifold

- Analogous to bulk conformal manifold
- Deforming a defect by defect **exactly marginal** operators $\mathbb{O}_i$ with $\Delta_{\mathbb{O}_i} = d$, correlation functions of any operator ϕ

$$\langle\!\langle \phi_1 \cdots \phi_n \rangle\!\rangle \to \langle\!\langle e^{-\int \zeta^i \mathbb{O}_i d^d x} \phi_1 \cdots \phi_n \rangle\!\rangle$$

$\langle\!\langle \cdots \rangle\!\rangle$: defect correlation function normalized by the expectation value of the defect.

- If the defect breaks $G \to G'$, the conservation equation is

$$\partial_\mu J^{\mu \underset{\uparrow}{a}} = \mathbb{O}_i(x_\parallel)\delta^{\underset{\uparrow}{ia}}\delta^{D-d}(x_\perp)$$

generators of G

generators broken by the defect

When a defect breaks a global symmetry, the resulting defect conformal manifold is the symmetry breaking coset $\mathcal{M} = G/G'$.

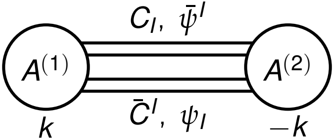
Examples: BPS Wilson Loops in ABJM Theory

ABJM theory: [Aharony, Bergman, Jafferis, Maldacena]

- 3d $\mathcal{N} = 6$ super Chern-Simons theory with gauge group $U(N_1) \times U(N_2)$.
- $\mathfrak{osp}(6|4) \supset \mathfrak{so}(4,1) \oplus \mathfrak{su}(4)$
- Field content

Gauge Field	$A_\mu^{(1)}$, $A_\mu^{(2)}$
Boson	C_I, $\bar{C}^I$
Fermion	ψ_I^α, $\bar{\psi}_\alpha^I$

where $\alpha = \pm$ are spinor indices, and $I = 1, \ldots, 4$ are $SU(4)$ R-symmetry indices.



Defect I: 1/2 BPS Loop in ABJM

- The 1/2 BPS Wilson loop $W = \text{Tr}\,\mathcal{P}e^{\int i\mathcal{L}d\tau}$ [Drukker, Trancanelli]

$$\mathcal{L}_1 = \begin{pmatrix} A_\tau^{(1)} + \alpha\bar{\alpha}(M_1)^I{}_J C_I \bar{C}^J & i\bar{\alpha}\bar{\psi}^1_+ \\ -i\alpha\psi_1^+ & A_\tau^{(2)} + \alpha\bar{\alpha}(M_1)^I{}_J \bar{C}^J C_I \end{pmatrix}$$

with $M_1 = \mathrm{diag}(-1, 1, 1, 1)$ and $\bar{\alpha}\alpha = -2\pi i/k$. The 3 pairs of defect exactly marginal operators $\mathbb{O}_i, \bar{\mathbb{O}}_{\bar{\imath}}$ are chiral.

Defect conformal manifold is $\mathbb{CP}^3 = SU(4)/SU(3)\times U(1)$

• Zamolodchikov Metric: $g_{i\bar{\jmath}} = \langle\!\langle \mathbb{O}_i(0)\bar{\mathbb{O}}_{\bar{\jmath}}(\infty)\rangle\!\rangle = 4B_{1/2}\delta_{i\bar{\jmath}}$
with the bremsstrahlung function $B_{1/2} = \sqrt{2\lambda}/4\pi + \ldots$

[Lewkowycz, Maldacena], [Bianchi, Griguolo, Mauri, Penati, Preti, Seminara], [Bianchi, Preti, Vescovi]

Curvature Tensor

- Curvature Tensor: [Friedan, Konechny]

$$R_{ijkl} = -\mathrm{RV}\int_{-\infty}^{+\infty} d\eta \log|\eta| \Big[\langle\!\langle \mathbb{O}_i(1)\mathbb{O}_j(\eta)\mathbb{O}_k(\infty)\mathbb{O}_l(0) \rangle\!\rangle_c + \langle\!\langle \mathbb{O}_i(0)\mathbb{O}_j(1-\eta)\mathbb{O}_k(\infty)\mathbb{O}_l(1) \rangle\!\rangle_c \Big]$$

- 4-pt functions [Bianchi, Bliard, Forini, Griguolo, Seminara]

$$\langle\!\langle \mathbb{O}_i(x_1)\bar{\mathbb{O}}_{\bar{j}}(x_2)\mathbb{O}_k(x_3)\bar{\mathbb{O}}_{\bar{l}}(x_4) \rangle\!\rangle = \frac{g_{i\bar{j}}g_{k\bar{l}}K_1(\eta) - g_{i\bar{l}}g_{k\bar{j}}K_2(\eta)}{x_{12}^2 x_{34}^2}$$

$$\langle\!\langle \mathbb{O}_i(x_1)\bar{\mathbb{O}}_{\bar{j}}(x_2)\bar{\mathbb{O}}_{\bar{k}}(x_3)\mathbb{O}_l(x_4) \rangle\!\rangle = \frac{g_{i\bar{j}}g_{l\bar{k}}H_1(\eta) - g_{i\bar{k}}g_{l\bar{j}}H_2(\eta)}{x_{12}^2 x_{34}^2}$$

Plugging into the curvature tensor

$$R_{ij\bar{k}\bar{l}} = (g_{i\bar{l}}g_{j\bar{k}} - g_{i\bar{k}}g_{j\bar{l}})\mathcal{R}_1, \quad R_{i\bar{j}k\bar{l}} = (g_{i\bar{l}}g_{k\bar{j}} + g_{i\bar{j}}g_{k\bar{l}})\mathcal{R}_2$$

where $\mathcal{R}_{1,2}$ are integrals of $K_{1,2}$, $H_{1,2}$.

Constraints

- Matching the components of curvature tensor

  $\Rightarrow \boxed{\text{The integrals } \mathcal{R}_1 = 0,\ \mathcal{R}_2 = 1/(4B_{1/2}).}$
- Check at tree level in the strong coupling limit:
  - Explicit expressions [Bianchi, Bliard, Forini, Griguolo, Seminara]

$$H_1 = \frac{1}{2\pi\sqrt{2\lambda}}\left(-\eta^2\log\eta + \left(\eta^2 - \frac{4}{\eta} + 3\right)\log(1-\eta) + \eta - 4\right)$$

$$H_2 = \frac{1}{2\pi\sqrt{2\lambda}}\left(\eta^2(3-4\eta)\log\eta + (\eta-1)(4\eta^2+\eta+1)\log(1-\eta) + 4\eta^2 - \eta\right)$$

$$K_1 = \frac{1}{2\pi\sqrt{2\lambda}}\left(-\frac{\eta^2}{(1-\eta)^2}\log\eta + \frac{\eta-4}{\eta}\log(1-\eta) - \frac{3\eta^2-7\eta+4}{(1-\eta)^2}\right)$$

$$K_2 = \frac{1}{2\pi\sqrt{2\lambda}}\left(\log(1-\eta) + \frac{\eta^2(\eta+3)}{(1-\eta)^3}\log\eta - \frac{\eta\left(3\eta^2-2\eta-1\right)}{(1-\eta)^3}\right)$$

  - $\mathcal{R}_1 = 0$, $\mathcal{R}_2 = \pi/\sqrt{2\lambda} \approx 1/(4B_{1/2})$.

Defect II: 1/3 BPS Loop in ABJM

- Another 1/2 BPS Wilson loop where

$$\mathcal{L}_4 = \begin{pmatrix} A_\tau^{(1)} + \alpha\bar{\alpha}(M_4)^I{}_J C_I \bar{C}^J & i\bar{\alpha}\bar{\psi}^4_- \\ -i\alpha\psi_4^- & A_\tau^{(2)} + \alpha\bar{\alpha}(M_4)^I{}_J \bar{C}^J C_I \end{pmatrix}$$

  with $M_4 = \mathrm{diag}(-1, -1, -1, 1)$.
- The simplest 1/3 BPS Wilson loop

$$\mathcal{L}' = \begin{pmatrix} \mathcal{L}_1 & 0 \\ 0 & \mathcal{L}_4 \end{pmatrix}$$

- Unique! [Drukker, Kong, Probst, Tenser, Trancanelli]

$$\Delta\mathcal{L}' = -i\mathcal{Q}G + \{H, G\} + \Pi G^2 + C$$

Defect conformal manifold is $SU(4)/SU(2) \times U(1) \times U(1)$

Other Examples

- 1/2 BPS surface operators in 6d $\mathcal{N} = (2, 0)$ theory
  [Drukker, Giombi, Tseytlin, Zhou]

$$S^4 = SO(5)/SO(4)$$

- Boundaries in free theories [Herzog, Schaub]

$$O(N)/\big(O(p) \times O(N-p)\big)$$
$$U(N)/\big(U(p) \times U(N-p)\big)$$

- Magnetic lines in $O(N)$ model
  [Gimenez-Grau, Lauria, Liendo, van Vliet]

$$S^{N-1} = O(N)/O(N-1)$$

Defect Conformal Manifold = Symmetry Breaking Coset

$$\mathcal{M}_{\text{dCFT}} = G/G'$$

Outlook

1. Non-homogenous defect conformal manifolds?

$$\mathcal{L} \to \mathcal{L} - i\mathcal{Q}G + \{H, G\} + \Pi G^2 + C$$

2. Generalization: A manifold comprised of
   - Defect marginal & non-marginal operators
   - Bulk & defect marginal operators
3. Integrated constraints for higher point functions?
4. $\cdots$

Thank you!