Twist noncommutative gauge theories

Tim Meier

Desy Theory Workshop

based on [hep-th:2301.08757] and [hep-th:2305.15470]

with Stijn van Tongeren

September 27, 2023

Motivation

$AdS_5 \times S^5$ String $\xleftrightarrow{\text{AdS/CFT correspondence}}$ $N = 4\ SYM$

Yang-Baxter deformation via classical r-matrix [1]

Twist psu(2,2|4) with the same twist [2]

deformed AdS_5 String $\xleftrightarrow{?}$ deformed $N = 4$ SYM

deformed AdS_5 String
- twisted psu(2,2|4)
- twist corresponds to r-matrix

deformed $N = 4$ SYM
- properties?
- observables?
- integrability?

[1][Klimcik 02, Delduc Magro Vicedo 13, Kawaguchi-Matsumoto-Yoshida 14, ...]
[2][van Tongeren 15]

Drinfel'd twist

- symmetry transformations act via Leibniz rule

$$\partial_\mu \left(f(x)g(x)\right) = \left(\partial_\mu f(x)\right) g(x) + f(x) \left(\partial_\mu g(x)\right)$$

- formalized by the coproduct

$$\begin{aligned} \mu(f \otimes g)(x) &= f(x)g(x) \\ X\mu(f \otimes g) &= \mu(\Delta(X)(f \otimes g)) \\ \Delta(x) &= X \otimes 1 + 1 \otimes X \end{aligned}$$

- Drinfel'd twist: $\mathcal{F} \in U(\mathfrak{g}) \otimes U(\mathfrak{g})$ introduces new coproduct $\Delta_{\mathcal{F}}(X) = \mathcal{F}\Delta(X)\mathcal{F}^{-1}$

$$\begin{aligned} \mu_{\mathcal{F}}(f \otimes g)(x) &= \mu \circ \mathcal{F}^{-1}(f \otimes g) \\ X\mu_{\mathcal{F}}(f \otimes g) &= \mu_{\mathcal{F}}(\Delta_{\mathcal{F}}(X)(f \otimes g)) \end{aligned}$$

The star product

- $\mu_{\mathcal{F}}$ is called star product

$$\mu_{\mathcal{F}}\left(f \otimes g\right)(x) = f(x) \star g(x) \neq g(x) \star f(x)$$

- remains associative

$$(f \star g) \star h = f \star (g \star h)$$

- generalize to forms

$$\omega \wedge_{\star} \chi = \wedge \left(\mathcal{F}^{-1}(\omega, \chi)\right)$$
$$\omega \wedge_{\star} \chi \neq (-1)^{|\omega||\chi|} \chi \wedge_{\star} \omega$$

Examples of deformations for $\mathcal{N} = 4$ SYM

- focus on spacetime deformations
- canonical deformation

$$\mathcal{F} = e^{-\frac{i}{2}\theta^{\mu\nu}\partial_\mu \otimes \partial_\nu} \qquad [x^\mu \stackrel{\star}{,} x^\nu] = i\theta^{\mu\nu}$$

  - acts trivial on forms

$$\mathcal{L}_{\partial_\mu}(\mathrm{d}x^\nu) = 0$$

- Lorentz deformation

$$\mathcal{F} = e^{-\frac{i}{2}\lambda M_{01} \wedge M_{23}}$$
$$x^0 \star x^2 = \cosh(\lambda) x^2 \star x^0 - i \sinh(\lambda) x^3 \star x^1$$

  - acts nontrivial on forms

$$\mathcal{L}_{M_{01}}(\mathrm{d}x^0) = -i\mathrm{d}x^1$$

Non commutative gauge theory

How can we define a deformed YM theory on these non commutative spaces?

- Idea: replace products by star products

Non commutative gauge theory

How can we define a deformed YM theory on these non commutative spaces?

- Idea: replace products by star products
- canonical deformation: [Szabo 2003]

Non commutative gauge theory

How can we define a deformed YM theory on these non commutative spaces?

- Idea: replace products by star products
- canonical deformation: [Szabo 2003]
- generalization not really known beyond canonical deformation
    - [Jonke,Dimitrijevic 2011; Ascieri et. al. 2006]

Non commutative gauge theory

How can we define a deformed YM theory on these non commutative spaces?

- Idea: replace products by star products
- canonical deformation: [Szabo 2003]
- generalization not really known beyond canonical deformation
  - [Jonke,Dimitrijevic 2011; Ascieri et. al. 2006]
  - gauge invariance: perturbatively in def. parameter

Non commutative gauge theory

How can we define a deformed YM theory on these non commutative spaces?

- Idea: replace products by star products
- canonical deformation: [Szabo 2003]
- generalization not really known beyond canonical deformation
    - [Jonke,Dimitrijevic 2011; Ascieri et. al. 2006]
    - gauge invariance: perturbatively in def. parameter
    - no all order gauge invariance!

star gauge symmetry

- naive star gauge symmetry

$$\begin{aligned} \phi &\to i\,[\epsilon \stackrel{\star}{,} \phi] \\ \partial_\mu \phi &\to i\partial_\mu\left([\epsilon \stackrel{\star}{,} \phi]\right) \\ &= i\mu_{\mathcal{F}}\left(\Delta_{\mathcal{F}}(\partial_\mu)\epsilon \otimes \phi\right) - i\mu_{\mathcal{F}}\left(\Delta_{\mathcal{F}}(\partial_\mu)\phi \otimes \epsilon\right) \\ &\neq i\,[\partial_\mu \epsilon \stackrel{\star}{,} \phi] + i\,[\epsilon \stackrel{\star}{,} \partial_\mu \phi] \end{aligned}$$

- cannot be made covariant with

$$\delta^\star_\epsilon A_\mu = \partial_\mu \epsilon + i\,[\epsilon \stackrel{\star}{,} A_\mu]$$

star gauge symmetry

- formulation via forms

$$\phi \rightarrow i\,[\epsilon \overset{\star}{,} \phi]$$

star gauge symmetry

- formulation via forms

$$\phi \to i\,[\epsilon \stackrel{\star}{,} \phi]$$
$$[\mathrm{d}, \mathcal{L}] = 0$$

star gauge symmetry

- formulation via forms

$$\phi \to i\,[\epsilon \stackrel{\star}{,} \phi]$$
$$[\mathrm{d}, \mathcal{L}] = 0$$

- means: exterior derivative commutes with star product

star gauge symmetry

- formulation via forms

$$\phi \to i\,[\epsilon \stackrel{\star}{,} \phi]$$
$$[\mathrm{d}, \mathcal{L}] = 0$$

- means: exterior derivative commutes with star product

$$\mathrm{d}\phi \to i\,[\mathrm{d}\epsilon \stackrel{\star}{,} \phi] + i\,[\epsilon \stackrel{\star}{,} \mathrm{d}\phi]$$
$$A \to \mathrm{d}\epsilon + i\,[\epsilon \stackrel{\star}{,} A]$$
$$\mathrm{D}\phi = \mathrm{d}\phi + i\,[A \stackrel{\star}{,} \phi]$$

- formulate manifestly via forms!

star gauge symmetry

- formulation via forms

$$\phi \to i\,[\epsilon \stackrel{\star}{,} \phi]$$
$$[\mathrm{d}, \mathcal{L}] = 0$$

- means: exterior derivative commutes with star product

$$\mathrm{d}\phi \to i\,[\mathrm{d}\epsilon \stackrel{\star}{,} \phi] + i\,[\epsilon \stackrel{\star}{,} \mathrm{d}\phi]$$
$$A \to \mathrm{d}\epsilon + i\,[\epsilon \stackrel{\star}{,} A]$$
$$\mathrm{D}\phi = \mathrm{d}\phi + i\,[A \stackrel{\star}{,} \phi]$$

- formulate manifestly via forms!
- formulation via forms of YM includes Hodge star operator

Deforming the Hodge star operator

- deformed algebra of forms causes hodge star operation to be deformed
- deformed top form gives deformed levi-civita symbol:

$$
\begin{aligned}
&\varepsilon^{\star\mu\nu\rho\sigma}\mathrm{d}x^0 \wedge_\star \mathrm{d}x^1 \wedge_\star \mathrm{d}x^2 \wedge_\star \mathrm{d}x^3 \\
=&\mathrm{d}x^\mu \wedge_\star \mathrm{d}x^\nu \wedge_\star \mathrm{d}x^\rho \wedge_\star \mathrm{d}x^\sigma
\end{aligned}
$$

Deforming the Hodge star operator

- deformed algebra of forms causes hodge star operation to be deformed
- deformed top form gives deformed levi-civita symbol:

$$\begin{aligned}&\varepsilon^{\star\mu\nu\rho\sigma}\mathrm{d}x^0\wedge_\star\mathrm{d}x^1\wedge_\star\mathrm{d}x^2\wedge_\star\mathrm{d}x^3\\=&\mathrm{d}x^\mu\wedge_\star\mathrm{d}x^\nu\wedge_\star\mathrm{d}x^\rho\wedge_\star\mathrm{d}x^\sigma\end{aligned}$$

- deformed hodge star:

$$\begin{aligned}&*\,\mathrm{d}x^{\mu_1}\wedge_\star\ldots\wedge_\star\mathrm{d}x^{\mu_k}\\=&\frac{(-1)^{\sigma(k)}}{(4-k)!}\varepsilon^\star_{\mu_{k+1}\cdots\mu_4}{}^{\mu_1\cdots\mu_k}\mathrm{d}x^{\mu_4}\wedge_\star\ldots\wedge_\star\mathrm{d}x^{\mu_{k+1}}\end{aligned}$$

- extending star linearly to arbitrary forms restricts twist to be in Poincaré algebra

Yang Mills action

- transformation of field strength $G = \mathrm{d}A - iA \wedge_\star A$

$$\delta_\epsilon (G) = i [\epsilon \stackrel{\star}{,} G]$$
$$\delta_\epsilon (*G) = i * [\epsilon \stackrel{\star}{,} G] = i [\epsilon \stackrel{\star}{,} *G]$$

- deformed YM action and its gauge transformations

$$S_{\text{NC-YM}} = -\frac{1}{2g_{\text{YM}}^2} \int \mathrm{tr}\ G \wedge_\star *G$$
$$\delta_\lambda S_{\text{NC-YM}} = -\frac{i}{2g_{\text{YM}}^2} \int \mathrm{tr}\ [\epsilon \stackrel{\star}{,} G \wedge_\star *G]$$

- gauge invariant if star product is cyclic under integration; requires the twist to correspond to a unimodular r matrix
- leaves 14 algebraically distinct cases in the Poincaré algebra

What we achieved

- constructed star gauge invariant YM action
- all kinds of fundamental and adjoint matter
- constructed deformed $\mathcal{N} = 4$ SYM for these twists
- proved twisted superconformal symmetry of the theory
- proved planar equivalence theorem to map from deformed to undeformed diagrams

Outlook

- construct and calculate two point functions of gauge invariant operators
- obtain twisted Bethe equations and integrable structures in deformed theory
- generalize construction to the full conformal algebra