Two-Point Functions in the D3-D5 Defect CFT

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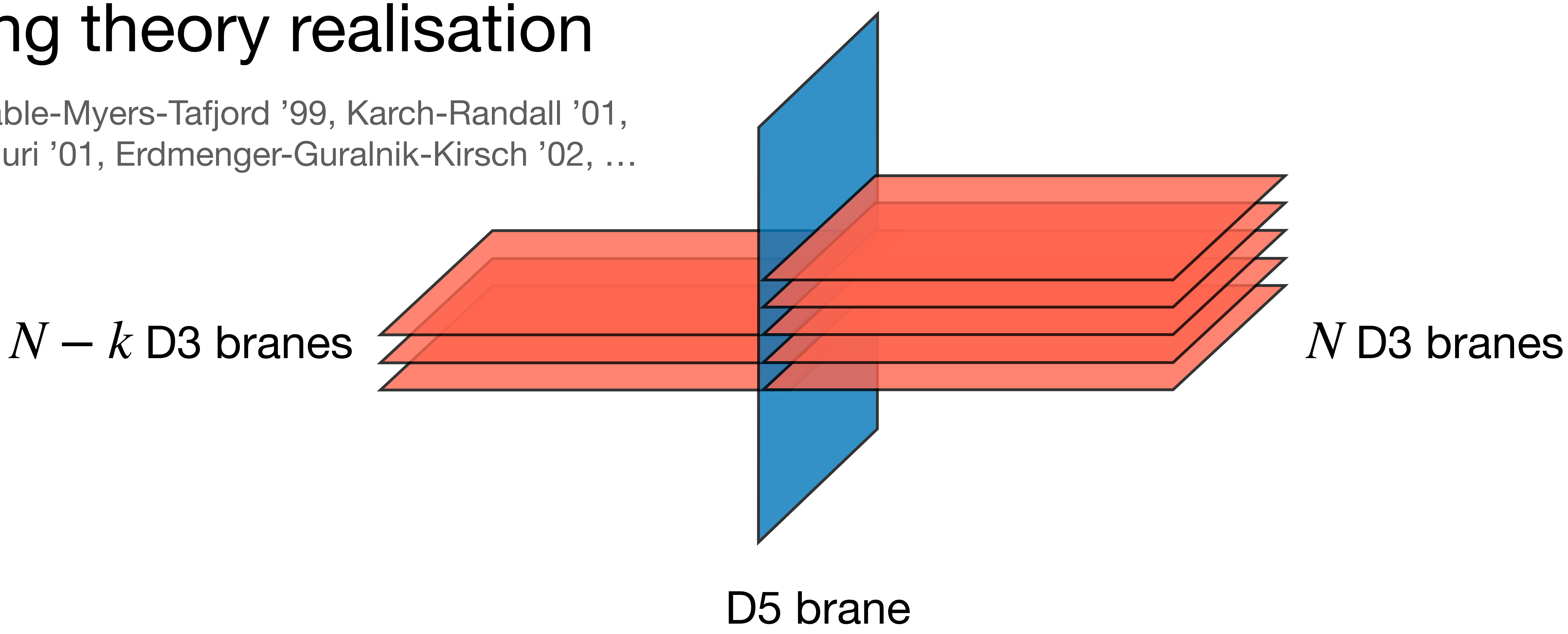
D3-D5 defect CFT with flux

- Type IIB string theory realisation

Diaconescu ’96, Constable-Myers-Tafjord ’99, Karch-Randall ’01, DeWolfe-Freedman-Ooguri ’01, Erdmenger-Guralnik-Kirsch ’02, …



- Large-N: probe D5 brane in $\mathrm{AdS}_5 \times \mathrm{S}^5$ with k units of flux
- Integrability

Buhl-Mortensen-de Leeuw-Kristjansen-Linardopoulos-Mori-Müller-Zarembo ’15-’20, Komatsu-Wang ’20, Gombor-Bajnok ‘20

4d N=4 SYM with Nahm pole boundary

- Consider N=4 SYM on half of Minkowski space $x_\perp > 0$ Gaiotto-Witten ’08

- Classification of 1/2-BPS BCs ⊃ Nahm pole BCs

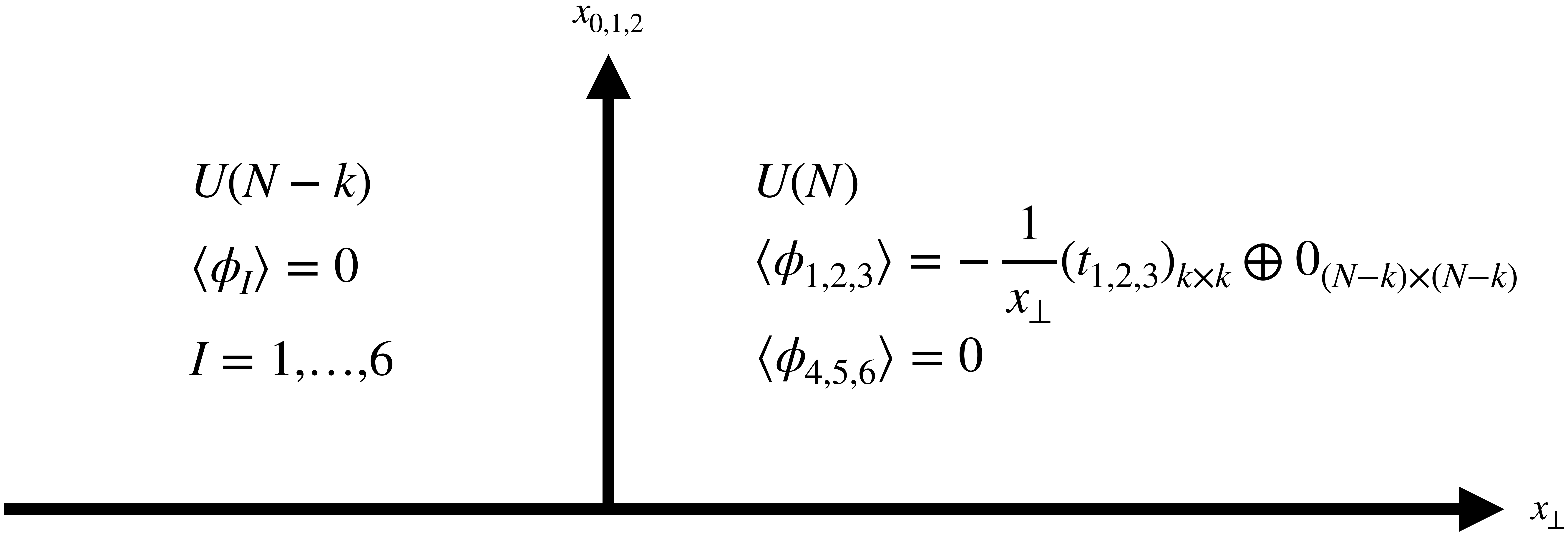
obey $SU(2)$
commutation relations

$$\frac{d\phi_1^{cl}}{dx_\perp} + i[\phi_2^{cl}, \phi_3^{cl}] = 0 \qquad \longrightarrow \qquad \phi_{1,2,3}^{cl} = -\frac{t_{1,2,3}}{x_\perp}, \;\; \phi_{4,5,6}^{cl} = 0$$

- Quantise around ϕ^{cl} $\longrightarrow$ boundary CFT

- Symmetry $\mathfrak{psu}(2,2|4) \to \mathfrak{osp}(4|4) \supset \mathfrak{so}(3,2) \times \mathfrak{so}(3)_+ \times \mathfrak{so}(3)_-$

4d N=4 SYM with Nahm pole defect



$U(N-k)$

$\langle \phi_I \rangle = 0$

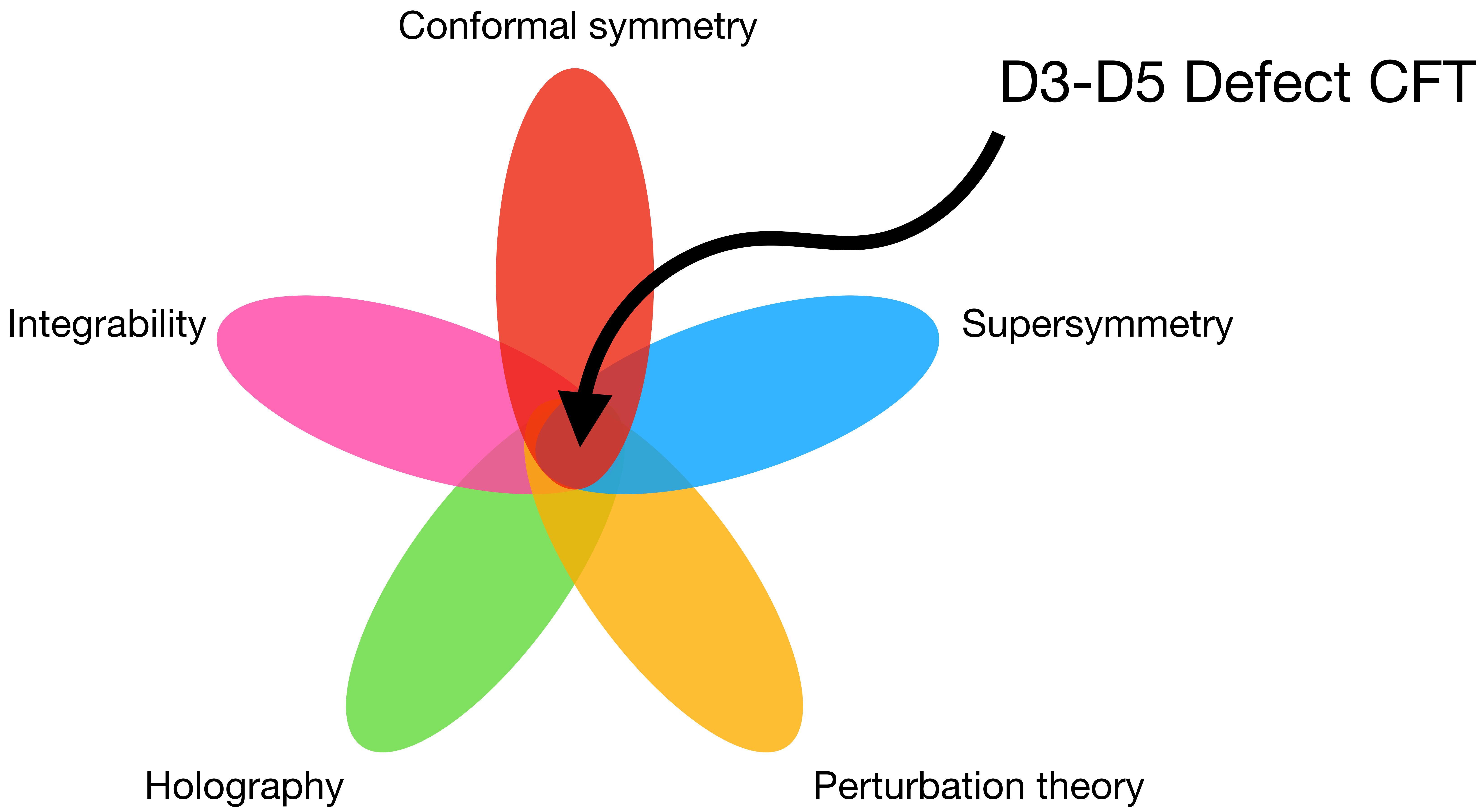
$I = 1,\ldots,6$

$U(N)$

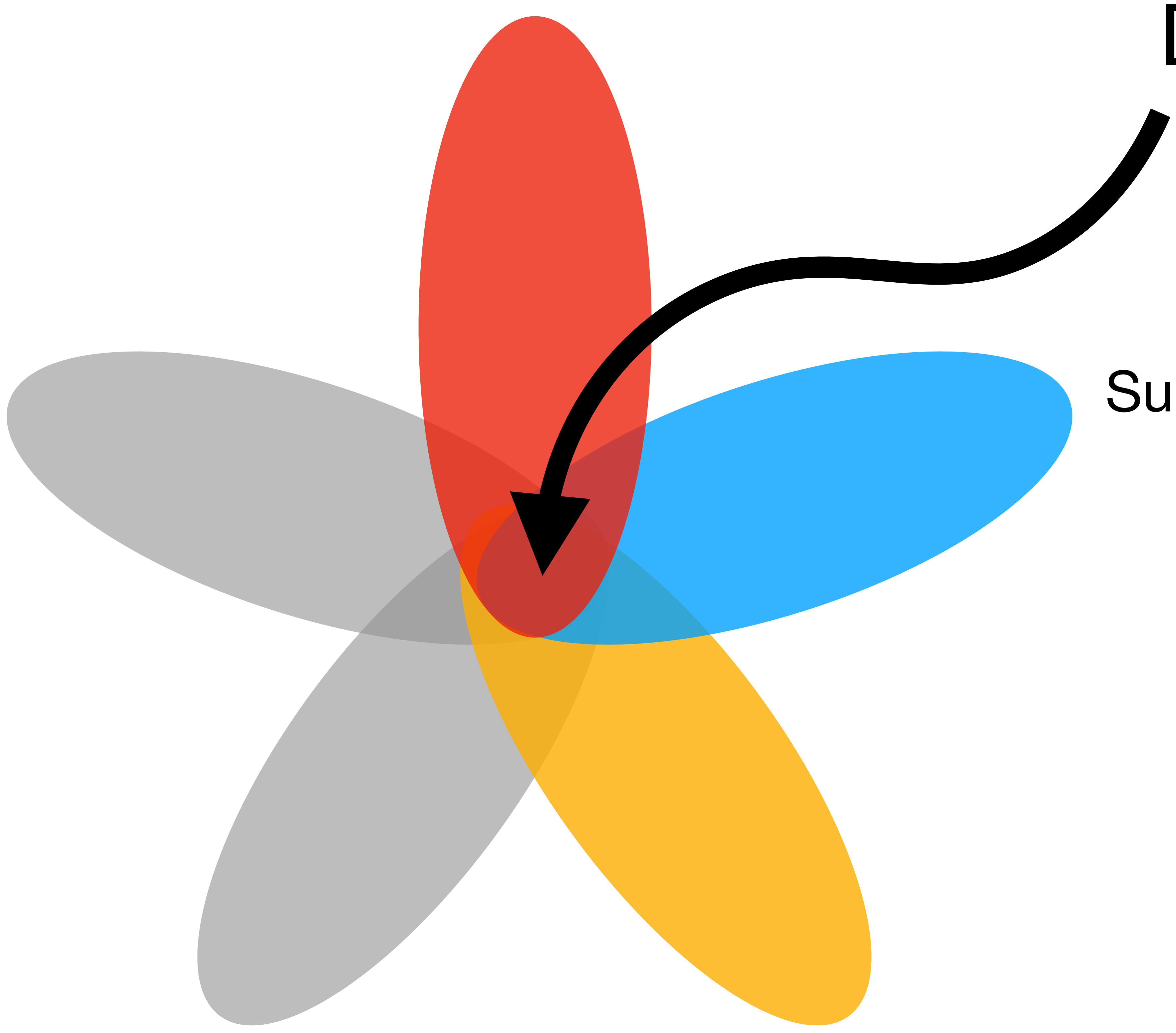
$\langle \phi_{1,2,3} \rangle = -\dfrac{1}{x_\perp}(t_{1,2,3})_{k\times k} \oplus 0_{(N-k)\times(N-k)}$

$\langle \phi_{4,5,6} \rangle = 0$

4d N=4 SYM with Nahm pole defect



4d N=4 SYM with Nahm pole defect



Conformal symmetry

Integrability

Supersymmetry

Holography

Perturbation theory

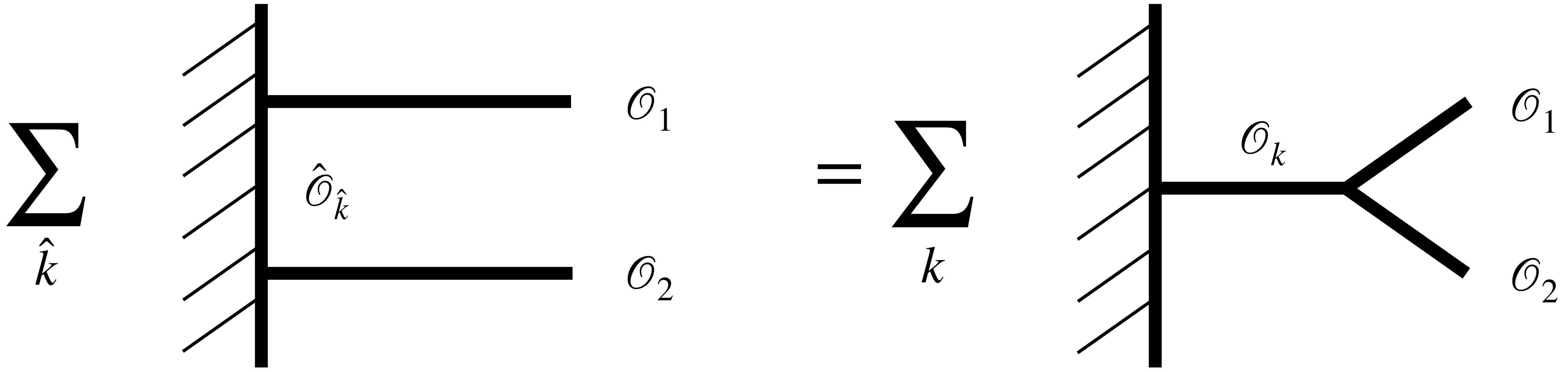
D3-D5 Defect CFT

Conformal symmetry

- In the presence of a conformal co-dimension one defect

$$\langle \mathcal{O}_1(x_\parallel, x_\perp)\mathcal{O}_2(y_\parallel, y_\perp)\rangle = \frac{F(\xi)}{(x_\perp)^{\Delta_1}(y_\perp)^{\Delta_2}}, \qquad \xi = \frac{(x-y)^2}{4x_\perp y_\perp}$$

- $F(\xi) = a_1 a_2 + \sum_{\hat{k}} b_{1\hat{k}} b_{2\hat{k}}\ \mathfrak{f}^{bdry}_{\delta_{\hat{k}}}(\xi) = \delta_{12} + \sum_{k} \lambda_{12k} a_k\ \mathfrak{f}^{blk}_{\Delta_k}(\xi)$



Liendo-Rastelli-van Rees ’12

Supersymmetry

- Bulk and boundary operators sit in superconformal representations

  E.g. $\mathscr{B}_{[0,2,0]} \supset T_{\mu\nu}, J^R_\mu, \mathscr{O}_2 = u^I u^J Tr(\phi_I \phi_J)$ with $u^2 = 0, \ldots$

  E.g. $(\mathscr{B}, \pm)_2 \supset D = T_{\perp\perp}|_\partial, \mathbb{O} = J^R_\perp|_\partial, \hat{O}_2, \ldots$

- Superblocks $\mathfrak{F}^{bdry}_\chi(\sigma, \bar{\sigma}, \xi)$, $\mathfrak{F}^{blk}_\chi(\sigma, \bar{\sigma}, \xi)$ capture exchanged superconf reps

  E.g. $\mathfrak{F}^{bdry}_{(\mathscr{B},\pm)_2} = \mathfrak{h}^{bdry}_{(2,0)}(\sigma, \bar{\sigma})\mathfrak{f}^{bdry}_2(\xi) + \#\mathfrak{h}^{bdry}_{(1,1)}(\sigma, \bar{\sigma})\mathfrak{f}^{bdry}_3(\xi) + \#\mathfrak{h}^{bdry}_{(0,0)}(\sigma, \bar{\sigma})\mathfrak{f}^{bdry}_4(\xi)$

Liendo-Meneghelli ‘16

Perturbation theory

- Different flavours and colours mix via position dependent mass terms
- Diagonalised by basis of fuzzy spherical harmonics $\tilde{\phi}_I = \sum_{\ell,m} \tilde{\phi}_{I,\ell,m} \hat{Y}_\ell^m$
- Propagator $\langle (\tilde{\phi}_I)_{\ell,m}(x)\, (\tilde{\phi}_J)^\dagger_{\ell',m'}(y) \rangle \sim \delta_{\ell\ell'} \frac{g_{YM}^2}{x_\perp y_\perp} \sum_{m^2} K_{AdS_4}^{m^2(\ell)}(x,y)$

Buhl-Mortensen-de Leeuw-Ipsen-Kristjansen-Wilhelm ‘16

2-pt functions of scalar 1/2-BPS operators

- Manifestly R-covariant $\langle \mathcal{O}_{J_1} \mathcal{O}_{J_2} \rangle$ in perturbation theory at $\mathcal{O}(g_{YM}^2)$,

  where $\mathcal{O}_J = u^{I_1} \ldots u^{I_J} Tr(\phi_{I_1} \ldots \phi_{I_J})$ with $u^2 = 0$ and normalisation $u \cdot \bar{u} = 2$

- Rich mathematical structure:

$$Tr((u^1 t_1 + u^2 t_2 + u^3 t_3)^L \hat{Y}_\ell^m) = \alpha_\ell^L \sqrt{\frac{(\ell - m)!}{(\ell + m)!}} \left(\frac{u^1 + i u^2}{\sqrt{1 - \left(u^3\right)^2}} \right)^m P_\ell^m(u^3)$$

$$\alpha_\ell^L = (-1)^k \sqrt{2\ell + 1} \sum_{n=1}^{k} \left(\frac{k+1}{2} - n \right)^L (-1)^n \begin{pmatrix} \frac{k-1}{2} & \ell & \frac{k-1}{2} \\ n - \frac{k+1}{2} & 0 & \frac{k+1}{2} - n \end{pmatrix}$$

2-pt functions of scalar 1/2-BPS operators

- Putting it all together:

$$\langle \mathcal{O}_{J_1}(x,u_1)\mathcal{O}_{J_2}(y,u_2)\rangle_c = \frac{J_1 J_2 g_{YM}^2}{8\pi^2 (x_3)^{J_1}(y_3)^{J_2}} \sum_{\ell=0}^{min(J_1-1,J_2-1)} \alpha_\ell^{J_1-1}\alpha_\ell^{J_2-1}\mathfrak{F}^{bdry}_{(B,+)_{\ell+1}}$$

- Generalises previous results in the literature

de Leeuw-Ipsen-Kristjansen-Vardinghus-Wilhelm ‘17

J=2: stress tensor and displacement

- Stress tensor supermultiplet two-point function

$$\langle \mathcal{O}_2(x,u_1)\mathcal{O}_2(y,u_2)\rangle_c = \frac{g_{YM}^2}{3\pi^2(x_3)^2(y_3)^2} B_3(\frac{k-1}{2})\mathfrak{F}^{bdry}_{(B,+)_2}$$

- Displacement two-point function $\langle D(x_{||})D(y_{||})\rangle \propto \dfrac{B_3(\frac{k-1}{2})}{|x_{||}-y_{||}|^8}$

de Leeuw-Kristjansen-Linardopoulos-Volk '23

- ∝ metric on defect conformal manifold $SU(4)/(SO(3)\times SO(3))$

Drukker-Kong-Sakkas '22, Herzog-Schaub '23

Summary

- Correlation functions for D3-D5 DCFT can be computed in perturbation theory
- R-covariant two-point function of scalar 1/2-BPS operators at $\mathcal{O}(g_{YM}^2)$
- Finite superconformal defect block expansion
- Only 1/2-BPS multiplets propagate in defect channel to $\mathcal{O}(g_{YM}^2)$
- Compact form for coefficients
- Relations b/w Wigner 3j symbols, and Bernoulli & central factorial polynomials

Lots more work to do

- Microbootstrap: 1/2-BPS sector only

Chester-Lee-Pufu-Yacoby ’14, Liendo-Meneghelli ’16

  - Equal R- & conformal cross-ratios $\longrightarrow$ superblocks simplify to constants
  - Bulk-to-defect couplings sum into a single Bernoulli polynomial in k
  - Use crossing to constrain 1/2-BPS defect CFT data at NLO
- Full bootstrap? Infinite number of bulk superblocks!

Lots more work to do

- Spectrum of defect operators
- Defect three-point functions, e.g. displacement $\longrightarrow$ fix defect Weyl anomaly
- Large N, k $\longrightarrow$ holography & integrability
- SUSY localisation & entanglement entropy
- Two-point functions for $k = 1$
- Other types of defects, e.g. D3-D7 interface, or defect in 3d N=6 ABJM theory