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The Lion, the Witch, and the Wormhole: Ensemble averaging the symmetric product orbifold

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with Joshua Kames-King, Bob Knighton, Mykhaylo Usatyuk

arXiv:2306.07321[hep-th]

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ONASSIS FOUNDATION

The Lion, the Witch, and the Wormhole: Ensemble averaging the symmetric product orbifold[1]

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[1]Title inspired by “Narain to Narnia” [Benjamin, Keller, Ooguri, Zadeh]

Motivation

In AdS/CFT: bulk theory dual to **a** specific boundary theory

$\longrightarrow$ $\mathcal{N} = 4$ SYM, IIB on $AdS_5 \times S^5$
$\longrightarrow$ Tensionless String: $\mathrm{Sym}^N(\mathbb{T}^4)$, IIB on $AdS_3 \times S^3 \times \mathbb{T}^4$

Another less intuitive duality: **Averaged** duality
The boundary theory has an interpretation as an ensemble average over boundary theories.

$\longrightarrow$ JT gravity and Matrix models [Saad, Shenker, Stanford]
$\longrightarrow$ "U(1)-gravity" and ensemble average of Narain CFTs [Maloney, Witten and N. Afkhami-Jeddi, H. Cohn, T. Hartman and A. Tajdini]

$$Z_{\text{bulk}} \sim \int_{\text{moduli } m} \mathrm{d}\mu(m) p(m) \cdot Z_{\text{boundary}}(m)$$

Aim

Extend the $\mathbb{T}^D/\mathrm{U}(1)$–gravity correspondence to $\mathrm{Sym}^N\left(\mathbb{T}^D\right)$ CFTs.
(see also [Eberhardt '21])

Provide a "stringy" embedding of ensemble–averaging.

$$\begin{tikzcd}[column sep=huge, row sep=huge]
2d & & 3d \\
\mathrm{Sym}^N\left(\mathbb{T}^4\right) \arrow[rr, leftrightarrow, "\text{duality}"] \arrow[d, "\text{Average}"'] & & \underset{\text{[Eberhardt, Gaberdiel, Gopakumar]}}{\text{Tensionless String}} \arrow[d, "\text{Average}"] \\
\int \mathrm{d}\mu\, \mathrm{Sym}^N\left(\mathbb{T}^4\right) \arrow[rr, leftrightarrow, "\text{duality}"] & & ?
\end{tikzcd}$$

Narain CFTs and Averaging

D free bosons taking values on $\mathbb{T}^D = \mathbb{R}^D/\Lambda_D$

$$I = \frac{1}{4\pi\alpha'} \int G_{pq}\delta^{\alpha\beta}\partial_\alpha X^p \partial_\beta X^q + iB_{pq}\epsilon^{\alpha\beta}\partial_\alpha X^p \partial_\beta X^q$$

Partition function $Z(m,\tau) = \frac{1}{|\eta(\tau)|^{2D}} \sum_{(\boldsymbol{p}_L,\boldsymbol{p}_R)\in\Gamma_{D,D}} q^{\frac{1}{2}\boldsymbol{p}_L^2}\bar{q}^{\frac{1}{2}\boldsymbol{p}_R^2} = \frac{\Theta(m,\tau)}{|\eta(\tau)|^{2D}}$, $q = e^{2\pi i\tau}$

Moduli space (metric G and B-field) is $\mathcal{M}_D = O(D,D;\mathbb{Z}) \setminus O(D,D)/(O(D)\times O(D))$

Deformations are described by marginal operators $\mathcal{O} = \left(\delta G_{pq}\delta^{\alpha\beta} + i\delta B_{pq}\epsilon^{\alpha\beta}\right)\partial_\alpha X^p \partial_\beta X^q$

These give a **measure** in moduli space averages [Zamolodchikov '86]

$$< \ldots >_m = \int_{\mathcal{M}_D} \mathrm{d}\mu(m)\,(\ldots)$$

Averaging and bulk

Siegel-Weil formula enables averaging over $\mathcal{M}_D$

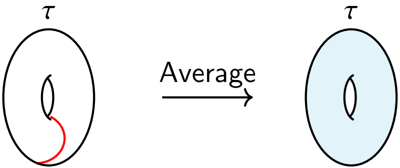
$$\int_{\mathcal{M}_D} \mathrm{d}\mu(m) Z(m,\tau) = \frac{E_{D/2}(\tau)}{(\mathrm{Im}(\tau))^{D/2} |\eta(\tau)|^{2D}} = \sum_{\gamma \in \Gamma_\infty \backslash \mathrm{SL}(2,\mathbb{Z})} \frac{1}{|\eta(\gamma \cdot \tau)|^{2D}}$$

$\rightarrow$ Sum over geometries of $\mathrm{U}(1)^D \times \mathrm{U}(1)^D$ Chern Simons

$$S_{CS} = \sum_{i=1}^{D} \int A_i \wedge \mathrm{d}A_i - B_i \wedge \mathrm{d}B_i$$

On a solid torus Y with boundary τ. Partition function $Z_{CS}(\tau) = \frac{1}{|\eta(\tau)|^{2D}}$

Averaging "sums over fillings of boundary torus"



Symmetric product orbifold

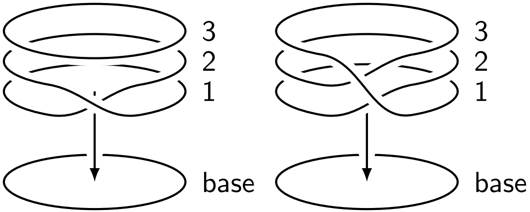
Generalize to $\left(\mathbb{T}^D\right)^{\otimes N}/S_N$. Take Nth tensor power of $\mathbb{T}^D$
$\left(\mathbb{T}^D\right)^{\otimes N} = \mathbb{T}^D \otimes \mathbb{T}^D \otimes \ldots \otimes \mathbb{T}^D$ and mod out permutations.
Symmetric group acts as

$$\pi \cdot (\boldsymbol{X}_1, \ldots, \boldsymbol{X}_N) = \left(\boldsymbol{X}_{\pi(1)}, \ldots, \boldsymbol{X}_{\pi(N)}\right)$$

Now the (twisted) boundary conditions are of the form

$$\boldsymbol{X}(a \cdot z) = \pi_a \cdot \boldsymbol{X}\ ,\ \boldsymbol{X}(b \cdot z) = \pi_b \cdot \boldsymbol{X}$$

To compute the partition function, go to the **covering space**.

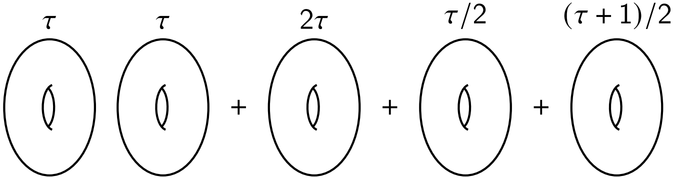


Symmetric product orbifold $\rightarrow N = 2$

The partition function for $N = 2$ reads (sum over covers corresponds to summing over boundary conditions)

$$Z_2(\tau) = \frac{1}{2} Z(\tau)^2 + \frac{1}{2} Z(\tfrac{\tau}{2}) + \frac{1}{2} Z(2\tau) + \frac{1}{2} Z(\tfrac{\tau+1}{2})$$

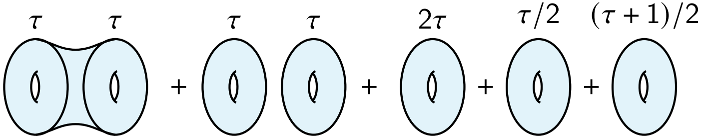
$Z(\tau)$ = partition function of "seed" theory (here $\mathbb{T}^D$) on genus $g = 1$ surface.



This can be averaged using Siegel-Weil formula!

$N = 2$ and averaging.

Averaging the previous expression "fills" cycles of boundary torus and we have a $3d$ $\mathrm{U}(1)^D \times \mathrm{U}(1)^D$ interpretation as

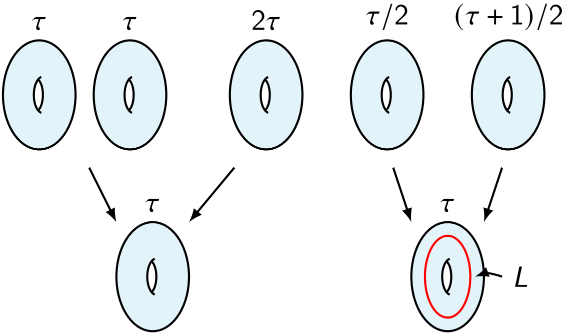


However this has **wrong boundaries and a $1/2$ factor...**
How can we interpret this as coming from **a single** bulk handlebody with boundary τ?
These are **covering** spaces of the bulk geometry!
Now $\mathrm{U}(1)^D \times \mathrm{U}(1)^D \wr S_2$ similarly to Sym^N. Now we have a discrete part in the gauge group (also in [Benjamin, Keller, Ooguri, Zadeh])

$$A_{(i)} \longrightarrow A_{(i)} + \mathrm{d}\lambda^A_{(i)}\,, \quad B_{(i)} \longrightarrow B_{(i)} + \mathrm{d}\lambda^B_{(i)}$$

$$A_{(i)} \longrightarrow A_{(\pi(i))},\, B_{(i)} \longrightarrow B_{(\pi(i))}$$

Bulk

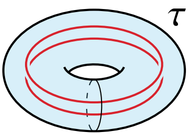


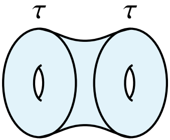
The **bulk partition** function is also computed by going to a covering space. The bulk theory includes **vortices**.
Taking the $\mathbb{Z}_2$ quotients of the covers, we find the bulk manifold.

Comparing the connected parts of bulk and boundary, we get (strictly speaking conjecture)

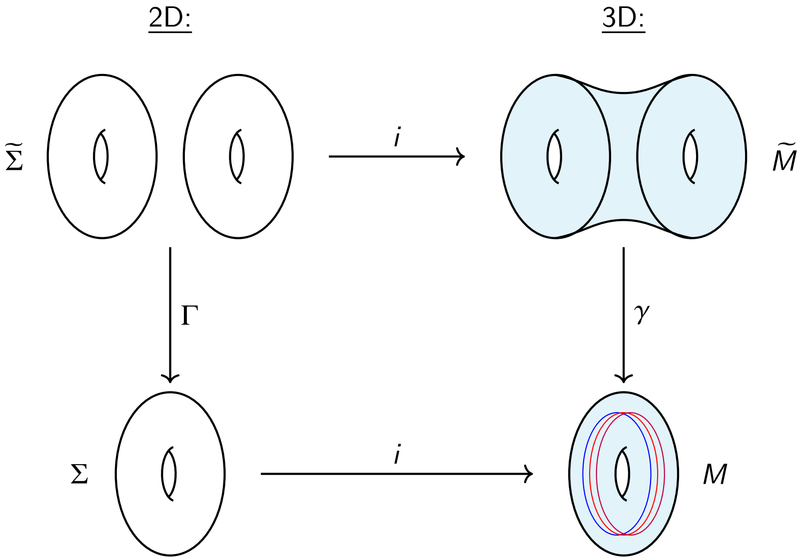
$$\sum_{\gamma\in\Gamma_\infty\backslash \mathrm{SL}(2,\mathbb{Z})}\left(\frac{1}{|\eta(2\,\gamma\cdot\tau)|^{2D}}+\frac{1}{\left|\eta(\frac{\gamma\cdot\tau}{2})\right|^{2D}}+\frac{1}{\left|\eta(\frac{\gamma\cdot\tau+1}{2})\right|^{2D}}\right)$$
$$=\sum_{\gamma\in\Gamma_\infty\backslash \mathrm{SL}(2,\mathbb{Z})}\left(\frac{1}{|\eta(\gamma\cdot(2\tau))|^{2D}}+\frac{1}{\left|\eta(\gamma\cdot(\frac{\tau}{2}))\right|^{2D}}+\frac{1}{\left|\eta(\gamma\cdot(\frac{\tau+1}{2}))\right|^{2D}}\right)$$

Average of disconnected part more tricky.





$= \mathbb{T}^2 \times I$


2D:
3D:
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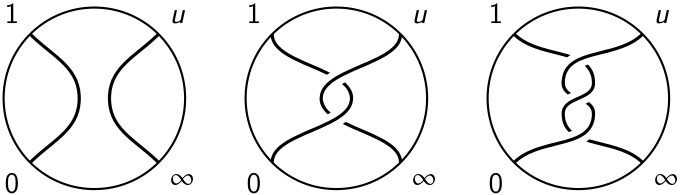
Correlators and SUSY

Also studied the average of $\mathrm{Sym}^2\left(\mathbb{T}^D\right)$ **correlators** (4 point function of twist fields). Interpretation as a sum over bulk vortex configurations in $\mathrm{U}(1)^D \times \mathrm{U}(1)^D \wr S_2$ (rational tangles, [Benjamin, Keller, Ooguri, Zadeh]).



Added **SUSY** to the $\mathbb{T}^D$-U(1) gravity correspondence.
Adding D fermions to the bosonic $\mathbb{T}^D$

$$\psi(A \cdot z) = e^{2\pi i \alpha}\psi(z)\,, \quad \psi(B \cdot z) = e^{2\pi i\beta}\psi(z)$$

The bulk theory is now $\mathcal{N} = (1,1)$ Chern-Simons ([Belyaev,Nieuwenhuizen]). We can also take the S_N orbifold.

Outlook

Some questions for the future:

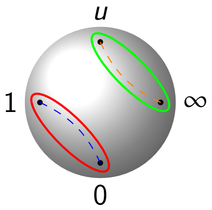
- Averaging Narain moduli with marginal deformation (towards SUGRA). How is the sum over geometries affected?
- Study OPE statistics of Narain theories (OPE randomness hypothesis [Belin, de Boer]).
- More complicate vortex configurations to reproduce average contributions.
- Averages of toroidal orbifolds on factorizable and non factorizable lattices (ongoing work [Förste, Jockers, Kames-King, Zadeh]). Different actions of $\mathbb{Z}_2$ orbifolds.
- Other examples where the averaging can be done in the context of string compactifications ([Benjamin, Keller, Ooguri, Zadeh])?

The end

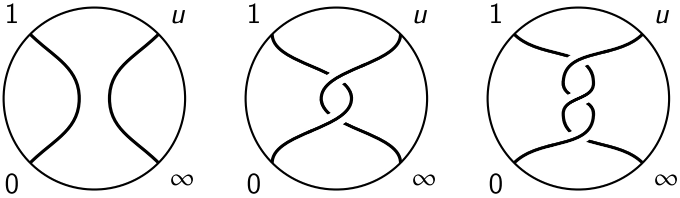
☺

Extra–Correlators

Average of $\mathrm{Sym}^2\left(\mathbb{T}^D\right)$ gives sum over bulk vortex configurations in $\mathrm{U}(1)^D \times \mathrm{U}(1)^D \wr S_2$ (rational tangles, [Benjamin, Keller, Ooguri, Zadeh])



$$\int_{\mathcal{M}_D} \mathrm{d}\mu(m)\, \left\langle \sigma_{(1\,2)}(0)\sigma_{(1\,2)}(1)\sigma_{(1\,2)}(u)\sigma_{(1\,2)}(\infty)\right\rangle \propto \sum_{\gamma\in\Gamma_\infty\backslash \mathrm{SL}(2,\mathbb{Z})} \frac{1}{|\eta(\gamma\cdot\tau)|^{2D}}$$

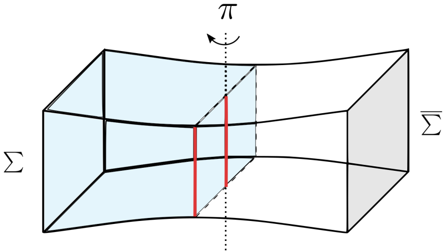


Extra

For a Riemann surface with period matrix Ω

$$\left\langle Z_{\mathbb{T}^D}(m,\Omega)\right\rangle \sim \sum_{\Gamma_0} \left(\det \mathrm{Im}\Omega_{\Gamma_0}\right)^{\frac{D}{2}}$$

Γ_0 Lagrangian sublattice.
Averaging dictates the bulk.
No independent Chern Simons calculation on $\mathbb{T}^2 \times I$ to match expected result from boundary averaging.



Averages of $\mathbb{T}^D$ converge when $D-1>g$ (genus of bdy).