Holographic SK contours and EFTs for Fluids

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Based on: T. He, R. Loganayagam, M. Rangamani, JV, S. Zhou [2108.03244], [2205.03415], [2211.07683], [2306.01055].

Introduction

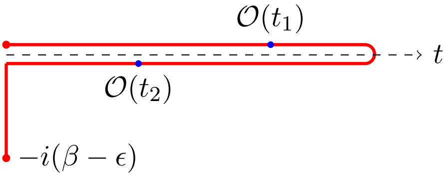
- Real-time thermal correlators are an important observable in QFT.
- They capture the dynamical response and fluid dynamical properties of the theory (Kubo formula-transport coefficients).
- They are key to understanding the chaotic properties of quantum systems (OTOCs).

A matter of order: the SK contour

- When computing real-time correlators, the order of operators is important (causality).
- A familiar choice is to compute time-ordered correlators: $\langle \mathcal{T}\mathcal{O}(t_1)\mathcal{O}(t_2)...\mathcal{O}(t_n)\rangle$.
- Other orderings are often useful to explore distinct physics. Example, retarded correlators: $\theta(t-t')\,\langle[\mathcal{O}(t),\mathcal{O}(t')]\rangle$.

- All the orderings can be taken into account by considering time-fold contours.
- Up to 3-point (thermal) correlators, one only needs a two-fold contour (SK contour):

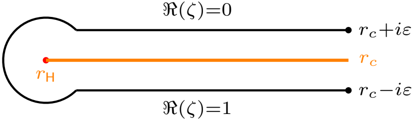
$$\langle \mathcal{O}(t_2)\mathcal{O}(t_1)\rangle_\beta =$$

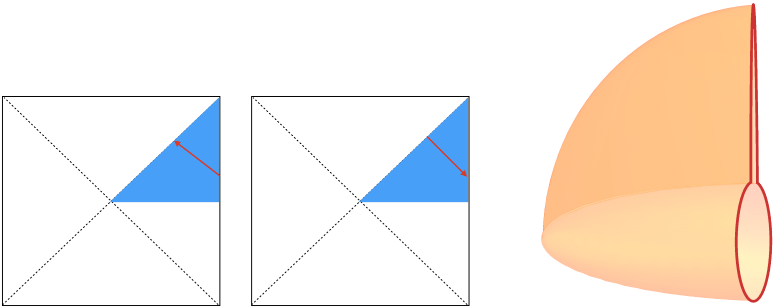


Holographis SK contours

- Long history of real-time holography [Herzog, Son, Starinets, Skenderis, van Rees...].
- Complex solution of Einstein equations whose boundary is the SK contour[Glorioso, Crossley, Liu] [Jana, Loganayagam, Rangamani].

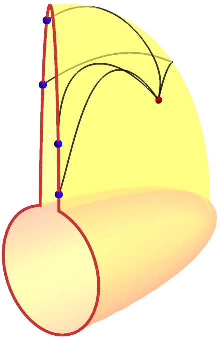
$$ds^2 = -r^2 f dv^2 + i\beta r^2 f dv d\zeta + r^2 d\vec{x}^2, \qquad \frac{dr}{d\zeta} = \frac{i\beta}{2} r^2 f$$





- Key ingredient: Joining conditions must impose the KMS relations for the bdy. correlators.

- Correlation functions are then computed as Witten diagrams in this geometry.



A useful scenario: Holographic fluids

- We can study real-time gravitational path integrals in a very controlled scenario.
- Conversely, we can use these techniques to systematically construct actions for fluids and learn more about them.

Repackaging gravity: designer scalars

- Consider Einstein-Maxwell (can include CS) gravity:

$$S_{\mathsf{Bulk}} = \int d^{d+1}x\sqrt{-G}\left(R + d(d-1) - \frac{1}{2}F_{AB}F^{AB}\right)$$

- We want to consider perturbations around equilibrium (RN black hole).
- These can be classified in terms of $SO(d-2)$ representations [Kodama, Ishibashi] [Gubser, Pufu].

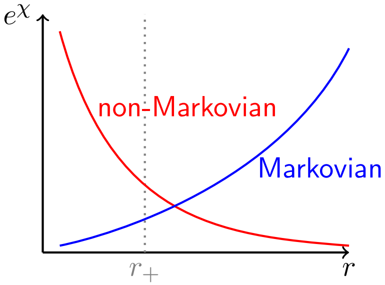
- Linear response in gravity is all packaged in a few scalars:

$$S_{\mathcal{M}} = -\frac{1}{2}\int d^{d+1}x\sqrt{-g}e^{\chi(r)}\nabla_A\Phi_{\mathcal{M}}\nabla^A\Phi_{\mathcal{M}}, \quad \sqrt{-g}e^{\chi(r)} \sim r^{\mathcal{M}}$$

- Tensors: $e^{\chi(r)} = 1$.
- Vectors: $e^{\chi(r)} = \frac{1}{r^{2(d-1)}}\,, \quad e^{\chi(r)} = \frac{h^2}{r^2}\,, \quad h(r) = 1 - \frac{\mathfrak{S}_Q}{r^{d-2}}$.
- Scalars: $e^{\chi(r)} = \frac{h^2}{r^{2(d-2)}\Lambda_k^2}\,, \quad e^{\chi(r)} = \frac{1}{h^2 r^{2(d-2)}}\,,$

  $\Lambda_k = k^2 + \frac{1}{2}(d-1)r^3 f'$.

- The dilaton makes all the difference:



- The asymptotic behavior determines whether a perturbation is long or short-lived.

Solutions and actions:

- Markovian:

$$\Phi_{\mathcal{M}}(\zeta, w, \vec{k}) = J_a G^{\text{in}}_{\mathcal{M}} + \left[\left(n_\beta + \frac{1}{2}\right) G^{\text{in}}_{\mathcal{M}} - n_\beta e^{\beta(\zeta-1)} G^{\text{rev}}_{\mathcal{M}}\right] J_d$$

$$S[\Phi_{\mathcal{M}}] = -\int_k J_d^\dagger \mathcal{K}^{\text{in}}_{\mathcal{M}} \left[J_a + \left(n_\beta + \frac{1}{2}\right) J_d\right]$$

$$\mathcal{K}_{\mathcal{M}} = -iw + \frac{k^2}{1-\mathcal{M}} + \Delta^{2,0}_{\mathcal{M}}(r_+) w^2 + \ldots$$

- Non-Markovian:

$$\Phi_{-\mathcal{M}}(\zeta, w, \vec{k}) = \check{\Phi}_a G^{\text{in}}_{-\mathcal{M}} + \left[\left(n_\beta + \frac{1}{2}\right) G^{\text{in}}_{-\mathcal{M}} - n_\beta e^{\beta(\zeta-1)} G^{\text{rev}}_{-\mathcal{M}}\right] \check{\Phi}_d$$

$$S[\Phi_{-\mathcal{M}}] = -\int_k \check{\Phi}_d^\dagger \mathcal{K}^{\text{in}}_{-\mathcal{M}} \left[\check{\Phi}_a + \left(n_\beta + \frac{1}{2}\right) \check{\Phi}_d\right]$$

$$\mathcal{K}_{-\mathcal{M}} = -iw + \frac{k^2}{1+\mathcal{M}} - \Delta^{2,0}_{\mathcal{M}}(r_+) w^2 + \ldots$$

- Sound is special:

$$\mathcal{K}_s = -w^2 + \frac{k^2}{d-1} + \nu_s k^2 \Gamma(w,k)\,.$$

Correlation functions:

$$\langle T_{xy}(-w,-k)T_{xy}(w,k)\rangle^{\text{Ret}} \sim i\mathcal{K}^{\text{in}}_{\mathcal{M}}(w,k)$$

$$\langle T_{vx}(-w,-k)T_{vx}(w,k)\rangle^{\text{Ret}} \sim i\left[\frac{a_1}{\mathcal{K}^{\text{in}}_{-\mathcal{M}}(w,k)} + a_2\mathcal{K}^{\text{in}}_{\mathcal{M}}(w,k)\right]$$

$$\langle J_{x}(-w,-k)J_{x}(w,k)\rangle^{\text{Ret}} \sim i\left[\frac{b_1}{\mathcal{K}^{\text{in}}_{-\mathcal{M}}(w,k)} + b_2\mathcal{K}^{\text{in}}_{\mathcal{M}}(w,k)\right]$$

$$\langle T_{vv}(-w,-k)T_{vv}(w,k)\rangle^{\text{Ret}} \sim i\left[\frac{c_1}{\mathcal{K}^{\text{in}}_{s}(w,k)} + \frac{c_2}{\mathcal{K}^{\text{in}}_{-\mathcal{M}}(w,k)}\right]$$

$$\langle J_{v}(-w,-k)J_{v}(w,k)\rangle^{\text{Ret}} \sim i\left[\frac{d_1}{\mathcal{K}^{\text{in}}_{s}(w,k)} + \frac{d_2}{\mathcal{K}^{\text{in}}_{-\mathcal{M}}(w,k)}\right]$$

Beyond the Gaussian level: Witten diagrams in SK geometry

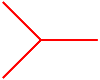
The ingredients

- Ingoing Bulk-to-Bdy Prop.: $G_{\text{in}}(\zeta, k)$.
- Outgoing Bulk-to-Bdy Prop.: $G_{\text{out}}(\zeta, k) = e^{-\beta\omega\zeta} G_{\text{in}}(\zeta, \bar{k})$
- Bulk-to-Bulk Prop:

  $G_{\text{bb}}(\zeta, \zeta'; k) = \mathcal{N}(k) e^{\beta\omega\zeta'} G_{\text{L}}(\zeta_>, k) G_{\text{R}}(\zeta'_<, k)$.

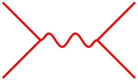
Important: $G_{\text{in}}(\zeta + 1, k) = G_{\text{in}}(\zeta, k)$.

- **Contact diagram:**



$$= \oint d\zeta\, \mathfrak{L}(\zeta) = \int_{r_{\mathsf{H}}}^{r_c} dr\, \left(\mathfrak{L}(\zeta(r)+1) - \mathfrak{L}(\zeta(r))\right)$$

- **Exchange diagram**



$$= \oint d\zeta \oint d\zeta' \left[F_1(\zeta,\zeta')\Theta(\zeta-\zeta') + F_2(\zeta,\zeta')\Theta(\zeta'-\zeta)\right]$$

$$= \int_{r_{\mathsf{H}}}^{r_c} dr \int_{r_{\mathsf{H}}}^{r_c} dr' \left[\mathfrak{F}_1\theta(r-r') + \mathfrak{F}_2\theta(r'-r)\right]$$

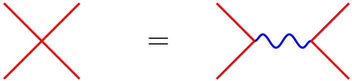
where

$$\mathfrak{F}_1 = F_1(\zeta,\zeta') - F_1(\zeta+1,\zeta') + F_2(\zeta+1,\zeta'+1) - F_2(\zeta,\zeta'+1)\,,$$

$$\mathfrak{F}_2 = F_1(\zeta+1,\zeta'+1) - F_1(\zeta+1,\zeta') + F_2(\zeta,\zeta') - F_2(\zeta,\zeta'+1)\,.$$

Analytic structure of Thermal correlators

- Explicit, closed-form, calculations are only possible in $d = 2$.
- General lesson: The analytic structure of higher order thermal correlators is determined by a single piece of data: $G_{\text{in}}(\zeta, k)$.
- Long-lived modes and factorization channels:



Outlook

- Effective action for fluids beyond the gaussian level:
  - SK Witten diagrams for $d > 3$.
  - Realistic vertices derived directly from gravity.
- Explicit computation of holographic OTOC (more time-folds).
- Connections to Thermal Bootstrap (block descomposition).
- Beyond holography: gravitational SK contours for more general spacetimes.

Thank You!