Constraints of Superconformal Symmetry

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Superconformal Symmetry and multi-point correlators

$$\text{CFT data: } \{\Delta, \lambda_{ijk}\}$$

$$\langle \mathcal{O}_{\Delta_i}(x_i)\,\mathcal{O}_{\Delta_j}(x_j)\rangle = \frac{1}{|x_{ij}|^{\Delta}}\,,\quad \Delta_i = \Delta_j = \Delta$$

$$\langle \mathcal{O}_{\Delta_i}(x_i)\,\mathcal{O}_{\Delta_j}(x_j)\,\mathcal{O}_{\Delta_k}(x_k)\rangle = \frac{\lambda_{ijk}}{|x_{ij}|^{\Delta_i+\Delta_j-\Delta_k}\,|x_{jk}|^{\Delta_j+\Delta_k-\Delta_i}\,|x_{ki}|^{\Delta_k+\Delta_i-\Delta_j}}$$

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$$\mathcal{O}(x)\mathcal{O}(0) \sim \sum_{\mathcal{O}'_{\Delta,l}} \lambda_{\mathcal{O}\mathcal{O}\mathcal{O}'}\, \mathcal{C}_{\mathcal{O}'}(x,\partial)\, \mathcal{O}'_{\Delta,l}$$

$$\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle \sim \sum_{\mathcal{O}'_{\Delta,l}} \lambda_{\mathcal{O}\mathcal{O}\mathcal{O}'}\, \langle \mathcal{O}\mathcal{O}\mathcal{O}'_{\Delta,l} \rangle$$

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$$\mathcal{O}(x)\mathcal{O}(0) \sim \sum_{\mathcal{O}'_{\Delta,l}} \lambda_{\mathcal{O}\mathcal{O}\mathcal{O}'}\,\mathcal{C}_{\mathcal{O}'}(x,\partial)\,\mathcal{O}'_{\Delta,l}$$

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$$\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}\rangle \sim \sum_{\mathcal{O}'_{\Delta,l}} \lambda_{\mathcal{O}\mathcal{O}\mathcal{O}'}\,\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O}'_{\Delta,l}\rangle$$

$$\langle \mathcal{B}\mathcal{B}\mathcal{B}\mathcal{B}\mathcal{B}\rangle \sim \sum_{\mathcal{A}} \lambda_{\mathcal{B}\mathcal{B}\mathcal{A}}\,\langle \mathcal{B}\mathcal{B}\mathcal{B}\mathcal{A}\rangle$$

Conformal Bootstrap

1. Constraints of superconformal symmetry (Superconformal Ward Identities)
2. Consistency conditions

Conformal Bootstrap

1. **Constraints of superconformal symmetry (Superconformal Ward Identities)**
2. Consistency conditions

PSU(2,2|4) and its analytic superspace

PSU(2,2|4)

- $\mathrm{SU}(2,2) \times \mathrm{SU}(4)_R$
- $\begin{cases} Q_{i\,\alpha}, \bar{Q}^{i}{}_{\dot\alpha} \\ S^{i}{}_{\alpha}, \bar{S}_{i\,\dot\alpha} \end{cases}, \; i = 1, ..., 4; \; \alpha, \dot\alpha = 1, 2$

$$\begin{pmatrix} D, P_\mu, K_\mu, M_{\mu,\nu} & Q, \bar{S} \\ \bar{Q}, S & R^{I}{}_{J} \end{pmatrix}$$

PSU(2,2|4) and its analytic superspace

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- $\begin{cases} Q_{i\,\alpha}, \bar{Q}^i{}_{\dot\alpha} \\ S^i{}_{\alpha}, \bar{S}_{i\,\dot\alpha} \end{cases}, \; i = 1, \dots, 4;\; \alpha, \dot\alpha = 1, 2$

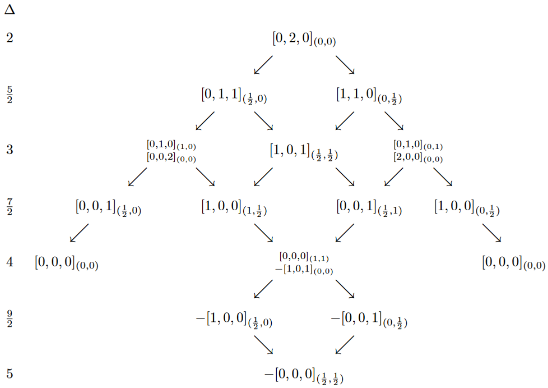
$$\begin{pmatrix} D, P_\mu, K_\mu, M_{\mu,\nu} & Q, \bar{S} \\ \bar{Q}, S & R^I{}_J \end{pmatrix}$$

Analytic superspace

$$\boldsymbol{X} = \begin{pmatrix} \boldsymbol{x}^{\alpha\dot\alpha} & \boldsymbol{\rho}^{\alpha\dot a} \\ \bar{\boldsymbol{\rho}}^{a\dot\alpha} & \boldsymbol{y}^{a\dot a} \end{pmatrix} \in \textbf{Mat(2|2)}$$

- $\alpha, \dot\alpha = 1, 2,\; a, \dot a = 1, 2$
- $x_{\alpha\,\dot\alpha} = (x^\mu \sigma_\mu)_{\alpha\,\dot\alpha}$: Minkowski
- $y_{a\,\dot a}$: Internal space
- $\rho_{\alpha\,\dot a}\,,\; \bar{\rho}_{\dot\alpha\,a}$: Grassmann-odd coord.

Stress tensor multiplet



$$\mathcal{W}_{p=2}(X) = \mathcal{O}_{20'}(x,y) + \rho_{\alpha\dot{a}}\Psi_{\alpha\dot{a}}(x,y) + \bar{\rho}^{a\dot{\alpha}}\bar{\Psi}_{a\dot{\alpha}}(x,y) + \rho^{\alpha\dot{a}}\bar{\rho}^{a\dot{\alpha}}A_{\alpha\dot{\alpha};a\dot{a}}(x,y) + \rho^2\bar{F}(x,y) + \bar{\rho}^2 F(x,y) + \rho^2\bar{\rho}^{a\dot{\alpha}}B_{a\dot{\alpha}}(x,y) + \ldots$$

[Dolan, Osborn '02]

4pt correlator of psu(2,2|4)

$$\langle \mathcal{W}_2(X_1)\,\mathcal{W}_2(X_2)\,\mathcal{W}_2(X_3)\,\mathcal{W}_2(X_4)\rangle = \langle \mathcal{O}_{20'}(x_1,y_1)\,\mathcal{O}_{20'}(x_2,y_2)\,\mathcal{O}_{20'}(x_3,y_3)\,\mathcal{O}_{20'}(x_4,y_4)\rangle$$

$$+\sum_{i=1}^{4}\rho_i^{\alpha\dot{a}}\,\langle \Psi_{\alpha\dot{a}}(x_i,y_i)\,\mathcal{O}_{20'}(x_k,y_k)\,\mathcal{O}_{20'}(x_l,y_l)\,\mathcal{O}_{20'}(x_m,y_m)\rangle$$

$$+\sum_{i=1}^{4}\bar{\rho}_i^{a\dot{\alpha}}\,\langle \bar{\Psi}_{a\dot{\alpha}}(x_i,y_i)\,\mathcal{O}_{20'}(x_k,y_k)\,\mathcal{O}_{20'}(x_l,y_l)\,\mathcal{O}_{20'}(x_m,y_m)\rangle$$

$$+\sum_{i=1}^{4}\rho_i^{\alpha\dot{a}}\bar{\rho}_i^{a\dot{\alpha}}\,\langle A_{\alpha\dot{\alpha};a\dot{a}}(x_i,y_i)\,\mathcal{O}_{20'}(x_k,y_k)\,\mathcal{O}_{20'}(x_l,y_l)\,\mathcal{O}_{20'}(x_m,y_m)\rangle + \dots$$

4pt correlator of psu(2,2|4)

$$
\begin{aligned}
&\langle \mathcal{W}_2(X_1)\,\mathcal{W}_2(X_2)\,\mathcal{W}_2(X_3)\,\mathcal{W}_2(X_4)\rangle \\
&\quad = \langle \mathcal{O}_{20'}(x_1,y_1)\,\mathcal{O}_{20'}(x_2,y_2)\,\mathcal{O}_{20'}(x_3,y_3)\,\mathcal{O}_{20'}(x_4,y_4)\rangle \\
&\quad + \sum_{i=1}^{4} \rho_i^{\alpha\dot a}\,\langle \Psi_{\alpha\dot a}(x_i,y_i)\,\mathcal{O}_{20'}(x_k,y_k)\,\mathcal{O}_{20'}(x_l,y_l)\,\mathcal{O}_{20'}(x_m,y_m)\rangle \\
&\quad + \sum_{i=1}^{4} \bar\rho_i^{a\dot\alpha}\,\langle \bar\Psi_{a\dot\alpha}(x_i,y_i)\,\mathcal{O}_{20'}(x_k,y_k)\,\mathcal{O}_{20'}(x_l,y_l)\,\mathcal{O}_{20'}(x_m,y_m)\rangle \\
&\quad + \sum_{i=1}^{4} \rho_i^{\alpha\dot a}\,\bar\rho_i^{a\dot\alpha}\,\langle A_{\alpha\dot\alpha;a\dot a}(x_i,y_i)\,\mathcal{O}_{20'}(x_k,y_k)\,\mathcal{O}_{20'}(x_l,y_l)\,\mathcal{O}_{20'}(x_m,y_m)\rangle \\
&\quad + \dots
\end{aligned}
$$

- determined by superprimary correlator
- proportional to only one arbitrary function of the conformal cross ratios (one channel)

Cross-ratios:

$$
u = \frac{x_{12}^2\,x_{34}^2}{x_{13}^2\,x_{24}^2}\,,\quad v = \frac{x_{14}^2\,x_{23}^2}{x_{13}^2\,x_{24}^2}
$$

[Heslop '22,...]

4pt correlator of psu(2,2|4)

$$\langle \mathcal{W}_2(X_1)\mathcal{W}_2(X_2)\mathcal{W}_2(X_3)\mathcal{W}_2(X_4)\rangle$$
$$= \langle \mathcal{O}_{\mathbf{20'}}(\mathbf{x_1},\mathbf{y_1})\mathcal{O}_{\mathbf{20'}}(\mathbf{x_2},\mathbf{y_2})\mathcal{O}_{\mathbf{20'}}(\mathbf{x_3},\mathbf{y_3})\mathcal{O}_{\mathbf{20'}}(\mathbf{x_4},\mathbf{y_4})\rangle$$
$$+ \Sigma_{i=1}^{4} \rho_i^{\alpha\dot{a}} \langle \Psi_{\alpha\dot{a}}(x_i,y_i)\mathcal{O}_{20'}(x_k,y_k)\mathcal{O}_{20'}(x_l,y_l)\mathcal{O}_{20'}(x_m,y_m)\rangle$$
$$+ \Sigma_{i=1}^{4} \bar{\rho}_i^{a\dot{\alpha}} \langle \bar{\Psi}_{a\dot{\alpha}}(x_i,y_i)\mathcal{O}_{20'}(x_k,y_k)\mathcal{O}_{20'}(x_l,y_l)\mathcal{O}_{20'}(x_m,y_m)\rangle$$
$$+ \Sigma_{i=1}^{4} \rho_i^{\alpha\dot{a}} \bar{\rho}_i^{a\dot{\alpha}} \langle A_{\alpha\dot{\alpha};a\dot{a}}(x_i,y_i)\mathcal{O}_{20'}(x_k,y_k)\mathcal{O}_{20'}(x_l,y_l)\mathcal{O}_{20'}(x_m,y_m)\rangle$$
$$+ \ldots$$

R-sym structures:

$$\langle \mathcal{O}_{20'}(x_1,y_1)\,\mathcal{O}_{20'}(x_2,y_2)\,\mathcal{O}_{20'}(x_3,y_3)\,\mathcal{O}_{20'}(x_4,y_4)\rangle$$
$$= y_{12}^4\,y_{34}^4 F_1(\{x\}) + y_{13}^4\,y_{24}^4 F_2(\{x\}) + y_{14}^4\,y_{23}^4 F_3(\{x\})$$
$$+ y_{12}^2\,y_{13}^2\,y_{24}^2\,y_{34}^2 F_4(\{x\}) + y_{12}^2\,y_{14}^2\,y_{23}^2\,y_{34}^2 F_5(\{x\})$$
$$+ y_{13}^2\,y_{14}^2\,y_{23}^2\,y_{24}^2 F_6(\{x\})$$

4pt correlator of psu(2,2|4)

$$\begin{aligned}
&\langle \mathcal{W}_2(X_1)\mathcal{W}_2(X_2)\mathcal{W}_2(X_3)\mathcal{W}_2(X_4)\rangle \\
&\quad = \langle \mathcal{O}_{\mathbf{20'}}(\mathbf{x_1},\mathbf{y_1})\mathcal{O}_{\mathbf{20'}}(\mathbf{x_2},\mathbf{y_2})\mathcal{O}_{\mathbf{20'}}(\mathbf{x_3},\mathbf{y_3})\mathcal{O}_{\mathbf{20'}}(\mathbf{x_4},\mathbf{y_4})\rangle \\
&\quad + \Sigma_{i=1}^4 \rho_i^{\alpha\dot{a}} \langle \Psi_{\alpha\dot{a}}(x_i,y_i)\mathcal{O}_{20'}(x_k,y_k)\mathcal{O}_{20'}(x_l,y_l)\mathcal{O}_{20'}(x_m,y_m)\rangle \\
&\quad + \Sigma_{i=1}^4 \bar{\rho}_i^{a\dot{\alpha}} \langle \bar{\Psi}_{a\dot{\alpha}}(x_i,y_i)\mathcal{O}_{20'}(x_k,y_k)\mathcal{O}_{20'}(x_l,y_l)\mathcal{O}_{20'}(x_m,y_m)\rangle \\
&\quad + \Sigma_{i=1}^4 \rho_i^{\alpha\dot{a}} \bar{\rho}_i^{a\dot{\alpha}} \langle A_{\alpha\dot{\alpha};a\dot{a}}(x_i,y_i)\mathcal{O}_{20'}(x_k,y_k)\mathcal{O}_{20'}(x_l,y_l)\mathcal{O}_{20'}(x_m,y_m)\rangle \\
&\quad + \dots
\end{aligned}$$

R-sym and spacetime structures:

$$\begin{aligned}
&\langle \mathcal{O}_{20'}(x_1,y_1)\,\mathcal{O}_{20'}(x_2,y_2)\,\mathcal{O}_{20'}(x_3,y_3)\,\mathcal{O}_{20'}(x_4,y_4)\rangle \\
&\quad = \Big(y_{12}^4\, y_{34}^4 + y_{13}^4\, y_{24}^4 + y_{14}^4\, y_{23}^4 + y_{12}^2\, y_{13}^2\, y_{24}^2\, y_{34}^2 \\
&\qquad + y_{12}^2\, y_{14}^2\, y_{23}^2\, y_{34}^2 + y_{13}^2\, y_{14}^2\, y_{23}^2\, y_{24}^2 \Big) \\
&\qquad \Big(\frac{1}{x_{12}^4\, x_{34}^4} f_1(u,v) + \frac{1}{x_{13}^4\, x_{24}^4} f_2(u,v) + \frac{1}{x_{14}^4\, x_{23}^4} f_3(u,v) \\
&\qquad + \dots + \frac{1}{x_{13}^2\, x_{14}^2\, x_{23}^2\, x_{24}^2} f_6(u,v) \Big)
\end{aligned}$$

4pt correlator of psu(2,2|4)

$$
\begin{aligned}
&\langle \mathcal{W}_2(X_1)\mathcal{W}_2(X_2)\mathcal{W}_2(X_3)\mathcal{W}_2(X_4)\rangle \\
&= \langle \mathcal{O}_{\mathbf{20'}}(\mathbf{x_1},\mathbf{y_1})\mathcal{O}_{\mathbf{20'}}(\mathbf{x_2},\mathbf{y_2})\mathcal{O}_{\mathbf{20'}}(\mathbf{x_3},\mathbf{y_3})\mathcal{O}_{\mathbf{20'}}(\mathbf{x_4},\mathbf{y_4})\rangle \\
&+ \Sigma_{i=1}^{4}\rho_i^{\alpha\dot{a}}\, \langle \Psi_{\alpha\dot{a}}(x_i,y_i)\mathcal{O}_{20'}(x_k,y_k)\mathcal{O}_{20'}(x_l,y_l)\mathcal{O}_{20'}(x_m,y_m)\rangle \\
&+ \Sigma_{i=1}^{4}\bar{\rho}_i^{a\dot{\alpha}}\, \langle \bar{\Psi}_{a\dot{\alpha}}(x_i,y_i)\mathcal{O}_{20'}(x_k,y_k)\mathcal{O}_{20'}(x_l,y_l)\mathcal{O}_{20'}(x_m,y_m)\rangle \\
&+ \Sigma_{i=1}^{4}\rho_i^{\alpha\dot{a}}\bar{\rho}_i^{a\dot{\alpha}}\, \langle A_{\alpha\dot{\alpha};a\dot{a}}(x_i,y_i)\mathcal{O}_{20'}(x_k,y_k)\mathcal{O}_{20'}(x_l,y_l)\mathcal{O}_{20'}(x_m,y_m)\rangle \\
&+ \dots
\end{aligned}
$$

R-sym and spacetime structures:

$$
\begin{aligned}
&\langle \mathcal{O}_{20'}(x_1,y_1)\,\mathcal{O}_{20'}(x_2,y_2)\,\mathcal{O}_{20'}(x_3,y_3)\,\mathcal{O}_{20'}(x_4,y_4)\rangle \\
&= \frac{y_{12}^4\, y_{34}^4}{x_{12}^4\, x_{34}^4} f_1(u,v) + \frac{y_{13}^4\, y_{24}^4}{x_{13}^4\, x_{24}^4} f_2(u,v) + \frac{y_{14}^4\, y_{23}^4}{x_{14}^4\, x_{23}^4} f_3(u,v) \\
&+ \frac{y_{12}^2\, y_{13}^2\, y_{24}^2\, y_{34}^2}{x_{12}^2\, x_{13}^2\, x_{24}^2\, x_{34}^2} f_5(u,v) + \frac{y_{12}^2\, y_{14}^2\, y_{23}^2\, y_{34}^2}{x_{12}^2\, x_{13}^2\, x_{24}^2\, x_{34}^2} f_5(u,v) \\
&+ \frac{y_{13}^2\, y_{14}^2\, y_{23}^2\, y_{24}^2}{x_{13}^2\, x_{14}^2\, x_{23}^2\, x_{24}^2} f_6(u,v)
\end{aligned}
$$

SUSY-invariance : $0 = \sum \frac{\partial}{\partial \rho}\langle \dots \rangle \, , \; 0 = \sum \frac{\partial}{\partial \bar{\rho}}\langle \dots \rangle$

Constraining the multiplet field expansion

Constraint for $\mathcal{W}_2(X)$:

$$0 \sim \left(\frac{\partial}{\partial X}\right)^3 \mathcal{W}_2(X)$$

Constraining the multiplet field expansion

Constraint for $\mathcal{W}_2\,(X)$:

$$0 \sim \left(\frac{\partial}{\partial X}\right)^3 \mathcal{W}_2\,(X)$$

$$A_{\alpha\dot{\alpha};a\dot{a}}(x,y) = J_{\alpha\dot{\alpha},a\dot{a}}(x,y) - \frac{1}{2}\frac{\partial}{\partial x^{\alpha\dot{\alpha}}}\frac{\partial}{\partial y^{a\dot{a}}}\mathcal{O}_{20'}(x,y)$$

Constraining the multiplet field expansion

Constraint for $\mathcal{W}_2(X)$:

$$0 \sim \left(\frac{\partial}{\partial X}\right)^3 \mathcal{W}_2(X)$$

$$A_{\alpha\dot{\alpha};a\dot{a}}(x,y) = J_{\alpha\dot{\alpha},a\dot{a}}(x,y) - \frac{1}{2}\frac{\partial}{\partial x^{\alpha\dot{\alpha}}}\frac{\partial}{\partial y^{a\dot{a}}}\mathcal{O}_{20'}(x,y)$$

SU(4)-current superprimary

Constraining the multiplet field expansion

Constraint for $\mathcal{W}_2(X)$

Correlator:

$$
\begin{aligned}
&\langle \mathcal{W}_2(X_1)\,\mathcal{W}_2(X_2)\,\mathcal{W}_2(X_3)\,\mathcal{W}_2(X_4)\rangle \\
&\quad = \langle \mathcal{O}_{20'}(x_1,y_1)\mathcal{O}_{20'}(x_2,y_2)\mathcal{O}_{20'}(x_3,y_3)\mathcal{O}_{20'}(x_4,y_4)\rangle \\
&\quad + \sum_{i=1}^{4} \rho_i^{\alpha\dot{a}}\bar{\rho}_i^{a\dot{\alpha}} \langle J_{\alpha\dot{\alpha};a\dot{a}}(x_i,y_i)\,\mathcal{O}_{20'}(x_k,y_k)\,\mathcal{O}_{20'}(x_l,y_l)\,\mathcal{O}_{20'}(x_m,y_m)\rangle \\
&\quad - \frac{1}{2}\sum_{i=1}^{4} \rho_i^{\alpha\dot{a}}\bar{\rho}_i^{a\dot{\alpha}} \frac{\partial}{\partial x_i^{\alpha\dot{\alpha}}}\frac{\partial}{\partial y_i^{a\dot{a}}} \langle \mathcal{O}_{20'}(x_1,y_1)\mathcal{O}_{20'}(x_2,y_2)\mathcal{O}_{20'}(x_3,y_3)\mathcal{O}_{20'}(x_4,y_4)\rangle \\
&\quad + \sum_{i=1}^{4}\sum_{j\neq i} \rho_i^{\alpha\dot{a}}\bar{\rho}_j^{a\dot{\alpha}} \langle \Psi_{\alpha\dot{a}}(x_i,y_i)\bar{\Psi}_{a\dot{\alpha}}(x_j,y_j)\mathcal{O}_{20'}(x_k,y_k)\mathcal{O}_{20'}(x_l,y_l)\rangle + \ldots
\end{aligned}
$$

Add constraints from SUSY

Constraint for $\mathcal{W}_2(X)$
Correlator:

$$\begin{aligned}
&\langle \mathcal{W}_2(X_1)\mathcal{W}_2(X_2)\mathcal{W}_2(X_3)\mathcal{W}_2(X_4)\rangle \\
&\quad = \langle \mathcal{O}_{20'}(x_1,y_1)\mathcal{O}_{20'}(x_2,y_2)\mathcal{O}_{20'}(x_3,y_3)\mathcal{O}_{20'}(x_4,y_4)\rangle \\
&\qquad + \Sigma_{i=1}^4 \rho_i^{\alpha\dot{a}}\bar{\rho}_i^{a\dot{\alpha}} \langle J_{\alpha\dot{\alpha};a\dot{a}}(x_i,y_i)\,\mathcal{O}_{20'}(x_k,y_k)\,\mathcal{O}_{20'}(x_l,y_l)\,\mathcal{O}_{20'}(x_m,y_m)\rangle \\
&\qquad - \frac{1}{2}\Sigma_{i=1}^4 \rho_i^{\alpha\dot{a}}\bar{\rho}_i^{a\dot{\alpha}} \frac{\partial}{\partial x_i^{\alpha\dot{\alpha}}}\frac{\partial}{\partial y_i^{a\dot{a}}} \langle \mathcal{O}_{20'}(x_1,y_1)\mathcal{O}_{20'}(x_2,y_2)\mathcal{O}_{20'}(x_3,y_3)\mathcal{O}_{20'}(x_4,y_4)\rangle \\
&\qquad + \Sigma_{i=1}^4 \Sigma_{j\neq i} \rho_i^{\alpha\dot{a}}\bar{\rho}_j^{a\dot{\alpha}} \langle \Psi_{\alpha\dot{a}}(x_i,y_i)\bar{\Psi}_{a\dot{\alpha}}(x_j,y_j)\mathcal{O}_{20'}(x_k,y_k)\mathcal{O}_{20'}(x_l,y_l)\rangle + \ldots
\end{aligned}$$

- Insert spacetime and R-sym structures x functions of cross ratios
- Impose Invariance under SUSY
- Solve constraints $\Rightarrow$ PDEs
- Solve PDEs

Solutions for 4pt functions

$$f_2(z,\bar z) = \frac{1}{z\,\bar z}\, f_1\,(z,\bar z) + \ldots$$

$$f_3(z,\bar z) = \frac{(1-z)(1-\bar z)}{z\,\bar z}\, f_1\,(z,\bar z) + \ldots$$

$$f_4(z,\bar z) = \frac{z+\bar z}{z\,\bar z}\, f_1\,(z,\bar z) + \ldots$$

$$f_5(z,\bar z) = \left(-2 + \frac{1}{z} + \frac{1}{\bar z}\right) f_1\,(z,\bar z) + \ldots$$

$$f_6(z,\bar z) = \frac{-2+z+\bar z}{z\bar z} f_1\,(z,\bar z) + \ldots$$

[Dolan, Gallot, Sokatchev '04,...]

$$u = \frac{x_{12}^2\,x_{34}^2}{x_{13}^2\,x_{24}^2} = z\bar z$$

$$v = \frac{x_{14}^2\,x_{23}^2}{x_{13}^2\,x_{24}^2} = (1-z)(1-\bar z)$$

Thank you for your attention!
Stay tuned for the 5pt results!

Explicit multiplet field expansion

$$\mathcal{W}_{p=2}(X) = \left(1 - \frac{1}{2}\rho^{\alpha\dot{a}}\bar{\rho}^{a\dot{\alpha}}\frac{\partial}{\partial x^{\alpha\dot{\alpha}}}\frac{\partial}{\partial y^{a\dot{a}}} + \frac{1}{16}\rho^{\alpha\dot{a}}\bar{\rho}^{a\dot{\alpha}}\rho^{\beta\dot{b}}\bar{\rho}^{b\dot{\beta}}\frac{\partial}{\partial x^{\alpha\dot{\alpha}}}\frac{\partial}{\partial y^{a\dot{a}}}\frac{\partial}{\partial x^{\beta\dot{\beta}}}\frac{\partial}{\partial y^{b\dot{b}}}\right)\mathcal{O}^{[0,2,0]_{(0,0)}}$$

$$+ \rho^{\alpha\dot{a}}\left(1 - \frac{1}{4}\rho^{\beta\dot{b}}\bar{\rho}^{b\dot{\beta}}\frac{\partial}{\partial x^{\beta\dot{\beta}}}\frac{\partial}{\partial y^{b\dot{b}}}\right)\mathcal{O}_{\alpha\dot{a}}^{[0,1,1]_{(\frac{1}{2},0)}}(x,y)$$

$$+ \bar{\rho}^{a\dot{\alpha}}\left(1 - \frac{1}{4}\rho^{\beta\dot{b}}\bar{\rho}^{b\dot{\beta}}\frac{\partial}{\partial x^{\beta\dot{\beta}}}\frac{\partial}{\partial y^{b\dot{b}}}\right)\mathcal{O}_{a\dot{\alpha}}^{[1,1,0]_{(0,\frac{1}{2})}}(x,y)$$

$$+ \rho^{\alpha\dot{a}}\bar{\rho}^{a\dot{\alpha}}\mathcal{O}_{\alpha\dot{a},a\dot{\alpha}}^{[1,0,1]_{(\frac{1}{2},\frac{1}{2})}}(x,y)$$

$$+ \ldots$$

Truncation in internal coordinates:

$$0 = \frac{\partial}{\partial y^{b\dot{b}}} \mathcal{O}^{[1,0,1]_{(\frac{1}{2},\frac{1}{2})}}_{\alpha\dot{a},a\dot{\alpha}}(x,y)\,|_{(ab),(\dot{a}\dot{b})}$$

$$0 = \frac{\partial}{\partial y^{c\dot{c}}} \mathcal{O}^{[1,0,0]_{(1,\frac{1}{2})}}_{\alpha\dot{a},\beta\dot{b};a\dot{\alpha}}(x,y)\,|_{(ac),(\dot{a}\dot{c})}$$

$$0 = \frac{\partial}{\partial y^{c\dot{c}}} \mathcal{O}^{[0,0,1]_{(\frac{1}{2},1)}}_{a\dot{\alpha},b\dot{\beta};\alpha\dot{a}}(x,y)\,|_{(ac),(\dot{a}\dot{c})}$$

Conservation equations:

$$0 = \epsilon^{\beta\alpha}\epsilon^{\dot{\beta}\dot{\alpha}} \frac{\partial}{\partial x^{\beta\dot{\beta}}} \mathcal{O}^{[1,0,1]_{(\frac{1}{2},\frac{1}{2})}}_{\alpha\dot{a},a\dot{\alpha}}(x,y)$$

$$0 = \epsilon^{\gamma\alpha}\epsilon^{\dot{\gamma}\dot{\alpha}} \frac{\partial}{\partial x^{\gamma\dot{\gamma}}} \mathcal{O}^{[0,0,1]_{(\frac{1}{2},1)}}_{a\dot{\alpha},b\dot{\beta};\alpha\dot{a}}(x,y)$$

$$0 = \epsilon^{\gamma\alpha}\epsilon^{\dot{\gamma}\dot{\alpha}} \frac{\partial}{\partial x^{\gamma\dot{\gamma}}} \mathcal{O}^{[1,0,0]_{(1,\frac{1}{2})}}_{\alpha\dot{a},\beta\dot{b};a\dot{\alpha}}(x,y)$$

$$0 = \epsilon^{\gamma\alpha}\epsilon^{\dot{\gamma}\dot{\alpha}} \frac{\partial}{\partial x^{\gamma\dot{\gamma}}} \mathcal{O}^{[0,0,0]_{(1,1)}}_{\alpha\dot{a},\beta\dot{b};a\dot{\alpha},b\dot{\beta}}(x,y)$$

PSU(1,1|2)

$$\langle J(x_1, y_1) J(x_2, y_2) J(x_3, y_3) J(x_4, y_4) J(x_5, y_5) \rangle$$
$$= \frac{y_{12}^2\, y_{34}\, y_{45}\, y_{53}}{x_{12}^2\, x_{34}\, x_{45}\, x_{53}} f_1(u,v) + \frac{y_{23}^2\, y_{14}\, y_{45}\, y_{51}}{x_{23}^2\, x_{14}\, x_{45}\, x_{51}} f_2(u,v) + \ldots + \frac{y_{12}\, y_{23}\, y_{34}\, y_{45}\, y_{51}}{x_{12}\, x_{23}\, x_{34}\, x_{45}\, x_{51}} f_6(u,v),$$

where $y_{ij} = y_i - y_j,\ x_{ij} = x_i - x_j$

Cross ratios : $u = \frac{x_{12}\, x_{34}}{x_{13}\, x_{24}},\ v = \frac{x_{23}\, x_{45}}{x_{24}\, x_{35}}$

Constraints : $\sum_{i=1}^{6} \frac{\partial}{\partial u} f_i(u,v) = 0,\ \sum_{i=1}^{6} \frac{\partial}{\partial v} f_i(u,v) = 0$