

Pure spinor superfield techniques in (twisted) supergravity

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This talk is based on joint work with Ingmar Saberi
(arXiv:2304.12371).

It builds on previous work with Richard Eager, Chris Elliott,
Simone Noja, Johannes Walcher and Brian R. Williams.

Introduction

One interesting thing about supersymmetric field theories is the presence of protected subsectors which are sensitive to topological and holomorphic structures on spacetime.

These subsectors are extracted by *twisting*: Let $Q \in \mathfrak{g}_{\text{odd}}$ with $[Q, Q] = 0$ and take invariants with respect to the odd abelian algebra spanned by Q .

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The twisted theories have many desirable properties:

- They are topological-holomorphic field theories (and thus much simpler than the full theory).
- They can typically be formulated in terms of geometric moduli problems on spacetime.
- Good behavior under quantization, nice results on symmetry enhancements...

Two natural questions arise:

1. How can we compute twists efficiently?
2. What can we learn from the twists about the full theory?

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*In superspace, the supersymmetries act geometric.
Twisting just means taking invariants in some odd
direction.*

Plan

I. How to produce universal superspace descriptions which are compatible with twisting?

→ Pure spinor superfields

II. What can we learn about the full theory and its twists?

→ Today: Eleven-dimensional supergravity

I. Pure spinor superfields and twisting

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Let $\mathfrak{p} = \mathfrak{p}_0 \ltimes \mathfrak{t}$ be a super Poincaré algebra. We call $\mathfrak{t}$ the supertranslation algebra and denote the associated super Lie group by T . ($T \sim$ superspace)

There are two actions on the free superfield $C^\infty(T)$

$$\mathcal{L}, \mathcal{R} : \mathfrak{p} \longrightarrow \text{Vect}(T)$$

given by the usual vector fields

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} + \gamma_{\alpha\beta}^\mu \theta^\beta \frac{\partial}{\partial x^\mu} \quad D_\alpha = \frac{\partial}{\partial \theta^\alpha} - \gamma_{\alpha\beta}^\mu \theta^\beta \frac{\partial}{\partial x^\mu}.$$

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Think of the superspace T as a supermanifold equipped with a distribution.

The nilpotence variety

$$Y = \{Q \in \mathfrak{t}_1 | [Q, Q] = 0\}$$

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The pure spinor superfield formalism constructs multiplets from $\mathcal{O}_Y$ -modules.

$$\begin{array}{ccc} \mathrm{Mod}_{\mathcal{O}_Y}^{\mathfrak{p}_0} & \xrightarrow{A^\bullet} & \mathrm{Mult}_{\mathfrak{p}} \\ \downarrow & \swarrow \nearrow & \\ \mathrm{Mod}_{C^\bullet(t)}^{\mathfrak{p}_0} & & \end{array}$$

The pure spinor construction is compatible with twisting.

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Choose $Q \in Y$. We can twist the multiplet by deforming the differential.

On the other hand, we can twist the input data for the formalism.

$$\mathfrak{p} \longrightarrow \mathfrak{p}_Q = H^\bullet(\mathfrak{p}, [Q, -])$$

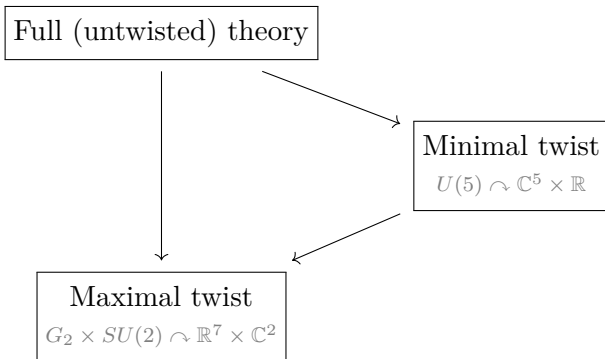
$\mathfrak{p}_Q$ is the residual symmetry algebra and has a new nilpotence variety Y_Q controlling further twists.

Both procedures are compatible [Saberì–Williams]:

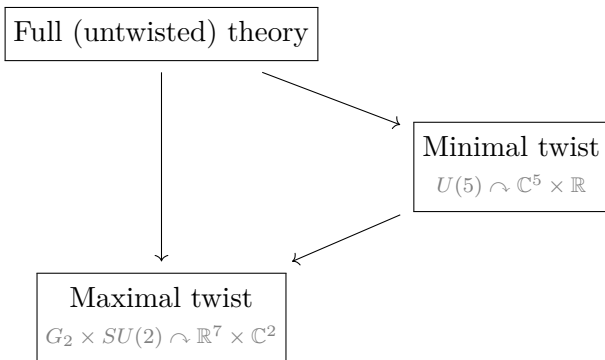
$$A^\bullet(\mathcal{O}_Y)^Q \cong A^\bullet(\mathcal{O}_{Y_Q})$$

II. (Twisted) eleven-dimensional supergravity

Eleven-dimensional supergravity has two distinct twists.



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The maximal twist is Poisson–Chern–Simons theory.

$$\left(\Omega^{0,\bullet}(\mathbb{C}^2) \otimes \Omega^\bullet(\mathbb{R}^7), \bar{\partial}_{\mathbb{C}^2} + d_{\mathbb{R}^7}, \{-, -\}_{PB} \right)$$

Poisson–Chern–Simons theory

Let X be a Calabi–Yau 2-fold. Think of the complex structure as an involutive distribution. Recall that the de Rham differential splits as $d_{\text{dR}} = \bar{\partial} + \partial$.

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Poisson–Chern–Simons theory on $X \times M$ is modeled by

$$(\Omega^{0,\bullet}(X), \bar{\partial}, \{-, -\}_{PB}) \otimes (\Omega^\bullet(M), d_{\text{dR}}),$$

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$$\{\alpha, \beta\}_{PB} = \pi(\partial\alpha \wedge \partial\beta).$$

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We can think about Poisson–Chern–Simons theory as modeling certain deformations of the complex structure, i.e. of a distribution.

Back to superspace

In eleven dimensions, we have three different superspaces:

$$\mathfrak{p} \rightsquigarrow T, \quad \mathfrak{p}_{Q_{\min}} \rightsquigarrow T_{Q_{\min}}, \quad \mathfrak{p}_{Q_{\max}} \rightsquigarrow T_{Q_{\max}}$$

All three are equipped with distributions spanned by odd left invariant vector fields. Except for the maximal twist, these are non-involutive.

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Consider differential forms $(\Omega^\bullet(T), d_{\text{dR}}) \sim \mathbb{C}[x, \theta, d\theta, dx]$.

In a left-invariant basis $\lambda = d\theta$ and $v = dx + \lambda\theta$:

$$d_{\text{dR}} = \underbrace{\lambda^\alpha \gamma_{\alpha\beta}^\mu \lambda^\beta \frac{\partial}{\partial v^\mu}}_{\gamma} + \underbrace{\lambda^\alpha \left(\frac{\partial}{\partial \theta^\alpha} - \theta^\beta \gamma_{\alpha\beta}^\mu \frac{\partial}{\partial x^\mu} \right)}_{\bar{\partial}} + \underbrace{v^\mu \frac{\partial}{\partial x^\mu}}_{\partial}$$

The differential γ reflects the non-involutivness of the distribution.

Taking cohomology with respect to γ gives the pure spinor multiplet $A^\bullet(\mathcal{O}_Y) = H^0(\Omega^\bullet(T), \gamma)$.

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Applying this procedure in eleven-dimensions constructs:

- Poisson–Chern–Simons theory on $\mathbb{R}^7 \times \mathbb{C}^2$ from $T_{Q_{\max}}$.
- A quartic action functional for the minimal twist on $\mathbb{C}^5 \times \mathbb{R}$ from $T_{Q_{\min}}$.
- Cederwall’s quartic action of eleven-dimensional supergravity in the pure spinor formalism from T .

Thank you!