Assessing the Role of Finite-Temperature Corrections in Dark Matter Freeze-In

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Based on ongoing work (2311.xxxxx) in collaboration with
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Emmy Noether-Programm
DFG Deutsche Forschungsgemeinschaft

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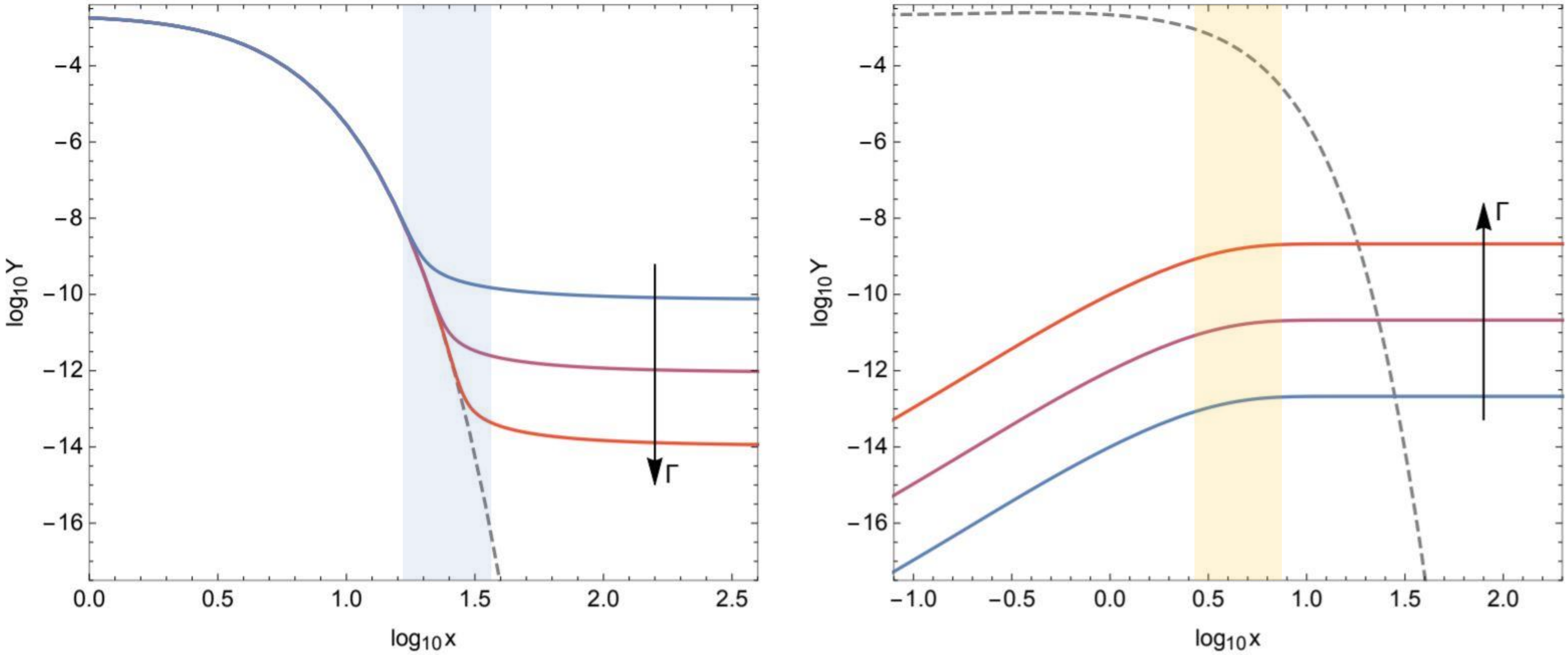
Motivation: Freeze-Out vs. Freeze-In

Adapted from: Bernal et al. (2017) arXiv:1706.07442



$$x_{\rm f.o.} = M/T_{\rm f.o.} \simeq 25 - 30$$

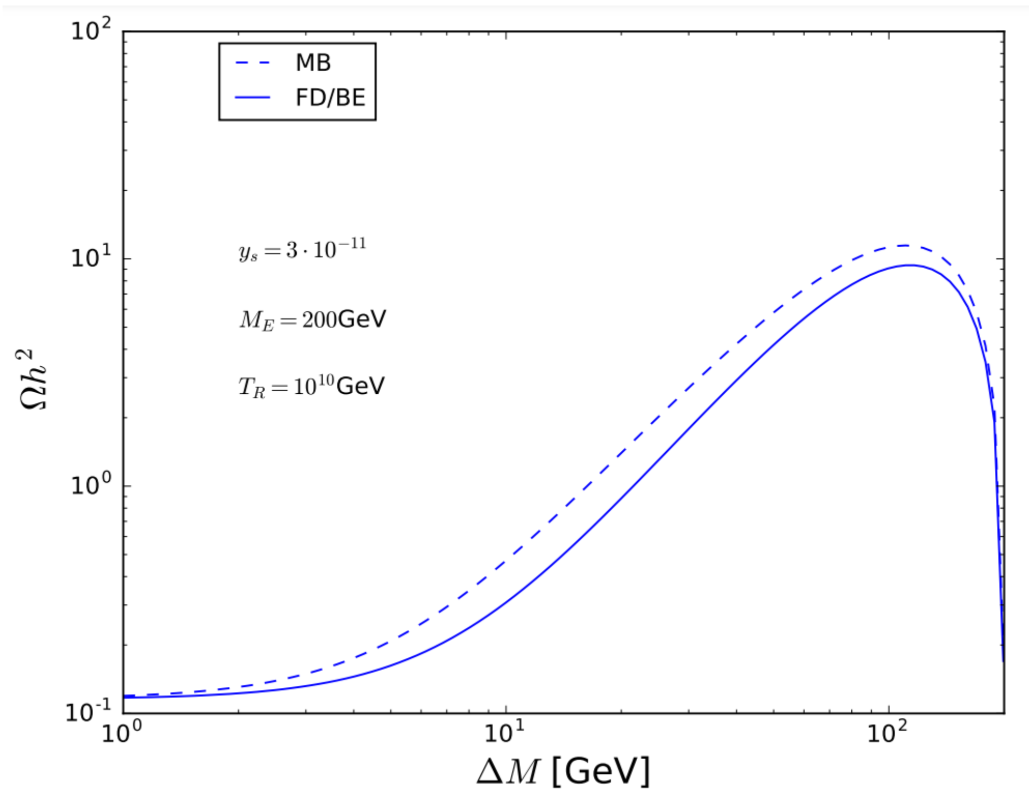
$$x_{\rm f.i.} = M/T_{\rm f.i.} \simeq 2 - 5$$

Motivation: relativistic dynamics

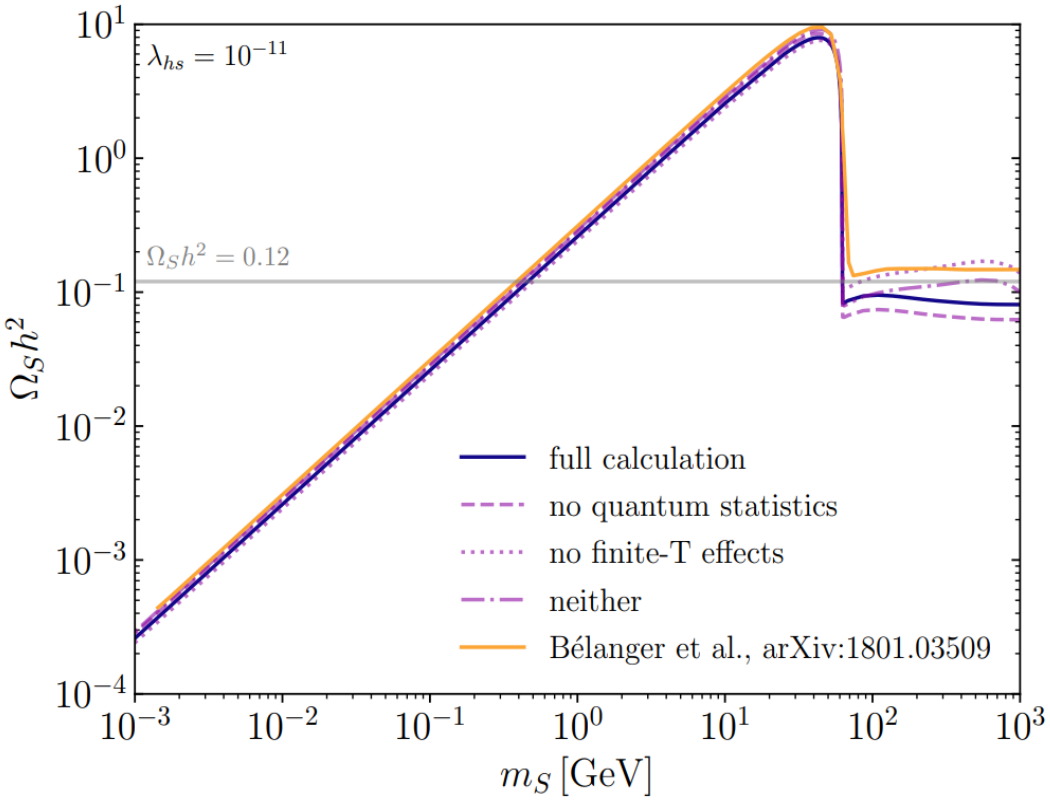
Vanilla Freeze-In $1 \pm f_{\mp}(E/T) \simeq 1$

misses Bose-enhancement and Pauli-blocking (relevant at high-T)!

Arcadi et al. (2019), arXiv:1906.07659
Lebedev&Toma (2019), arXiv:1908.05491
De Romeri et al. (2020), arXiv:2003.12606



Credit: Bélanger et al. arXiv:1801.03509



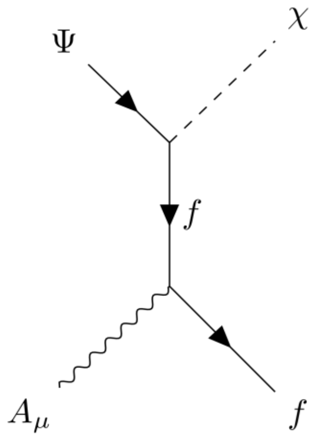
Credit: Bringmann et al. arXiv:2111.14871

Motivation: thermal regulators

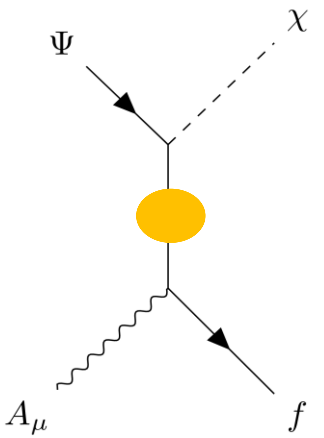
$$\mathcal{L}_{\rm int} \supset -y_{\rm DM}\bar{\Psi}\, f_{\rm SM}\chi + h.c.$$

"Vector-like portal"

Giacchino et al. (2013), arXiv:1307.6480
Colucci et al. (2018), arXiv:1805.10173
Bélanger et al. (2018), arXiv:1811.05478
Arina et al. (2020), arXiv:2001.05024

$|\mathcal{M}|^2 \sim \Big|$  $\Big|^2 \propto \frac{1}{t}$

$$\sigma \propto \int \mathrm{d}t\, |\mathcal{M}|^2 \sim \int \frac{\mathrm{d}t}{t} \sim \ln\left(\frac{m_f}{T}\right)$$

$\Big|$  $\Big|^2 \propto \frac{1}{t - m_f^2(T)}$

$$\sigma \propto \ln g$$

$$m_f(T) \sim gT$$

No et al. (2020), arXiv:1908.11387
Bélanger et al. (2020), arXiv:2005.06294
Calibbi et al. (2021), arXiv:2102.06221
Grzadkowski et al. (2022), arXiv:2108.01757
Decant et al. (2022), arXiv:2111.09321
Bringmann et al. (2022), arXiv:2111.14871

What we aim at

- Calculate the DM production rate in the Closed Time Path formalism of TQFT
- Accounting for gauge interactions with 1-loop fully-resummed propagators
- Compare with:
  - Semi-classical Boltzmann approach with thermal masses
  - Rate derived with Hard Thermal Loop approximated propagators
- Provide best practices for the DM pheno community

Closed Time Path formalism in a nutshell

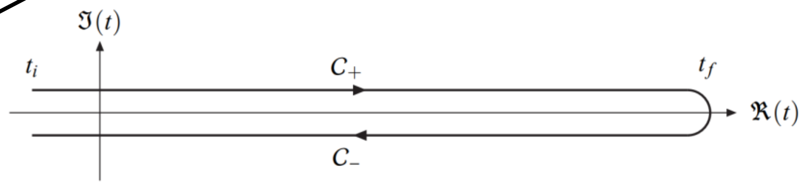
"CTP" = "Keldysh-Schwinger" = "real-time" = "in-in" formalism

Construct 2-particle irreducible (2PI) effective action Γ

Compute propagators on the closed time path

Find Schwinger-Dyson equations of motion

$$\frac{\delta\Gamma}{\delta G^{ab}} = 0$$



$$(G)^{ab} = \begin{pmatrix} G^{++} & G^{+-} \\ G^{-+} & G^{--} \end{pmatrix}$$

Kadanoff-Baym equations

Kadanoff-Baym ansatz
and some manipulations...

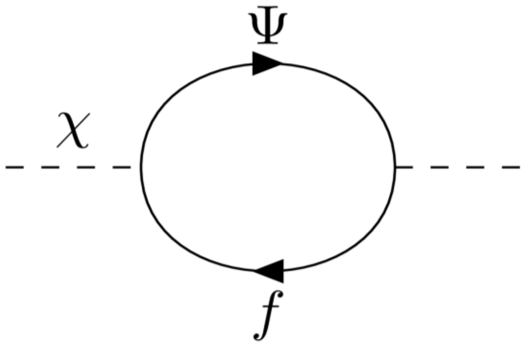
Boltzmann-like evolution

Calzetta&Hu, PRD 37 (1988) 2878
Berges (2004), arXiv:hep-ph/0409233
Prokopec et al. (2004), arXiv:hep-ph/0312110
Garny et al. (2010), arXiv:0911.4122
Carrington&Guo (2010), arXiv:hep-ph/1010.2978
Anisimov et al. (2010), arXiv:1012.5821
Drewes et al. (2013), arXiv:1305.0267
Garbrecht et al. (2013), arXiv:1303.5498
Drewes et al. (2015), arXiv:1510.05646
Garbrecht (2019),arXiv:1812.02651

DM rate equation in Thermal QFT

$$\mathcal{L}_{\rm int} \supset -y_{\rm DM}\bar{\Psi}\, f_{\rm SM}\chi + h.c.$$

$$\dot{n}_\chi + 3Hn_\chi = \gamma_{\rm DM} \sim \int \mathrm{d}^3\mathbf{p}\,\frac{\Pi^{\mathcal{A}}_\chi}{\omega_\chi} f_\chi(\omega_\chi)$$

$\Pi^{\mathcal{A}}_\chi = -{\rm Im}\Pi^{\rm R}_\chi =$ 

$+ \ldots$

$$\Pi^{\mathcal{A}}_\chi(P) \sim \int \mathrm{d}^4K\, S^{\mathcal{A}}_\Psi(K) S^{\mathcal{A}}_f(P-K) + \ldots$$

Approximations:

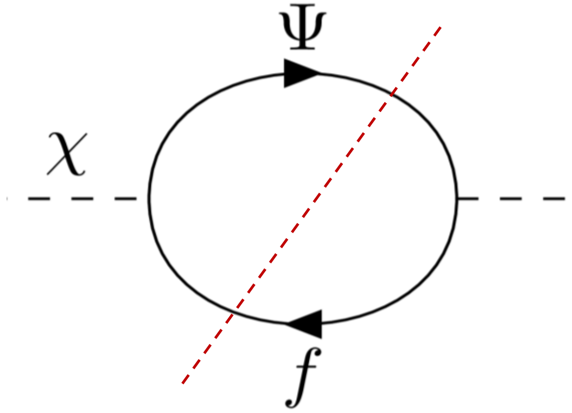
1. **Loop order** of DM self-energy: LO (1-loop), **NLO** (2-loop) etc…
2. Propagators appearing in the DM self-energy:

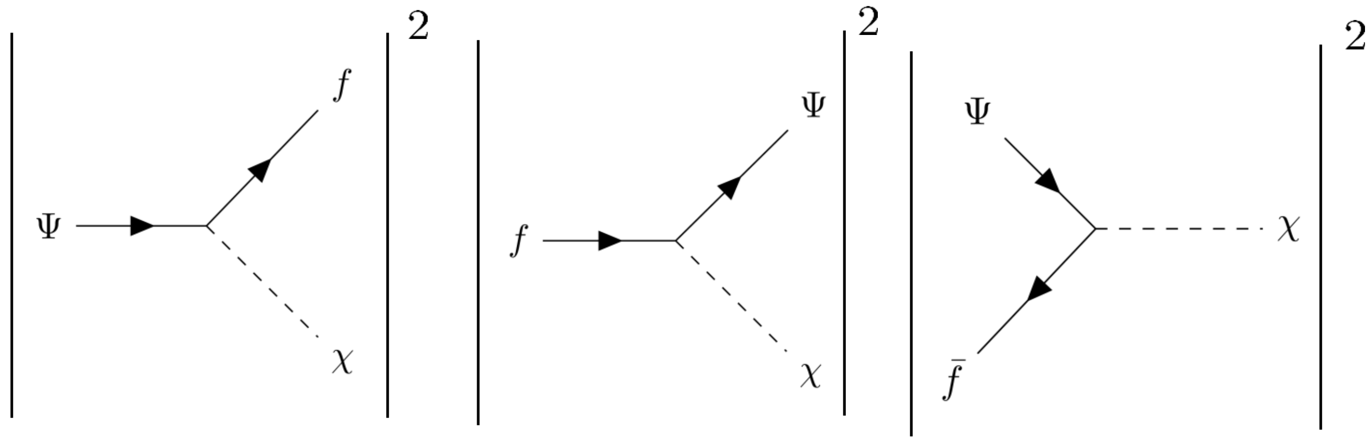
   Tree-level, Perturbative 1-loop, HTL-resummed, Fully-resummed

Example: tree-level propagators

$$\Pi_\chi^{\mathcal{A}}(p_0,\vec{p}) = 2y_{\mathrm{DM}}^2 \int \mathrm{d}^4k\,\mathrm{Tr}\Big\{P_L \slashed{S}_\Psi^{\mathcal{A}}(k) P_R \slashed{S}_f^{\mathcal{A}}(p-k)\Big\}\big(1-f_+(k_0)-f_+(p_0-k_0)\big)\,\mathrm{sign}\big(k_0(p_0-k_0)\big)$$

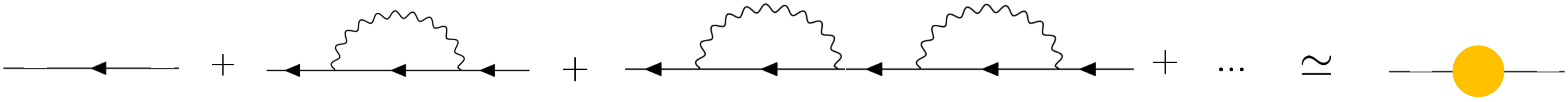
Tree-level $\slashed{S}_{\Psi,f}^{\mathcal{A}}(k) \propto (\slashed{k} + m_{\Psi,f})\,\delta\left(k^2 - m_{\Psi,f}^2\right)$ → $k^0 = \pm\sqrt{\vec{k}^2 + m_{\Psi,f}^2}$

$\Pi_\chi^{\mathcal{A},\mathrm{tree}} =$ 

→ 

Decays and inverse decays!

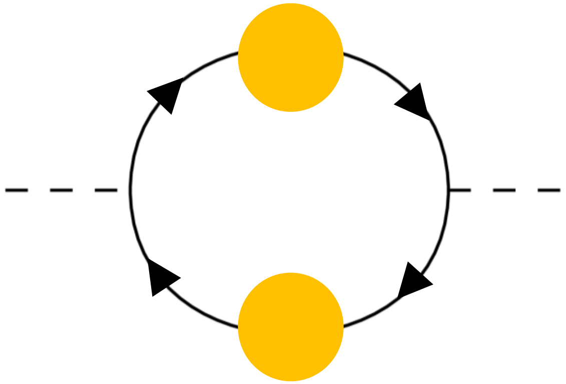
One-loop resummed propagators



Resummed

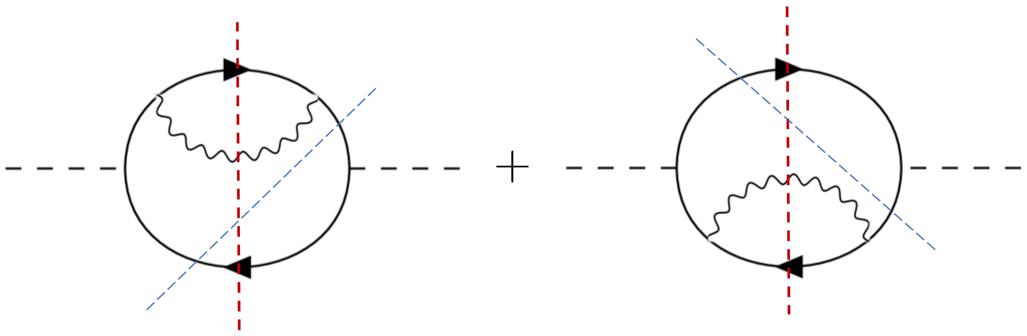
$$= \frac{i}{\not p - m - \Sigma(p)}$$

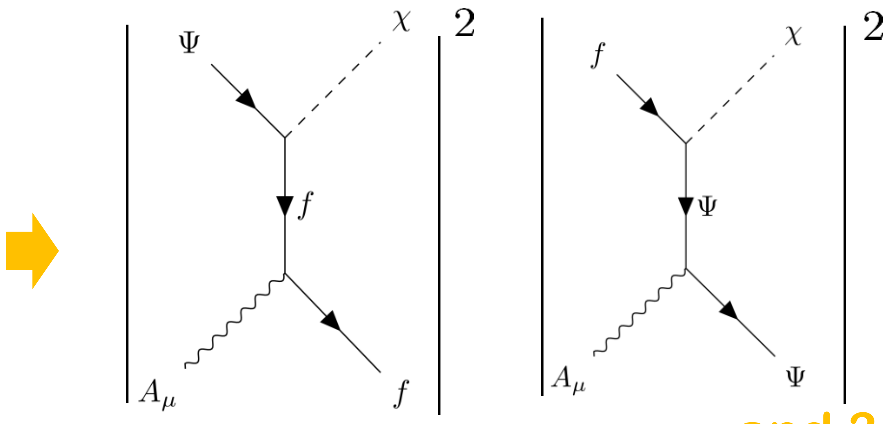
$$\Pi_\chi^{\mathcal{A},\mathrm{res}} =$$



Perturbative approximation

$\approx$





$+ \ldots$

Scatterings and 3-body decays!

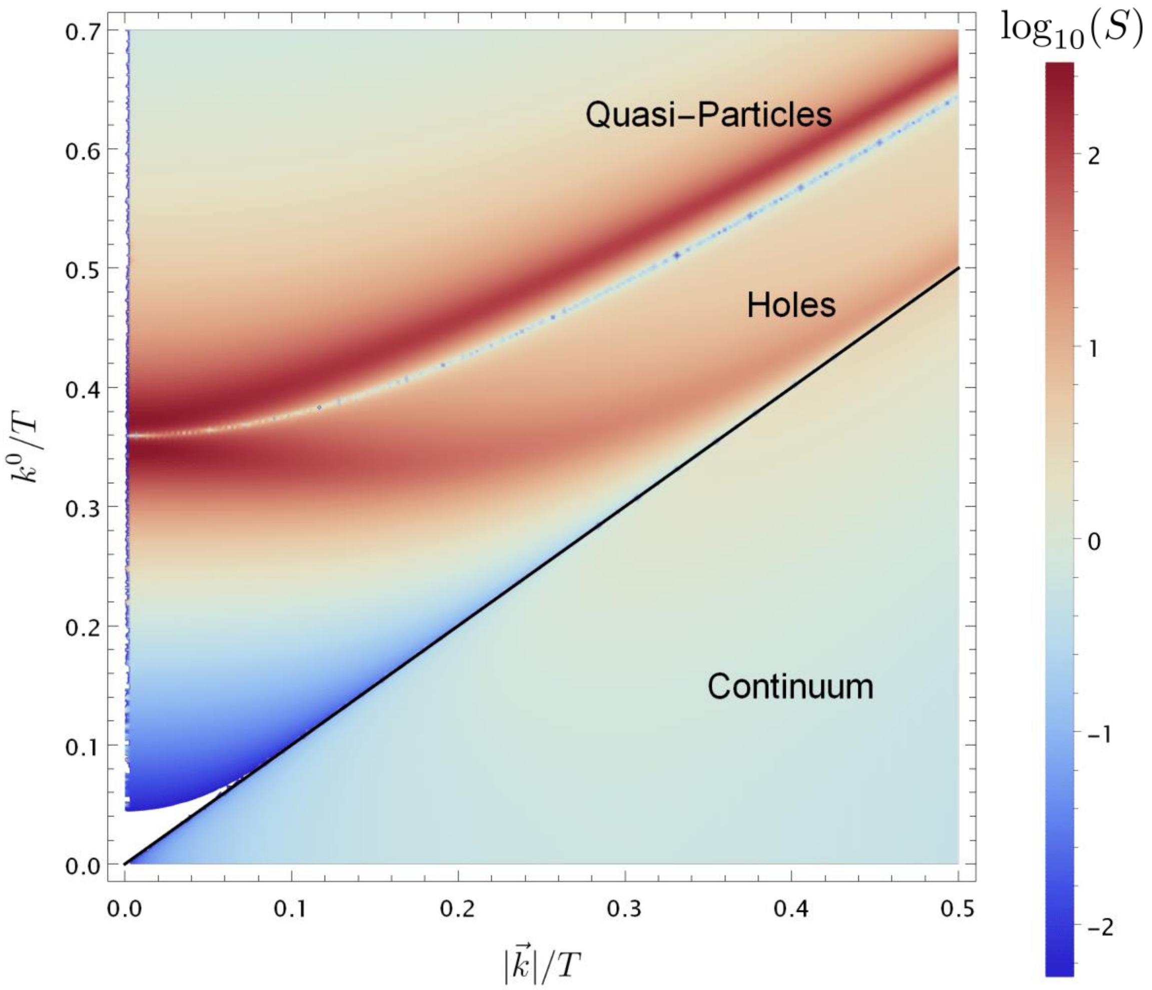
One-loop resummed propagators

$$\not{S}^{\mathcal{A}}(K) \sim \frac{\Gamma(K)}{\left(\left(\not{K} - \not{\Sigma}^{\mathcal{H}}(K)\right)^2 - m^2\right)^2 + \Gamma^2(K)}$$

$$\lim_{\Gamma \to 0} \not{S}^{\mathcal{A}} \propto \delta(k_0 - \omega_+(|\vec{k}|)) + \quad \text{“Quasi-particles”}$$

$$\delta(k_0 - \omega_-(|\vec{k}|)) + \quad \text{“Holes”}$$

$$\rho_{\text{con}}(k_0)\theta(-k_0^2 + \vec{k}^2) \quad \text{“Continuum”}$$



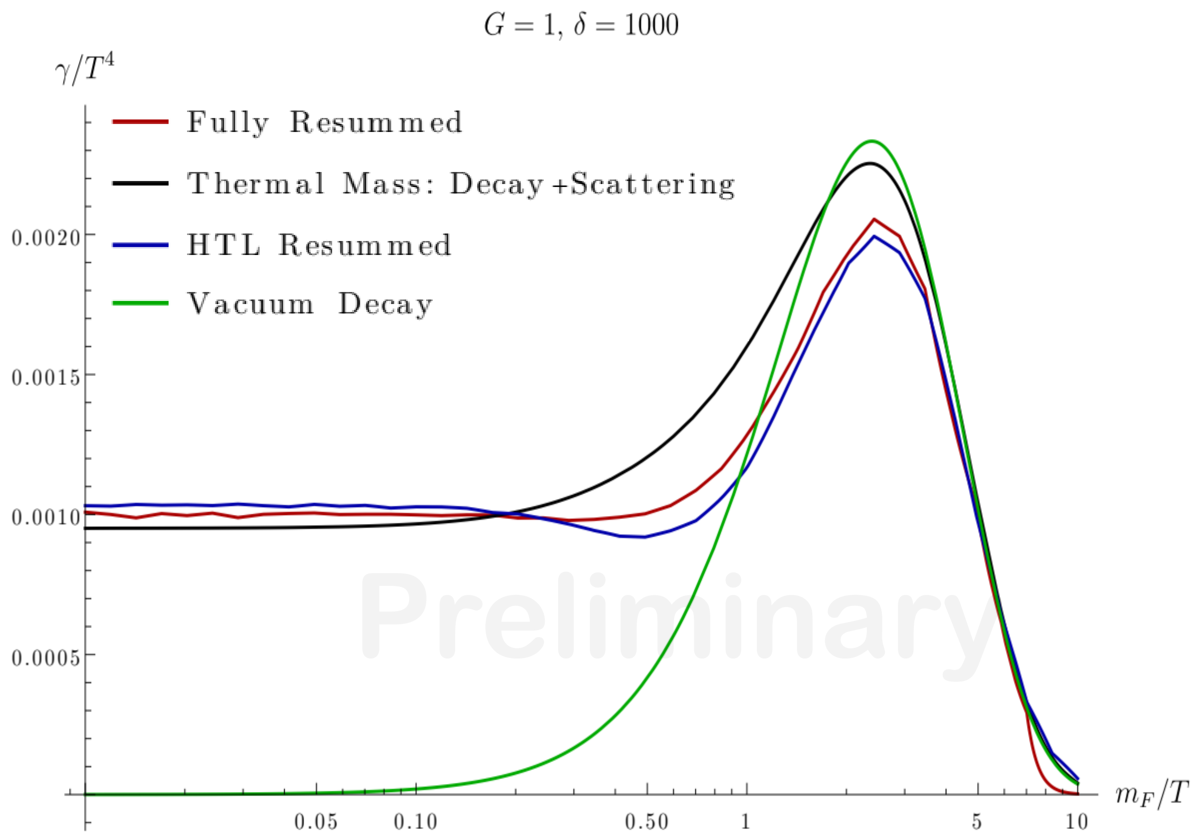
Literature: either **HTL** (high-T) or an **interpolation** between **UR** ($T \gg M$) and **NR** ($T \ll M$) regimes. Might not capture the relevant scale $T \sim M$ well!

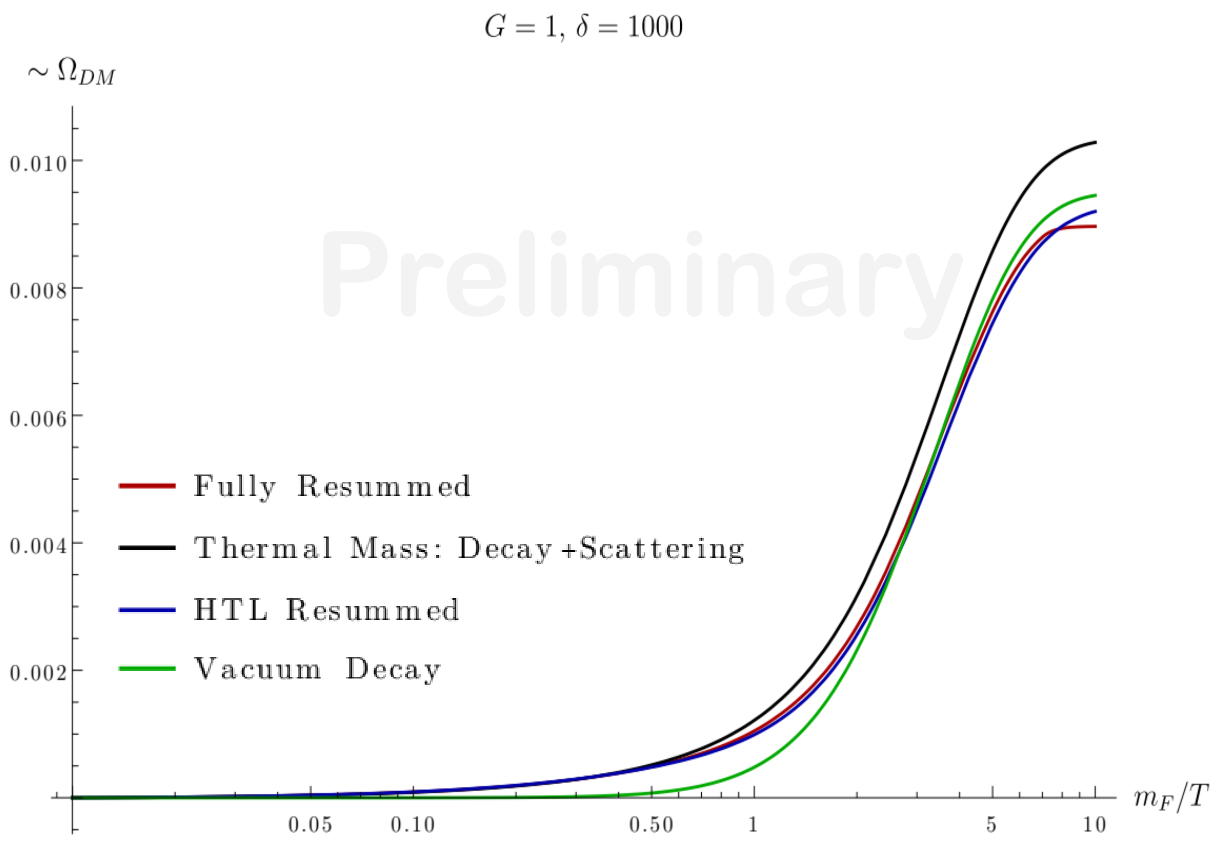
Becker, **EC**, Harz, Tamarit (in preparation)

E.g., Garbrecht et al. (2019), Biondini&Ghiglieri (2020) and references therein...

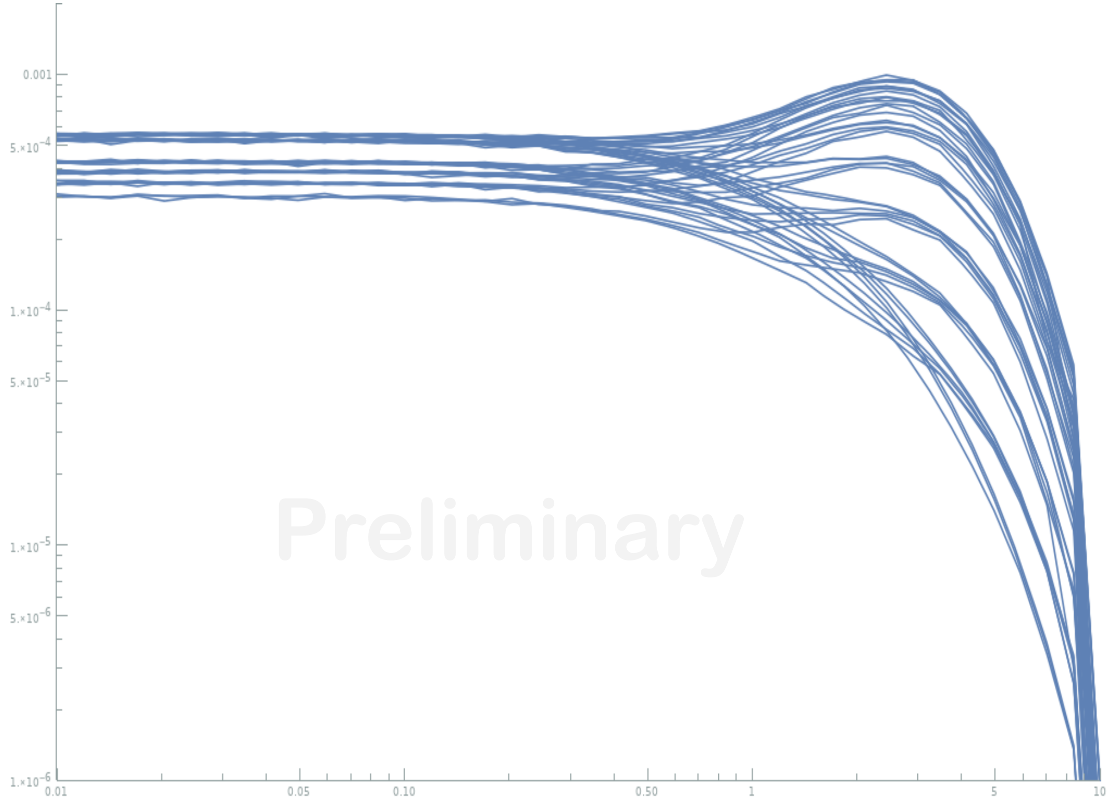
Some (preliminary) results

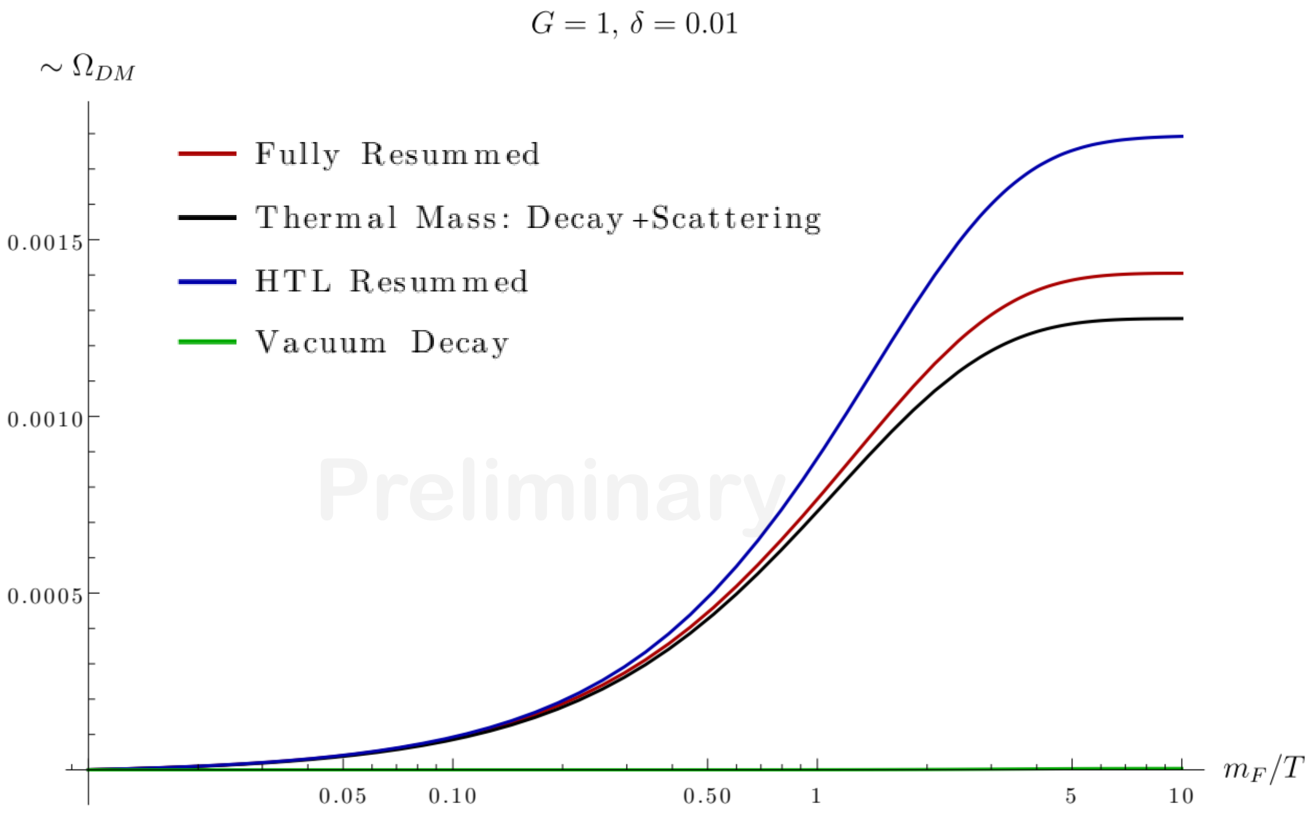
Becker, **EC**, Harz, Tamarit (in preparation)





Some (preliminary) results

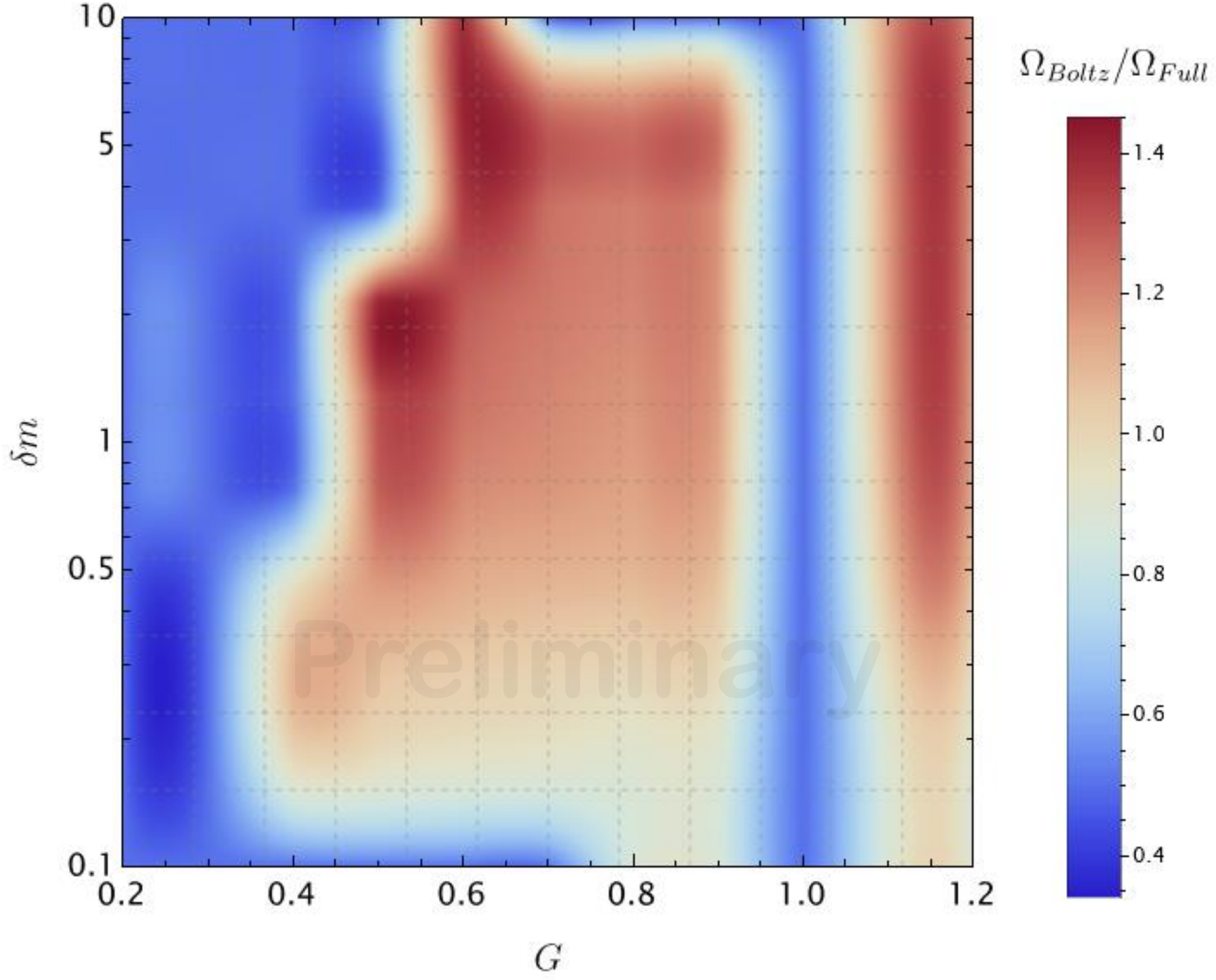




Becker, **EC**, Harz, Tamarit (in preparation)

Some results

Becker, **EC**, Harz, Tamarit (in preparation)



Disclaimer: some points are still to be computed! (Darker blue points)

$O(10\%) - O(40\%)$ differences

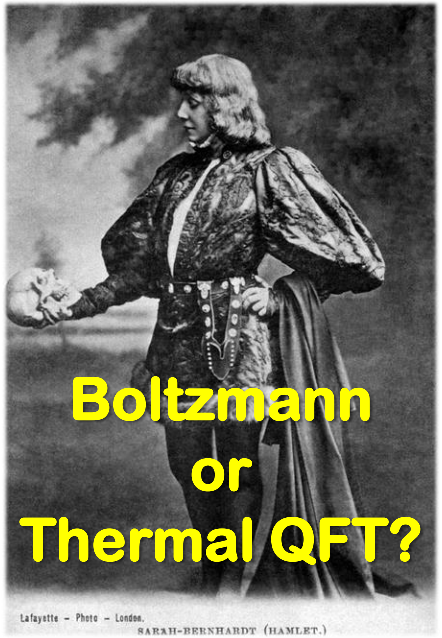
Take home messages

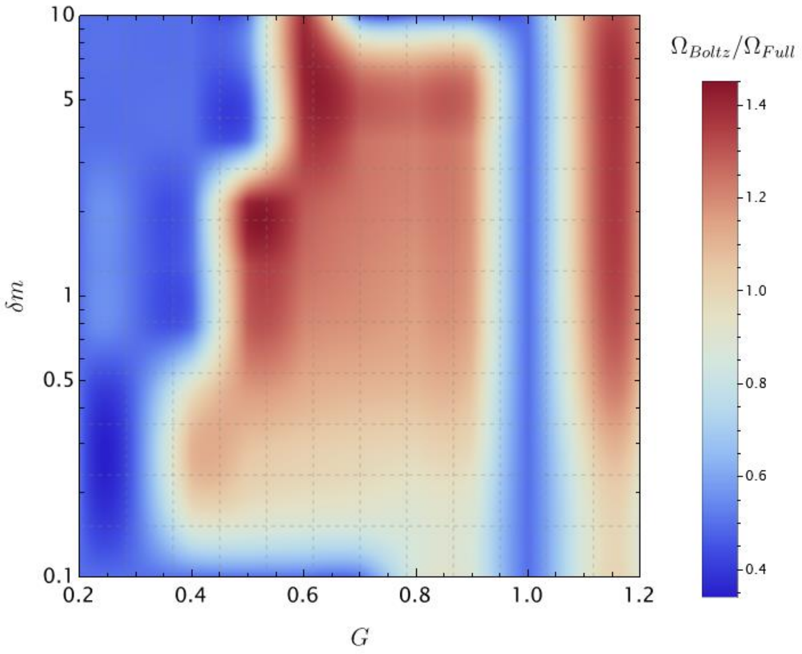
1 $T_{f.i.} \sim M$ finite-T corrections are relevant for freeze-in

2 Freeze-in from 2PIEA in the CTP formalism

- Rate equation for DM within finite-density medium
- Thermal effects with fully-resummed propagators → all T vs. M regimes

3





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What remains to do

1) Complete comparison with Boltzmann and HTL.

2) Determine theoretical uncertainty for different $G \text{ v.s. } \delta m$

- Consistent description at finite-T (at **LO** in the 2PIEA and NLO eventually)

- Encompass all regimes consistently,
  especially bulk production at $T = T_{\text{fi}} \sim M$

Thank you for your attention!

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