Inflection Point Inflation in SuperGravity

Preliminary Results with Prof.Dr. Manuel Drees

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Outline

Introduction

- Non-trivial to incorporate inflation within SUGRA:
  - η problem.
  - Overproduction of Gravitino.

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- Non-trivial to incorporate inflation within SUGRA:
  - η problem.
  - Overproduction of Gravitino.
- Modern solution needs additional degree of freedom, symmetry and special form of Kähler potential.
- How about the simplest case? Polynomial super potential and canonical Kähler potential.
- Moreover, how the SUSY breaking term effects our inflation scenario?

Renormalizable Inflection Point Model:Setup

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$$V(\phi) = b\phi^2 + c\phi^3 + d\phi^4 \, .$$

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- Non-flatness is measured by β:

$$V(\phi) = d\left(\phi^4 - \frac{8}{3}\phi_0(1-\beta)\phi^3 + 2\phi_0^2\phi^2\right)\,.$$

- Three free parameter, d, ϕ_0, β, together with $\phi_{\rm cmb}$, to match $P_\zeta, n_s, N_{\rm cmb}$.

Renormalizable Inflection Point Model: Results

Analytic results for $\phi_0 < 1, N_{\rm cmb} = 65$: M. Drees, Y. Xu, 2104.03977

$$d = 6.61 \times 10^{-16}\phi_0^2,\ \beta = 9.73 \times 10^{-7}\phi_0^4$$

With following predictions ($M_{\rm pl} = 1$):

$$\begin{aligned} b &= 1.3 \times 10^{-15}\,\phi_0^4\,; \quad c = 1.8 \times 10^{-15}\,\phi_0^3\,; \\ H_{\rm inf} &= 8.6 \times 10^{-9}\,\phi_0^3\,; \quad m_\phi = 5.2 \times 10^{-8}\,\phi_0^2\,; \\ r &= 7.1 \times 10^{-9}\,\phi_0^6\,; \quad \alpha = -1.4 \times 10^{-3}\,. \end{aligned}$$

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Note: $b\,\phi_0^2, c\,\phi_0^3, d\,\phi_0^4$ are the same order $\mathcal{O}(\phi_0^6)$. How about higher order terms $(\phi_0^5, \phi_0^6, ...)$?

Inflection Point Model in SUGRA: Setup

We choose following superpotential and Kähler potential:

$$W(\Phi) = D(B\Phi^2 + C\Phi^3 + \Phi^4)\,, \quad K = \Phi\Phi^*$$

$$V(\phi) = e^K[|D_\phi W|^2 - 3|W|^2]\,.$$

- W starts from Φ^2, no vev for Φ, inflaton do not decay into gravitinos.
- B,C,D are real coefficients. Imaginary part of Superfield stays at origin.
- Inflection point conditions require: $B \approx 3\phi_0^2, C \approx -2\sqrt{2}\phi_0$.
- First five terms (up to ϕ^6) contribute equally to scalar potential, $V(\phi_0) = 2D^2\phi_0^6$.
- $\phi = 0$ is the true minimum of the potential. Positive semi-definite potential.

Inflection Point Model in SUGRA: Calculation Procedure

Our general procedure goes as follows (For given $n_s, P_\zeta, N_{\rm cmb}$):

- We choose ϕ_0 as a free parameter and solve the inflection point conditions $V' = V'' = 0$ to find corresponding B, C solutions.

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- We introduce non-flatness of the potential and make Taylor expansion around inflection point by substituting $B \to B(1+\beta)$ and $\phi = \phi_0(1-\delta\phi)$:

$$\epsilon = \frac{1}{2}\left(\frac{6\beta + 6\delta\phi^2}{\phi_0}\right)^2 \; ; \quad \eta = \frac{15.75\phi_0^2\,\beta - 12\,\delta\phi}{\phi_0^2} \, .$$

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- We determine the start and end point of observable inflation by resolve η ($\epsilon \ll \eta$):

$$\delta\phi = -\frac{\phi_0^2\eta - 15.75\phi_0^2\,\beta}{12} \quad \eta_{\rm cmb} = \frac{n_s - 1}{2}\,, \eta_{\rm end} = -1\,.$$

Inflection Point Model in SUGRA: Calculation Procedure

- The number of e-folds of corresponding inflation is given by:

$$\begin{aligned} N_{\rm cmb} &= \int_{\phi_{\rm cmb}}^{\phi_{\rm end}} \frac{1}{\sqrt{2\epsilon}} d\phi \\ &= \frac{\phi_0^2}{6\sqrt{\beta}} \left(\arctan\left(\frac{\delta\phi_{\rm end}}{\sqrt{\beta}}\right) - \arctan\left(\frac{\delta\phi_{\rm cmb}}{\sqrt{\beta}}\right) \right) , \end{aligned}$$

For $N_{\rm cmb} = 65$, we have:

$$\beta = 8.67 \cdot 10^{-6} \phi_0^4 \, , \quad \delta\phi \propto \phi_0^2 \, .$$

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- It is then easy to calculate $\epsilon_{\rm cmb}$, find normalization of superpotential and all the relevant scales. Note:

$$\alpha \approx -2\xi^2 \propto \frac{\beta + \delta\phi^2}{\phi_0^4} = c.$$

Inflection Point Model in SUGRA: Results

When $\phi_0 < 1$ we find ($M_{\rm pl} = 1$):

$$\beta = 8.67 \cdot 10^{-6} \phi_0^4 \,; \quad \epsilon_{\rm cmb} = 1.59 \cdot 10^{-9} \phi_0^6 \,;$$
$$D^{-2} = 2.53 \times 10^{15} \,; \quad H = 1.62 \cdot 10^{-8} \phi_0^3 \,;$$
$$m_\phi = 1.19 \cdot 10^{-7} \phi_0^2 \,; \quad \alpha = -0.0015 \,.$$

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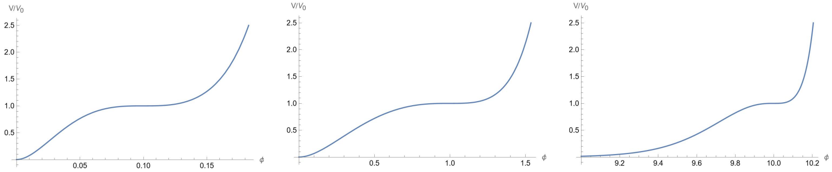
When $\phi_0 > 1$, we have a qualitative understanding. In this case, the slow roll parameter reads:

$$V(\phi) = e^{\frac{1}{2}\phi^2} P(\phi)\,, \quad \eta \propto \phi^2 \cdot f(\delta\phi)\,, \quad N_{\rm cmb} = \int_{\phi_{\rm cmb}}^{\phi_{end}} \frac{1}{\sqrt{2\epsilon}} d\phi$$

Note $\eta_{end} - \eta_{\rm cmb} \approx 1$, thus $\phi_0 \uparrow \xrightarrow{\eta} \delta\phi \downarrow \xrightarrow{N} \epsilon \downarrow \xrightarrow{P} V \downarrow \xrightarrow{exp} H \downarrow, m_\phi \downdownarrows$.

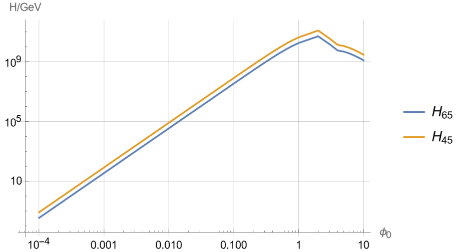
Inflection Point Model in SUGRA: Potential



(a) $\phi_0 = 0.1$. (b) $\phi_0 = 1$. (c) $\phi_0 = 10$.

- Shape of potential is controlled by ϕ_0.
- Length of the flat plateau is shorted by $e^{\frac{1}{2}\phi_0^2}$.

Inflection Point Model in SUGRA: Hubble Scale and Inflaton Mass



- Maximal around $\phi_0 \approx 1$. $H_{\rm max} \sim \mathcal{O}(10^{10}\text{GeV})$.
- Difference between $N = 65$ and $N = 45$ is less than a factor of 3.

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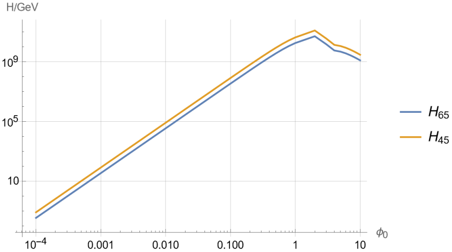
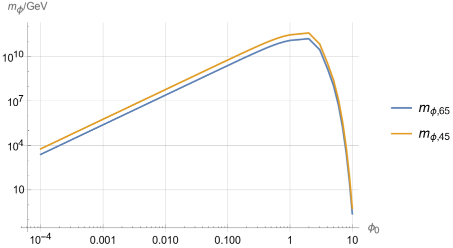
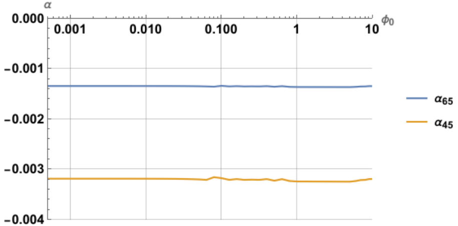




- Maximal around $\phi_0 \approx 1$. $H_{\max} \sim \mathcal{O}(10^{10}\text{GeV})$.
- Difference between $N = 65$ and $N = 45$ is less than a factor of 3.

- Maximal inflaton mass $m_{\max} \sim \mathcal{O}(10^{11}\text{GeV})$.
- Exponential drop when $\phi_0 > 1$.

Inflection Point Model in SUGRA: Running of spectral index α and tensor to scalar ratio r



- Constant α as long as N_{cmb} is fixed.
- Can be seen in next generation CMB experiments.

J. B. Muñoz, E. D. Kovetz, A. Raccanelli, M. Kamionkowski, and J. Silk, 1611.05883

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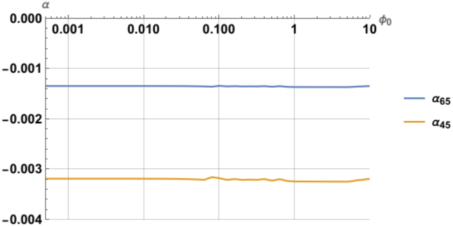

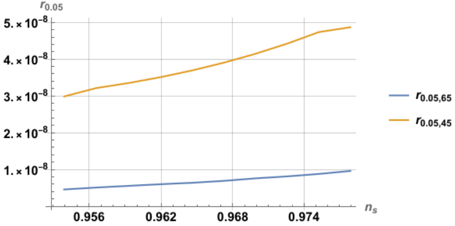

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- Maximal tensor to scalar ratio r for given spectral index n_s.
- $r \sim 10^{-8}$. Unlikely to be seen.

SUSY Breaking: Setup

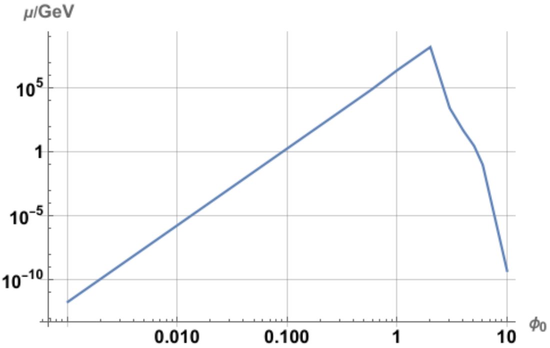
We consider the classical Polony field to break SUSY:

$$
\begin{aligned}
W &= D\left(\left(B\Phi^2 + C\Phi^3 + \Phi^4\right) + \tilde{\mu}(Z + \tilde{\beta})\right) \\
&= W_{\mathrm{I}} + W_{\mathrm{P}}
\end{aligned}
$$

The successful SUSY breaking needs $\tilde{\beta} = 2 - \sqrt{3}$ with a final gravitino mass: $m_{3/2} = D\tilde{\mu}e^{2-\sqrt{3}}$.

- We get additional contribution to slow roll parameter from $\tilde{\mu}$ term.
- Assume $W_{\mathrm{I}} \gg W_{\mathrm{P}}$, treat polony term as a perturbation. Solutions for B, C remains.
- Requiring additional contribution $\delta\epsilon, \delta\eta \ll \epsilon, \eta$.

SUSY Breaking: Inflaton Dominated Case

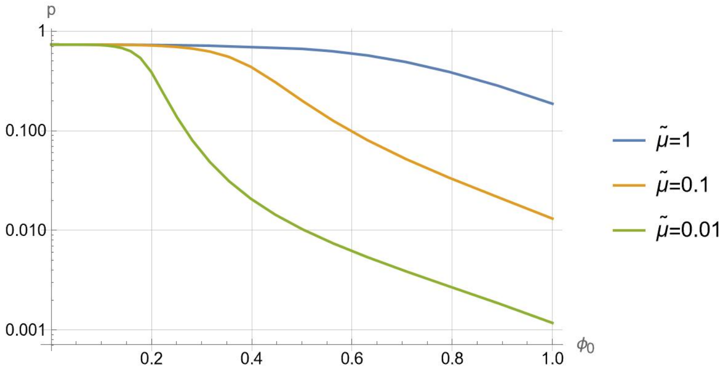


- For $\mu \sim 1\text{TeV}$, we need $0.3 < \phi_0 < 3$, $H > 10^8$ GeV.
- Polony term stays around origin during inflation. After inflation, correction to inflaton mass term:

$$m_\phi^2|_z = e^{4-2\sqrt{3}} m_\phi^2 \left(1 + (2-\sqrt{3})\frac{\mu}{m_\phi}\right) \approx (1.3 m_\phi)^2$$

SUSY Breaking: Polony Field

Above we always assume $W_{\rm I} \gg W_{\rm P}$. We can also treat them as a whole.



- We find the position of Polony field during inflation by solve $\frac{\partial V}{\partial Z} = 0$.
- When $\tilde{\mu} \gg \phi^2$, $W_{\rm P}$ dominates, $\mathrm{Real}(Z) \approx \sqrt{3} - 1$.

SUSY Breaking: Polony Dominated Case

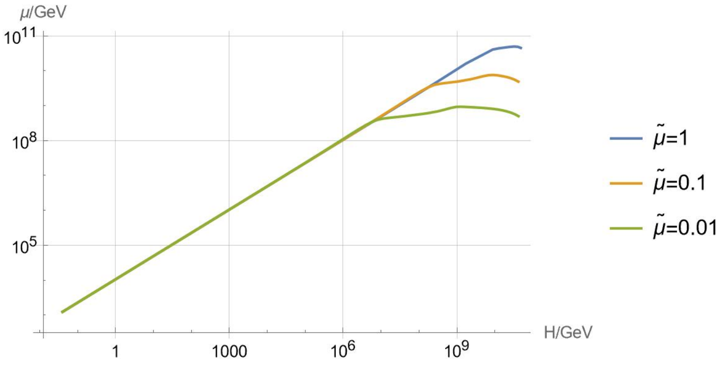
If $W_{\rm P} \gg W_{\rm I}$, another set of solutions for inflection point conditions:

$$B \approx 0.4952\tilde{\mu}\,, \quad C \approx -\frac{0.4996}{\phi_0}\tilde{\mu}\,, \quad V \approx 0.187183\,\mu^2\phi^2.$$

In this case, relevant scales reads:

$$\mu = \tilde{\mu}D = 4.82 \cdot 10^{-8}\phi_0^2\,, \quad H = 1.2 \cdot 10^{-8}\phi_0^3\,, \quad m_\phi \approx 2\mu\,,$$

- The choice of $\tilde{\mu}$ do not effects the final SUSY breaking scale.
- The inflaton mass directly proportional to SUSY breaking scale.
- A universal relationship between μ and H.



Only the lower part of the figure is reachable!

$$\mu < 3 \cdot 10^4 \left(\frac{H}{\mathrm{GeV}}\right)^{2/3} \mathrm{GeV}\,.$$

For our normal choice $\mu > 1\mathrm{TeV}$, we must have $H > 1\mathrm{GeV}$.

Summary

- One chiral superfield is enough to generate inflection point inflation in SUGRA without SUSY breaking.
- The special form of the scalar potential limits the possible Hubble value up to $\mathcal{O}(10^{10})$ GeV and the inflaton mass to $\mathcal{O}(10^{11})$ GeV.
- The tensor to scalar ratio $r \sim \mathcal{O}(10^{-9})$ is always suppressed, while the running of spectral index $\alpha \sim \mathcal{O}(10^{-3})$ will be testable.
- In terms of SUSY breaking, a general upper bound on SUSY breaking scale. If we know more about inflation, we might also know more about SUSY.

Backup: Scalar Potential

Full potential from our setup:

$$W(\Phi) = D(B\Phi^2 + C\Phi^3 + \Phi^4)\,, \quad K = \Phi\Phi^*$$

$$V(\phi) = e^K[|D_\phi W|^2 - 3|W|^2]\,.$$

reads:

$$\begin{aligned} V(\phi) = e^{\frac{\phi^2}{2}} \Big[& 2B^2\phi^2 + 3\sqrt{2}BC\phi^3 + \frac{1}{4}\phi^4\left(B^2 + 16BD + 9C^2\right) + \frac{1}{2}\phi^5\left(\sqrt{2}BC + 6\sqrt{2}CD\right) \\ & + \frac{1}{8}\phi^6\left(B^2 + 6BD + 3C^2 + 16D^2\right) + \frac{1}{8}\phi^7\left(\sqrt{2}BC + 4\sqrt{2}CD\right) \\ & + \frac{1}{16}\phi^8\left(2BD + C^2 + 5D^2\right) + \frac{CD\phi^9}{8\sqrt{2}} + \frac{D^2\phi^{10}}{32} \Big], \end{aligned}$$

Backup: Gravitino Production

There are three main gravitino production channels:

- Thermal Production after reheating.
- Inflation decay into gravitinos (absent in the model).
- Non-thermal production during reheating.

Thermal production is unavoidable, with:

V. S. Rychkov and A. Strumia, hep-ph/0701104

$$\frac{n^t_{3/2}}{s} \approx 6.11 \times 10^{-12} \left(\frac{T_R}{10^{10}\text{GeV}} \right) .$$

Non-thermal production can be estimated by oscillation frequency and amplitude of gravitino mass term:

Y. Ema, K. Mukaida, K. Nakayama, and T. Terada, 1609.04716

$$n_\psi(t) \approx \frac{\zeta}{16\pi} \Omega^2 \tilde{m}^2 t \, ,$$

$$\frac{n^t_{3/2}}{s} \approx 8 \times 10^{-16} \zeta \left(\frac{H}{10^{13}\text{GeV}} \right) \left(\frac{T_R}{10^{10}\text{GeV}} \right) ,$$

Backup: Gravitino Production

For large field case, we estimate the maximal gravitino production by assuming oscillation happens right after inflation.

$$V \approx e^{1/2\phi^2}\frac{D^2}{32}\phi^{10}\,, \quad m_{3/2} \approx e^{1/4\phi^2}D\phi^4 \to m_{3/2}^2 \approx 32\frac{V}{\phi_0^2}\,.$$

Gravitino abundance reads:

$$\frac{n_{3/2}^t}{s} \approx 0.022\frac{1}{T_R^3}\frac{m_\phi^2 H_{\rm inf}}{\phi_0^2}\,,$$

For instance, when $\phi_0 = 5(7)$:

$$\frac{n_{3/2}^t}{s} \approx 8.8\times 10^{-6}(5.77\times 10^{-13})\left(\frac{10^{10}\mathrm{GeV}}{T_R}\right)^3$$

Abundance decrease when ϕ_0 increase. Need careful investigation.