Technische Universität München
T31 SFB 1258 Projekt D04
„Neutrinos and Dark Matter in Astro- and Particle Physics“

Excited bound states and their role in Dark Matter production

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In collaboration with:
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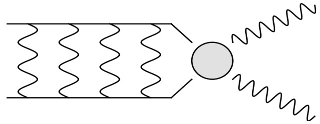
@ DESY theory workshop 2023

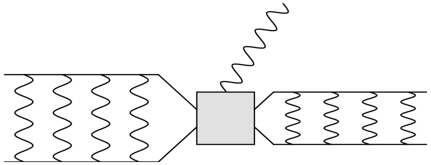
Introduction

Long-range force effects:

- Bound state decay, Sommerfeld enhanced annihilation



- Scattering-to-bound, bound-to-bound transitions



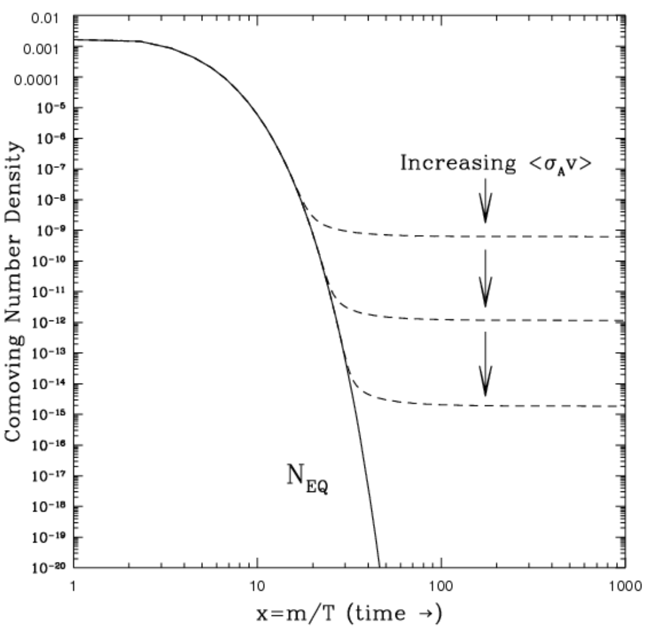
Examples:

- WIMPs at the TeV scale and above:
  Wino, Minimal Dark Matter,
  colored co-annihilation scenarios
  [Hisano et al. 03,05,06, Ellis et al. 15, Mitridate et al. 17, Harz & Petraki 18, ...]

- Self-interacting Dark Matter with light mediators

- Dark Sectors, e.g., SU(N)

Role of highly excited states in Dark Matter production?

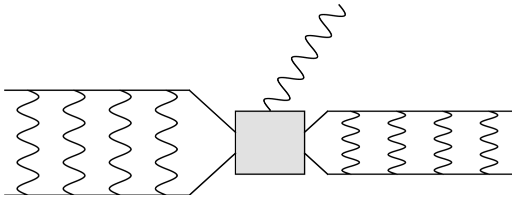


Dipole transition matrix elements

- Focus on dipole transitions of the form:

$$\langle\psi_f|\,\mathbf{r}\,|\psi_i\rangle = \int \mathrm{d}^3 r\ \psi_f^\star(\mathbf{r})\ \mathbf{r}\ \psi_i(\mathbf{r}).$$

$$\mathrm{V}_{i/f} = -\frac{\alpha_{i/f}^{\mathrm{eff}}}{r}$$



- E.g.: (chromo-) electric dipole transitions of pairs in unbroken U(1) and SU(N) gauge theories

- Analytic result in terms of recursive relations allows for efficient and stable evaluation up to:

  - $\mathrm{n} \leq 1000, l \leq n-1$ for scattering-to-bound

  - $\mathrm{n} \leq 100, l \leq n-1$ for bound-to-bound

Bound-state formation in U(1) gauge theory

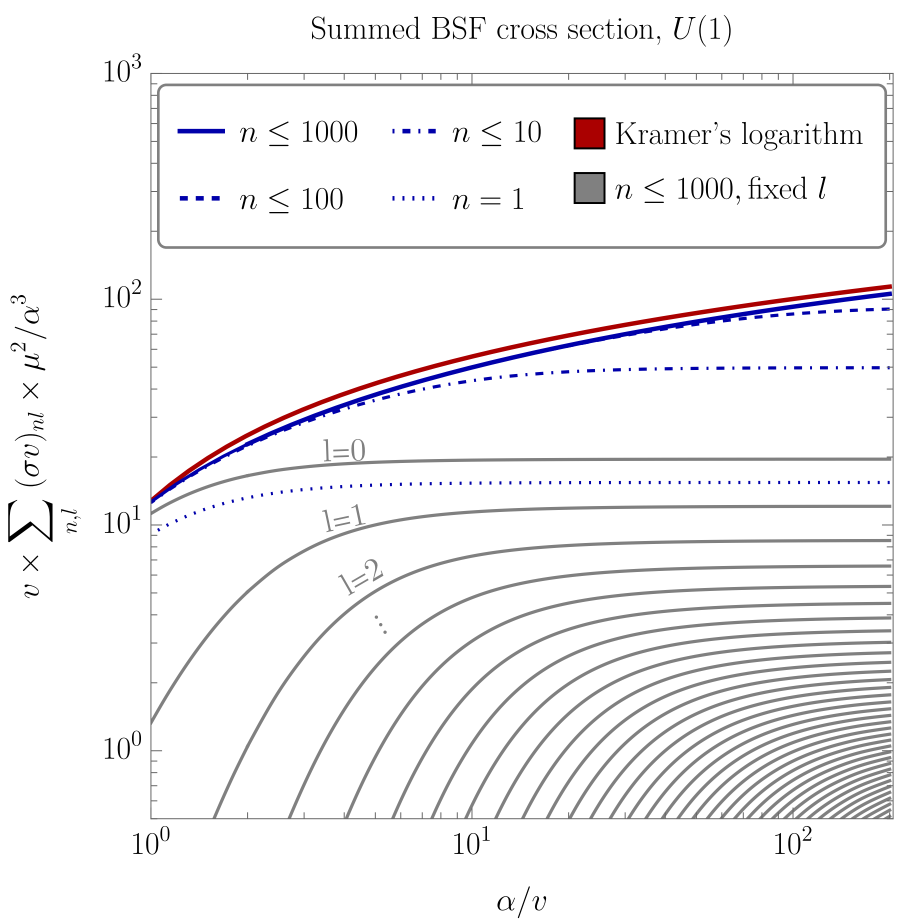
$$\mathcal{S}(\chi\bar{\chi}) \to \mathcal{B}(\chi\bar{\chi})_{nl} + \gamma$$

$$(\sigma v)_{nl} = \frac{4\alpha}{3}\Delta E^3 |\langle\psi_{nl}|\,\mathbf{r}\,|\psi_{\mathbf{p}}\rangle|^2$$

(e.g. hydrogen, (dark) positronium, complex scalars)

- Up to half a million bound states: all $\mathrm{n} \leq 1000, l \leq n-1$.
- Confirm Kramer's logarithm within expected error as a check:

$$\sum_{n,\ell} (\sigma v)_{n\ell} \simeq \frac{32\pi}{3\sqrt{3}} \frac{\alpha^2}{\mu^2} \frac{\alpha}{v} \left[\log(\alpha/v) + \gamma_E\right], \text{ for } v \ll \alpha.$$



Bound-state formation in SU(3) gauge theory

$3 \otimes \bar{3} = 1 \oplus 8$

$\mathcal{S}(\chi\bar{\chi})^8 \to \mathcal{B}(\chi\bar{\chi})^1_{nl} + g$

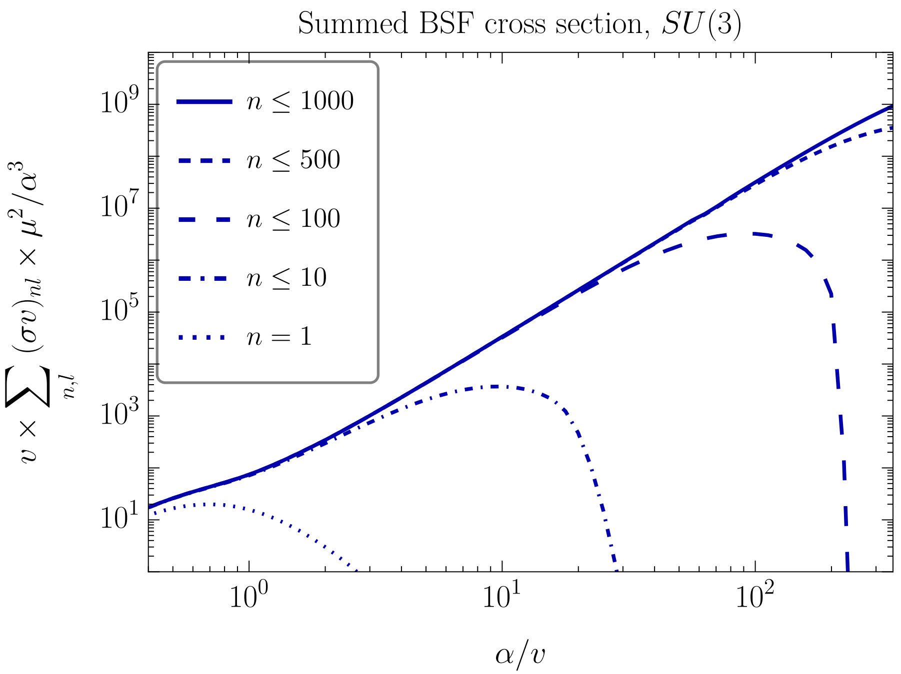
$(\sigma v)_{nl} = \frac{C_F}{N_c^2} \frac{4\alpha}{3} \Delta E^3 |\langle \psi^1_{nl} | \mathbf{r} | \psi^8_{\mathbf{p}} \rangle|^2$

(e.g. quarkonium, squark)

- Assume constant coupling
- Low velocity scaling much stronger:

  $\sum_{nl} (\sigma v)_{nl} \propto v^{-4}$ for $v \ll \alpha$

- Raises concerns about partial wave-unitarity violation



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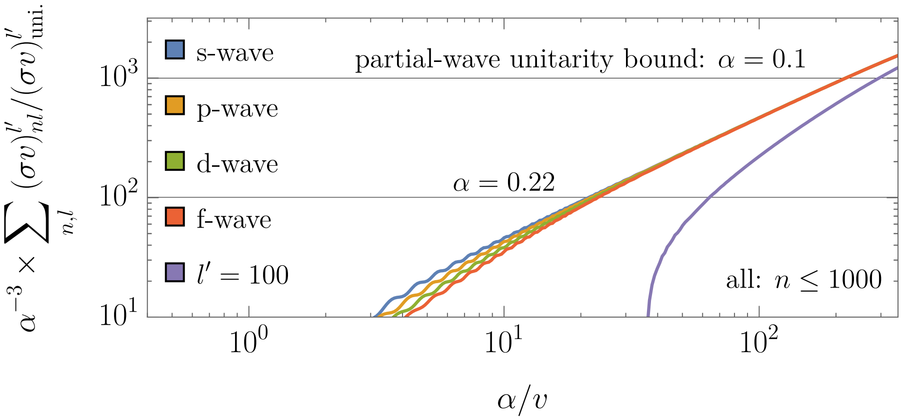
(e.g. quarkonium, squark)

- Unitarity condition:

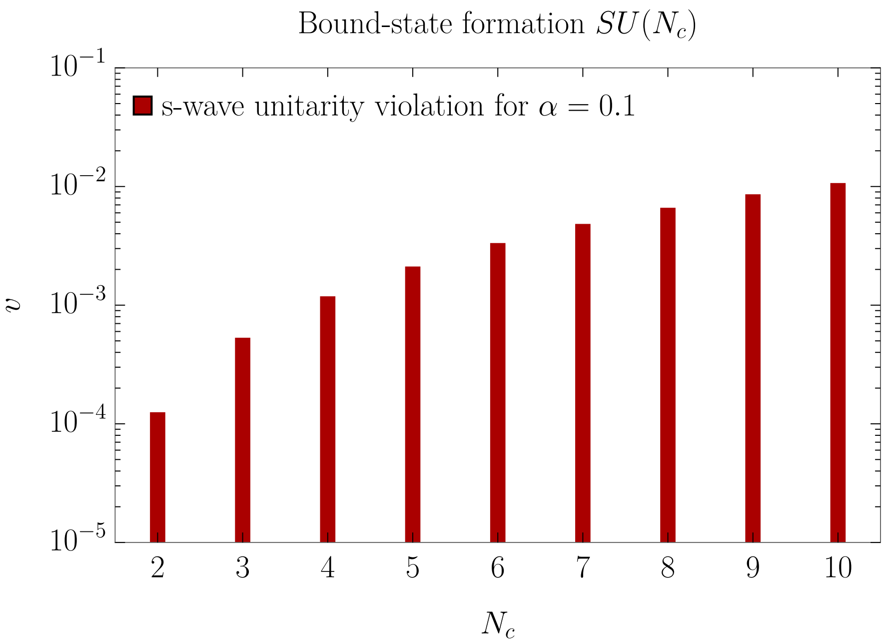
$$\sum_{nl}(\sigma v)^{l'}_{nl} \leq (\sigma v)^{l'}_{\text{uni.}} = \frac{\pi(2l'+1)}{\mu^2 v}$$

- Observe partial-wave unitarity violation in the perturbative regime

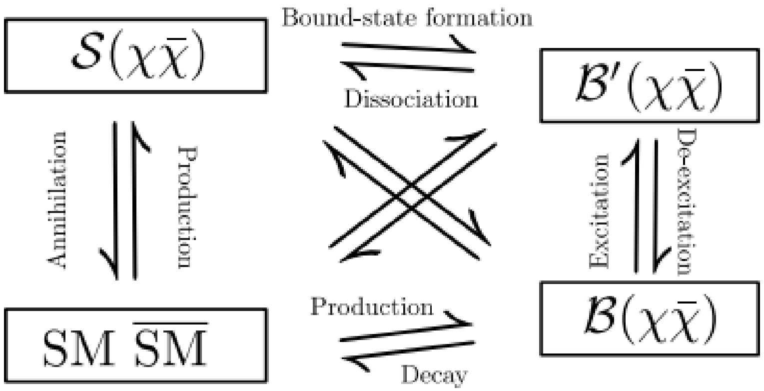


Partial wave unitarity violation in SU(N)

- We observe partial wave unitarity violation in SU(N) gauge theories for perturbatively small couplings
- Mechanism behind unitarization unknown
- In the following, focussing on the regime consistent with perturbativity and unitarity



Effective cross section & critical scaling



Quasi steady-state

$$\dot{n} + 3Hn = -\langle\sigma v\rangle_{\rm eff}(n^2 - n_{\rm eq}^2)$$

$$\langle\sigma v\rangle_{\rm eff} = \langle\sigma v\rangle_{\rm ann} + \sum_{\mathcal{B}} \langle\sigma v\rangle_{\mathcal{B}} R_{\mathcal{B}}$$

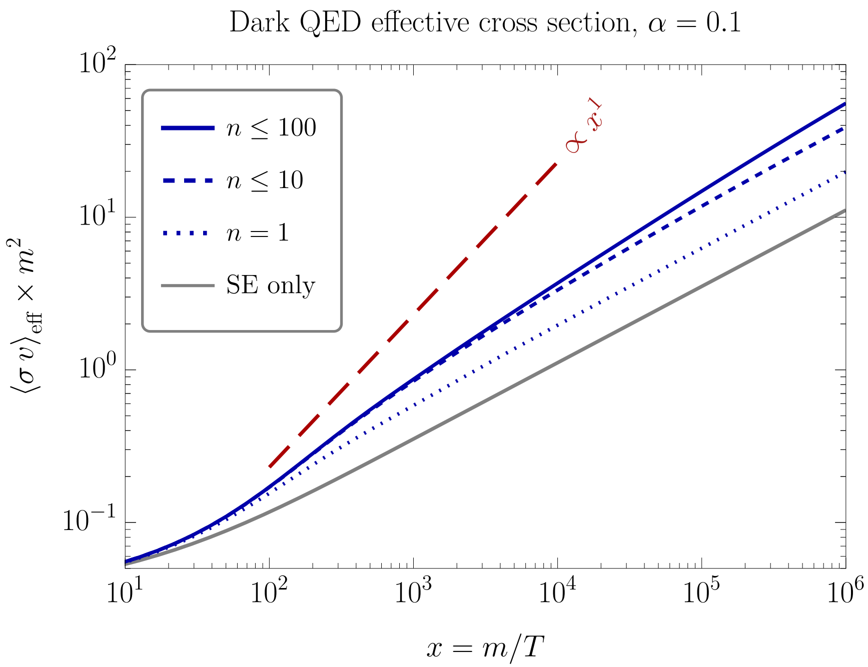
[Ellis et al. 15, Mitridate et al. 17,
Binder et al. 21, Garny & Heisig 21]

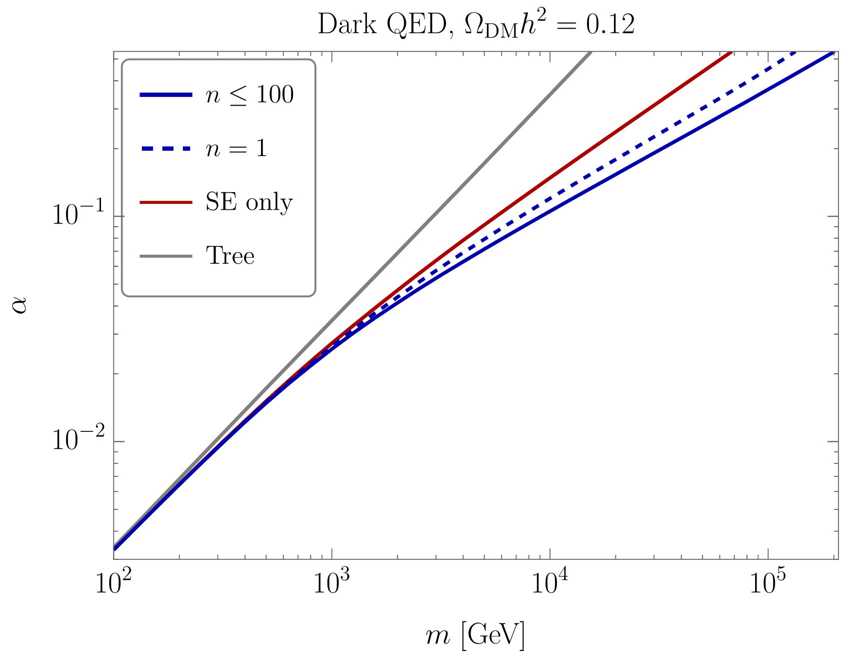
- Effective cross section encodes complex interplay between annihilation, scattering-to-bound, bound-to-bound and bound-state decay processes.

- *Super critical scaling:* $\langle\sigma v\rangle_{\rm eff} \propto (m/T)^\gamma$, where $\gamma \geq 1$.

  i.e. particles do not freeze-out but continue depletion.

Effective cross section: Dark QED sector

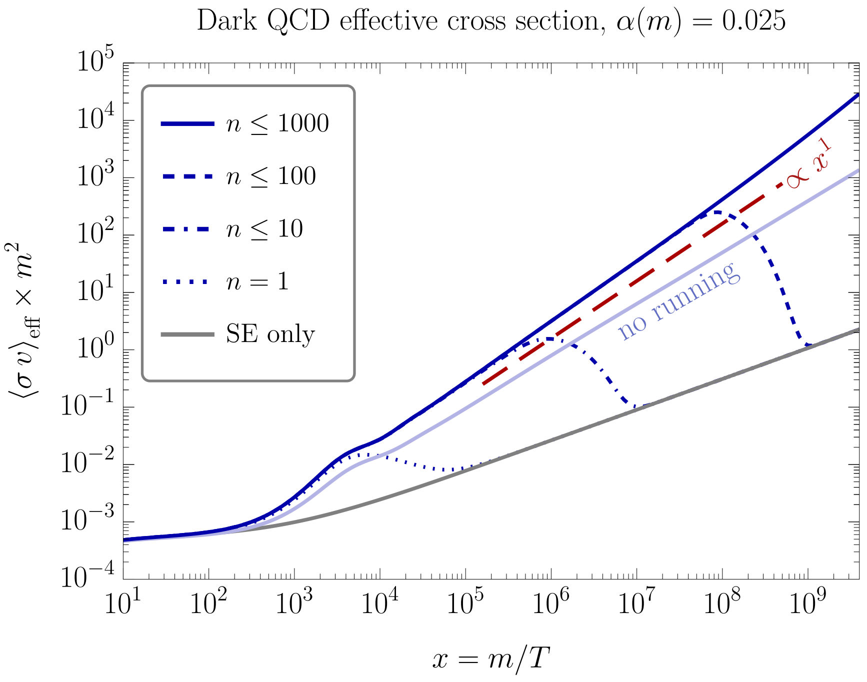




- Includes about 5000 bound states and all possible transitions (about a million).
- Dark QED indeed freezes out.
- Upper bound on DM mass consistent with perturbative unitarity is 0.2 PeV.

Effective cross section: Dark QCD sector

- Only s-wave bound states included, which are dominant decay channel.
- Transitions among color singlet bound states not possible via chromo-electric dipole operator.
- Running of the coupling leads to supercritical behaviour, i.e., no freeze-out in the perturbative regime.
- SU(N) with N>3: all supercritical
- Simultaneous SU(N) and U(1) charge allows for transitions among singlets → enhanced scaling

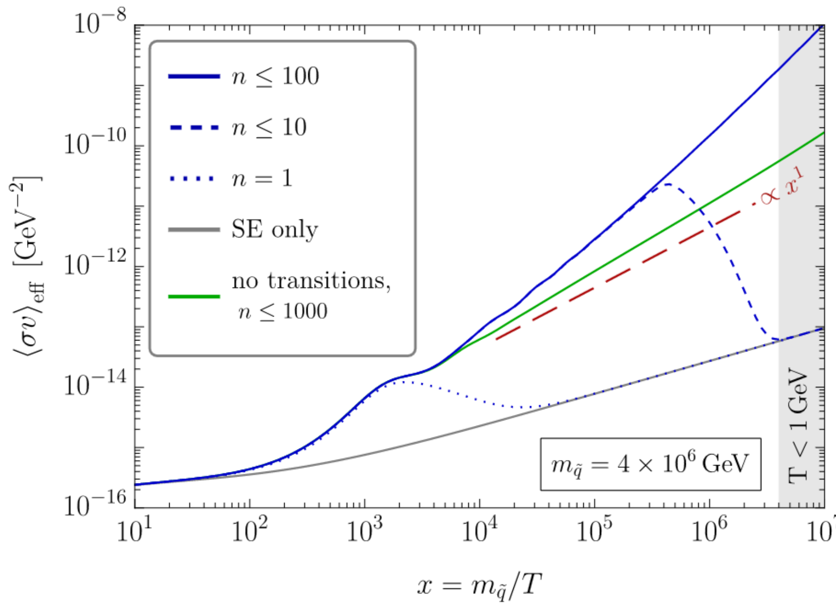


SU(3) and U(1) charged mediator model

„t-channel“ simplified model:

$$\mathcal{L} \supset \lambda_\chi \tilde{q} \bar{q}_R \chi + h.c.$$

- $\tilde{q}$: scalar mediator, carries SM electric and color charge
- q_R : right handed SM quark
- χ : Majorana Fermion Dark Matter



DM production can be classified into:

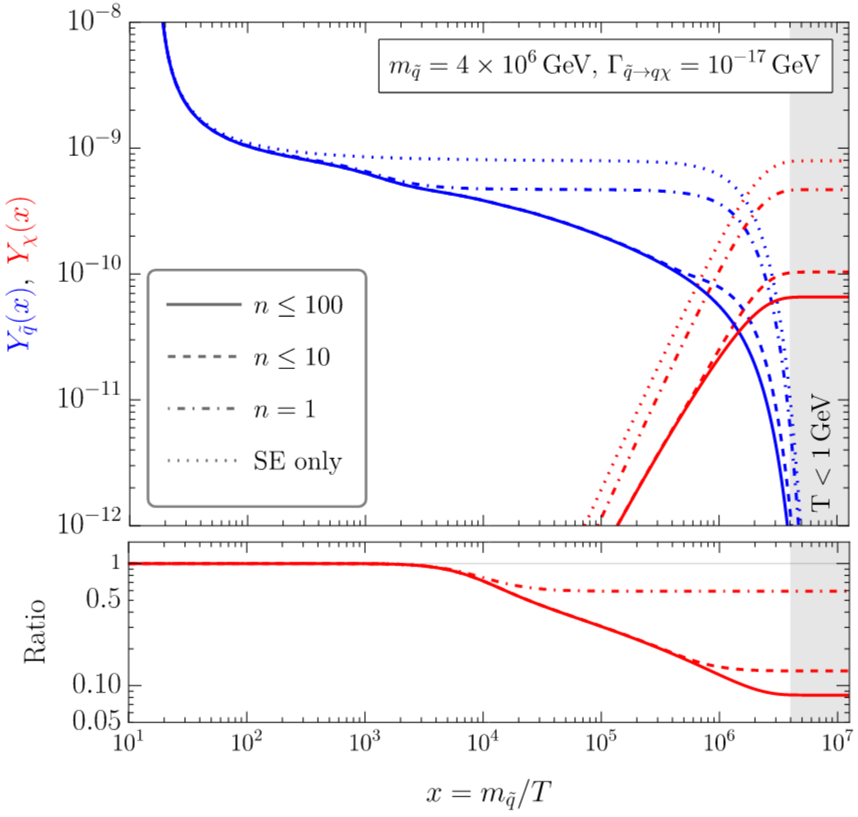
$$\begin{aligned}
\Gamma_{\text{conv}}^{\chi\to\tilde{q}} &\gg H(m_{\tilde{q}}) & &\text{coannihilation}\,,\\
\Gamma_{\text{conv}}^{\chi\to\tilde{q}} &\sim H(m_{\tilde{q}}) & &\text{conversion-driven}\,,\\
\Gamma_{\text{conv}}^{\chi\to\tilde{q}} &\ll H(m_{\tilde{q}}) & &\text{superWIMP/freeze-in}\,.
\end{aligned}$$

$$\frac{\mathrm{d}Y_{\tilde{q}}}{\mathrm{d}x} = \frac{1}{3H}\frac{\mathrm{d}s}{\mathrm{d}x}\left[\frac{1}{2}\langle\sigma_{\tilde{q}\tilde{q}^\dagger}v\rangle_{\text{eff}}\left(Y_{\tilde{q}}^2 - Y_{\tilde{q}}^{\text{eq}\,2}\right) + \langle\sigma_{\chi\tilde{q}}v\rangle\left(Y_\chi Y_{\tilde{q}} - Y_\chi^{\text{eq}}Y_{\tilde{q}}^{\text{eq}}\right) + \frac{\Gamma_{\text{conv}}^{\tilde{q}\to\chi}}{s}\left(Y_{\tilde{q}} - Y_\chi\frac{Y_{\tilde{q}}^{\text{eq}}}{Y_\chi^{\text{eq}}}\right)\right], \tag{19}$$

$$\frac{\mathrm{d}Y_{\chi}}{\mathrm{d}x} = \frac{1}{3H}\frac{\mathrm{d}s}{\mathrm{d}x}\left[\langle\sigma_{\chi\chi}v\rangle\left(Y_{\chi}^2 - Y_{\chi}^{\text{eq}\,2}\right) + \langle\sigma_{\chi\tilde{q}}v\rangle\left(Y_\chi Y_{\tilde{q}} - Y_\chi^{\text{eq}}Y_{\tilde{q}}^{\text{eq}}\right) - \frac{\Gamma_{\text{conv}}^{\tilde{q}\to\chi}}{s}\left(Y_{\tilde{q}} - Y_\chi\frac{Y_{\tilde{q}}^{\text{eq}}}{Y_\chi^{\text{eq}}}\right)\right], \tag{20}$$

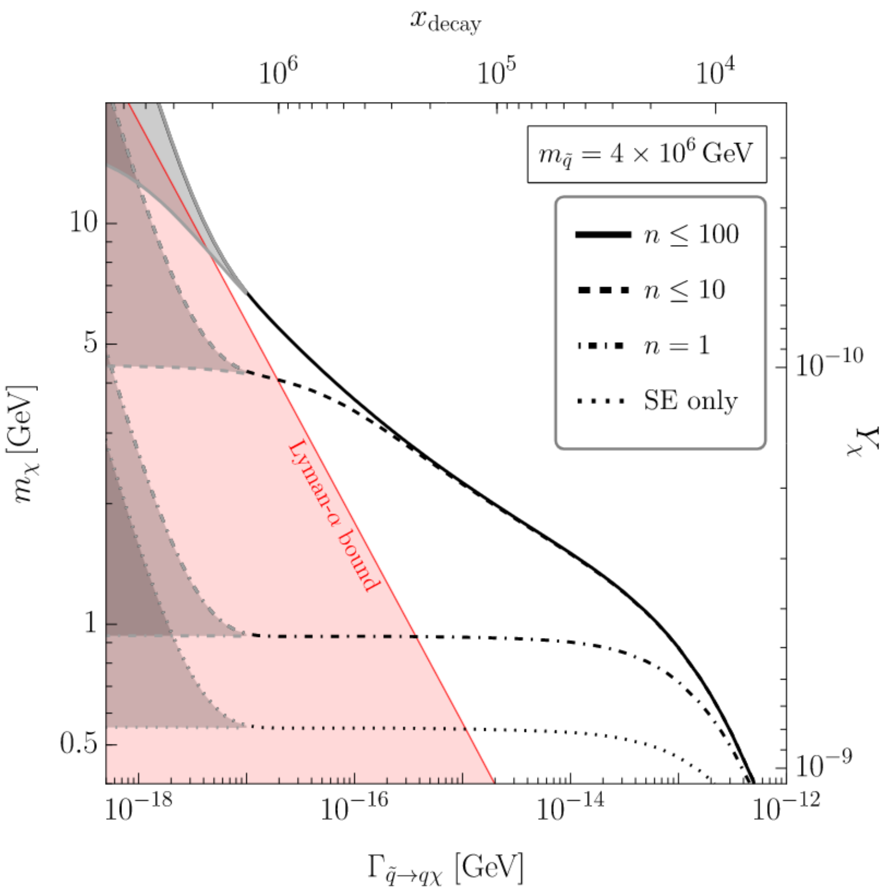
SuperWIMP regime

- Typical superWIMP regime:
  late decay of mediator into DM, final DM yield independent of actual size of the conversion rate.
- Continous depletion of mediator yield from bound state effects.
- → introduces a dependence of the DM yield on the conversion rate as a novel feature.



Constraints

- DM produced relativistically from heavy mediator decay
- DM can be „too hot“, i.e., substructure can be erased by free-streaming effect
- Substructure probed by Ly-alpha observations
- Bound state effects open up parameter space
- Corrections to the DM mass up to an order of magnitude



Outlook

- Bound-state formation cross section & de-excitation rate *factorize* :

$$(\sigma v)_{nl} \sim g^2 |\langle\psi_{nl}|\,\mathbf{r}\,|\psi_v\rangle|^2 \times \int \frac{\mathrm{d}^3 p}{(2\pi)^3} D^{-+}(P^0 = \Delta E, \mathbf{p})$$

$$\Gamma_{nl}^{n'l'} \sim g^2 |\langle\psi_{nl}|\,\mathbf{r}\,|\psi_{n'l'}\rangle|^2 \times \int \frac{\mathrm{d}^3 p}{(2\pi)^3} D^{-+}(P^0 = \Delta E, \mathbf{p})$$

Plasma contact encoded in *Electric Field Correlator*:

$D(x,y) \equiv \langle T_{\mathcal{C}} E(x) E(y)\rangle$, where

$\langle ...\rangle \propto \mathrm{Tr}[e^{-H_{\mathrm{env}}/T} ...]$.

- Results of electric field correlator at NLO zero and finite temperature for U(1) and SU(N) already avaiable [Binder et al. 20, Binder et al. 21]

- How „stable“ is the (chomo-) electric dipole operator picture? (further corrections)

- Unitarization of bound-state formation

Summary & Conclusion

- Highly excited bound states can play an important role for predicting the DM relic abundance precisely.

- Can lead to „eternal freeze-out“ in unbroken non-abelian gauge theories

- SuperWIMP regime:

  - bound state effects can introduce a dependence of the DM yield on the mediator lifetime as a novel feature

  - corrections by up to an order of magnitude in the DM mass

- unitarization of bound state formation in unbroken non-Abelian gauge theories within the regime of perturbatively small couplings (?)

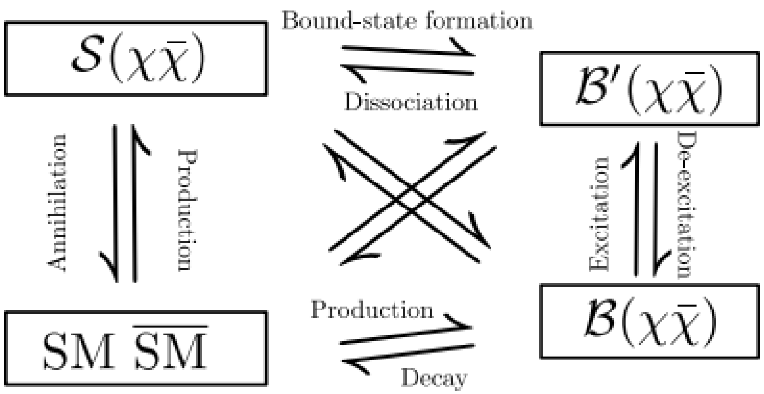
pNREFT

$$\mathcal{L}^{\text{pNRQED}} \supset \int \mathrm{d}^3 r\ S^\dagger \left[i\partial_t - H + \ \mathbf{r} \cdot g\mathbf{E}(\mathbf{x}, t) \right] S$$

$$\boldsymbol{R} \otimes \bar{\boldsymbol{R}} = \mathbf{1} \oplus \boldsymbol{adj} \oplus \cdots$$

$$\mathcal{L}_{\text{pNREFT}} \supset \int \mathrm{d}^3 r\ \mathrm{Tr} \Big[\mathrm{S}^\dagger (i\partial_0 - H_s)\mathrm{S} + \mathrm{Adj}^\dagger (iD_0 - H_{\text{adj}})\mathrm{Adj} \ \ - V_A (\mathrm{Adj}^\dagger \boldsymbol{r} \cdot g\boldsymbol{E}\mathrm{S} + \mathrm{h.c.})$$

Effective cross section details



$$\dot{n} + 3Hn = -\langle\sigma v\rangle_{\text{eff}}(n^2 - n_{\text{eq}}^2)$$

$$\langle\sigma v\rangle_{\text{eff}} = \langle\sigma v\rangle_{\text{ann}} + \sum_{\mathcal{B}} \langle\sigma v\rangle_{\mathcal{B}} R_{\mathcal{B}}$$

$$R_{\mathcal{B}} \equiv 1 - \sum_{\mathcal{B}'} M^{-1}_{\mathcal{B}\mathcal{B}'} \frac{\Gamma^{\mathcal{B}'}_{\text{ion}}}{\Gamma^{\mathcal{B}'}}$$

$$M_{\mathcal{B}\mathcal{B}'} \equiv \delta_{\mathcal{B}\mathcal{B}'} - \frac{\Gamma^{\mathcal{B}\to\mathcal{B}'}_{\text{trans}}}{\Gamma^{\mathcal{B}}}$$

$$\Gamma^{\mathcal{B}} \equiv \Gamma^{\mathcal{B}}_{\text{ion}} + \Gamma^{\mathcal{B}}_{\text{dec}} + \sum_{\mathcal{B}'\neq\mathcal{B}} \Gamma^{\mathcal{B}\to\mathcal{B}'}_{\text{trans}}$$