

# Gravitational waves from QCD-triggered conformal symmetry breaking

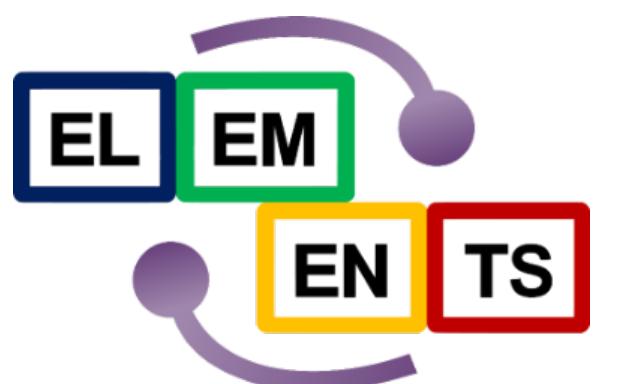
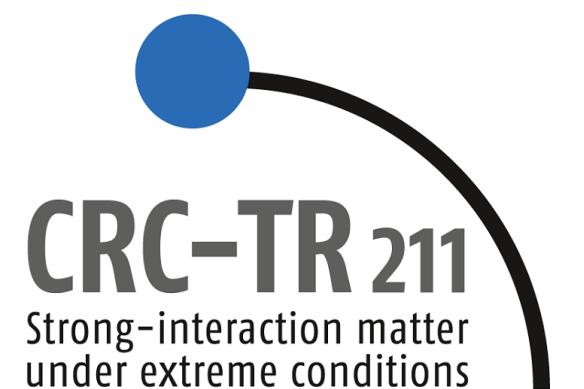
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**Collaborators:** L. Sagunski, P. Schicho

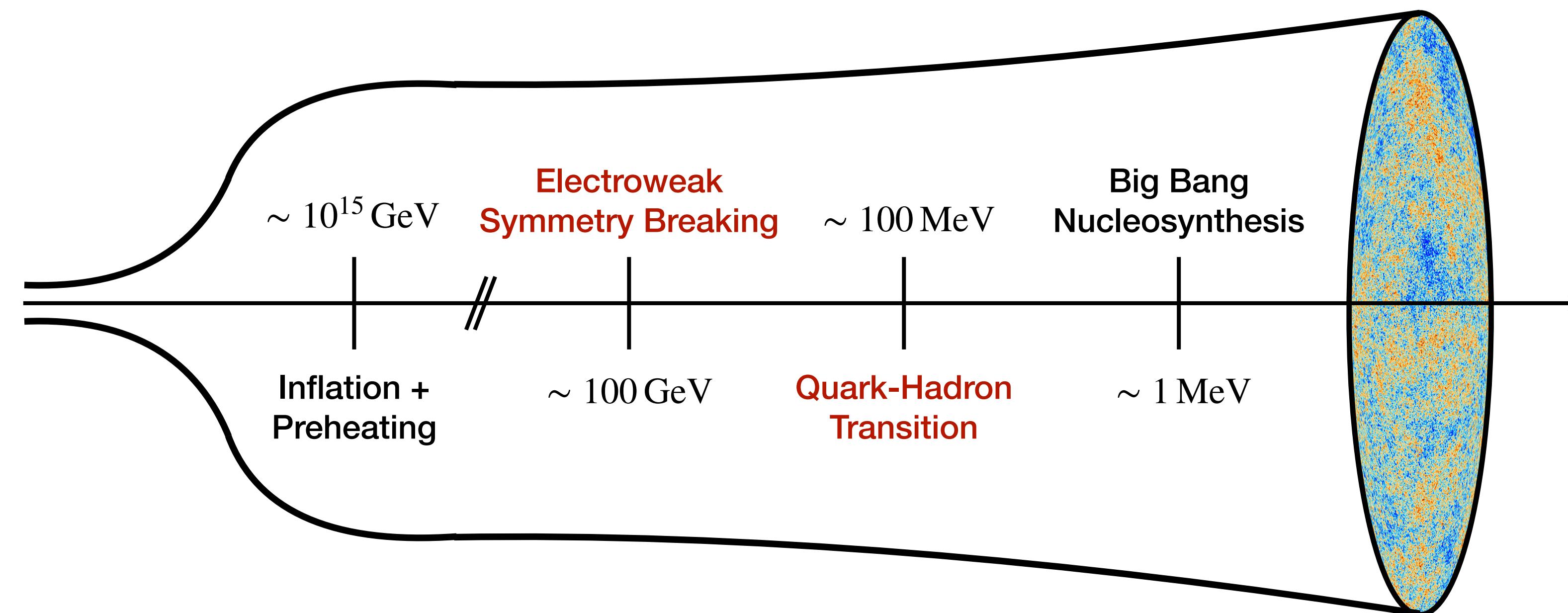
Based on: PRD 107, 123512 (2023) [[arXiv:2302.02450](https://arxiv.org/abs/2302.02450)]



# Overview

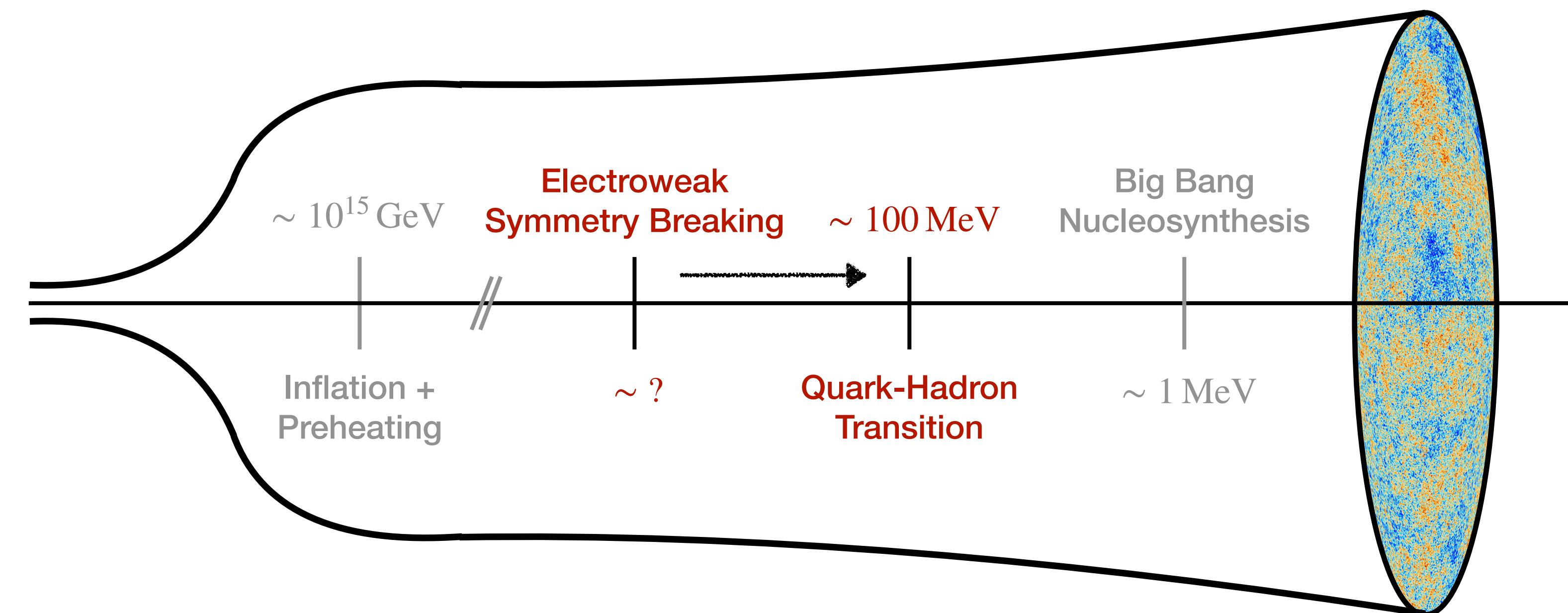
Standard Model (SM): QCD + electroweak (EW) transition **separated**

Crossover transitions<sup>1</sup>: no gravitational wave (GW) emission



Scale-invariant SM extensions: EW transition delayed<sup>2</sup>

Possible outcome: strong first-order QCD transition



# **1. The Supercooled Universe**

# Classically Conformal (CC) Models

- Two main principles:
  1. Extend SM by **additional gauge symmetry** with scalar field  $\Phi$  (e.g.  $U(1)_{B-L}$ )
  2. Impose **scale invariance** at tree level<sup>3</sup>

# Classically Conformal (CC) Models

- Two main principles:
  1. Extend SM by additional gauge symmetry with scalar field  $\Phi$  (e.g.  $U(1)_{B-L}$ )
  2. Impose scale invariance at tree level<sup>3</sup>

$$V(H, \Phi) = \lambda_\Phi \Phi^4 + \lambda H^4 - \lambda_p \Phi^2 H^2$$

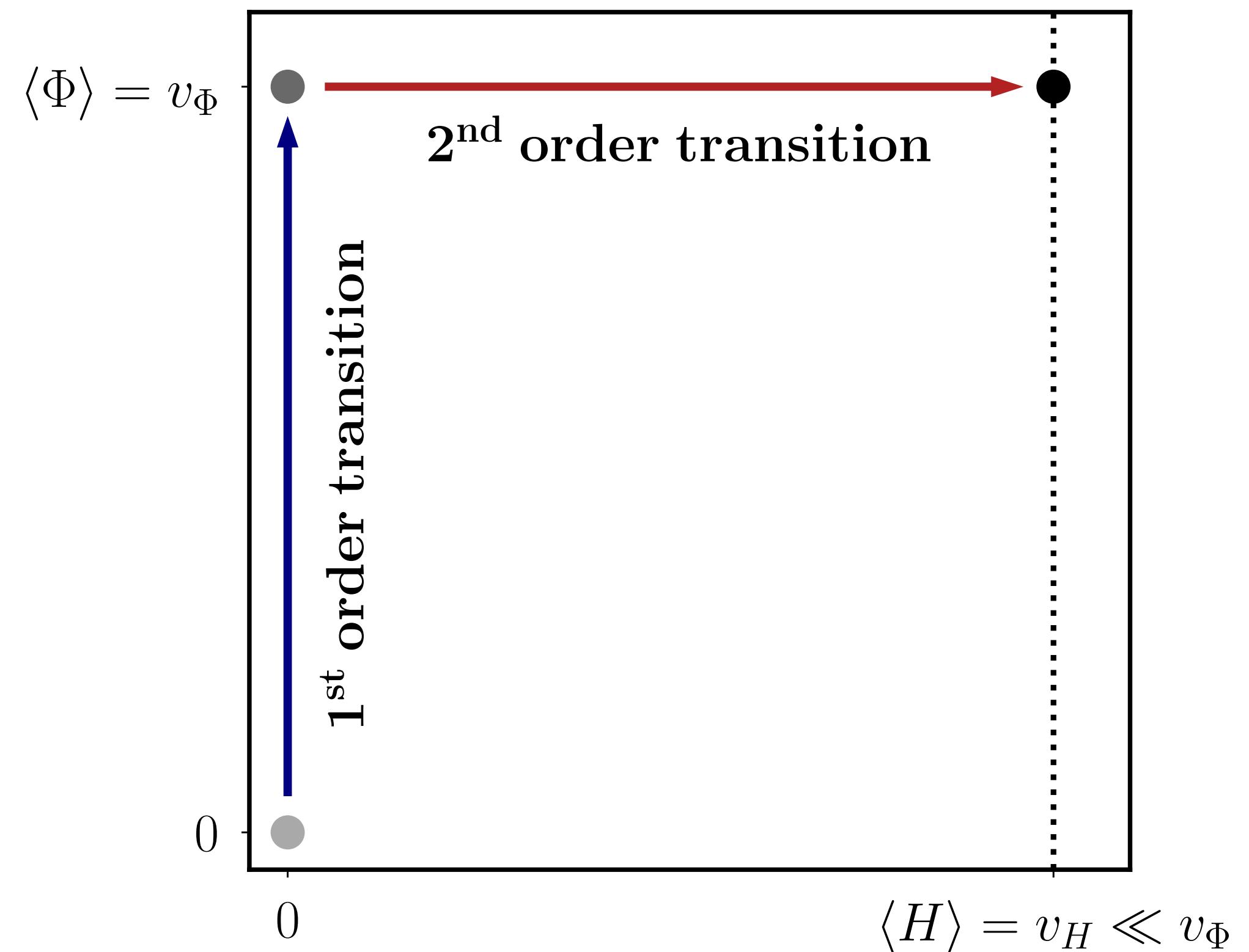
Higgs mass term replaced by **portal coupling**:  $-\mu^2 H^2 \rightarrow -\lambda_p \Phi^2 H^2$

# Symmetry Breaking Pattern

Conformal symmetry broken **radiatively**

$$\langle \Phi \rangle = v_\Phi \quad \rightarrow \quad \mu_H^2 = \lambda_p v_\Phi^2$$

Higgs mass **generated dynamically**



# Conformal Symmetry Breaking

Flat potential around origin

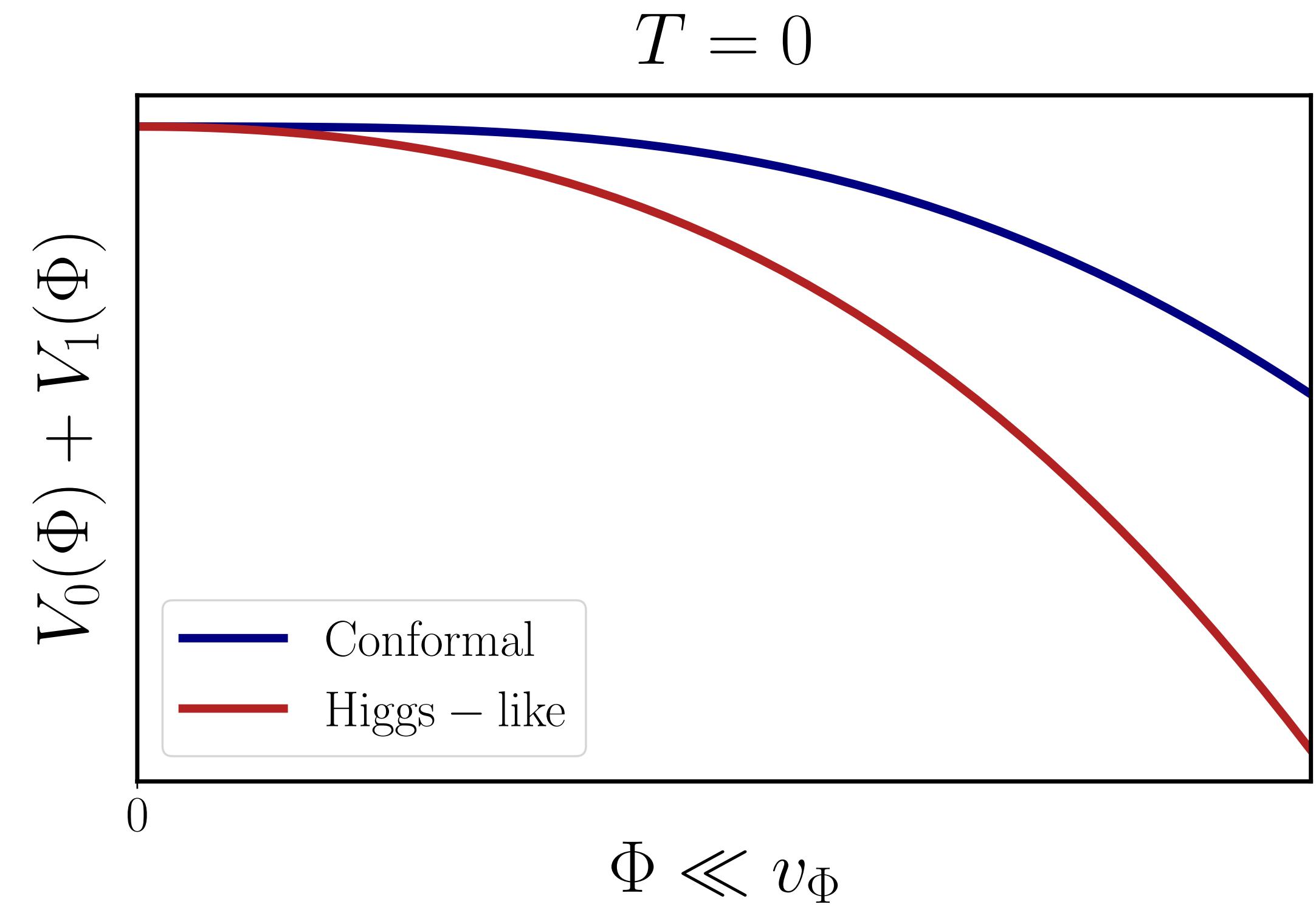


Thermal barrier **remains until  $T \rightarrow 0$**



Universe trapped in false vacuum

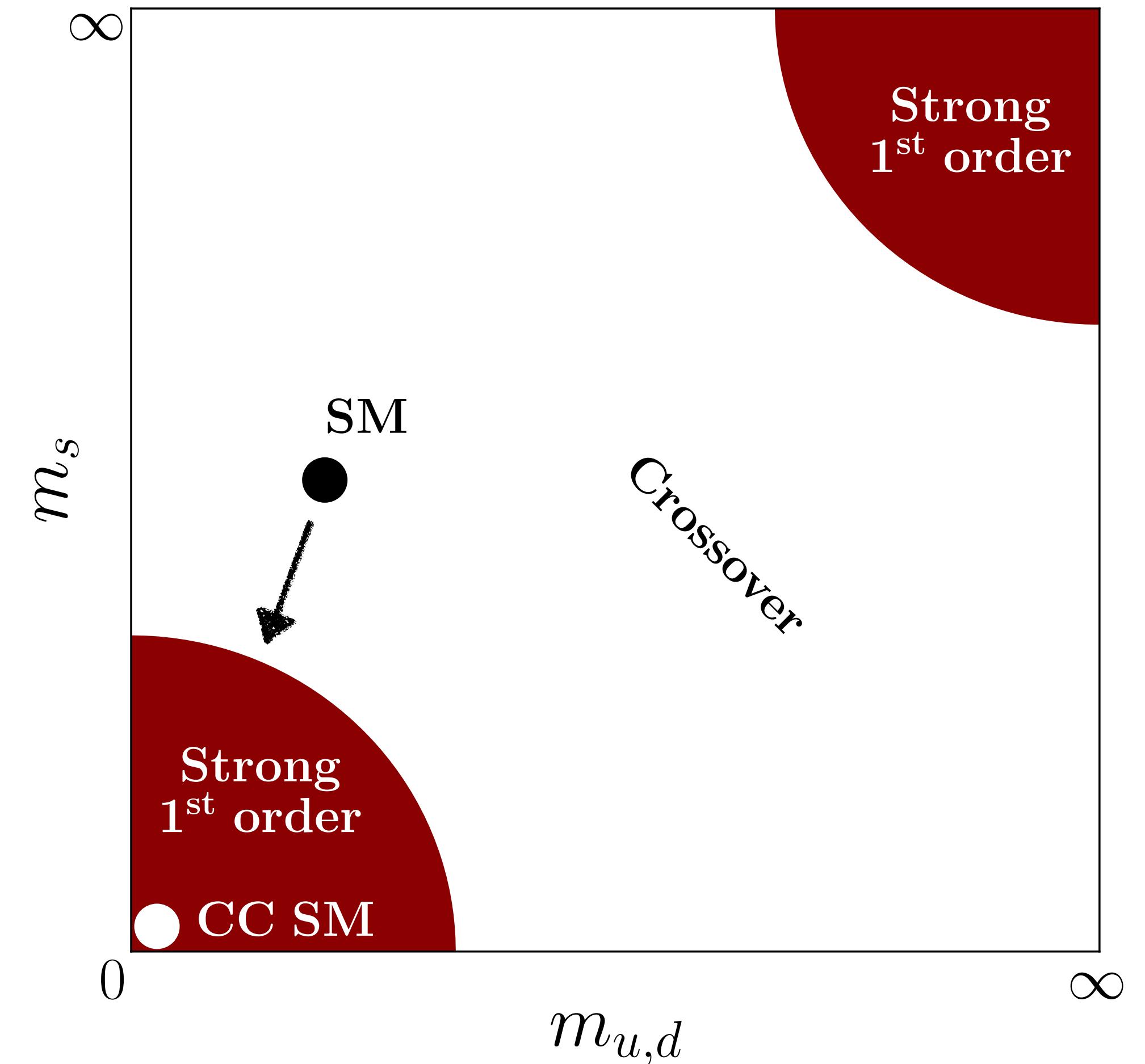
Possible Supercooling<sup>4</sup> to  $T_p \ll T_{\text{EW}}$



# Supercooling and the QCD Transition

EWPT can be **supercooled** to QCD scale

Quarks remain massless



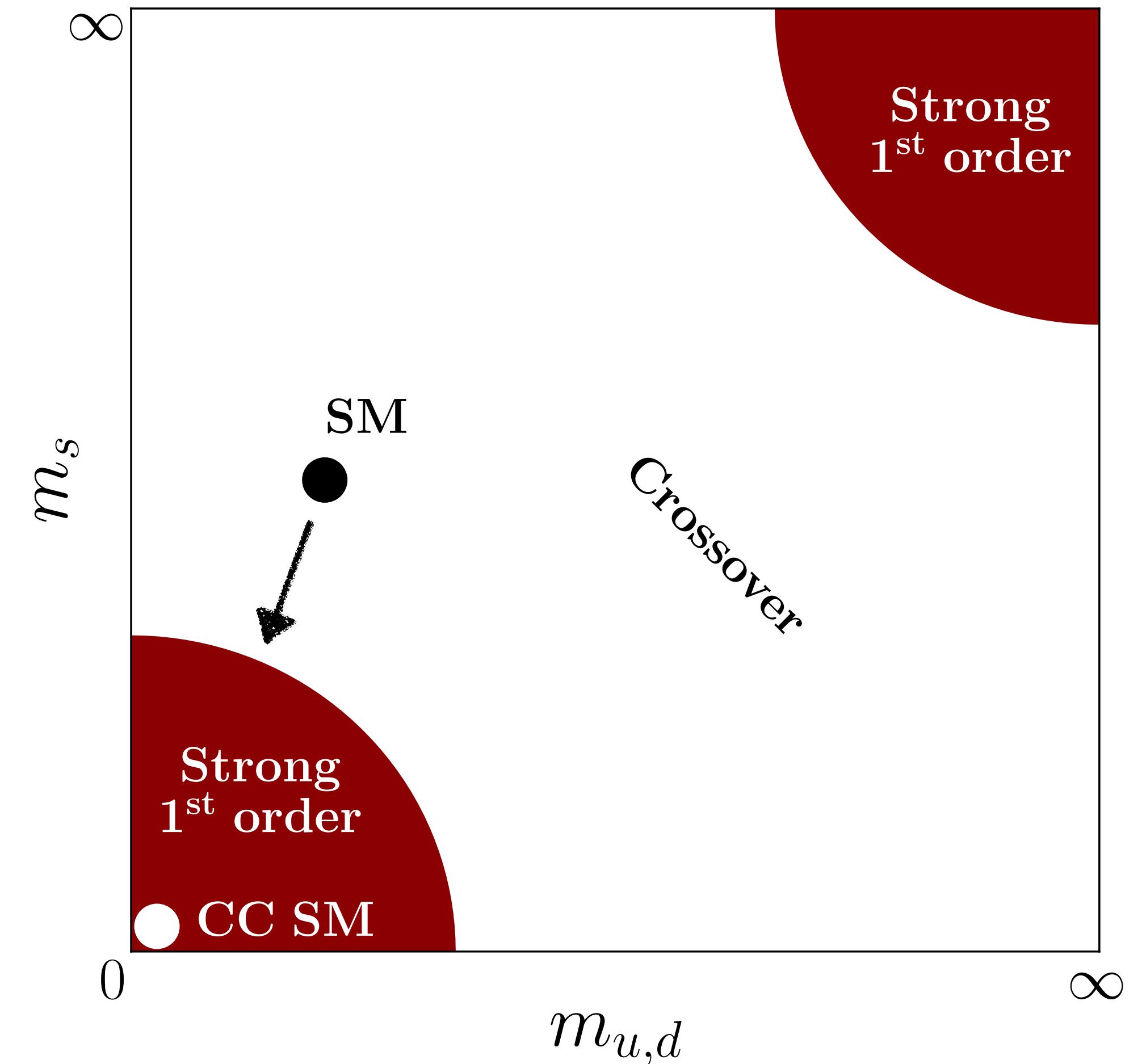
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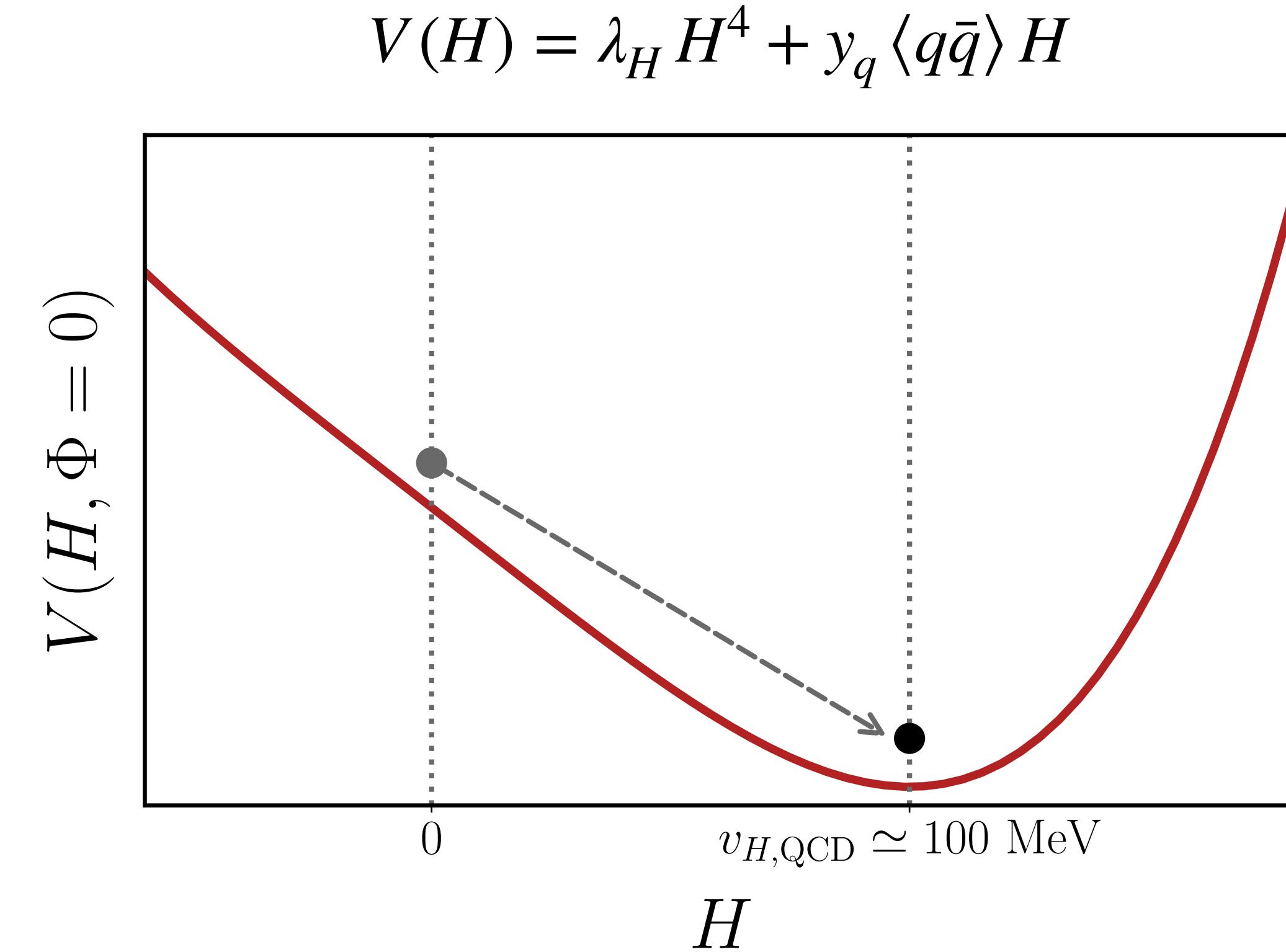
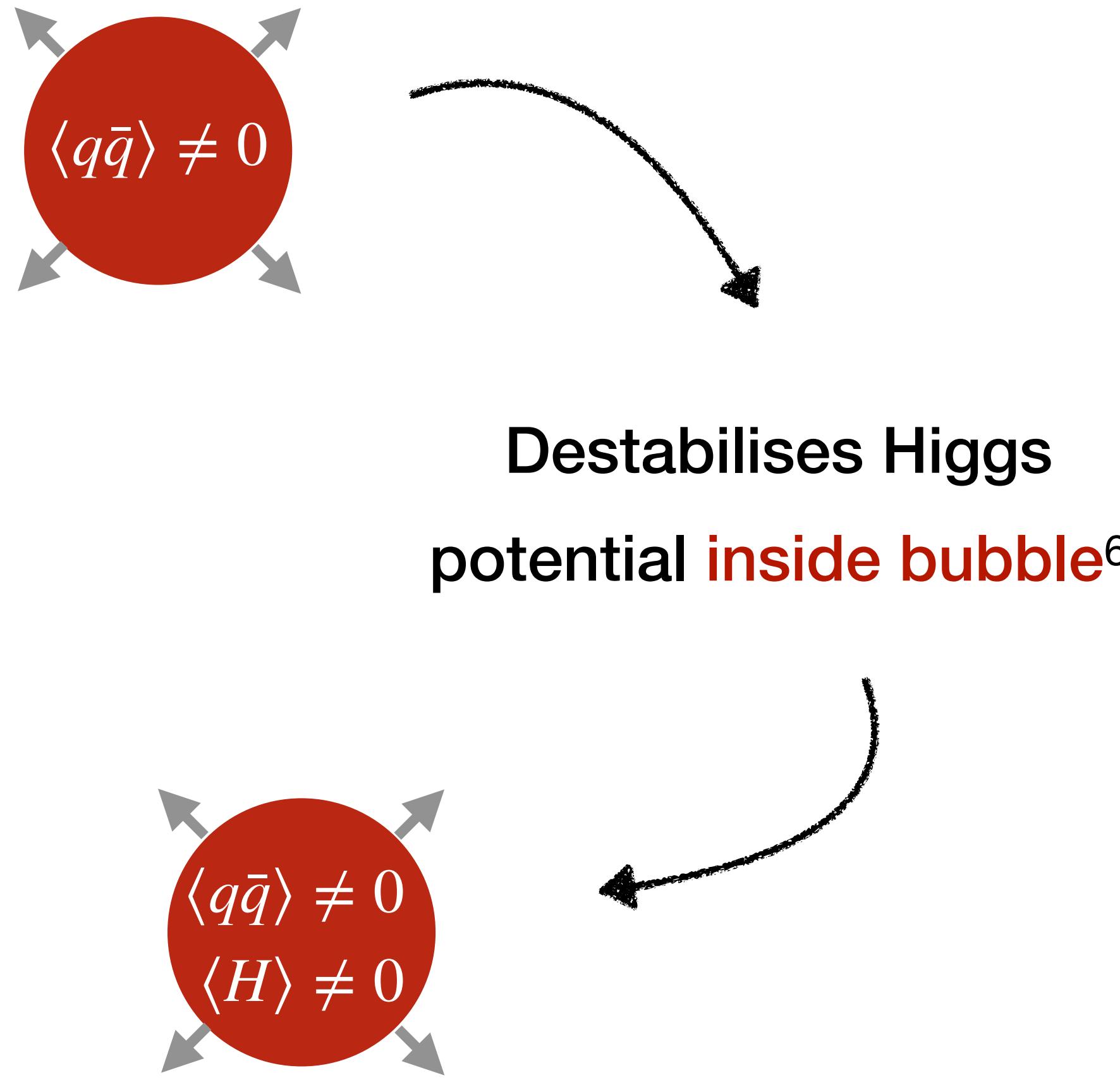
First-order chiral quark-hadron transition<sup>5</sup>

$$\langle q\bar{q} \rangle \neq 0$$



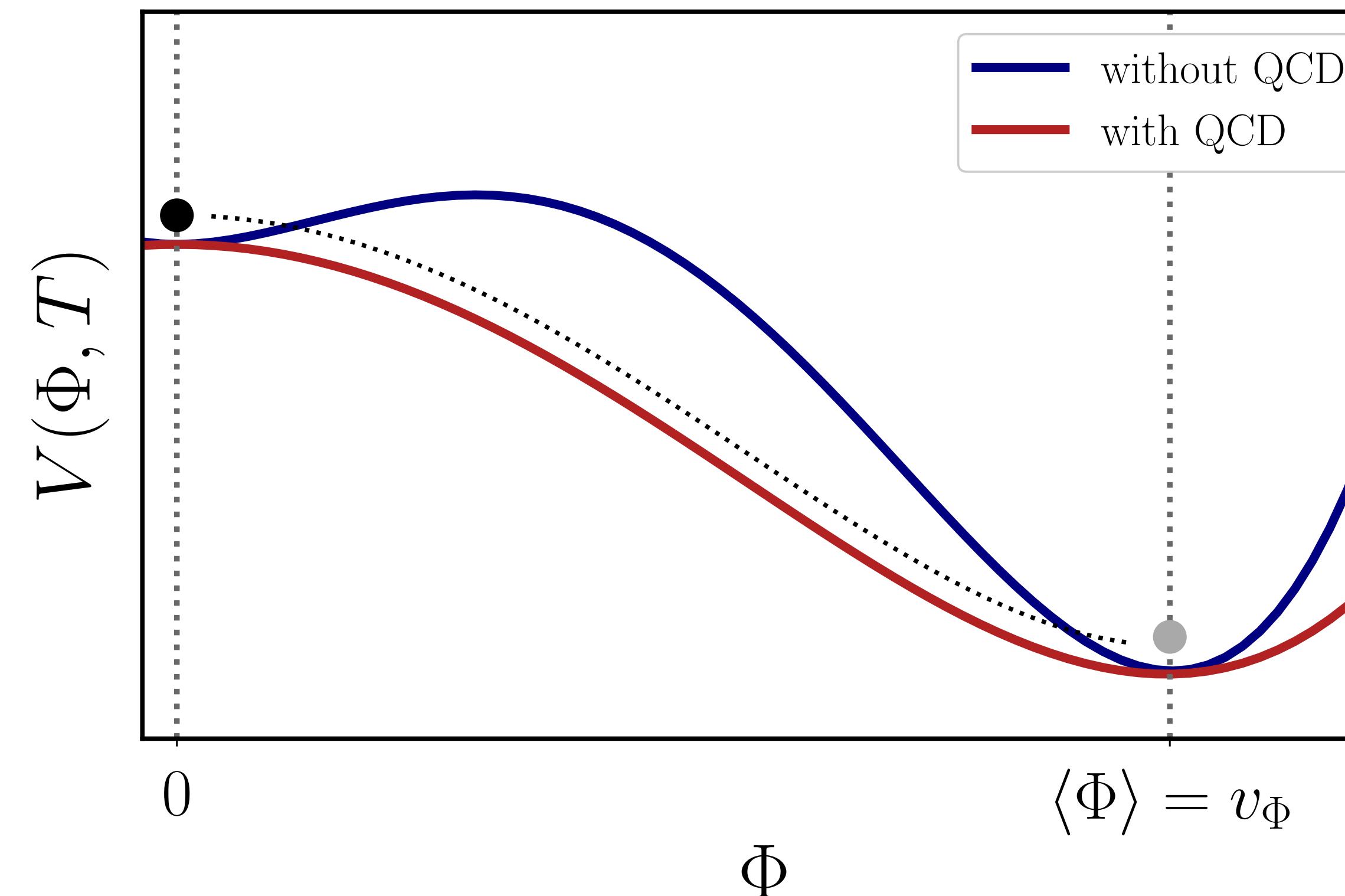
<sup>5</sup>Pisarski, Wilczek [1984]; Brown, Butler, Chen et al. [1990]

# Combined QCD - EW Phase Transition



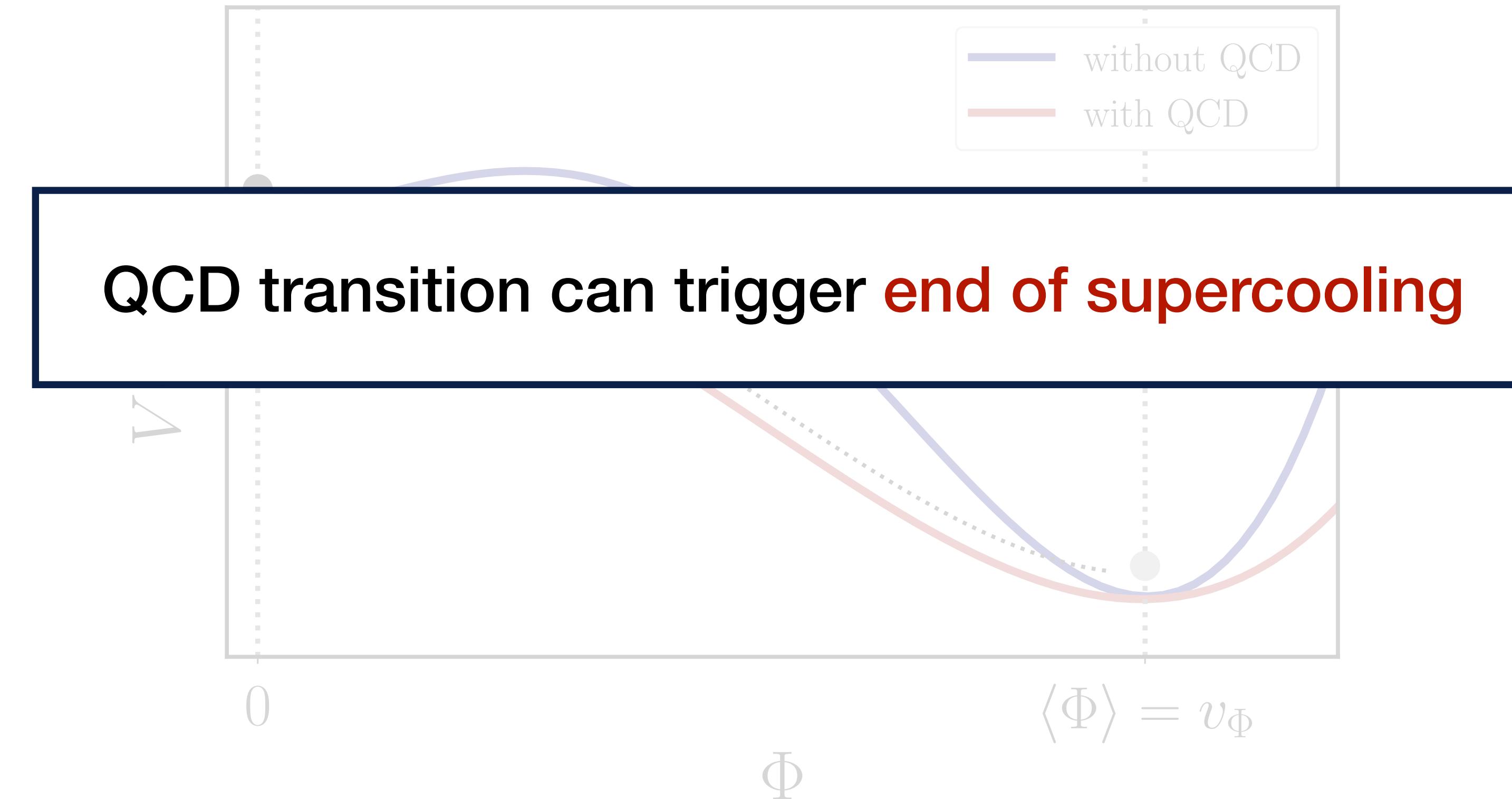
# Supercool Exit

**QCD-triggered EWSB breaks scale invariance**  $V_{\text{QCD}}(\Phi) = \lambda_p \langle H \rangle_{\text{QCD}}^2 \Phi^2$



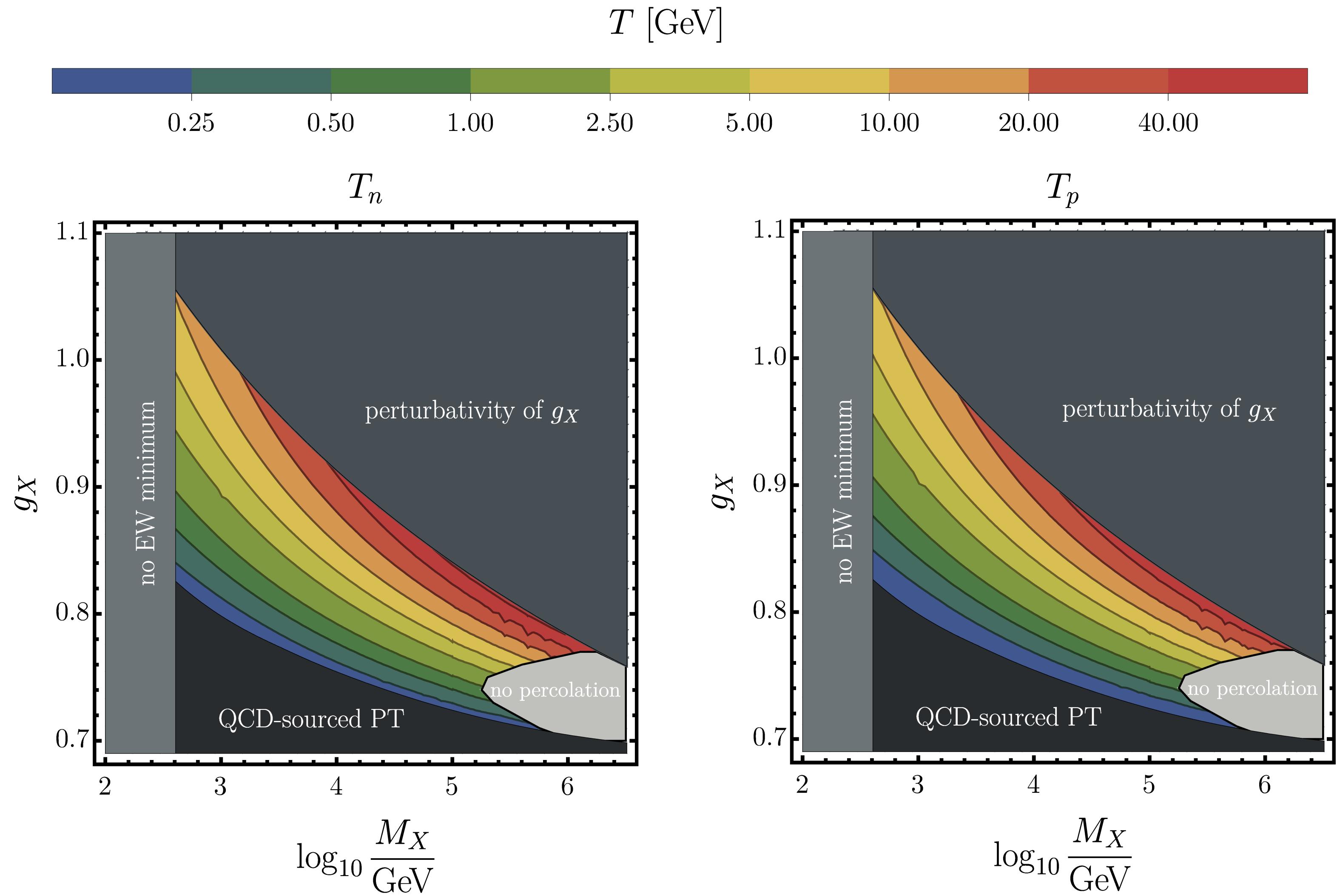
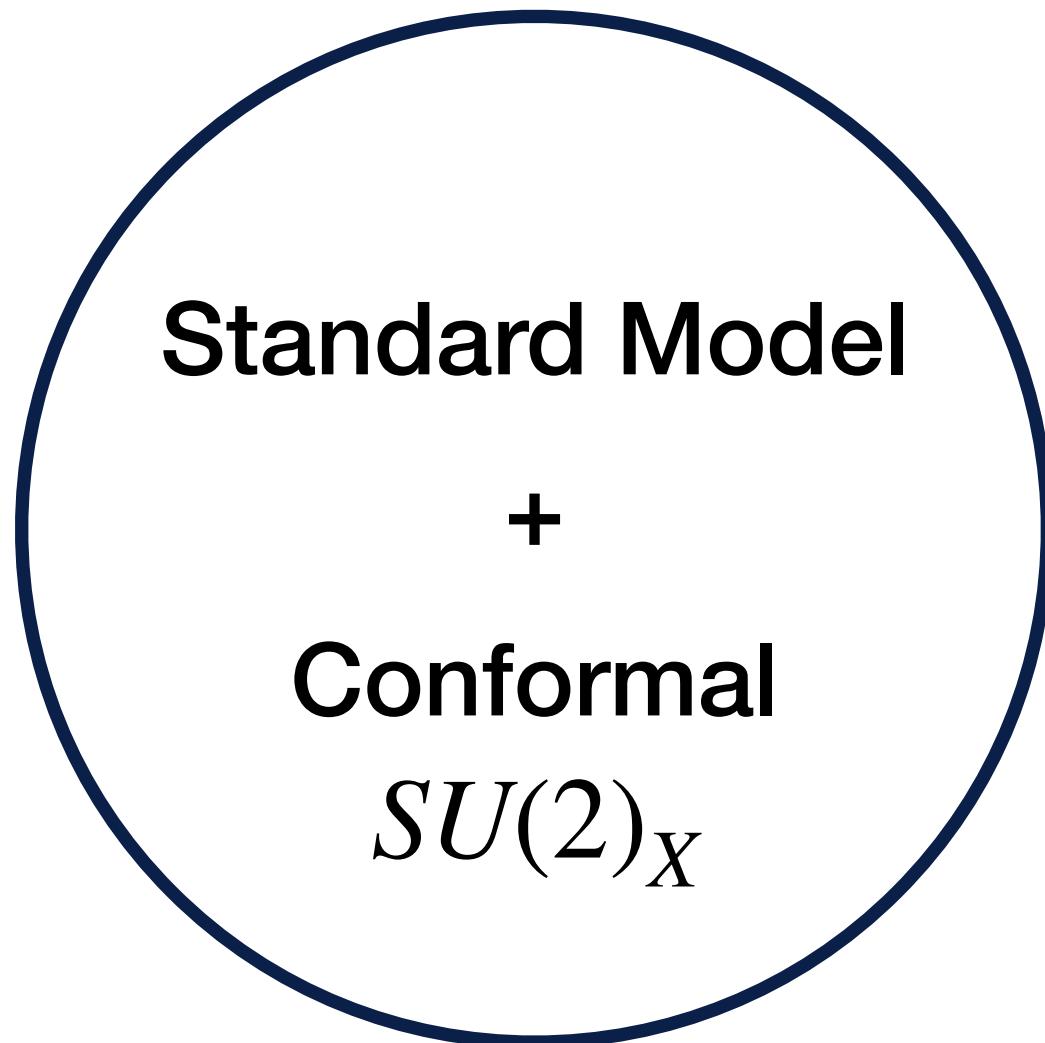
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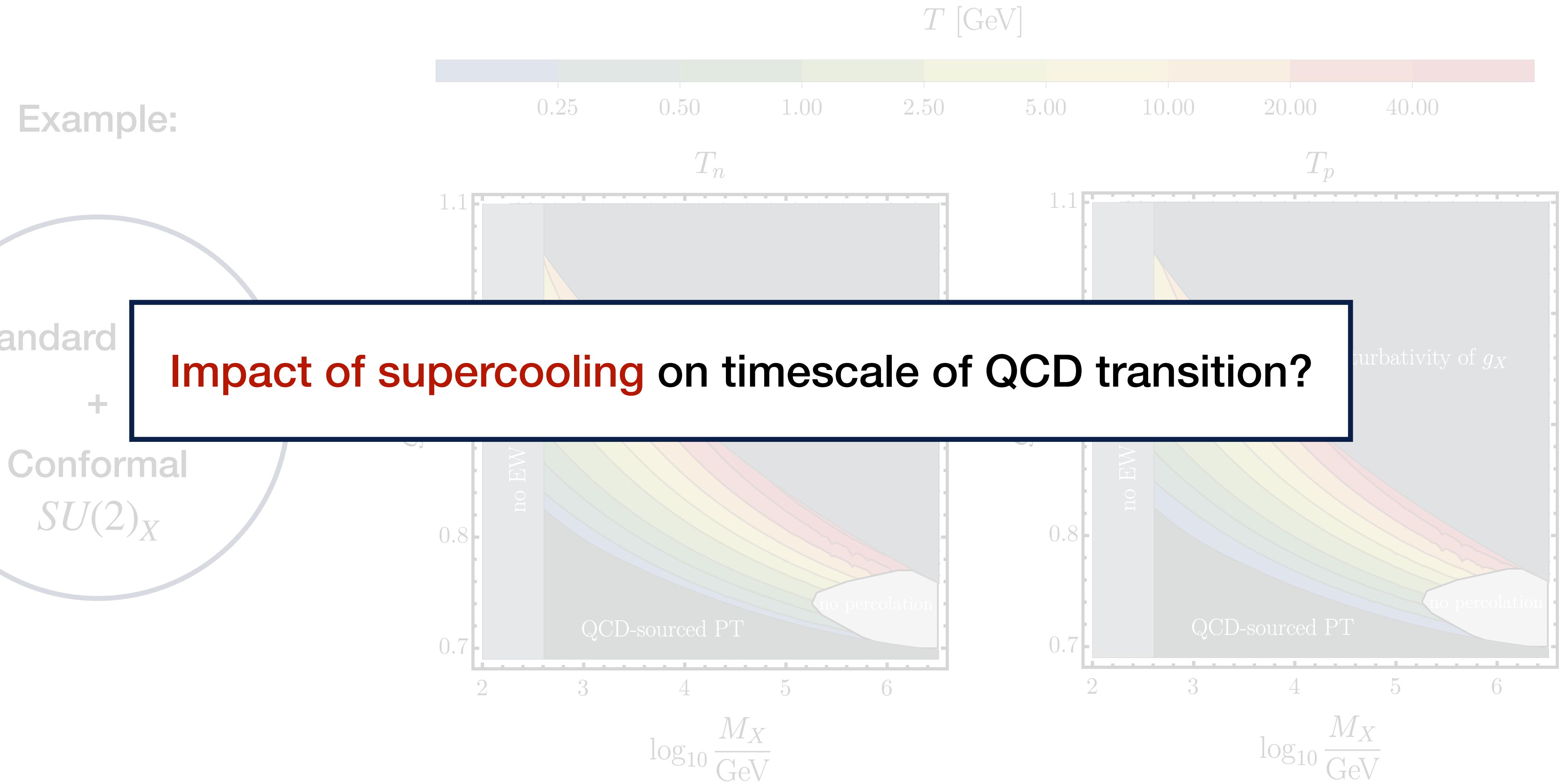


# Viable Parameter Space

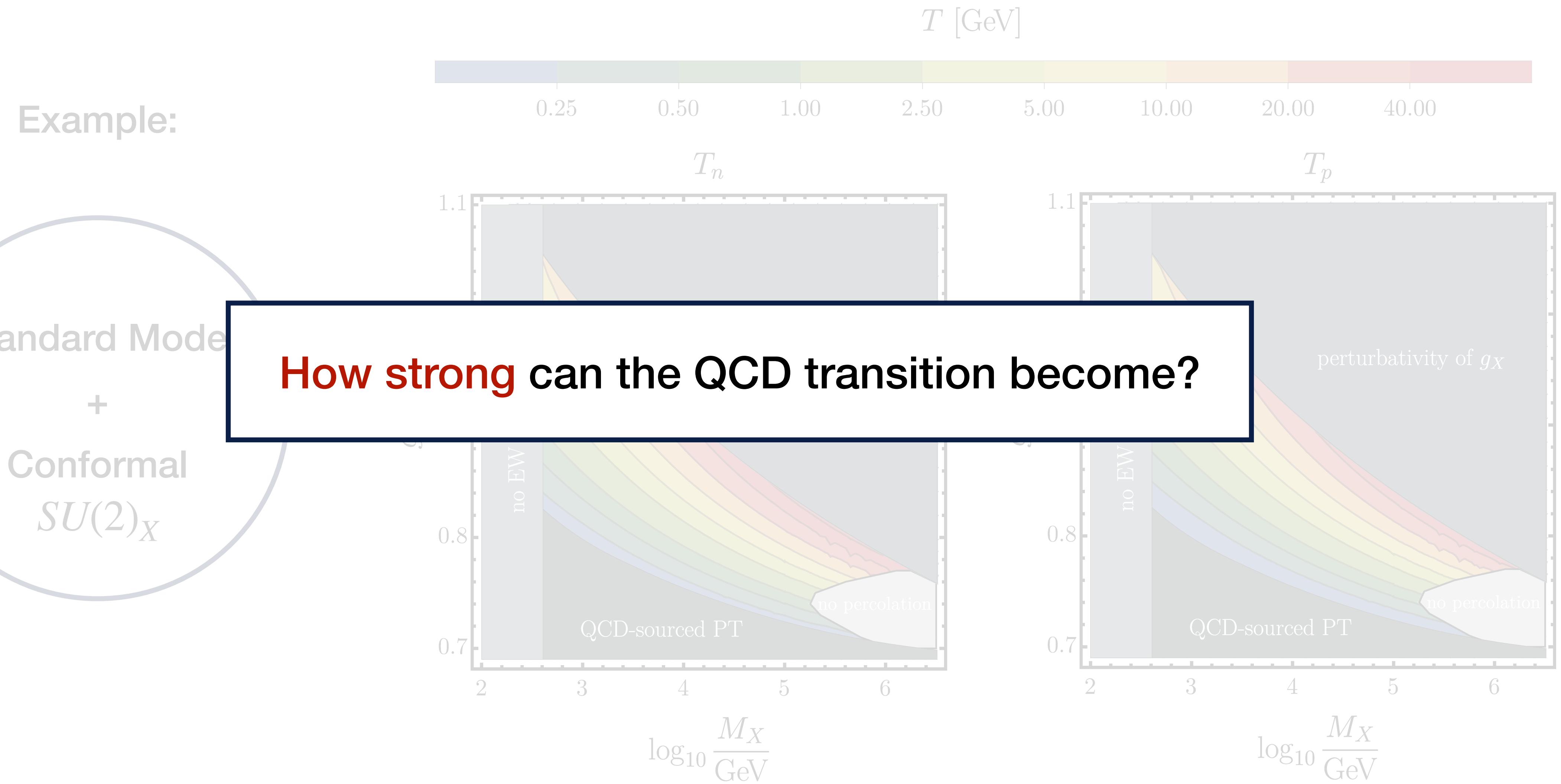
Example:



# Viable Parameter Space

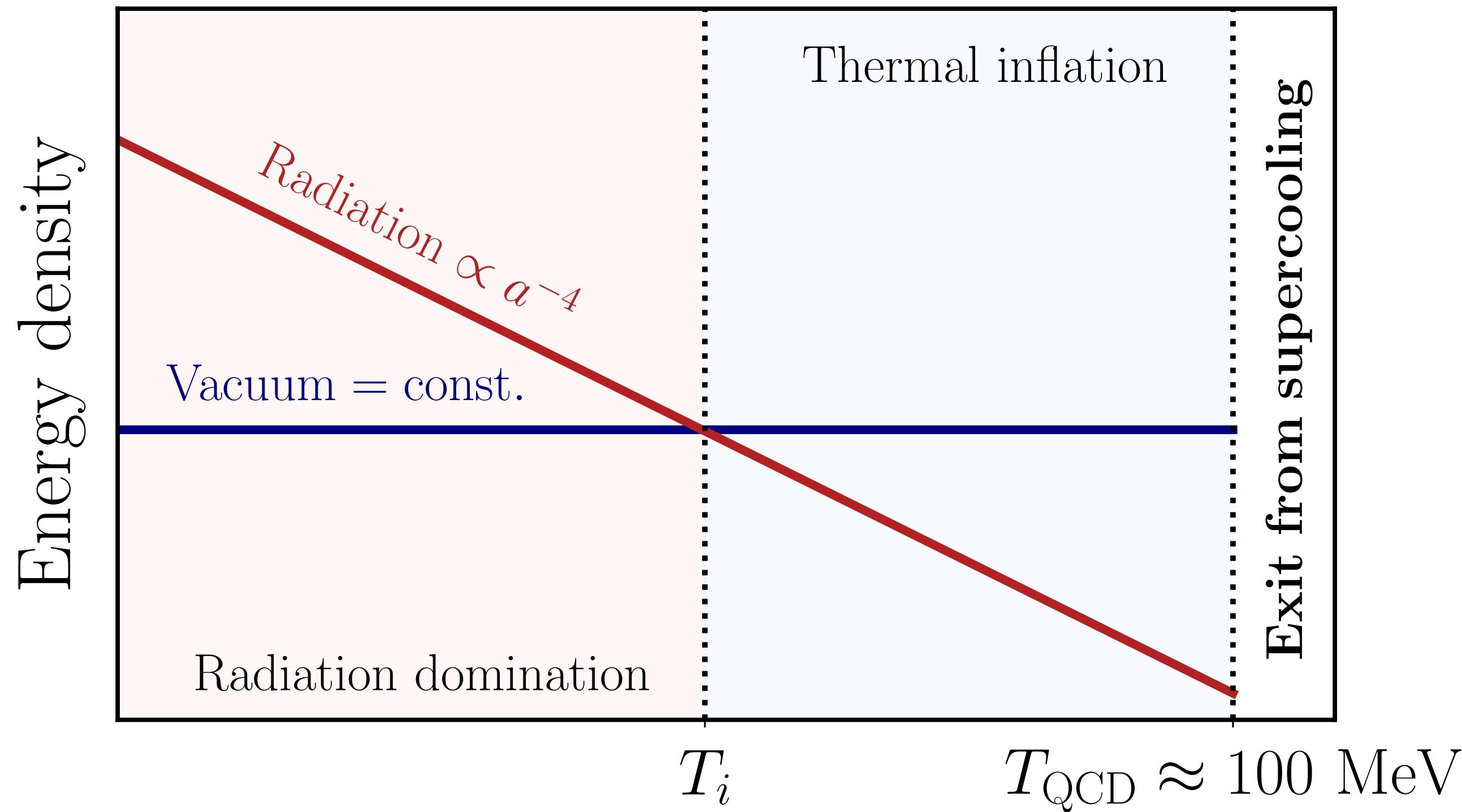


# Viable Parameter Space



## **2. Supercooled QCD Phase Transition**

# Our Setup

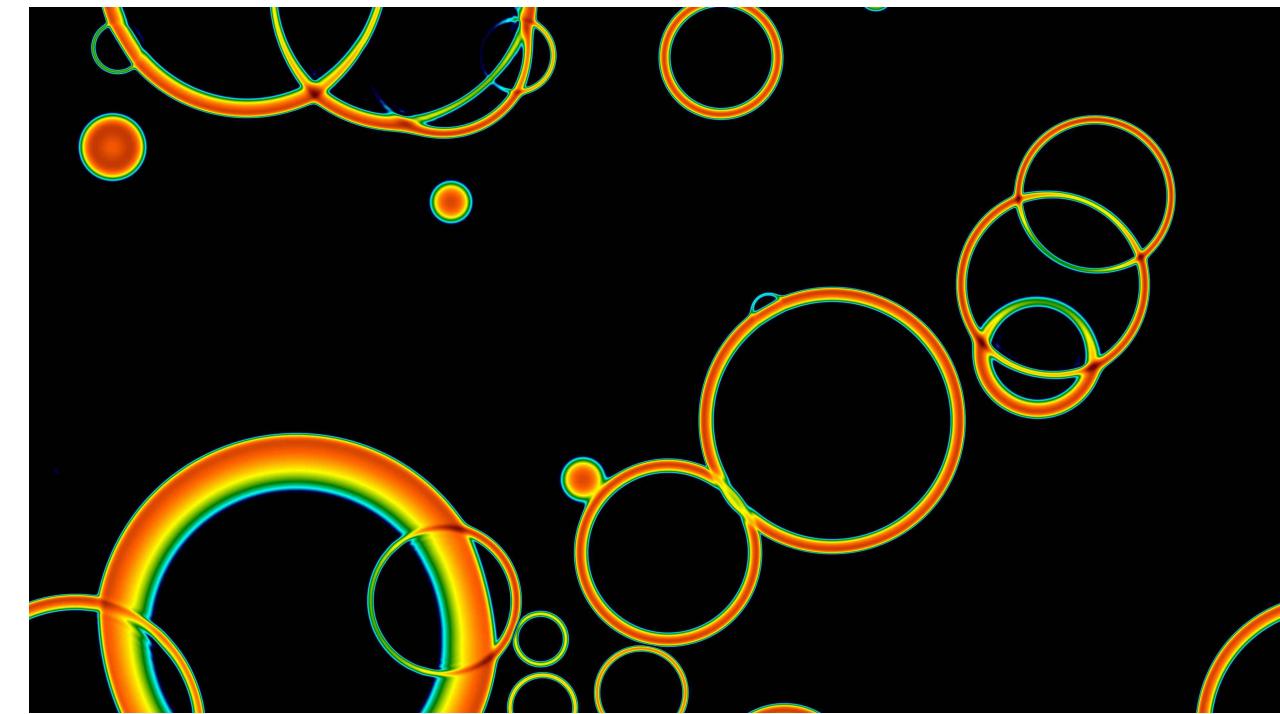


**Input: temperature of thermal inflation  $T_i$**

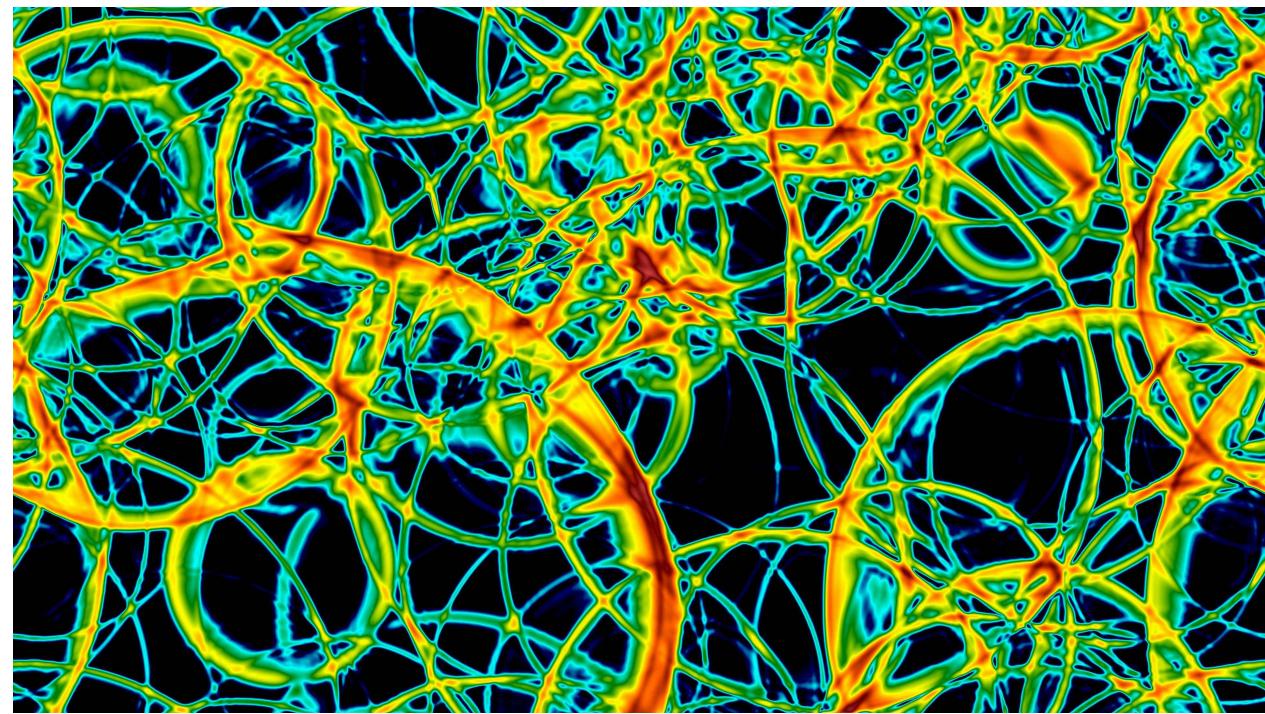
$$3M_{\text{Pl}}^2 H^2 \simeq \Delta V_{\text{CCSM}} = \frac{\pi^2}{30} g_\star T_i^4$$

**Compute transition dynamics via  
effective QCD models<sup>7</sup>**

# Gravitational Waves from First-Order PTs

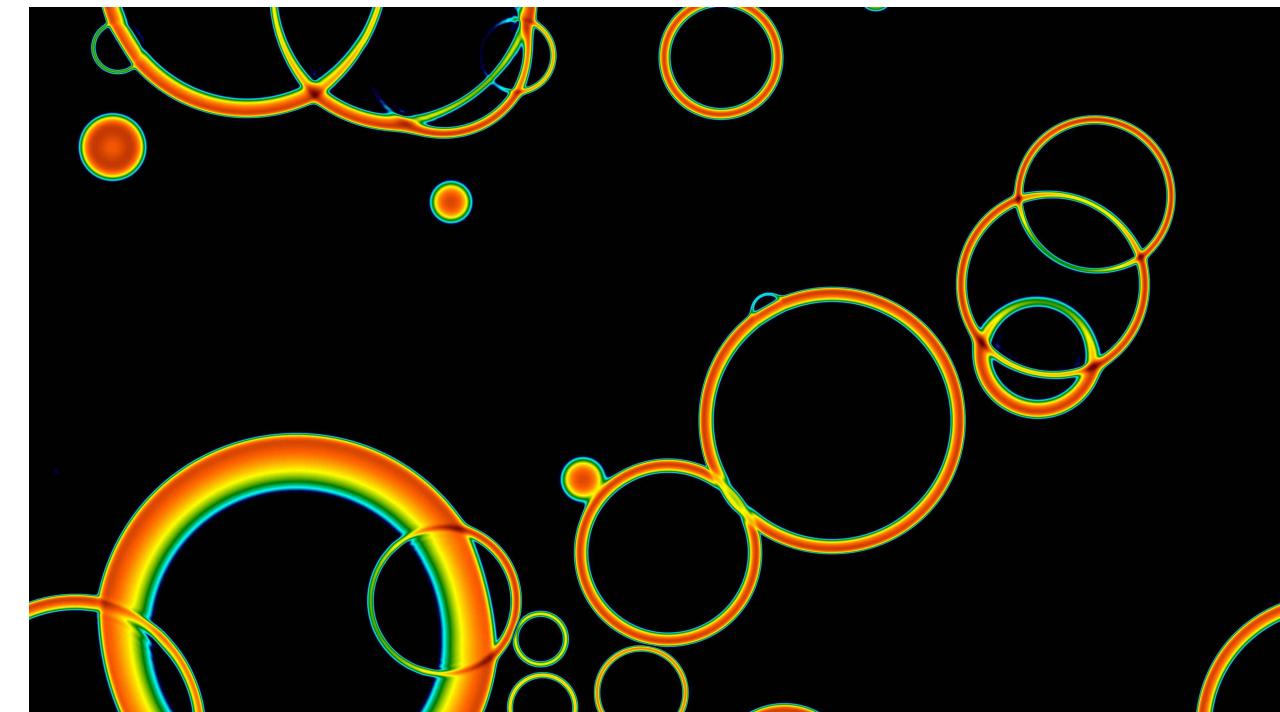


$$\Omega_{\text{GW}} \propto \frac{\alpha}{1 + \alpha} \times \left( \frac{H}{\beta} \right)^2$$

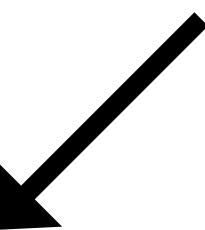


Figures: Weir [2018]

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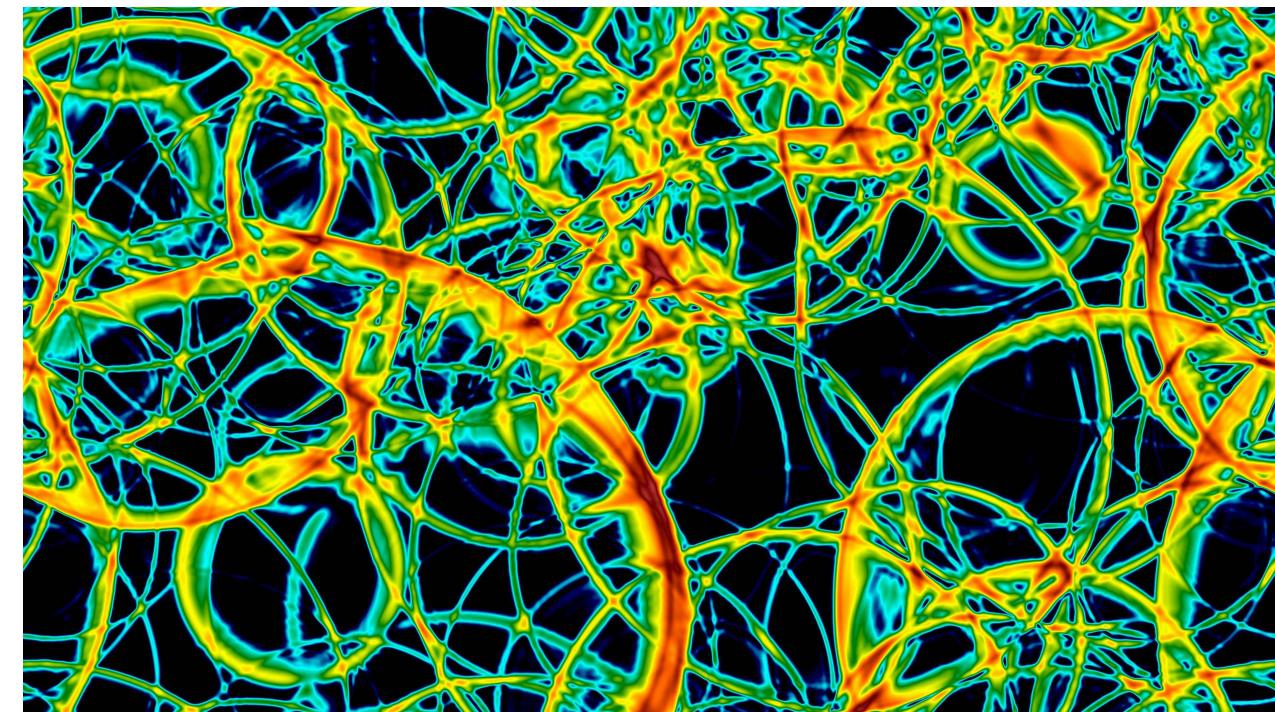


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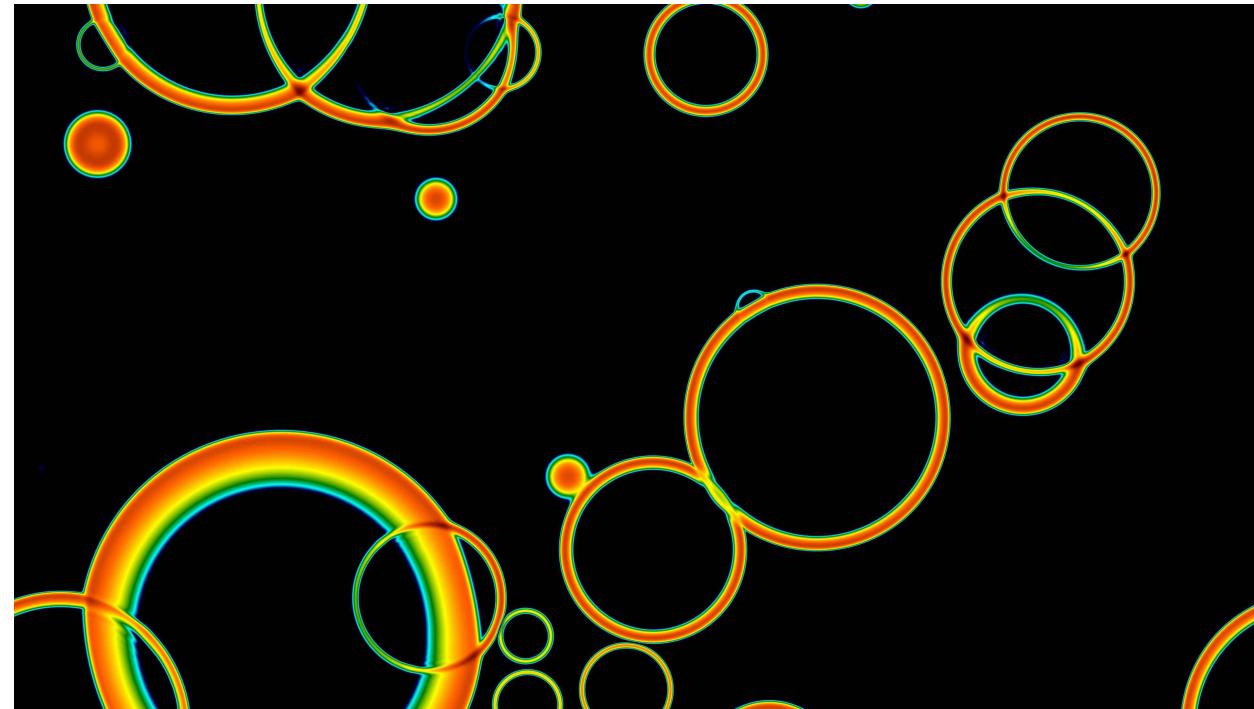
Transition strength  $\alpha$

$$\alpha = \frac{\Delta V(\Phi)}{\rho_{\text{rad}}(T_{\text{QCD}})} \propto \underbrace{\left( \frac{T_i}{T_{\text{QCD}}} \right)^4}_{\text{BSM physics}} \gg 1$$

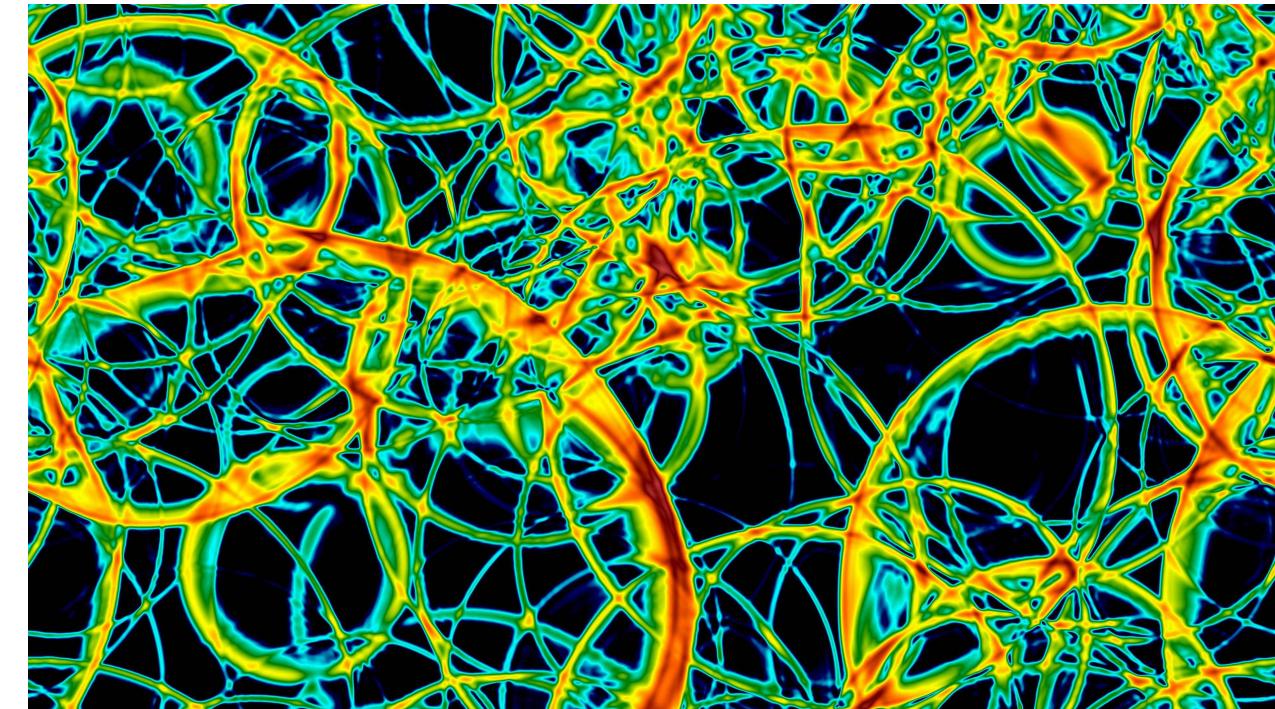


Figures: Weir [2018]

# Gravitational Waves from First-Order PTs



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Figures: Weir [2018]



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Inverse timescale  $\beta$

**Nucleation of hadronic bubbles**

# Nambu–Jona-Lasinio (NJL) Model

- Quark-based effective theory<sup>8</sup>

$$\mathcal{L} = \sum_i \bar{q}_i (i\cancel{\partial} - m_i) q_i + \underbrace{\mathcal{L}_{4F}}_{\times} + \underbrace{\mathcal{L}_{6F}}_{\times}$$

$\sim G$        $\sim G_D$

- Fit model parameters to recover properties of QCD  $\rightarrow$  Take chiral limit  $m_i \rightarrow 0$

# What about gluons?

Quark confinement



Diverging free energy

# What about gluons?

- Fundamental traced Polyakov loop<sup>9</sup>  $\ell$

$$\ell(\mathbf{x}) = \frac{1}{N_c} \text{Tr}_c \mathbf{L} = \exp \left( -\beta F_q(r) \right), \quad \text{where} \quad \mathbf{L} = \mathcal{P} \exp \left[ i g_s \int_0^{\beta=1/T} d\tau \underbrace{i A_0}_{i A_0} A_4(\mathbf{x}, \tau) \right]$$

Quark confinement



Diverging free energy

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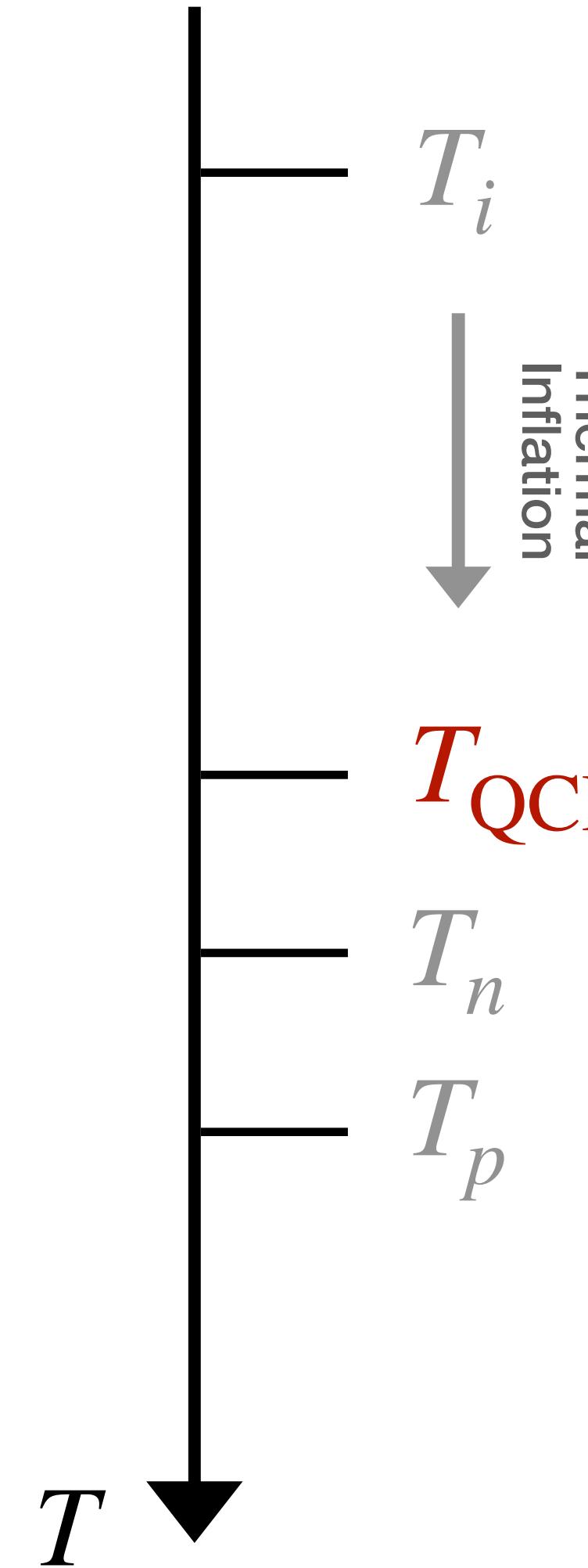
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- Thermodynamics fitted against lattice data
  - Pure Yang-Mills: Polyakov loop extended NJL (**PNJL**) model
  - QCD: Improved PNJL model<sup>10</sup>

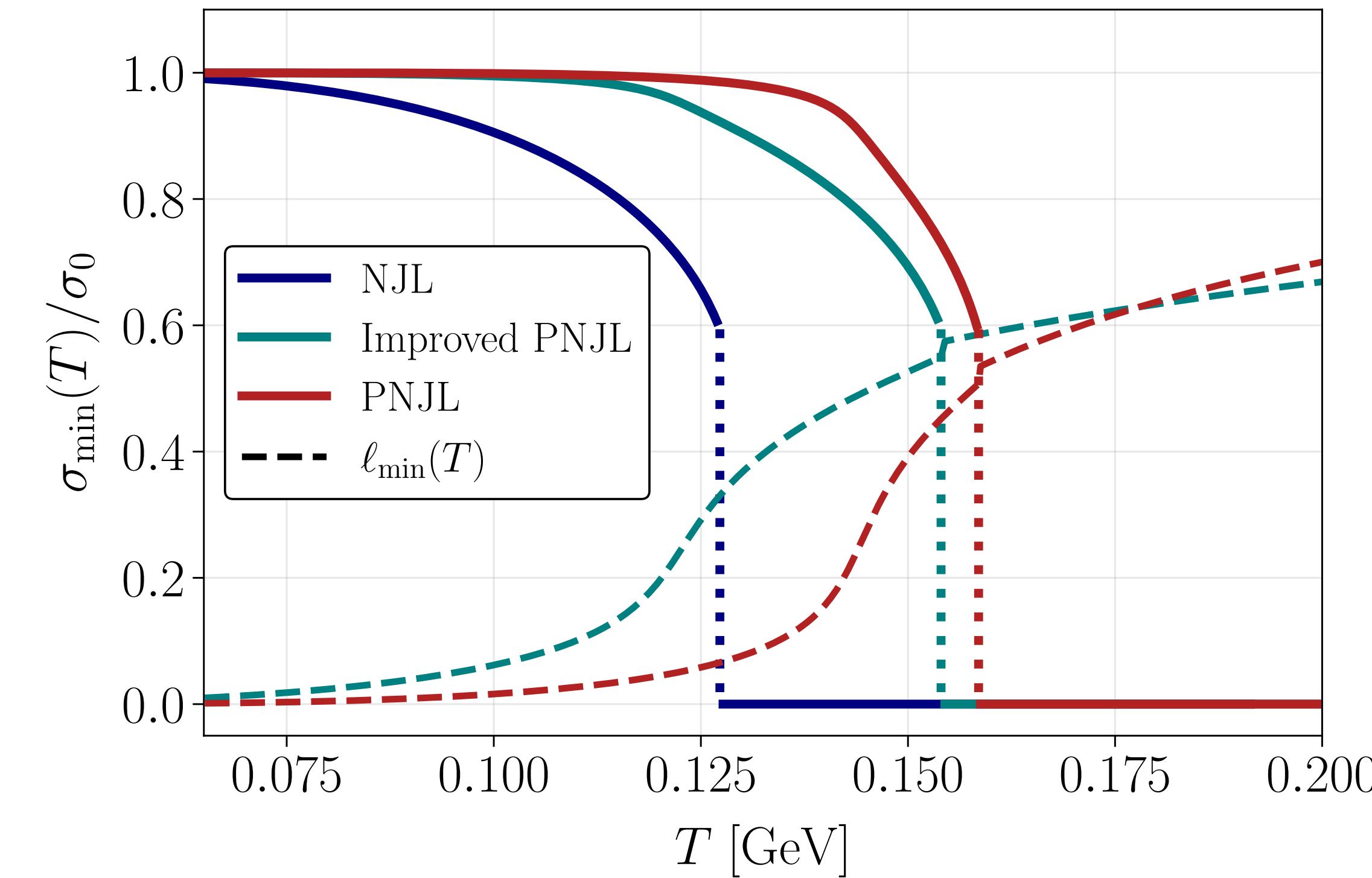
<sup>9</sup>Fukushima [2004]; Fukushima, Skokov [2017]

<sup>10</sup>Haas, Stiele, Braun et al. [2013]

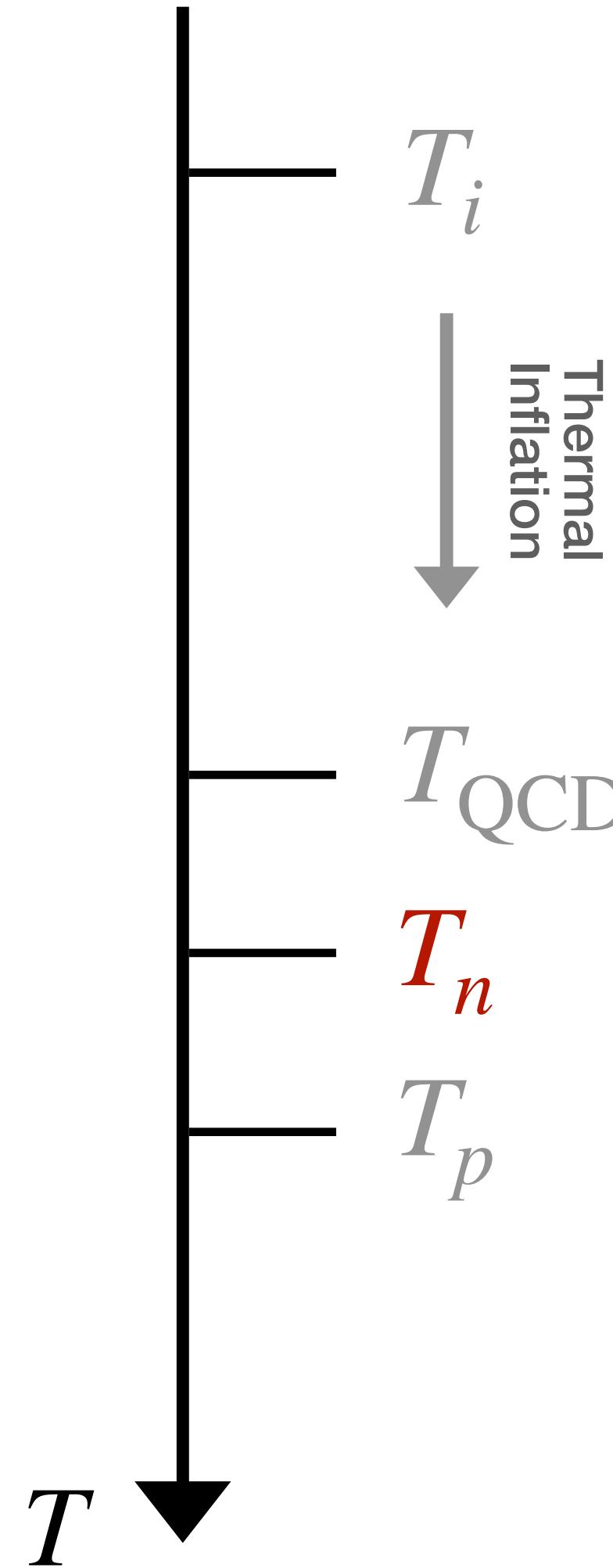
# Critical Temperature



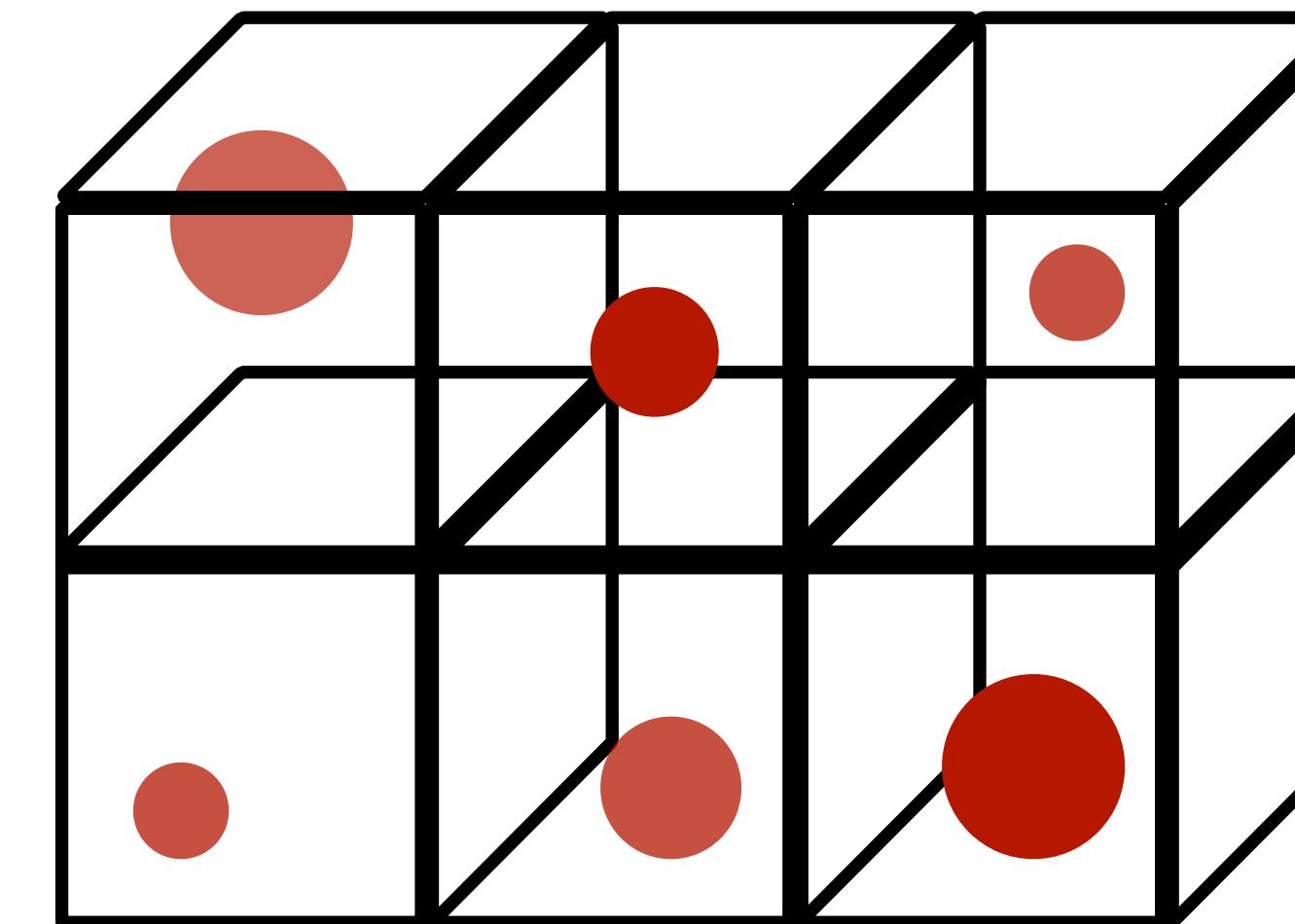
Quark condensation becomes energetically favorable



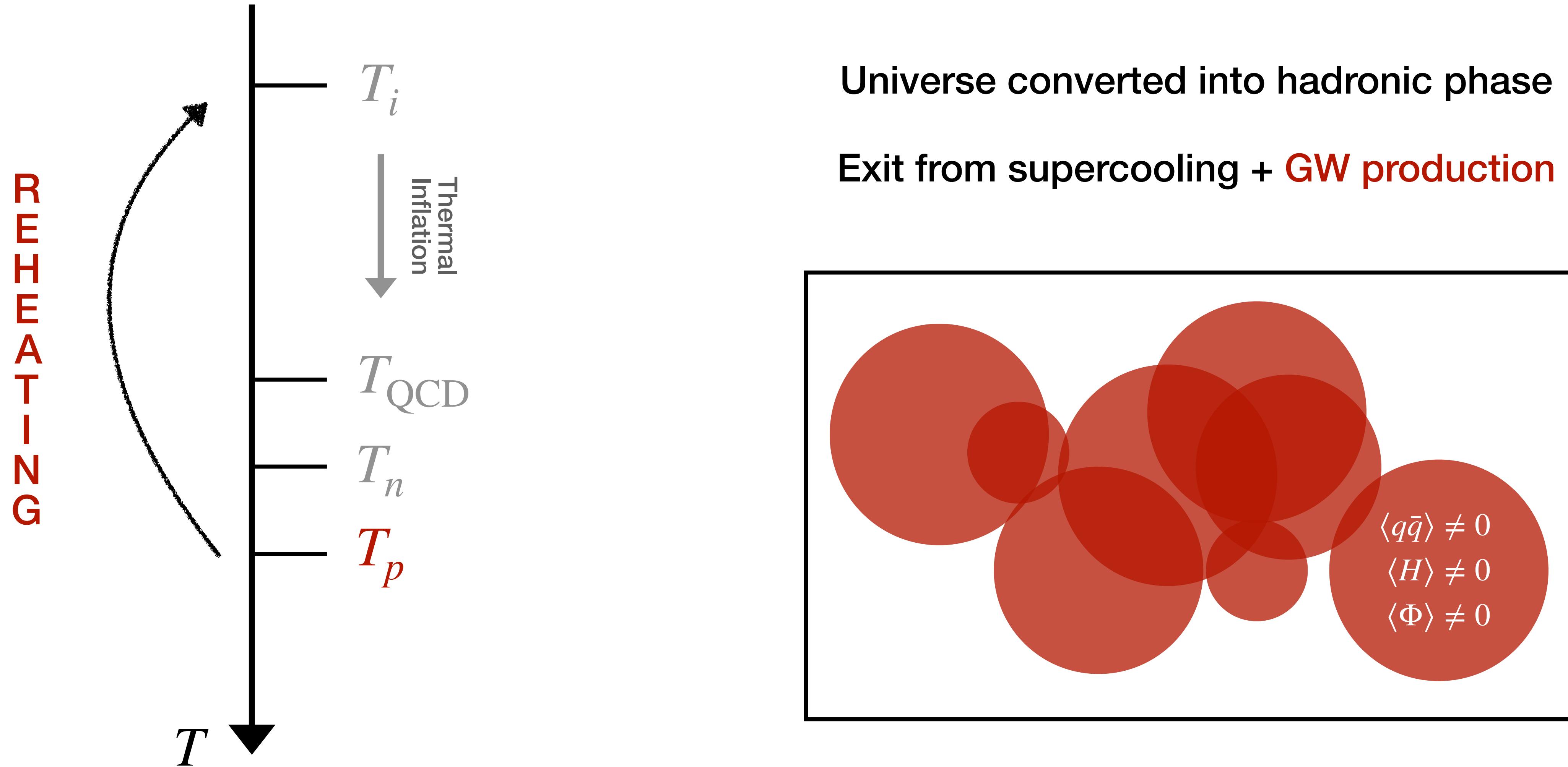
# Nucleation Temperature



One hadronic bubble nucleated per Hubble patch

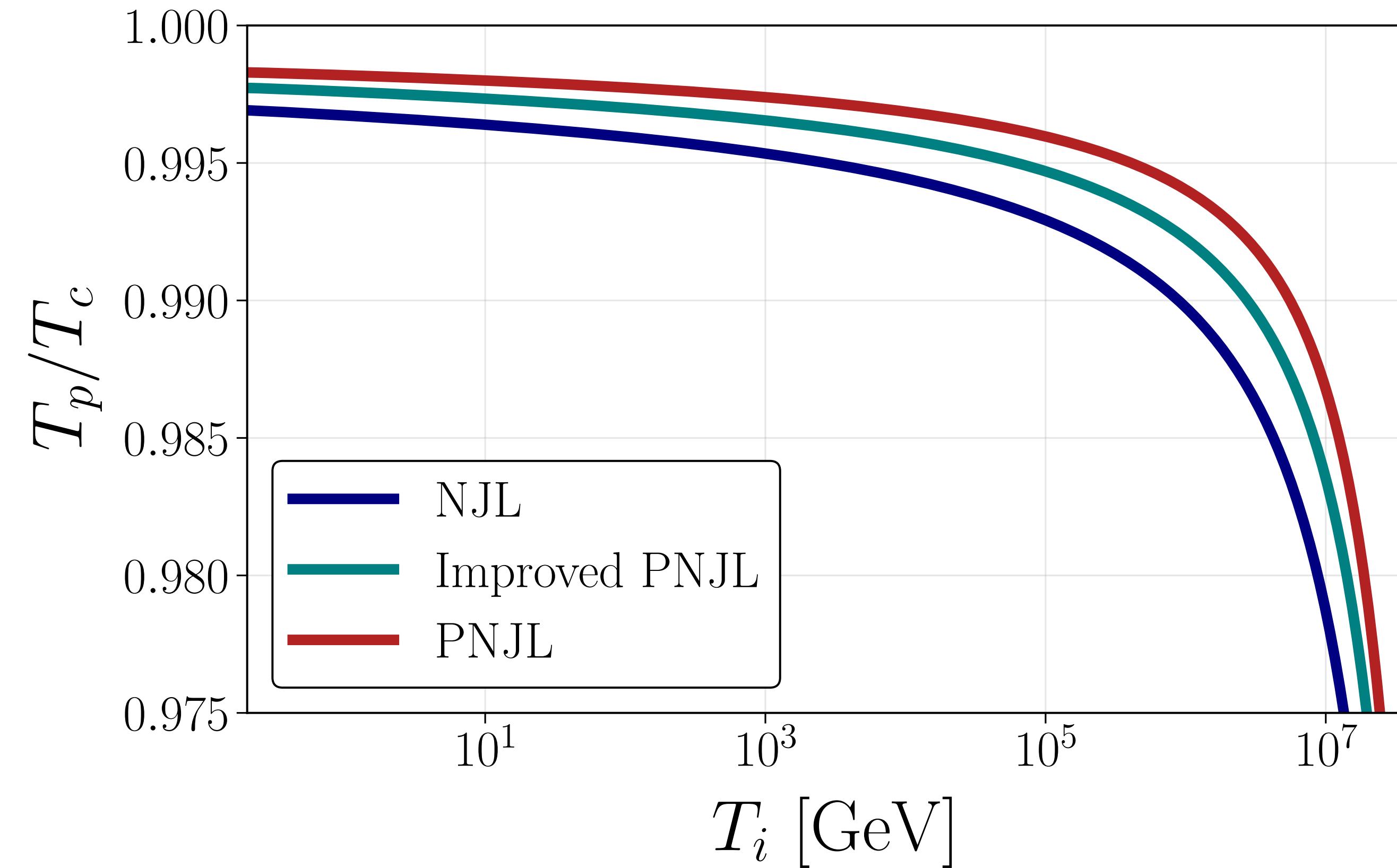


# Percolation Temperature



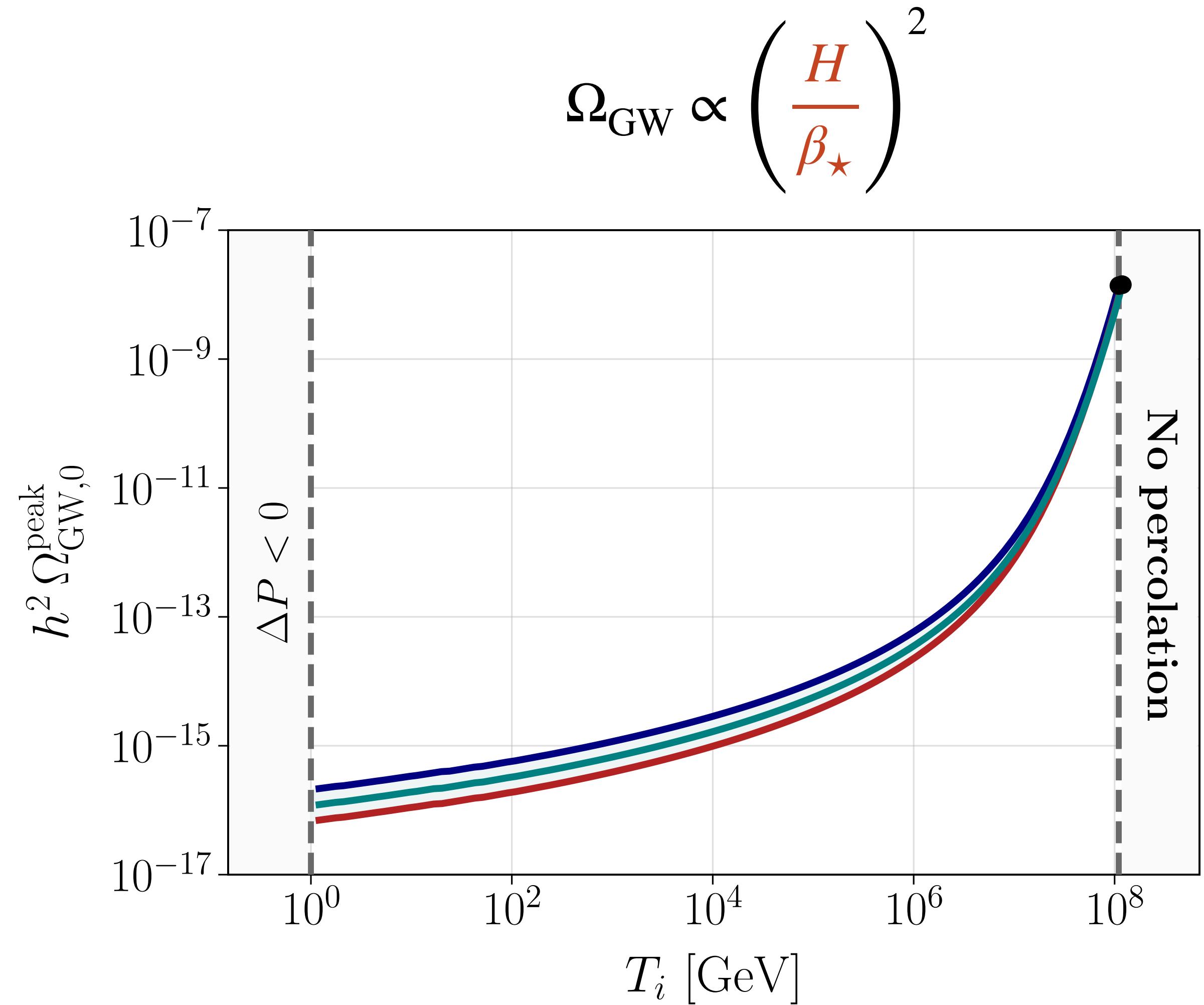
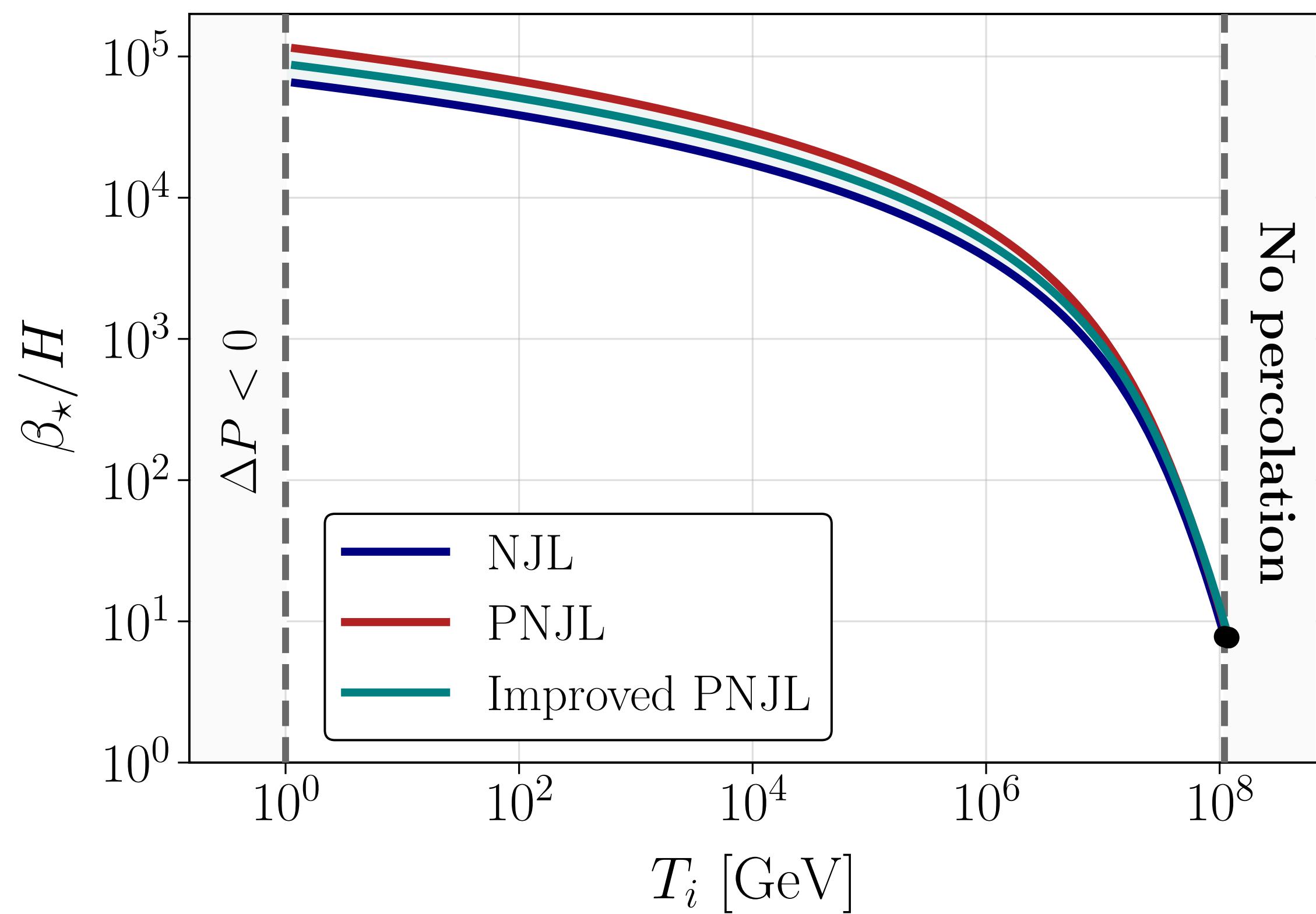
# Percolation Temperature

Percolation temperature decreases with increasing amount of supercooling

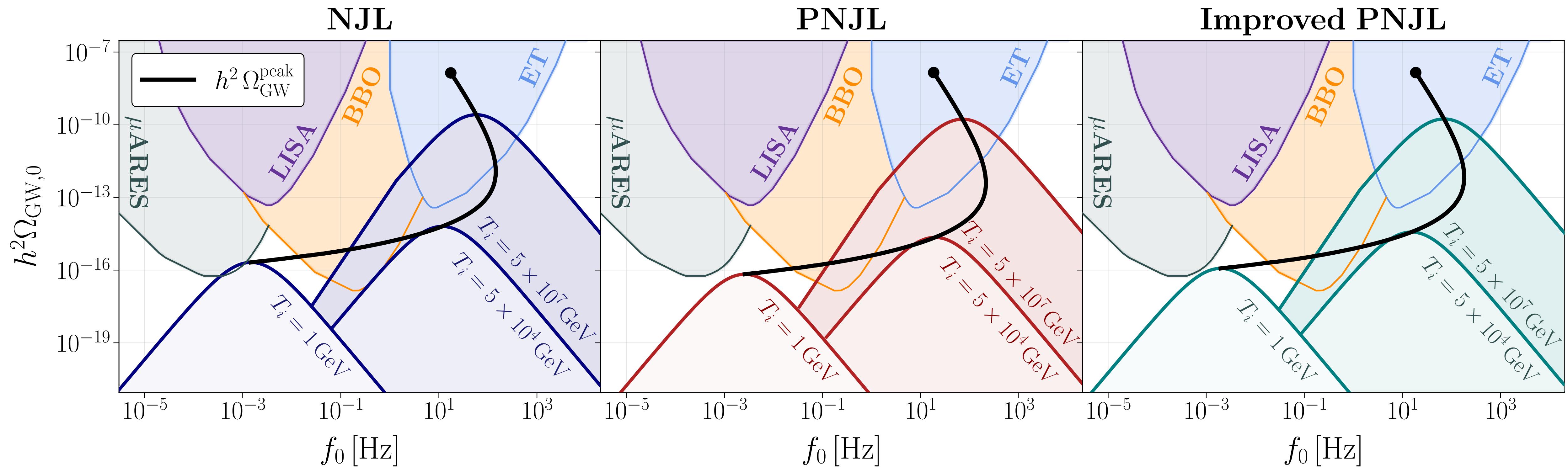


# Gravitational Wave Amplitude

How fast does the PT proceed?



# Observational Prospects



# Summary

**Classically scale-invariant SM extensions well motivated BSM scenario**

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**Natural outcome: Universe is supercooled down to QCD scale**

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**QCD can trigger the end of thermal inflation**

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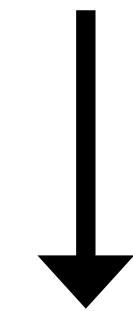
**Resulting GW amplitude grows with amount of supercooling**

# **Backup Slides**

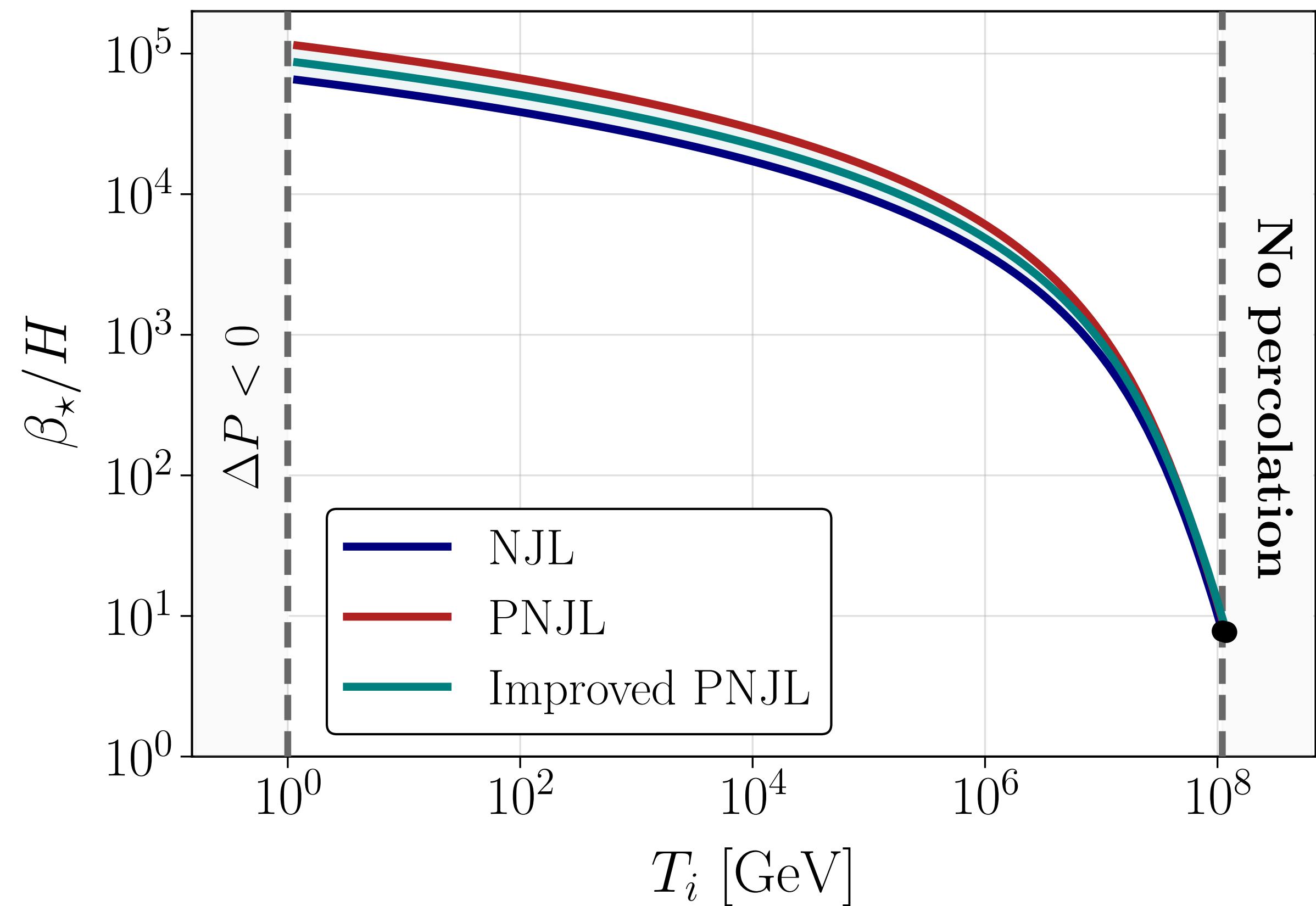
# GWs - Frequency Dependence

$$f_0 \propto \beta \frac{a_\star}{a_0} \propto \frac{\beta}{H_\star} H_\star \frac{a_\star}{a_0} \propto \frac{\beta}{H_\star} T_i$$

Large  $T_i$  : inverse timescale drops  
faster than increase in  $T_i$



Frequency decreases again



# Temperature Limits

**Minimum Temperature:** Consider leading order friction on bubble wall

$$\Delta V = P_{1 \rightarrow 1} \propto \sum_i \Delta m_i^2 T_{\text{QCD}}^2$$

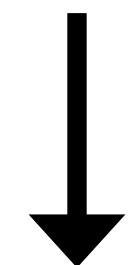
$$\downarrow i = \{t, W^\pm, Z\}$$

$$T_{i,\min} = \mathcal{O}(1) \text{ GeV}$$

# Temperature Limits

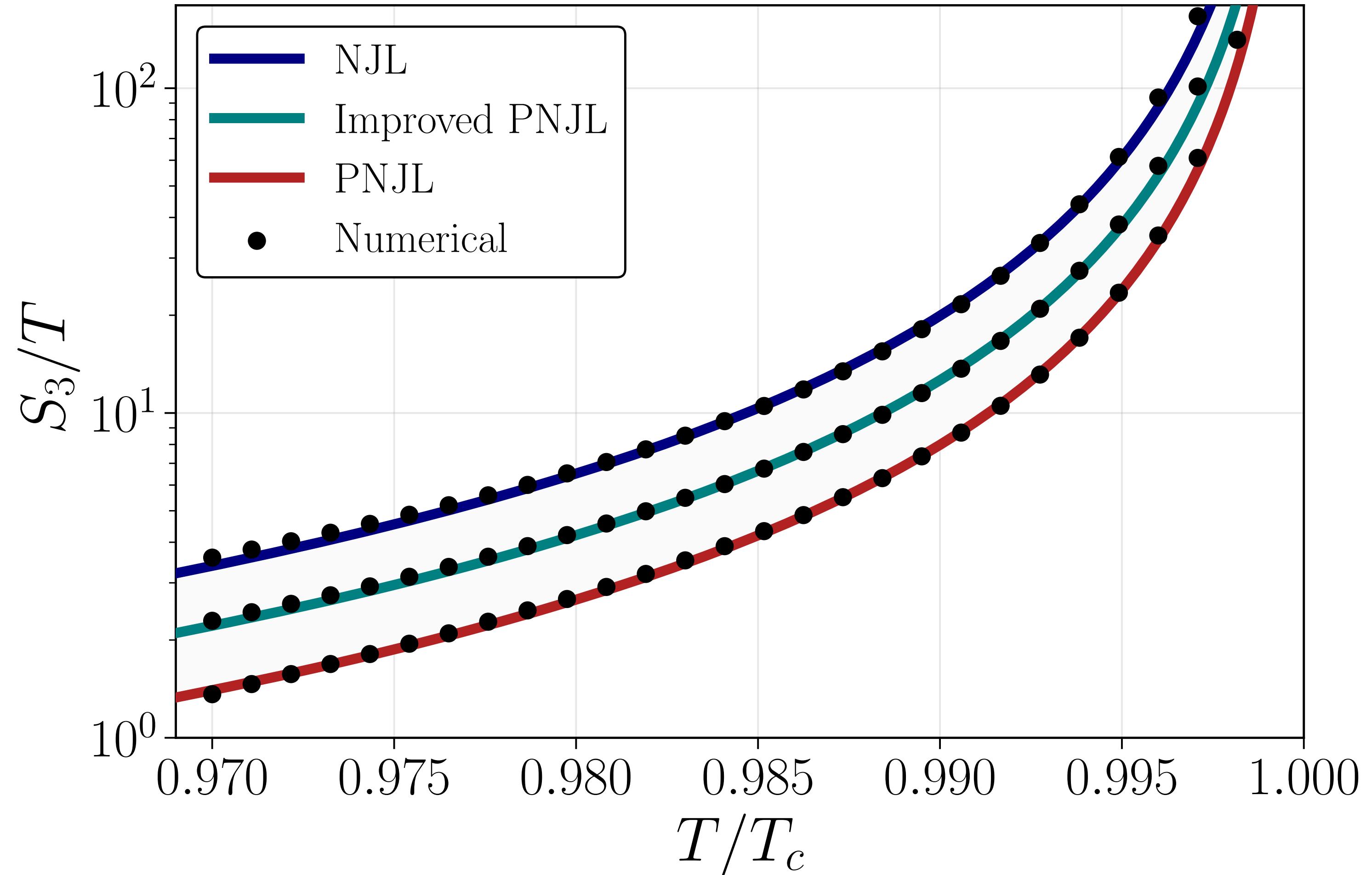
**Maximum Temperature:** Consider volume trapped in false vacuum

$$\frac{1}{V_{\text{false}}} \frac{dV_{\text{false}}}{dt} = H(T) \left( 3 + T \frac{dI(T)}{dT} \right) < 0$$



$$T_{i,\max} = \mathcal{O}(10^8) \text{ GeV}$$

# Bounce Action



Employ fitting template

$$\frac{S_3}{T} \simeq b \left( 1 - \frac{T}{T_c} \right)^{-\gamma}$$

$$\Gamma \propto \exp \left( -\frac{S_3}{T} \right)$$

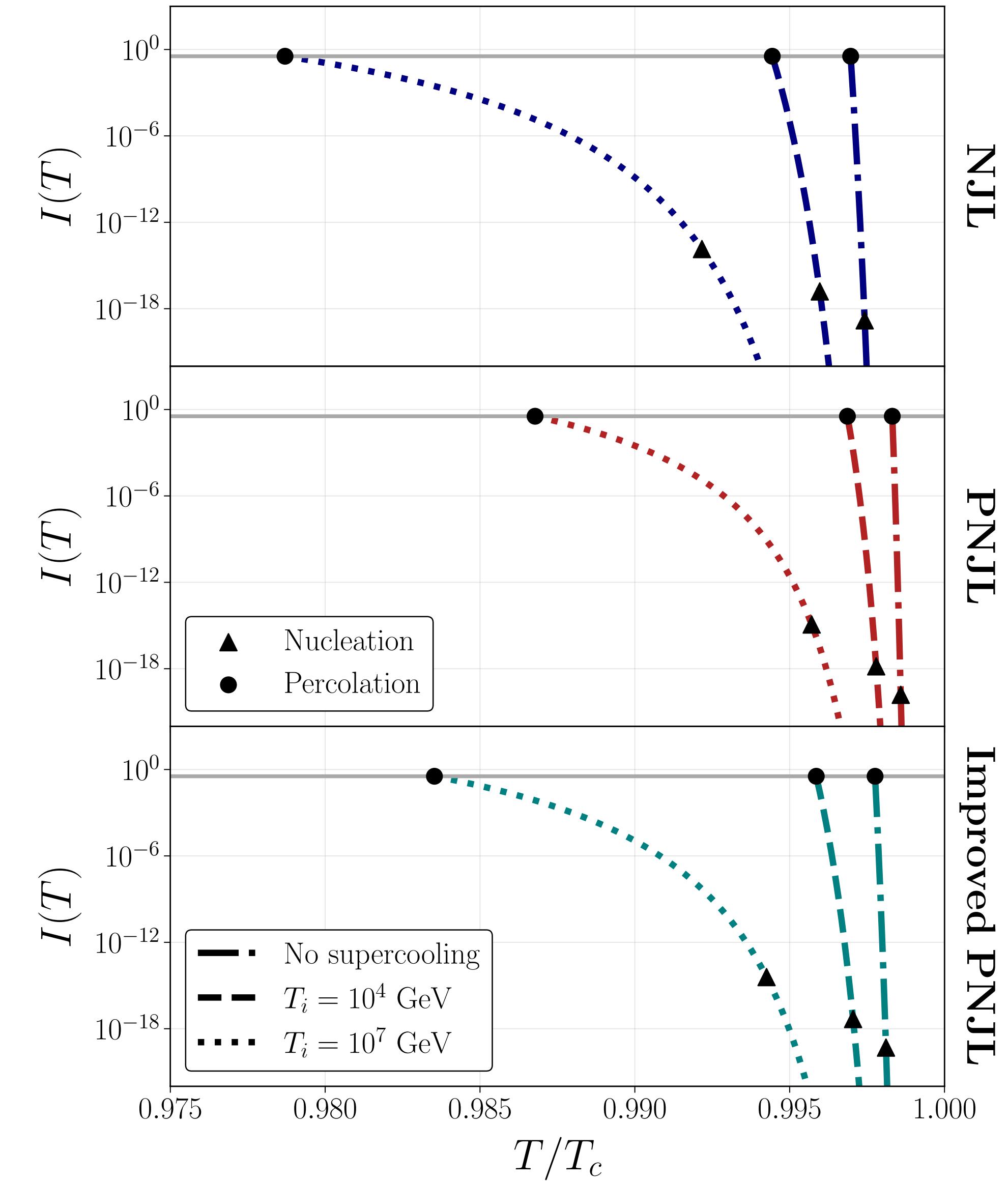
# Completing the Phase Transition

Probability to remain in false vacuum

$$P = \exp[-I(T)]$$

Percolation temperature<sup>11</sup>

$$I(T_p) = 0.34$$



<sup>11</sup>Ellis, Lewicki, No [2018]

# Bounce Action

Solve equation of motion

$$\frac{d^2\sigma}{dr^2} + \frac{2}{r} \frac{d\sigma}{dr} - \frac{1}{2} \frac{\partial \log Z_\sigma}{\partial \sigma} \left( \frac{d\sigma}{dr} \right)^2 = Z_\sigma \frac{dV_{\text{eff}}(\sigma, \ell, T)}{d\sigma}$$

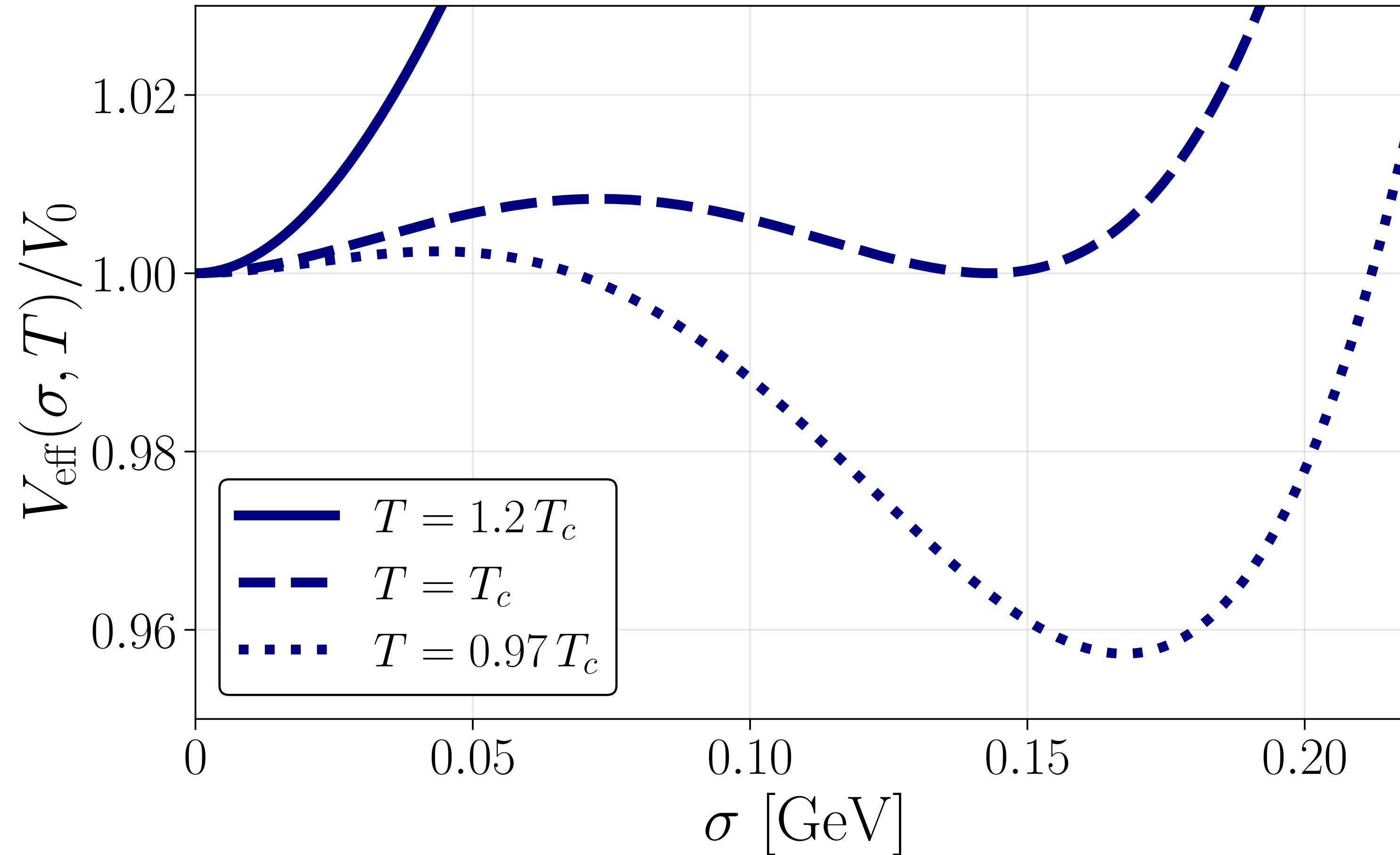
Compute 3D bounce action numerically

$$S_3 = 4\pi \int dr r^2 \left[ \frac{Z_\sigma^{-1}}{2} \left( \frac{d\sigma}{dr} \right)^2 + V_{\text{eff}}(\sigma, \ell, T) \right]$$

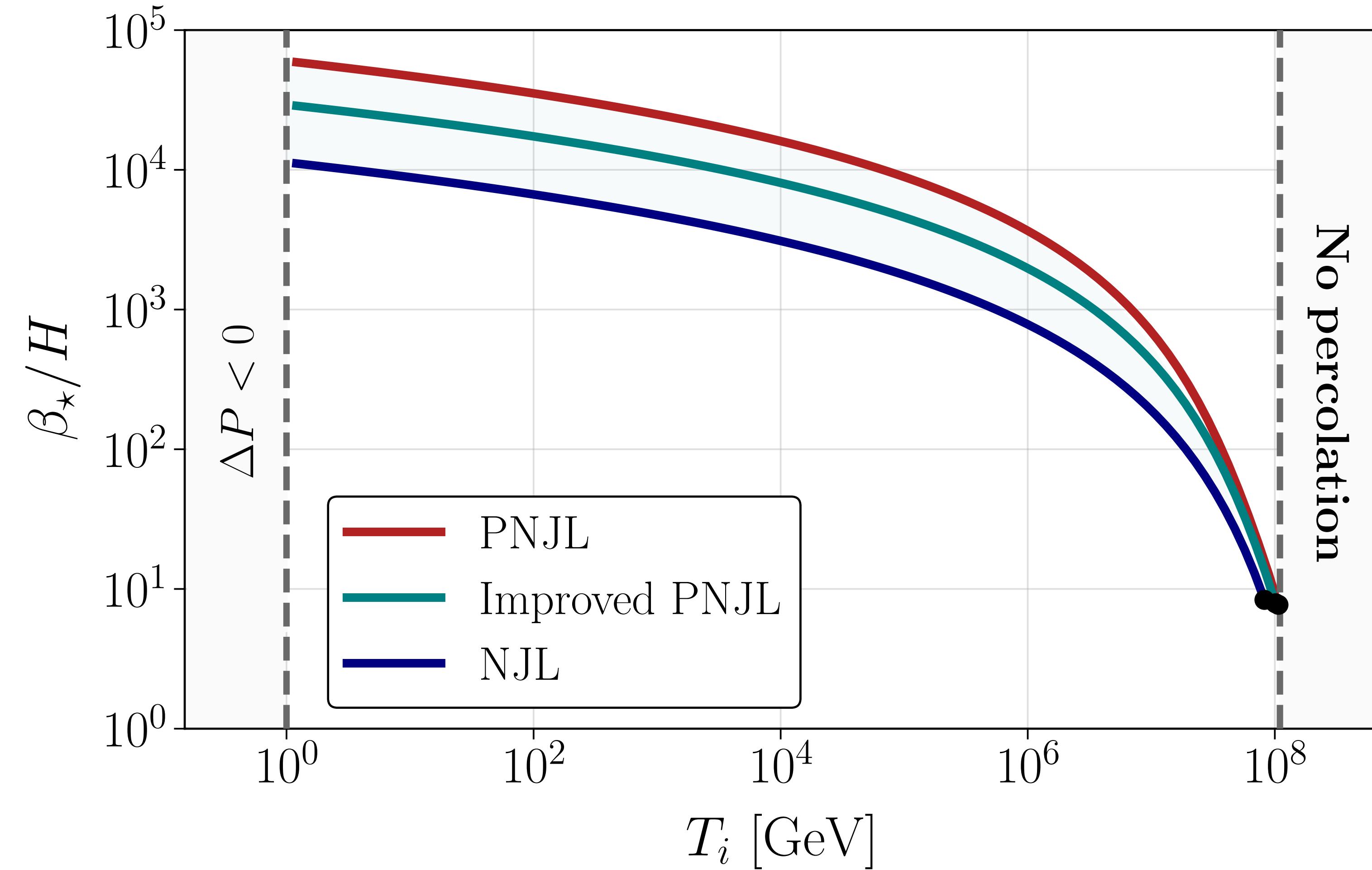
$Z_\sigma^{-1}$ : Wave function renormalization

# NJL Effective Potential

$$V_{\text{eff}}(\sigma, T) = V_0(\sigma) + V_1(\sigma) + V_T(\sigma, T)$$



# Transition Timescale - 4D Cutoff



# Cutoff Scheme Dependence

