The warm inflation scalar power spectrum

Alejandro Pérez Rodríguez
Universidad Autónoma de Madrid, IFT UAM-CSIC

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Outline

Based on 2208.14978[1] and 2304.05978[2]:

1. Introduction and motivation to warm inflation

2. Warm inflation scalar power spectrum

3. Gravitational waves from warm inflation
    3.1 Consequences for 2nd-order scalar induced GWB
    3.2 Consequences for CMB tensor-to-scalar ratio

[1] In collaboration in G. Ballesteros, M.A.G. García, M. Pierre and J. Rey

[2] In collaboration in G. Ballesteros and M. Pierre

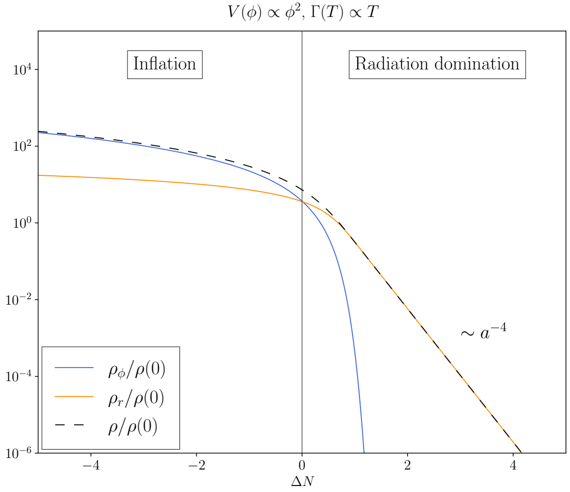
Main idea: **dissipation** from the inflaton ϕ into a **thermalised** bath with $\rho_r \propto T^4$.

$$\ddot{\phi} + (3H + \Gamma) + V_{,\phi} = 0\,,$$
$$\dot{\rho}_r + 4H\rho_r = \Gamma\dot{\phi}^2\,.$$

- The effective dissipative coefficient Γ encloses information about (indirect) coupling of inflaton to light degrees of freedom [Kamali, Motaharfar, Ramos '23].
- Useful notation: $Q = \frac{\Gamma}{3H}$.
  $Q = 0$: cold inflation. $Q \ll 1$: weak dissipation; $Q \gg 1$: strong dissipation.

If Q is large enough:

- **Smooth transition** into radiation domination
- Evade tensions with **distance and dS conjectures**



Perturbation equations in warm inflation substantially differ from cold inflation:

- Extra **dissipation** terms $\propto \Gamma$ (to be expected)
- **Additional** radiation component $T^{\mu\nu} = T^{\mu\nu}_{(\phi)} + T^{\mu\nu}_{(r)}$ coupled to the inflaton and the metric.
- **Stochastic** white noise acts as **thermal source** for $\delta\phi$ (fluctuation dissipation theorem) $\langle \xi_{\boldsymbol{k}}(t) \rangle = 0, \quad \langle \xi_{\boldsymbol{k}}(t) \xi_{\boldsymbol{k'}}(t') \rangle = \delta(t - t')\delta(\boldsymbol{k} + \boldsymbol{k'})$

In order to gain some intuition, consider the following simplified, decoupled equation

$$\delta\ddot{\phi}_{\boldsymbol{k}} + (3H + \Gamma)\delta\dot{\phi}_{\boldsymbol{k}} + \frac{k^2}{a^2}\delta\phi_{\boldsymbol{k}} = \sqrt{\frac{2\Gamma T}{a^3}}\,\xi_{\boldsymbol{k}}(t)\,.$$

Homogeneous solution (**IC** dependent, recovers **cold limit** when $\Gamma \to 0$):

$$\delta\ddot{\phi}^{(h)}_{\boldsymbol{k}} + (3H + \Gamma)\delta\dot{\phi}^{(h)}_{\boldsymbol{k}} + \frac{k^2}{a^2}\delta\phi^{(h)}_{\boldsymbol{k}} = 0.$$

Inhomogeneous solution (vanishes when $\Gamma \to 0$):

$$\delta\ddot{\phi}_{\boldsymbol{k}} + (3H+\Gamma)\delta\dot{\phi}_{\boldsymbol{k}} + \frac{k^2}{a^2}\delta\phi_{\boldsymbol{k}} = \sqrt{\frac{2\Gamma T}{a^3}}\,\xi_{\boldsymbol{k}}(t) \implies \delta\phi^{(i)}_{\boldsymbol{k}} \propto \Gamma_* T_* \int^t dt' \, \frac{G^{(\mathrm{ret})}(t,t')}{a^{3/2}}\xi_{\boldsymbol{k}}(t')\,.$$

$$\boxed{\delta\hat{\phi}_{\boldsymbol{k}} = \underbrace{\delta\phi^{(h)}_{k}\,\hat{a}_{\boldsymbol{k}} + \mathrm{h.c.}}_{\text{quantum}} \quad + \quad \underbrace{\delta\phi^{(i)}_{\boldsymbol{k}}\,\mathbb{I}}_{\text{thermal (classical)}}}$$

$$\delta(\boldsymbol{k}+\boldsymbol{k}')\,\mathcal{P}_{\delta\phi}(k,t) = \frac{k^3}{2\pi^2}\,\langle\langle \delta\hat{\phi}_{\boldsymbol{k}}(t)\,\delta\hat{\phi}_{\boldsymbol{k}'}(t)\rangle\rangle\,, \quad \delta\hat{\phi}_{\boldsymbol{k}} = \delta\phi_k^{(h)}\,\hat{a}_{\boldsymbol{k}} + \text{h.c.} + \delta\phi_{\boldsymbol{k}}^{(i)}\,\mathbb{I}$$

- **Quantum** expectation value: $\langle a_{\boldsymbol{k}} a^\dagger_{\boldsymbol{k}'} + \text{h.c.}\rangle = \Theta\,\delta(\boldsymbol{k}+\boldsymbol{k}')$.

  E.g. vacuum ($\Theta = 1$), thermal equilibrium ($\Theta = 1 + 2n_{BE}$) ☞ next slide
- **Ensemble** average of **thermal** noise: $\langle \xi_{\boldsymbol{k}}(t)\xi_{\boldsymbol{k}'}(t')\rangle = \delta(t-t')\delta(\boldsymbol{k}+\boldsymbol{k}')$
- No cross terms

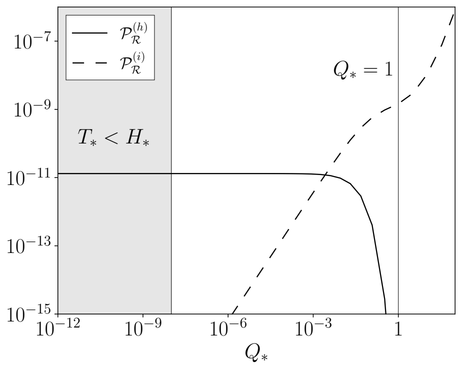
Summary:

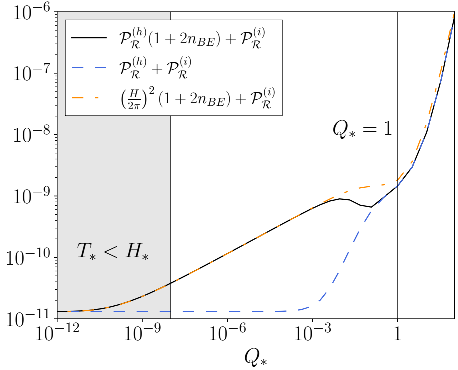
$\mathcal{P}_{\delta\phi} = \mathcal{P}_{\delta\phi}^{(h)} + \mathcal{P}_{\delta\phi}^{(i)}$, with $\mathcal{P}_{\delta\phi}^{(h)} = \frac{k^3}{2\pi^2}|\delta\phi_k^{(h)}|^2\,\Theta$ and $\mathcal{P}_{\delta\phi}^{(i)} = \Gamma_* T_* \int^t dt'\,\frac{G(t,t')^2}{a^3}$.

$Q_* \ll 1$: $\mathcal{P}_{\delta\phi}^{(h)}$ dominates (and recovers $H^2/(2\pi)$ for $Q_* \to 0$)

$Q_* > 10^{-2}$ (approx.): $\mathcal{P}_{\delta\phi}^{(i)}$ dominates .

1. Θ is a sizeable, microphysics-dependent correction.
2. $H^2/(2\pi)$ is not a good approximation for $\mathcal{P}^{(h)}_{\delta\phi}$ for large Q_*





3. Implications for GWB | CMB tensor-to-scalar ratio

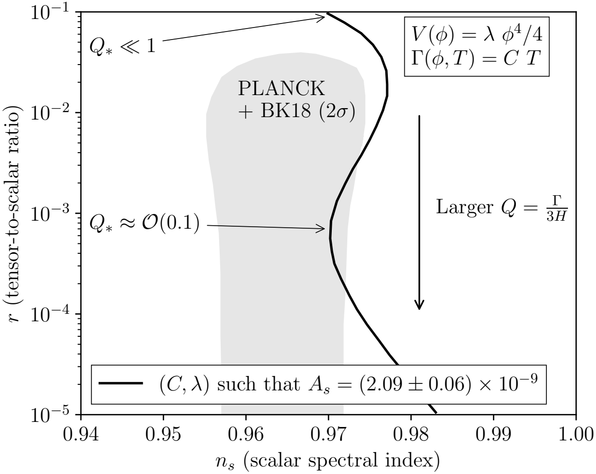
Recall:

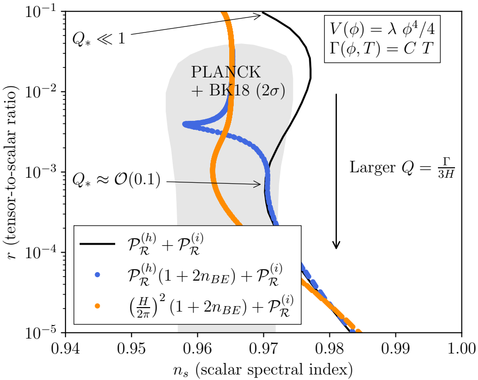
$\mathcal{P}_{\delta\phi} = \mathcal{P}_{\delta\phi}^{(h)} + \mathcal{P}_{\delta\phi}^{(i)}$, with $\mathcal{P}_{\delta\phi}^{(h)} = \frac{k^3}{2\pi^2}|\delta\phi_k^{(h)}|^2\,\Theta$ and $\mathcal{P}_{\delta\phi}^{(i)} = \Gamma_* T_* \int^t dt' \, \frac{G(t,t')^2}{a^3}$.

$Q_* \ll 1$: $\mathcal{P}_{\delta\phi}^{(h)}$ dominates (and recovers $H^2/(2\pi)$ for $Q_* \to 0$)

$Q_* > 10^{-2}$ (approx.): $\mathcal{P}_{\delta\phi}^{(i)}$ dominates .

- **Main idea**: thermal fluctuations **enhance** scalar perturbations while leaving tensor perturbations **unaffected**.
- At CMB scales, this naturally suppresses the predictions for the tensor-to-scalar ratio r.
- Scalar spectral index is transformed in a non-trivial way.


10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
r (tensor-to-scalar ratio)
0.94
0.95
0.96
0.97
0.98
0.99
1.00
n_s (scalar spectral index)
$Q_* \ll 1$
$Q_* \approx \mathcal{O}(0.1)$
PLANCK
+ BK18 (2σ)
$V(\phi) = \lambda\ \phi^4/4$
$\Gamma(\phi, T) = C\ T$
Larger $Q = \frac{\Gamma}{3H}$
(C, λ) such that $A_s = (2.09 \pm 0.06) \times 10^{-9}$



[Ballesteros, APR, Pierre '23]

Compatible with CMB?
(assuming $\Theta = 1$)

	ϕ^6	ϕ^4	ϕ^2
T	Yes	Yes	No
T^3	No	Yes	No
T^3/ϕ^2	No*	No*	No

Rows: $\Gamma(\phi, T)$, cols.: $V(\phi)$.

*"Yes" if $\Theta = 1 + 2n_{BE}$ (critical model dependence)

[Bartrum et al. '14]
[Benetti, Ramos '17]

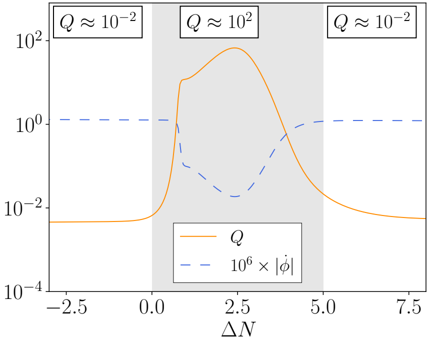
Recall:

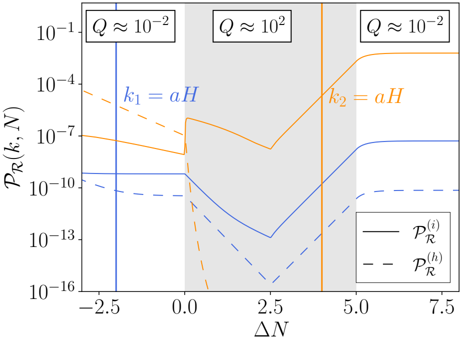
$\mathcal{P}_{\delta\phi} = \mathcal{P}_{\delta\phi}^{(h)} + \mathcal{P}_{\delta\phi}^{(i)}$, with $\mathcal{P}_{\delta\phi}^{(h)} = \frac{k^3}{2\pi^2}|\delta\phi_k^{(h)}|^2\,\Theta$ and $\mathcal{P}_{\delta\phi}^{(i)} = \Gamma_* T_* \int^t dt' \, \frac{G(t,t')^2}{a^3}$.

$Q_* \ll 1$: $\mathcal{P}_{\delta\phi}^{(h)}$ dominates (and recovers $H^2/(2\pi)$ for $Q_* \to 0$)

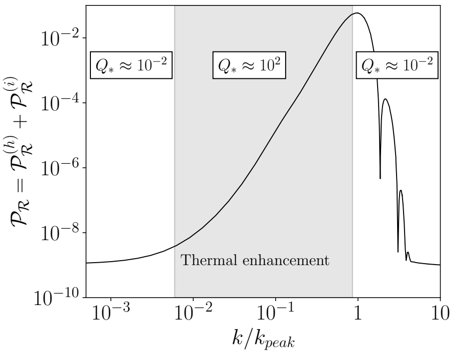
$Q_* > 10^{-2}$ (approx.): $\mathcal{P}_{\delta\phi}^{(i)}$ dominates .

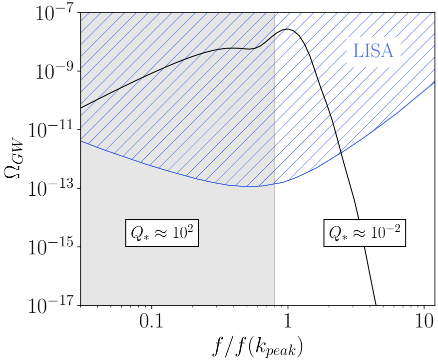
- **Main idea**: a **transient** phase of strong dissipation enhances selectively the modes which exit the horizon during it due to the enhancement of $\mathcal{P}_{\mathcal{R}}^{(i)}$.
- This leads to a peak in the $\mathcal{P}_{\mathcal{R}}$, with the subsequent peak in Ω_{GW}





Cf. inflection point potentials: quantum fluctuations are **suppressed**.


10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^{-10}
$\mathcal{P}_{\mathcal{R}} = \mathcal{P}_{\mathcal{R}}^{(h)} + \mathcal{P}_{\mathcal{R}}^{(i)}$
$Q_* \approx 10^{-2}$
$Q_* \approx 10^{2}$
$Q_* \approx 10^{-2}$
Thermal enhancement
10^{-3} 10^{-2} 10^{-1} 1 10
k/k_{peak}


10^{-7}
10^{-9}
10^{-11}
10^{-13}
10^{-15}
10^{-17}
Ω_{GW}
LISA
$Q_* \approx 10^{2}$
$Q_* \approx 10^{-2}$
0.1 1 10
$f/f(k_{peak})$

Conclusions

- Power spectrum in warm inflation: **quantum** + **thermal** component

$$\boxed{\mathcal{P}_{\delta\phi} = \mathcal{P}_{\delta\phi}^{(h)} + \mathcal{P}_{\delta\phi}^{(i)}\,.}$$

  For $Q_* > 10^{-2}$, $\mathcal{P}_{\delta\phi}^{(i)}$ is (largely) dominant

- Implications for GWB:
  - Dissipation $\implies$ suppression of tensor-to-scalar ratio. Some models reconciled with CMB
  - Transient strong dissipation leads to peak in scalar power spectrum $\implies$ peak in Ω_{GW} from scalar-induced GWs.

Comments and questions: `alejandro.perezrodriguez@uam.es`