$\Delta S=2$ transitions beyond the Standard Model

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K^0-Meson Oscillations

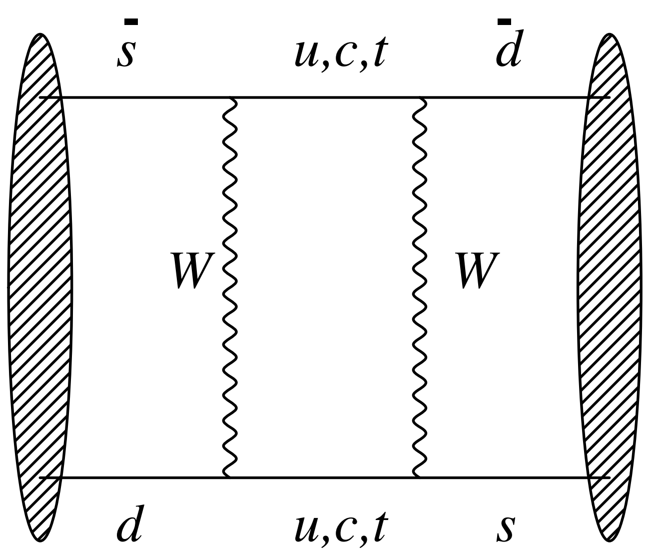
B_K

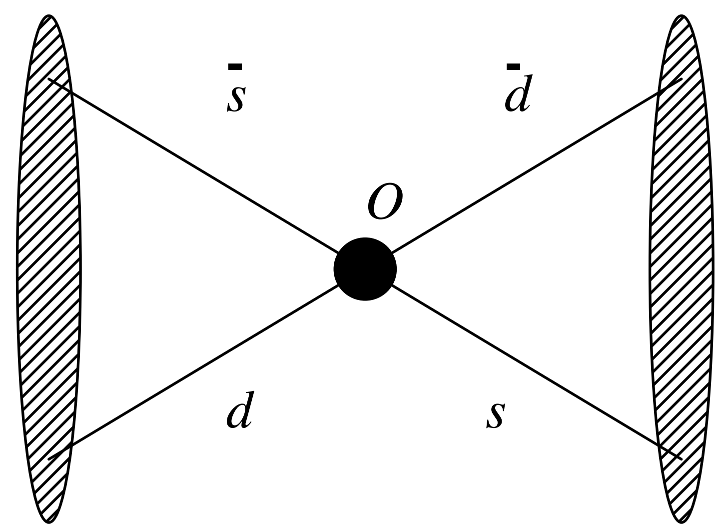
in the SM

B_K in the SM

- indirect CP-violation $$\epsilon_K = \frac{\mathcal{A}[K_L \to (\pi\pi)_{I=0}]}{\mathcal{A}[K_S \to (\pi\pi)_{I=0}]} = [2.282(17) \times 10^{-3}]\, \exp(i\pi/4)$$

can also be expressed in terms of neutral K-oscillations: dominant EW process is FCNC (2-W exchange)





QCD effects consist in gluon and internal quark-loop exchanges (not shown here)

- $\Delta S = 2$ oscillations are governed by the transition amplitude of an effective Hamiltonian, obtained by successively integrating out W's and t- (b-) and c-quarks

- We are left with an OPE with a single, dim-6, 4-fermion, $L \otimes L$ operator, in a 3-quark approximation of QCD ($N_f = 3$)

$$Q^{\Delta S=2} = [\bar{s}\gamma_\mu(1-\gamma_5)d]\,[\bar{s}\gamma_\mu(1-\gamma_5)d] \equiv O_{\mathrm{VV+AA}} - O_{\mathrm{VA+AV}}$$

B_K in the SM

- $\Delta S = 2$ transitions are governed by the transition amplitude of the effective Hamiltonian:

$$\langle \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 \rangle = \frac{G_F^2 M_{\rm W}^2}{16\pi^2} \left[\lambda_c^2 S_0(x_c)\eta_1 + \lambda_t^2 S_0(x_t)\eta_2 + 2\lambda_c\lambda_t S_0(x_c,x_t)\eta_3 \right]$$
$$\times \left(\frac{\bar{g}(\mu)^2}{4\pi} \right)^{-\gamma_0/(2\beta_0)} \left\{ 1 + \frac{\bar{g}(\mu)^2}{(4\pi)^2} \left[\frac{\beta_1\gamma_0 - \beta_0\gamma_1}{2\beta_0^2} \right] \right\} \boxed{\langle \bar{K}^0 | Q_{\rm R}^{\Delta S=2}(\mu) | K^0 \rangle} + \text{ h.c.}$$

- Four fermion $L \otimes L$ operator of dim=6 ($Q^{\Delta S=2}{}_{\rm R}$: renormalized; parity-even part contributes):

$$Q^{\Delta S=2} = [\bar{s}\gamma_\mu(1-\gamma_5)d]\,[\bar{s}\gamma_\mu(1-\gamma_5)d] \equiv O_{\rm VV+AA} - O_{\rm VA+AV}$$

- Computed on the lattice through its B_K-parameter:

$$B_{\rm K}(\mu) = \frac{\left\langle \bar{K}^0 \left| Q_{\rm R}^{\Delta S=2}(\mu) \right| K^0 \right\rangle}{\frac{8}{3} f_K^2 m_K^2}$$

B_K in the SM

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- Computed on the lattice through its B_K-parameter:

RGI (scale μ-independent at NLO)

$$B_{\rm K}(\mu) = \frac{\langle \bar{K}^0 \left| Q_{\rm R}^{\Delta S=2}(\mu) \right| K^0 \rangle}{\frac{8}{3} f_K^2 m_K^2}$$

$$\hat{B}_{\rm K}$$

B_K in the SM

No new results since previous version

Agreement between estimates at different N_f

~ 1.5% overall accuracy for N_f = 2+1

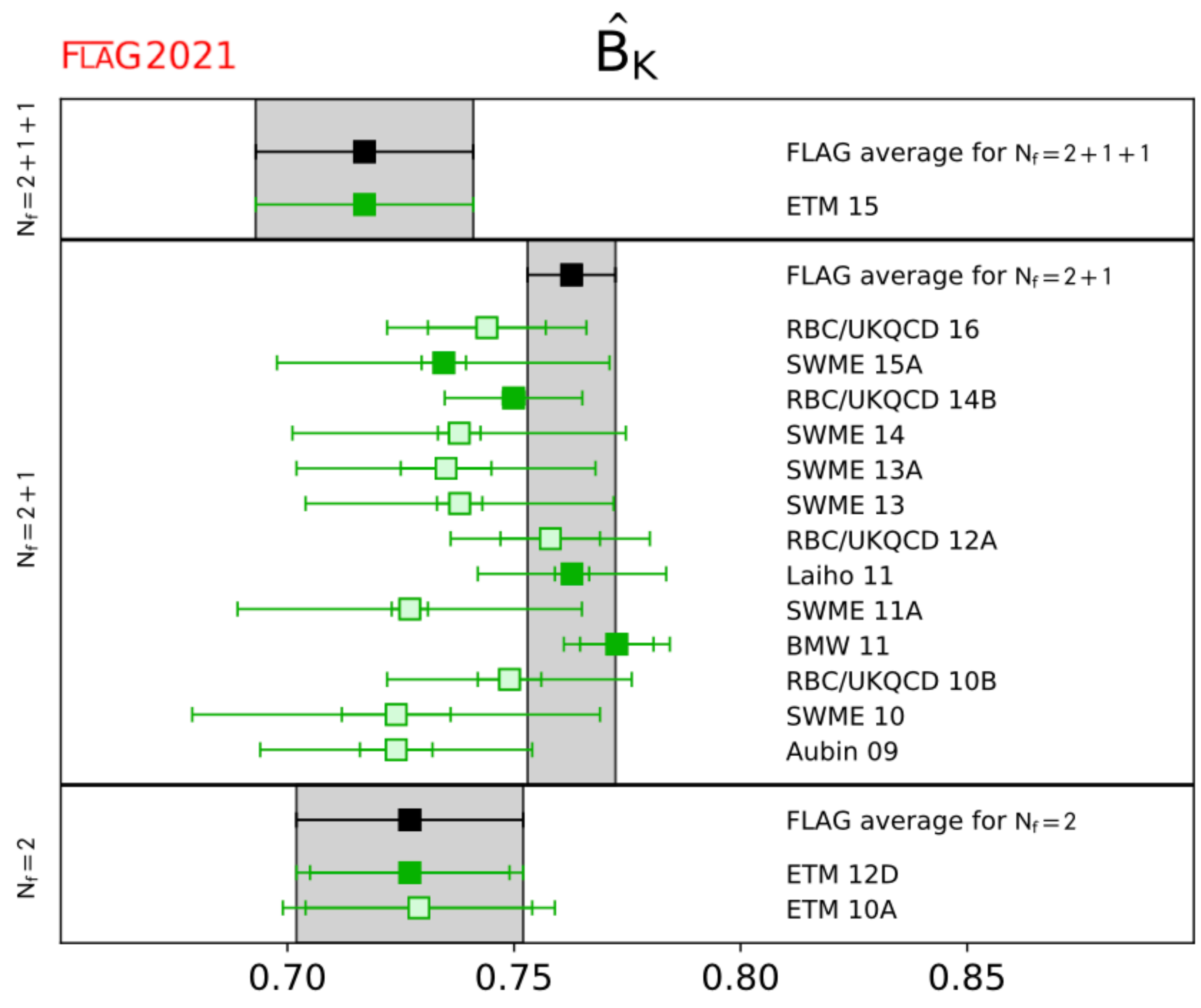
Y. Aoki et al., "FLAG Review 2021", Eur.Phys.J. C82 (2022) 869

no new results since last update (Dec. 2016)

$$N_f = 2+1+1: \quad \hat{B}_K = 0.717(18)(16) \quad [3.4\%], \qquad B_K^{\overline{MS}}(2\,\mathrm{GeV}) = 0.524(13)(12)$$

$$N_f = 2+1: \quad \hat{B}_K = 0.7625(97) \quad [1.3\%], \qquad B_K^{\overline{MS}}(2\,\mathrm{GeV}) = 0.5570(71)$$

$$N_f = 2: \quad \hat{B}_K = 0.727(22)(12) \quad [3.4\%], \qquad B_K^{\overline{MS}}(2\,\mathrm{GeV}) = 0.531(16)(19)$$



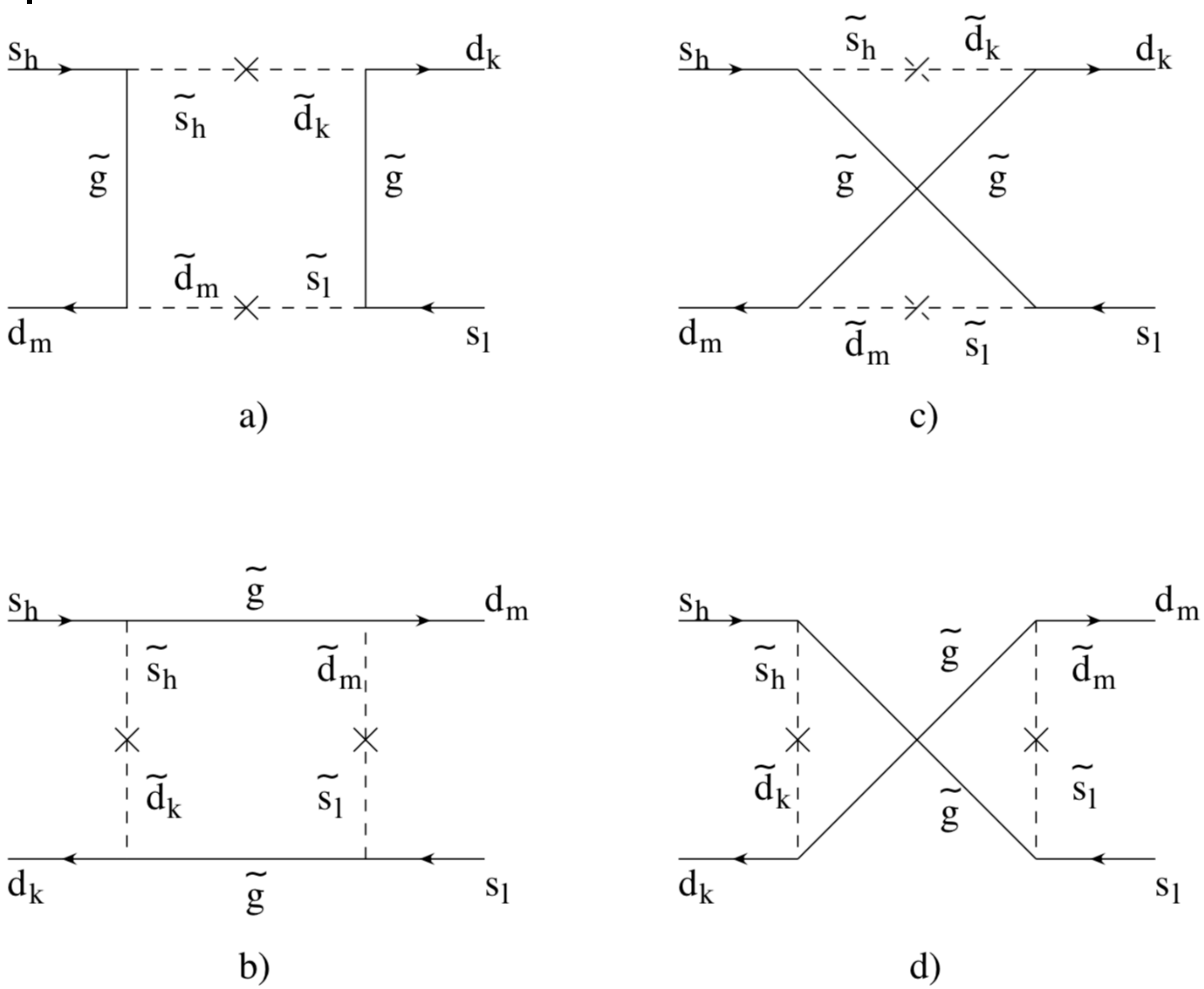
K^0-Meson Oscillations

B_K

beyond the SM

B_K beyond the SM

- Analyse New Physics (NP) effects in a model-independent way: assume a generalisation of the effective $\Delta S = 2$ Hamiltonian which contains operators beyond the SM one; the amplitude is:



SM contribution

BSM contributions

$$< \bar{K}^0 | \mathcal{H}_{\text{eff}}^{\Delta S=2} | K^0 > = C_1 < \bar{K}^0 | O_1 | K^0 > + \sum_{i=2}^{5} C_i < \bar{K}^0 | O_i | K^0 >$$

B_K beyond the SM

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$$\begin{aligned}
O_1 &= [\bar{s}^\alpha \gamma_\mu (1-\gamma_5) d^\alpha][\bar{s}^\beta \gamma_\mu (1-\gamma_5) d^\beta] \\
O_2 &= [\bar{s}^\alpha (1-\gamma_5) d^\alpha][\bar{s}^\beta (1-\gamma_5) d^\beta] \\
O_3 &= [\bar{s}^\alpha (1-\gamma_5) d^\beta][\bar{s}^\beta (1-\gamma_5) d^\alpha] \\
O_4 &= [\bar{s}^\alpha (1-\gamma_5) d^\alpha][\bar{s}^\beta (1+\gamma_5) d^\beta] \\
O_5 &= [\bar{s}^\alpha (1-\gamma_5) d^\beta][\bar{s}^\beta (1+\gamma_5) d^\alpha]
\end{aligned}$$

SM contributions

BSM contributions

- Square brackets imply closure of spin indices; colour indices are explicit and those of O_3 and O_5 are "lopsided".

- The above is known as the SUSY operator basis, habitually used in phenomenological studies. Other bases, which are linear combinations of the above, are more suitable for other aims (e.g. NP renormalisation on the lattice).

- Only the parity-even part of the operators contributes in the above K^0-K^0 matrix elements (parity is conserved in QCD!).

B_K beyond the SM

- Analyse New Physics (NP) effects in a model-independent way: assume a generalisation of the effective $\Delta S = 2$ Hamiltonian which contains operators beyond the SM one; the amplitude is:

$$< \bar{K}^0|\mathcal{H}_{\rm eff}^{\Delta S=2}|K^0 > = C_1 < \bar{K}^0|O_1|K^0 > + \sum_{i=2}^{5} C_i < \bar{K}^0|O_i|K^0 >$$

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O_5 &= [\bar{s}^\alpha(1-\gamma_5)d^\beta][\bar{s}^\beta(1+\gamma_5)d^\alpha]
\end{aligned}$$

B-parameters (with $B_1 \equiv B_K$) are defined in accord with VSA (historical)

$$\begin{aligned}
B_1 &\equiv \frac{\langle\bar{K}^0|O_1|K^0\rangle}{\frac{8}{3}f_{\rm K}^2 m_{\rm K}^2} \\
B_i &\equiv \frac{\langle\bar{K}^0|O_i|K^0\rangle}{N_i\langle\bar{K}^0|\bar{s}\gamma_5 d|0\rangle\langle 0|\bar{s}\gamma_5 d|K^0\rangle} \qquad i = 2,\cdots,5 \\
N_i &\equiv \{-5/3, 1/3, 2, 2/3\}
\end{aligned}$$

B_K beyond the SM

- Analyse New Physics (NP) effects in a model-independent way: assume a generalisation of the effective $\Delta S = 2$ Hamiltonian which contains operators beyond the SM one; the amplitude is:

$$< \bar{K}^0 | \mathcal{H}_{\rm eff}^{\Delta S=2} | K^0 > \, = \, C_1 < \bar{K}^0 | O_1 | K^0 > \, + \, \sum_{i=2}^{5} C_i < \bar{K}^0 | O_i | K^0 >$$

$$\begin{aligned}
O_1 &= [\bar{s}^\alpha \gamma_\mu (1-\gamma_5) d^\alpha][\bar{s}^\beta \gamma_\mu (1-\gamma_5) d^\beta] & (27,1) \\
O_2 &= [\bar{s}^\alpha (1-\gamma_5) d^\alpha][\bar{s}^\beta (1-\gamma_5) d^\beta] & \\
O_3 &= [\bar{s}^\alpha (1-\gamma_5) d^\beta][\bar{s}^\beta (1-\gamma_5) d^\alpha] & (6,\bar{6}) \\
O_4 &= [\bar{s}^\alpha (1-\gamma_5) d^\alpha][\bar{s}^\beta (1+\gamma_5) d^\beta] & \\
O_5 &= [\bar{s}^\alpha (1-\gamma_5) d^\beta][\bar{s}^\beta (1+\gamma_5) d^\alpha] & (8,8)
\end{aligned}$$

$SU_{\rm L}(3) \otimes SU_{\rm R}(3)$

- BSM operators renormalise in pairs; renormalisation matrix is a block diagonal 5×5 with three blocks: 1×1, 2×2 and 2×2. In practice we are looking at the parity-even part of the operators.

$$\begin{pmatrix} O_1^{\rm R} \\ O_2^{\rm R} \\ O_3^{\rm R} \\ O_4^{\rm R} \\ O_5^{\rm R} \end{pmatrix} = \begin{pmatrix} Z_{11} & 0 & 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} & 0 & 0 \\ 0 & Z_{32} & Z_{33} & 0 & 0 \\ 0 & 0 & 0 & Z_{44} & Z_{45} \\ 0 & 0 & 0 & Z_{54} & Z_{55} \end{pmatrix} \begin{pmatrix} O_1 \\ O_2 \\ O_3 \\ O_4 \\ O_5 \end{pmatrix}$$

B_K beyond the SM

- Analyse New Physics (NP) effects in a model-independent way: assume a generalisation of the effective $\Delta S = 2$ Hamiltonian which contains operators beyond the SM one; the amplitude is:

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O_2 &= [\bar{s}^\alpha (1-\gamma_5) d^\alpha][\bar{s}^\beta (1-\gamma_5) d^\beta] && \\
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O_5 &= [\bar{s}^\alpha (1-\gamma_5) d^\beta][\bar{s}^\beta (1+\gamma_5) d^\alpha] && (8,8)
\end{aligned}$$

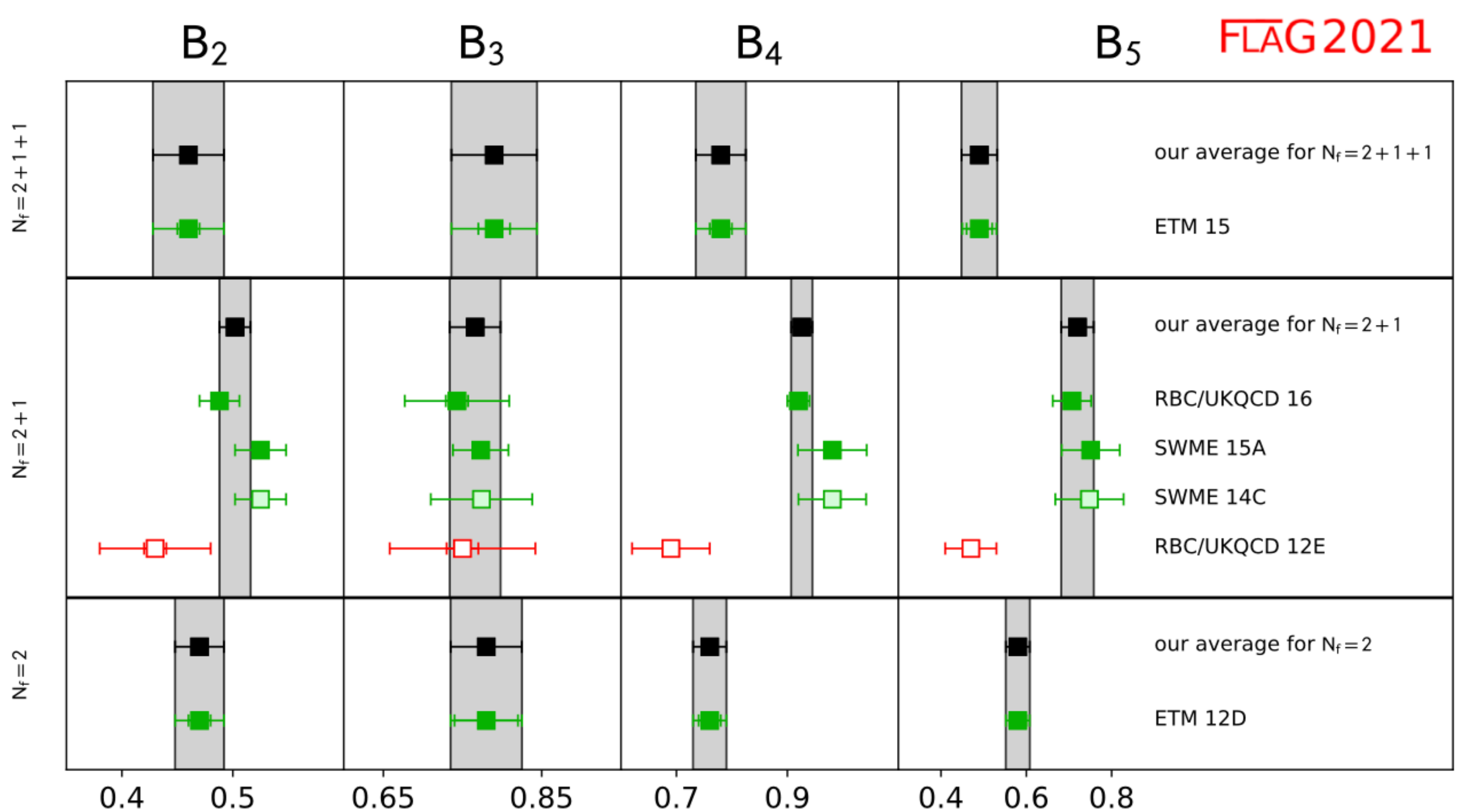
$$SU_{\rm L}(3) \otimes SU_{\rm R}(3)$$

- To the extent that chirality is broken by the (lattice) regularisation, the "off-block-diagonal-elements" are non-zero; this effect is slight with domain walls and strong with Wilson fermions.

$$\begin{pmatrix} O_1^{\rm R} \\ O_2^{\rm R} \\ O_3^{\rm R} \\ O_4^{\rm R} \\ O_5^{\rm R} \end{pmatrix} = \begin{pmatrix} Z_{11} & 0 & 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} & 0 & 0 \\ 0 & Z_{32} & Z_{33} & 0 & 0 \\ 0 & 0 & 0 & Z_{44} & Z_{45} \\ 0 & 0 & 0 & Z_{54} & Z_{55} \end{pmatrix} \begin{pmatrix} O_1 \\ O_2 \\ O_3 \\ O_4 \\ O_5 \end{pmatrix}$$

B_K beyond the SM

- Y. Aoki et al., FLAG 2021, Eur.Phys.J. C82 (2022) 869 (no new results since previous edition)
- Results are shown at reference scale 3GeV in $\overline{\mathrm{MS}}$
- Agreement is not as good as in the SM quantity $B_1 \equiv B_K$
- In particular no discrepancy between various B_2 and B_3 results, but those for B_4 and B_5 show a marked, apparent N_f dependence



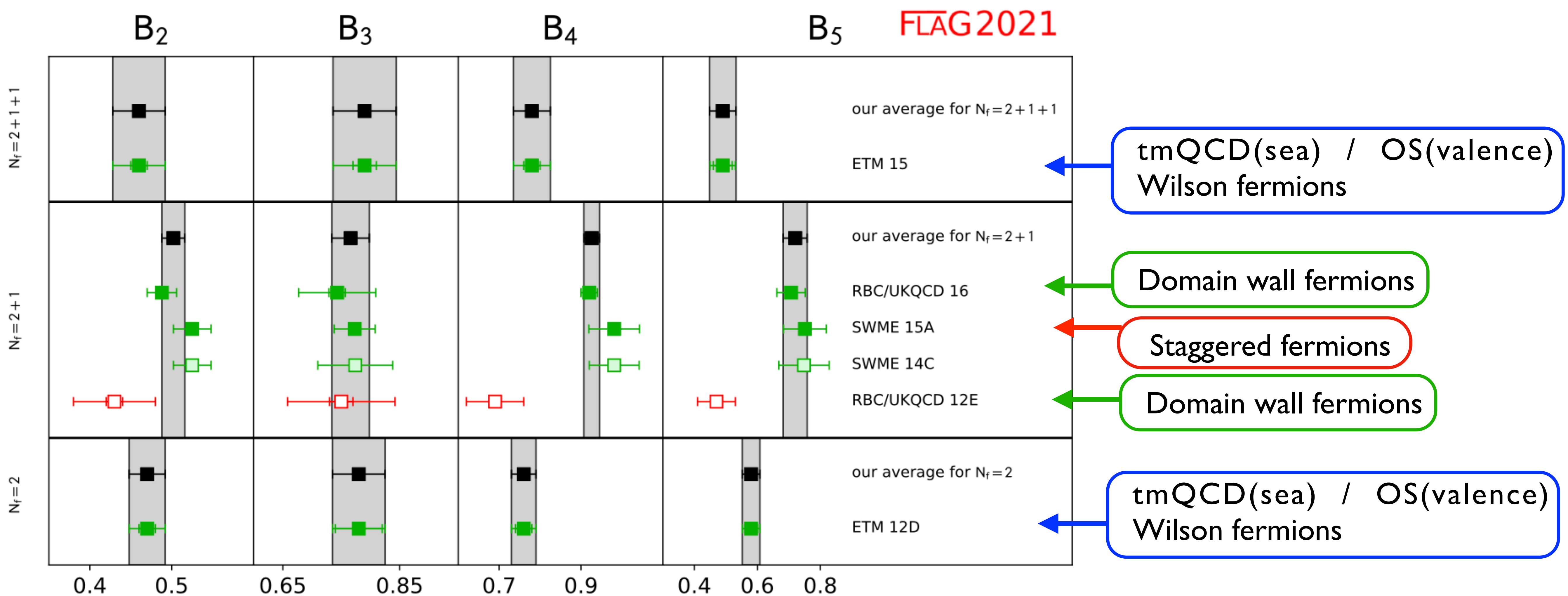
$N_f = 2+1$ vs. $N_f = 2+1+1$:

- B_4 : $\sim$ 19%, 3σ
- B_5 : $\sim$ 47%, 4σ

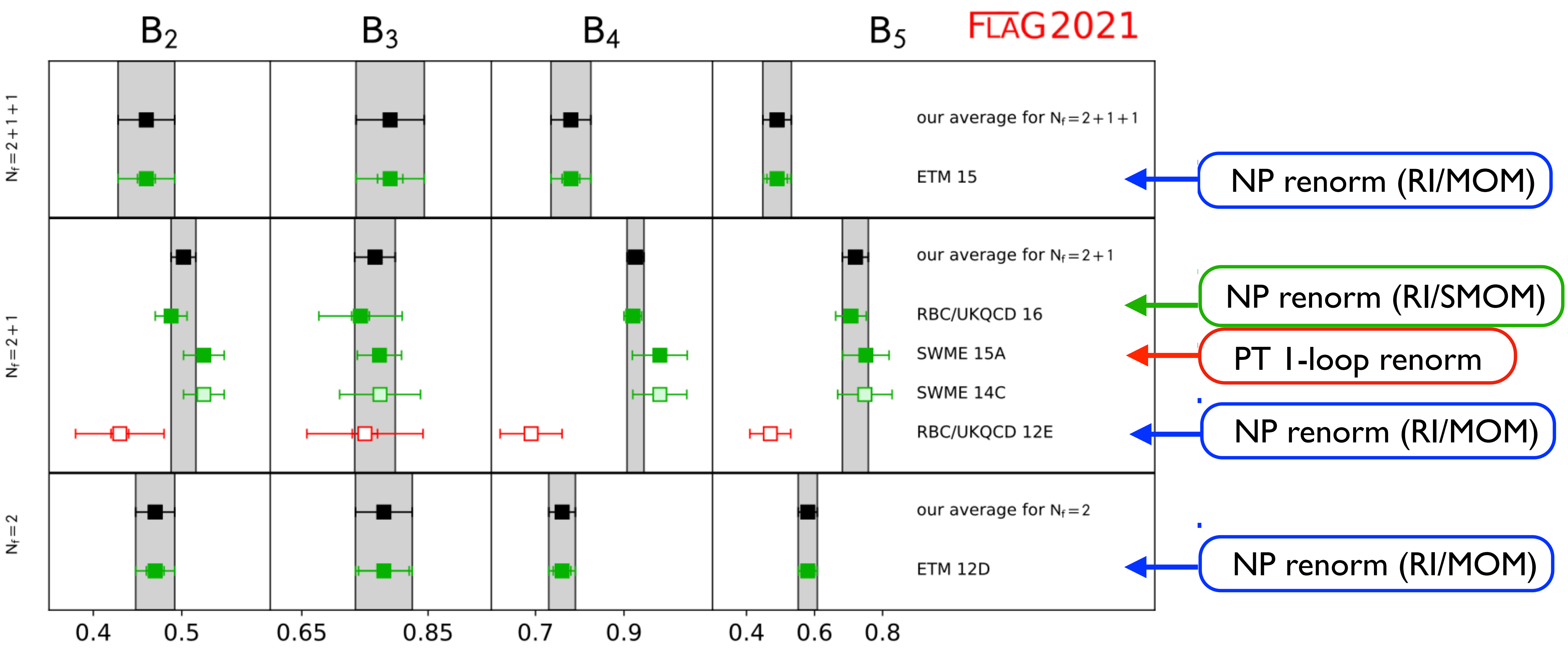
note: $\mu = 3$ GeV

- ETM results are obtained with same methods for N_f=2 and N_f=2+1+1. Any small differences between these may be attributed to N_f effects

B_K beyond the SM

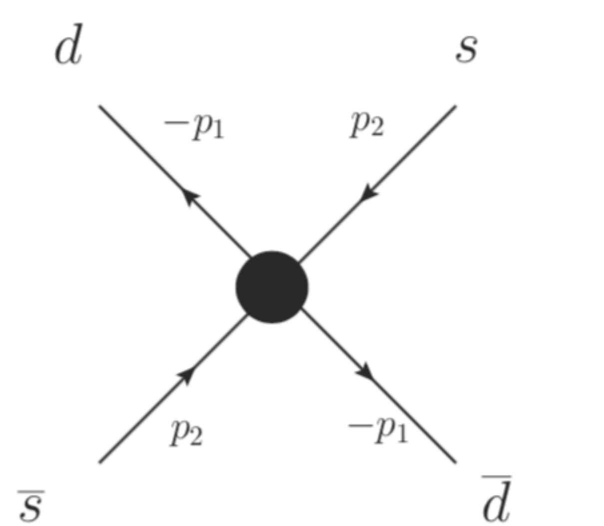


B_K beyond the SM



B_K beyond the SM

- P.A.Boyle, N.Garron, R.J.Hudspith, C. Lehner, A.Lytle, JHEP10 (2017) 054



$\boxed{\text{(RI/MOM)}}$ $$-p_1 = p_2$$

exceptional momenta imply bad infrared behaviour; at low energies pion poles contaminate the chiral behaviour of the Green functions

$\boxed{\text{NP renorm (RI/SMOM)}}$ $$-p_1 \neq p_2$$

$$p_1^2 = p_2^2 = (p_1 - p_2)^2$$

single renormalisation scale; no exceptional momenta

RBC/UKQCD has done extensive research with RI/SMOM over the years

B_K beyond the SM

- RBC/UKQCD also applied RI/MOM to the same ensembles and found results similar to ETM

- P.A.Boyle, N.Garron, R.J.Hudspith, C. Lehner, A.Lytle, JHEP11 (2016) 001
- P.A.Boyle, N.Garron, R.J.Hudspith, C. Lehner, A.Lytle, JHEP10 (2017) 054
- P.A.Boyle, N.Garron, J.Kettle, A.Khamseh, A. Jüttner, J.Kettle, A. Khamseh, J.T.Tsang , POS(LATTICE2018)285

- Results are shown at reference scale 3GeV in $\overline{\mathrm{MS}}$

	ETM 12	ETM 15	RBC – UKQCD 12	SWME 15	This work	
n_f	2	$2+1+1$	$2+1$	$2+1$	$2+1$	$2+1$
interm. scheme	RI-MOM	RI-MOM	RI-MOM	1-loop	RI-SMOM	RI-MOM
B_2	0.47(2)	0.46(3)(1)	0.43(5)	0.525(1)(23)	0.488(7)(17)	0.417(6)(2)
B_3	0.78(4)	0.79(5)(1)	0.75(9)	0.772(5)(35)	0.743(14)(65)	0.655(12)(44)
B_4	0.76(3)	0.78(4)(3)	0.69(7)	0.981(3)(61)	0.920(12)(16)	0.745(9)(28)
B_5	0.58(3)	0.49(4)(1)	0.47(6)	0.751(8)(68)	0.707(8)(44)	0.555(6)(53)

- Alpha can provide a completely independent check, with a third NP renormalisation scheme, namely one of the Schrödinger Functional family

- Moreover, RG-running can also be done NPly

B-parameters from tmQCD fermions, SF- and χSF- renormalisation schemes

K^0 oscillations and operator renormalisation

- The four-fermion renormalisation problem (with Wilson fermions) is best analysed by considering the more abstract problem of renormalising d=6 four-fermion operators with four distinct flavours; physical flavours are assigned at the end of the day.

- The NPR operator basis is now more convenient

$$\mathcal{O}^{\pm}_{\Gamma_1\Gamma_2} = \frac{1}{2}\left[(\bar\psi_1\Gamma_1\psi_2)(\bar\psi_3\Gamma_2\psi_4) \pm (\bar\psi_1\Gamma_1\psi_4)(\bar\psi_3\Gamma_2\psi_2)\right]$$

- Occasionally, simplify the notation by dropping ± superscript

parity even

$$\begin{aligned} Q_1 &= \mathcal{O}_{\rm VV+AA} \\ Q_2 &= \mathcal{O}_{\rm VV-AA} \\ Q_3 &= \mathcal{O}_{\rm SS-PP} \\ Q_4 &= \mathcal{O}_{\rm SS+PP} \\ Q_5 &= -2\ \mathcal{O}_{\rm TT} \end{aligned}$$

$$\begin{aligned} \mathcal{Q}_1 &= \mathcal{O}_{\rm VA+AV} \\ \mathcal{Q}_2 &= \mathcal{O}_{\rm VA-AV} \\ \mathcal{Q}_3 &= \mathcal{O}_{\rm PS-SP} \\ \mathcal{Q}_4 &= \mathcal{O}_{\rm PS+SP} \\ \mathcal{Q}_5 &= -2\ \mathcal{O}_{\rm T\tilde{T}} \end{aligned}$$

parity odd

- The ± operators do not mix under renormalisation, as they are symmetric/antisymmetric under flavour exchange $2 \leftrightarrow 4$

- Parity-even operators do not mix with parity-odd ones

K0 oscillations and operator renormalisation

- Naively one expects that with Wilson fermions all operators mix under renormalisation, with continuum-like mixing related to divergences, and the remaining mixing related to lattice subtractions

5x5 matrices

5x1 columns

$$[Q^{\pm}]_R = Z^{\pm}\,[I + \Delta^{\pm}]\,Q^{\pm}$$

$$[\mathcal{Q}^{\pm}]_R = \mathcal{Z}^{\pm}\,[I + \bar{\Delta}^{\pm}]\,\mathcal{Q}^{\pm}$$

5x1 columns

$$\mathbf{Z} = \begin{pmatrix} Z_{11} & 0 & 0 & 0 & 0 \\ 0 & Z_{22} & Z_{23} & 0 & 0 \\ 0 & Z_{32} & Z_{33} & 0 & 0 \\ 0 & 0 & 0 & Z_{44} & Z_{45} \\ 0 & 0 & 0 & Z_{54} & Z_{55} \end{pmatrix}, \qquad \mathbf{\Delta} = \begin{pmatrix} 0 & \Delta_{12} & \Delta_{13} & \Delta_{14} & \Delta_{15} \\ \Delta_{21} & 0 & 0 & \Delta_{24} & \Delta_{25} \\ \Delta_{31} & 0 & 0 & \Delta_{34} & \Delta_{35} \\ \Delta_{41} & \Delta_{42} & \Delta_{43} & 0 & 0 \\ \Delta_{51} & \Delta_{52} & \Delta_{53} & 0 & 0 \end{pmatrix}$$

- Z's are scale-dependent; they remove UV divergences; most operators have two anomalous dimensions

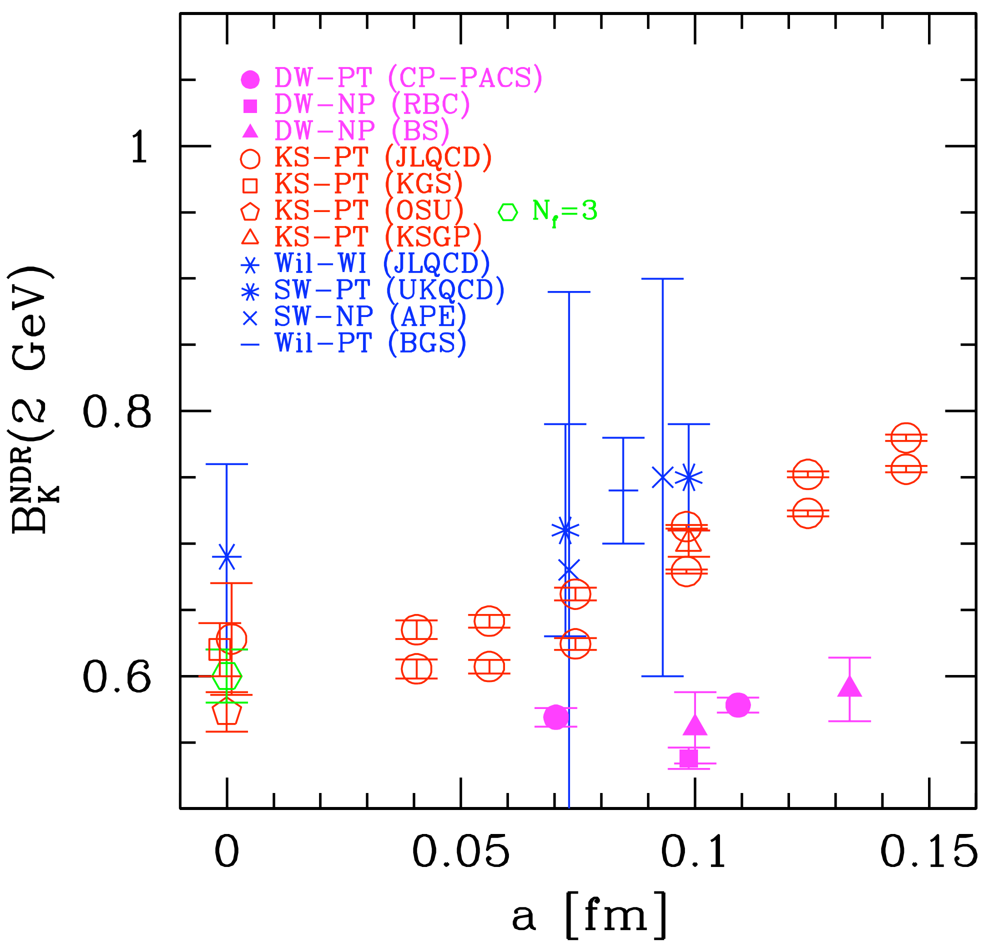
- Δ's are scale-independent $O(a^0)$, $O(g^2)$ artefacts due to chiral symmetry breaking from Wilson term

- Their presence renders the Wilson-fermion determination of B-parameters much noisier that other determinations

- Moreover there are too many Symanzik-improvement counterterms to be subtracted (dim-7 operators) in order to remove $O(a)$-effects

K^0 oscillations and operator renormalisation

Subtractions flaw the quality of Wilson fermion results



L. Lellouch Nucl.Phys.Proc.Suppl.94(2001)142

There are two important sources of systematic error which would better be removed if Wilson fermion B_K determinations are to be on the same footing as the others:

1. Additive renormalization;
2. $O(a)$ discretization errors

K^0 oscillations and operator renormalisation

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5x1 columns

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- Same pattern for parity-odd Z's but due to CPS symmetries the parity-odd Δ's vanish!!!!
- A paradoxical situation: parity-odd operators appear to be "useless" for neutral meson weak matrix elements, but they display better renormalisation patterns
- A. Donini, V. Gimenenz, G. Martinelli, M. Talevi, A.V. Eur.Phys.J.C10(1999)121

K^0 oscillations and operator renormalisation

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5x5 matrices

5x1 columns

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5x1 columns

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- Both the renormalisation problem and the improvement one are solved through some chirally-twisted QCD variant(s) with fully-twisted angle π/2

$$\langle \bar{K}^0 | Q_R | K^0 \rangle = \mathcal{Z}\,\langle \bar{K}^0 | \mathcal{Q} | K^0 \rangle + O(a^p)$$

physical, parity-even ME

bare, parity-odd ME in OS fully tmQCD; $O(a^2)$

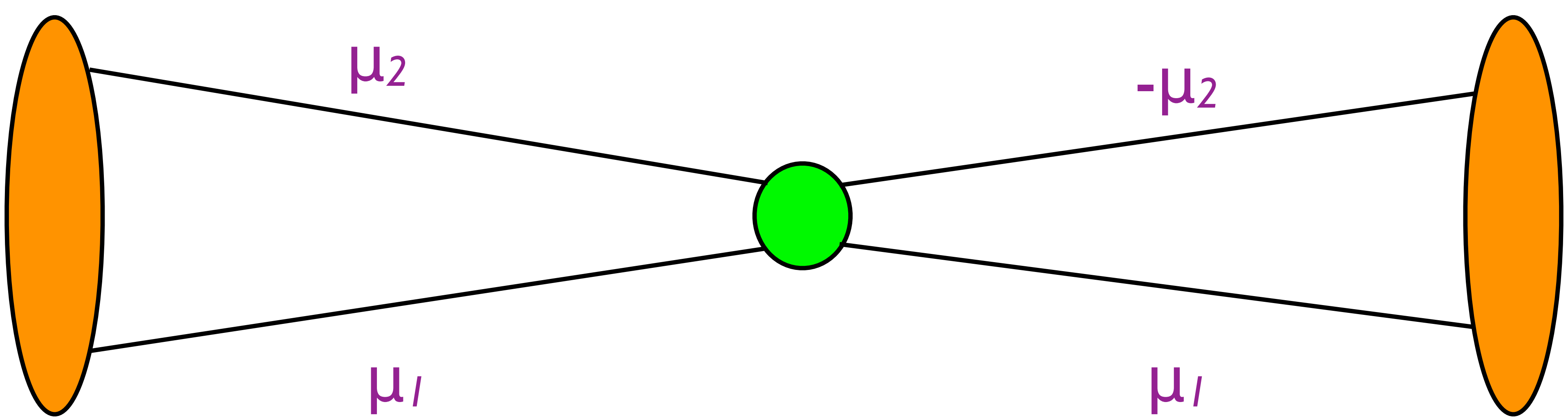
parity-odd Z in massless QCD with SF boundaries; $O(a)$

parity-odd Z in massless QCD with χSF boundaries; $O(a^2)$

K^0 oscillations and operator renormalisation

- PROBLEM: impossible to have Wilson fully-twisted parity-odd fermions which map onto parity-even counter-terms C. Pena, S. Sint, , A.V, JHEP09 (2004) 069
- WAY OUT: mixed-action approach R. Frezzotti, G.C. Rossi, JHEP10 (2004) 070
- treat sea quarks in fully twisted tmQCD fashion, or Wilson-Clover (our choice as CLS)
- treat valence quarks in OS fashion (i.e. use a mixed action formulation)
- Each **valence** quark flavour is regularised by the Osterwalder-Seiler (OS) variant of tmQCD
- Valence quarks enter with a distinct action for each flavour, which is **fully twisted**.
    - quark fields are not organised in isospin doublets (i.e. no τ^3)
    - twisted masses are $\mu_1=\mu_3=\mu_s$; $\mu_2=\mu_d$; $\mu_4=-\mu_d$ (i.e. twist angle $\alpha = \pm\pi/2$)

ETM V. Bertone net al., JHEP03 (2013) 089



$$\mathcal{L}_{OS} = \bar{\psi}_f \left[D_W + i\mu_f\gamma_5 \right] \psi_f \qquad f = u, d, s \cdots$$

K^0 oscillations and operator renormalisation

parity even

Wilson/ Clover valence quarks

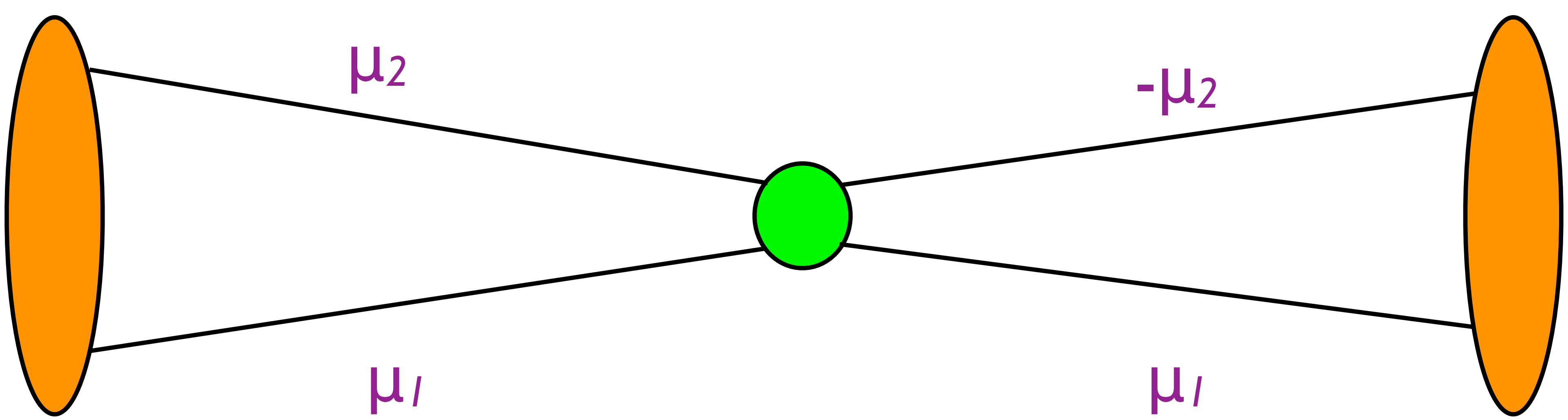
$$
\begin{aligned}
Q_1^{\pm} &\rightarrow -i\,\mathcal{Q}_1^{\pm} \\
Q_2^{\pm} &\rightarrow -i\,\mathcal{Q}_2^{\mp} \\
Q_3^{\pm} &\rightarrow -i\,\mathcal{Q}_3^{\mp} \\
Q_4^{\pm} &\rightarrow -i\,\mathcal{Q}_4^{\pm} \\
Q_5^{\pm} &\rightarrow -i\,\mathcal{Q}_5^{\pm}
\end{aligned}
$$

parity odd

valence quarks in tmQCD

- complicated subtraction pattern (mixing)
- complicated $O(a)$ improvement pattern (too many d=7 Symanzik counterterms)

- “chiral”-like subtraction pattern
- automatic improvement pattern



NB: all this has to do with (massive) weak matrix elements

K^0 oscillations and operator renormalisation

$$
\begin{aligned}
Q_1^{\pm} &\rightarrow -i\,\mathcal{Q}_1^{\pm} \\
Q_2^{\pm} &\rightarrow -i\,\mathcal{Q}_2^{\mp} \\
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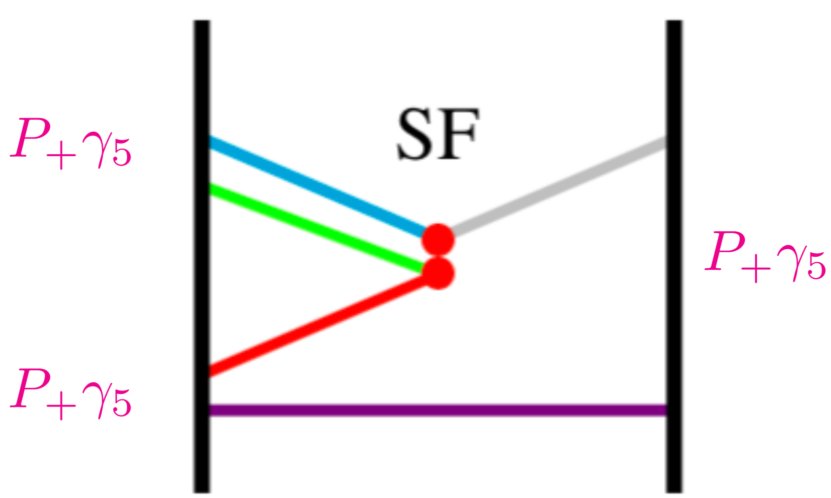
Q_4^{\pm} &\rightarrow -i\,\mathcal{Q}_4^{\pm} \\
Q_5^{\pm} &\rightarrow -i\,\mathcal{Q}_5^{\pm}
\end{aligned}
$$

parity odd

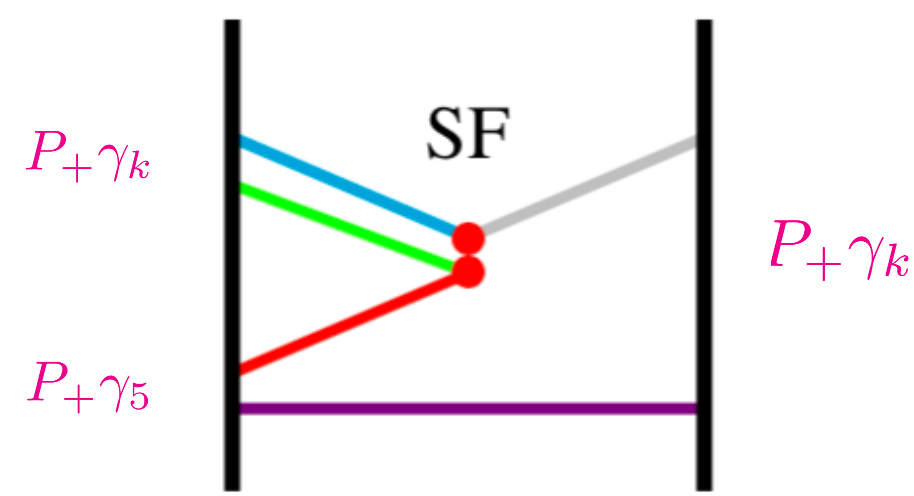
SF renormalisation involves massless fermions, parity odd operators and SF boundaries

- This is massless lattice (Wilson-Clover) QCD with SF boundaries; there is no twisting.

- To preserve parity *P* we must introduce correlation functions with 5 distinct flavours and **two** combinations of wall sources



NOISY!



$$
\begin{aligned}
\overline{Q}_2^{+} &= -i\left[\mathcal{Z}_{22}^{-}\ \mathcal{Q}_2^{-} + \mathcal{Z}_{23}^{-}\ \mathcal{Q}_3^{-}\right] \\
\overline{Q}_3^{+} &= -i\left[\mathcal{Z}_{32}^{-}\ \mathcal{Q}_2^{-} + \mathcal{Z}_{33}^{-}\ \mathcal{Q}_3^{-}\right]
\end{aligned}
$$

K^0 oscillations and operator renormalisation

- The definitions of renormalisation parameters of multiplicatively renormalisable operators in the SF scheme and their RG-running have been extended in the case of operator bases which mix under renormalisation
- NB: renormalisation constants, anomalous dimensions and SSF's are matrices

M.Papinutto, C.Pena, D.Preti Eur.Phys.J.C77(2017)376
P.Dimopoulos, G.Herdoiza, M.Papinutto, C.Pena, D.Preti, A.V., Eur.Phys.J.C78(2018)579

renormalisation pattern
$$\mathbf{Q}_{\mathrm{R}} = \lim_{g_0^2 \to 0} \mathbf{Z}(a\mu, g_0^2)\ \mathbf{Q}(g_0^2)$$

RG equation
$$\mathbf{Q}_{\mathrm{R}}\ \gamma(g_{\mathrm{R}}) = \mu \frac{d\mathbf{Q}_{\mathrm{R}}}{d\mu}$$

evolution matrix
$$\mathbf{Q}_{\mathrm{R}}(\mu_2) = \mathbf{U}(\mu_2, \mu_1)\ \mathbf{Q}_{\mathrm{R}}(\mu_1)$$

evolution matrix factorises
$$\mathbf{U}(\mu_2, \mu_1) = [\tilde{\mathbf{U}}(\mu_2)]^{-1}\ \tilde{\mathbf{U}}(\mu_1)$$

RGI evolution
$$\tilde{\mathbf{U}}(\mu) = \left[\frac{g_{\mathrm{R}}^2(\mu)}{4\pi}\right]^{-\gamma^{(0)}/(2b_0)} \mathbf{W}(\mu)$$
LO × beyond LO (W)

RG-running in the continuum

P.Dimopoulos, G.Herdoiza, M.Papinutto, C.Pena, D.Preti, A.V., Eur.Phys.J.C78(2018)579

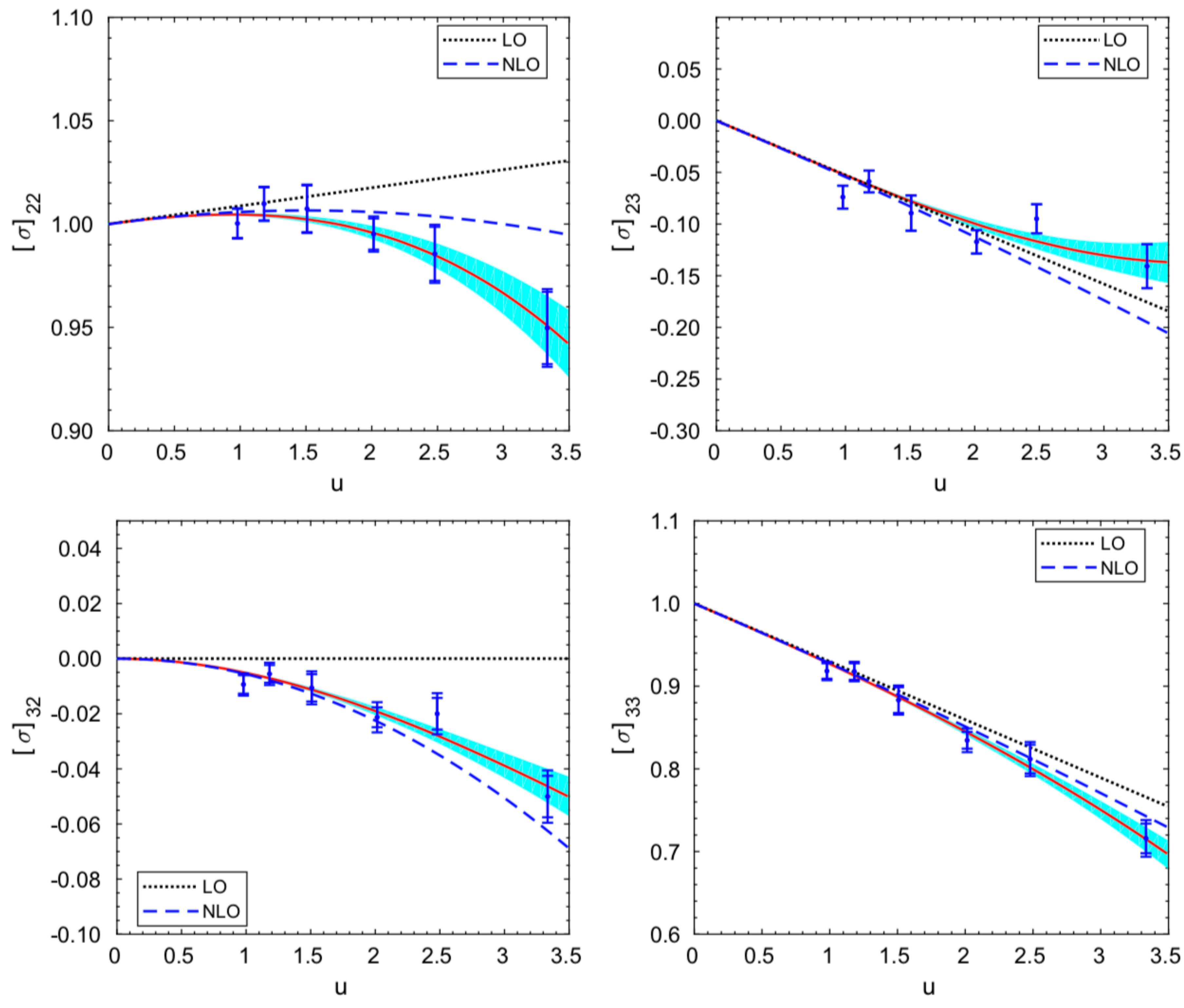
continuum matrix SSF

as a function of the

renormalised coupling

for operators Q_2^- , Q_3^-

N_f = 2 RESULTS



RG-running in the continuum

P.Dimopoulos, G.Herdoiza, M.Papinutto, C.Pena, D.Preti, A.V., Eur.Phys.J.C78(2018)579

continuum matrix SSF

as a function of the

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for operators Q_4^+ , Q_5^+

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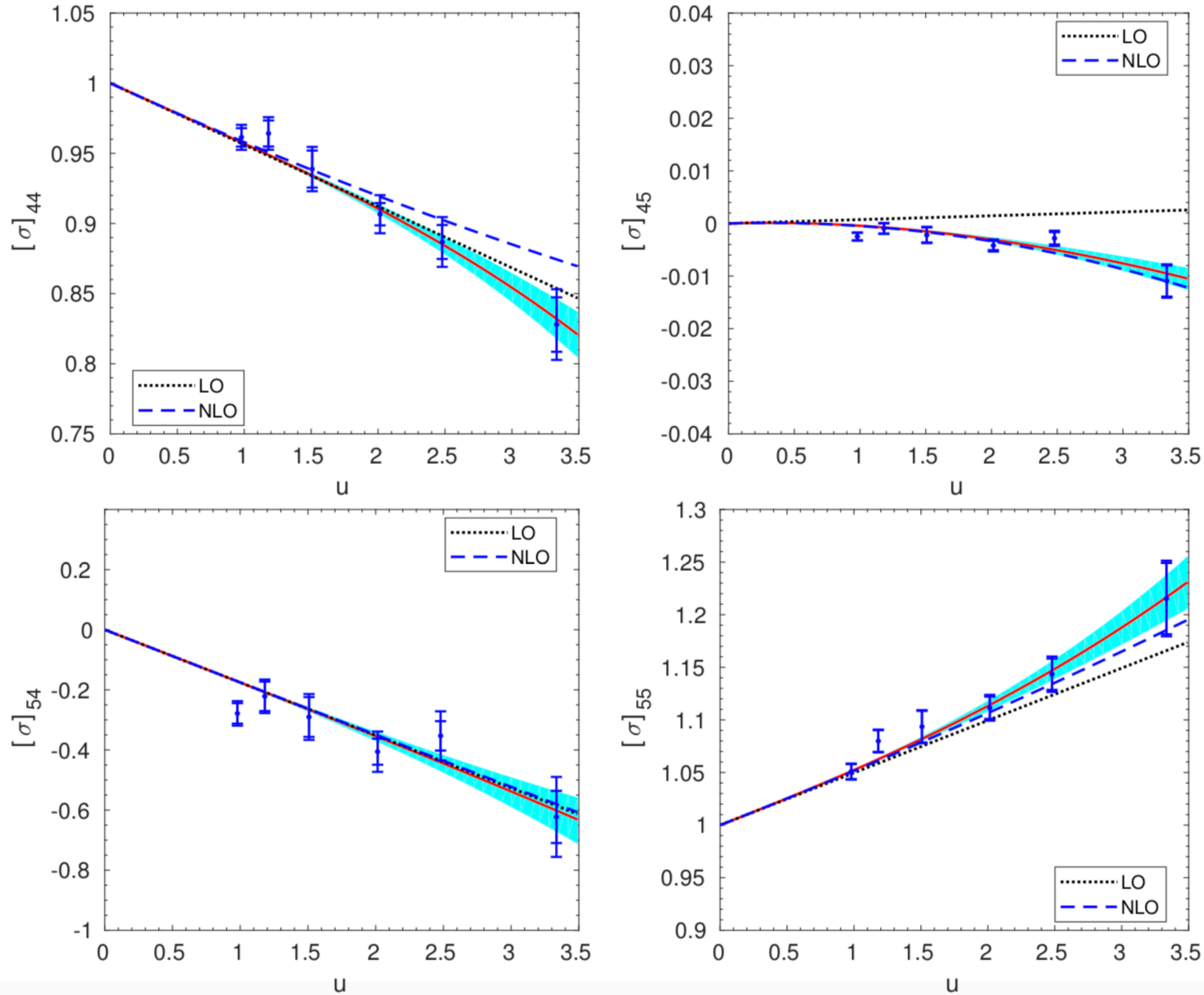


RG-running in the continuum

P.Dimopoulos, G.Herdoiza, M.Papinutto, C.Pena, D.Preti, A.V., Eur.Phys.J.C78(2018)579

RGI evolution

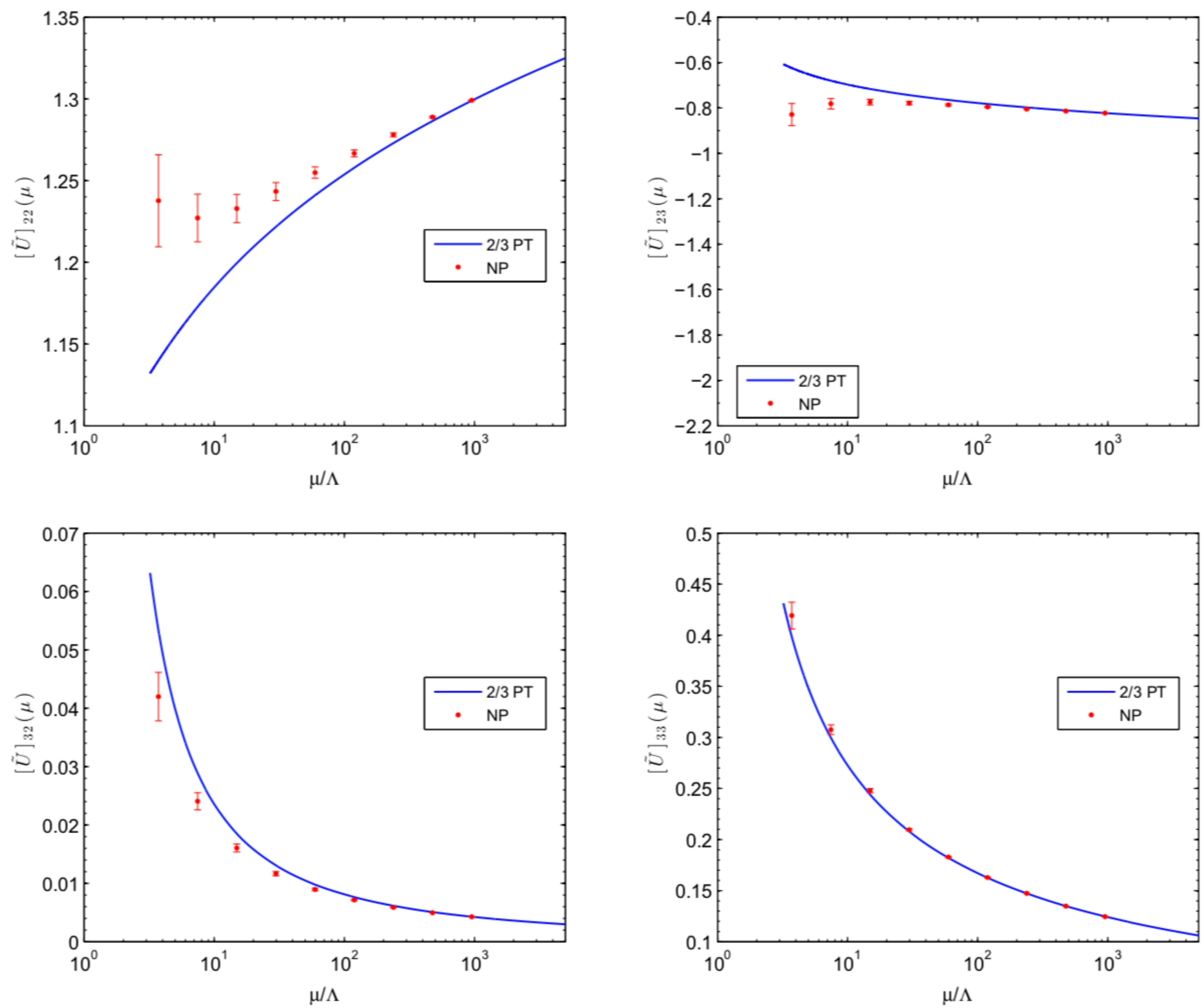
as a function of the

renormalised coupling

for operators Q_2^- , Q_3^-

$$\mathbf{Q}^{\mathrm{RGI}} \equiv \left[\frac{g_{\mathrm{R}}(\mu)}{4\pi}\right]^{-\gamma^{(0)}/2b_0} \mathbf{W}(\mu)\mathbf{Q}(\mu)$$

$$\boxed{\tilde{U}(\mu)}$$



RG-running in the continuum

P.Dimopoulos, G.Herdoiza, M.Papinutto, C.Pena, D.Preti, A.V., Eur.Phys.J.C78(2018)579

RGI evolution

as a function of the

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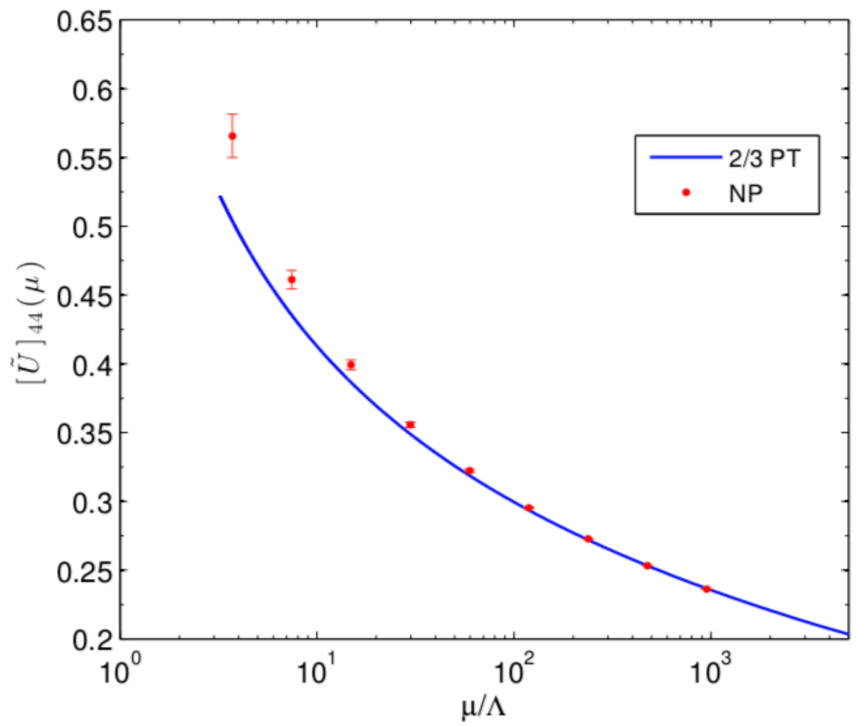
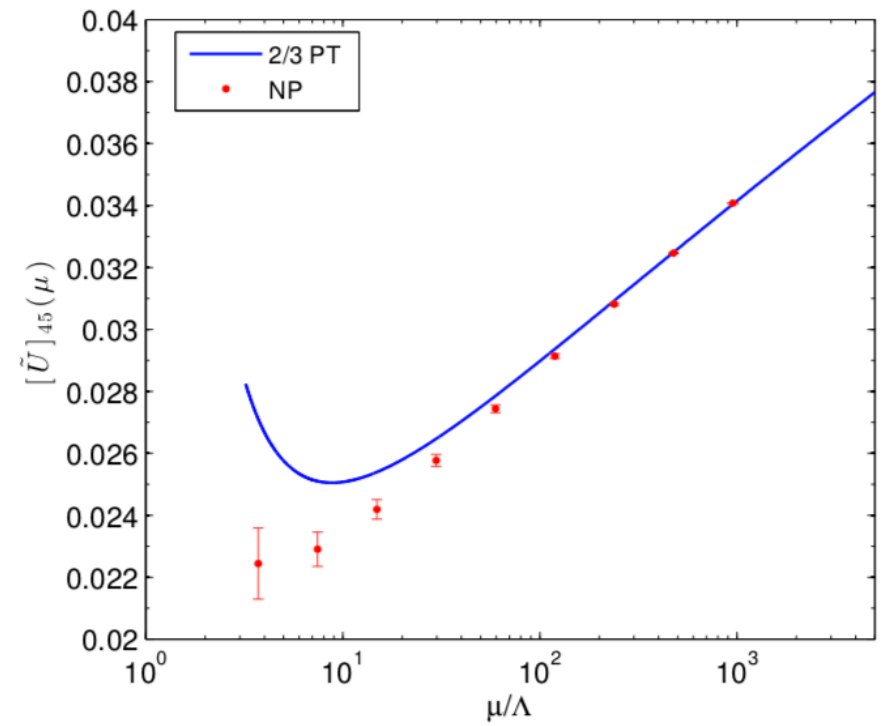
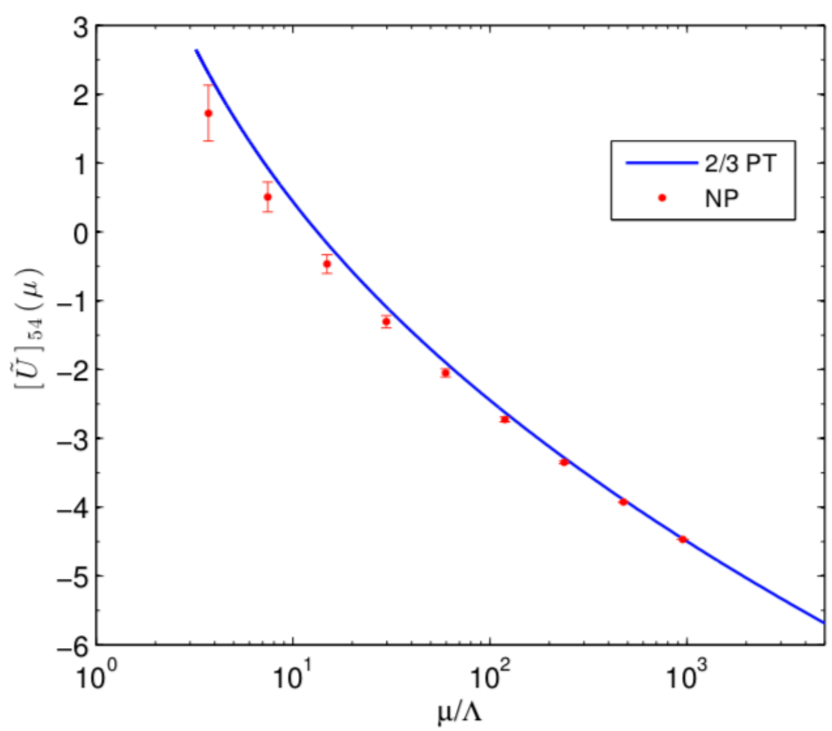
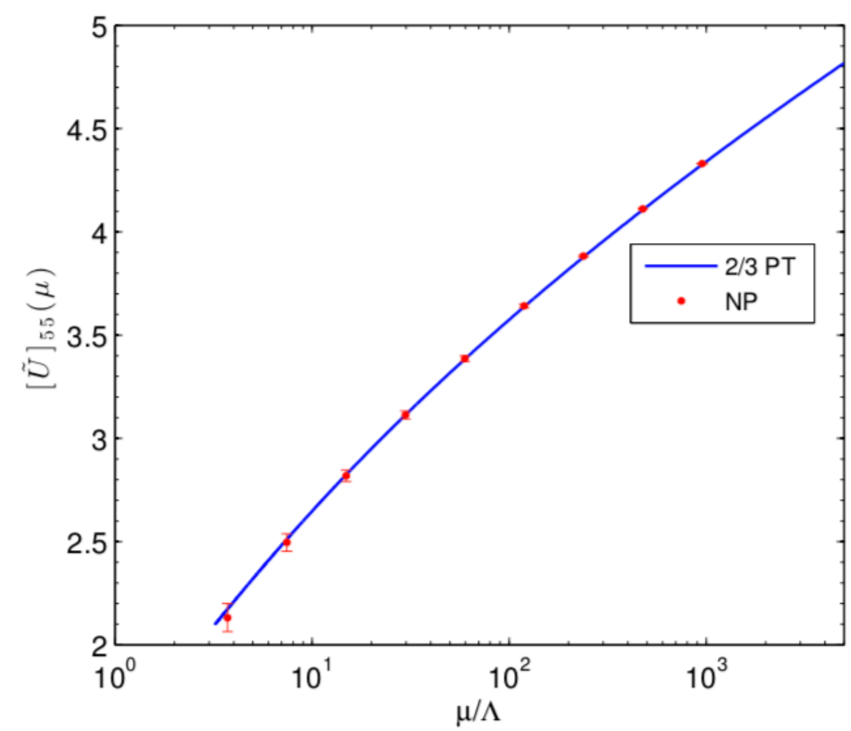




$$\mathbf{Q}^{\rm RGI} \equiv \left[\frac{g_{\rm R}(\mu)}{4\pi}\right]^{-\gamma^{(0)}/2b_0} \mathbf{W}(\mu)\mathbf{Q}(\mu)$$

$\tilde{U}(\mu)$





K⁰ oscillations and operator renormalisation

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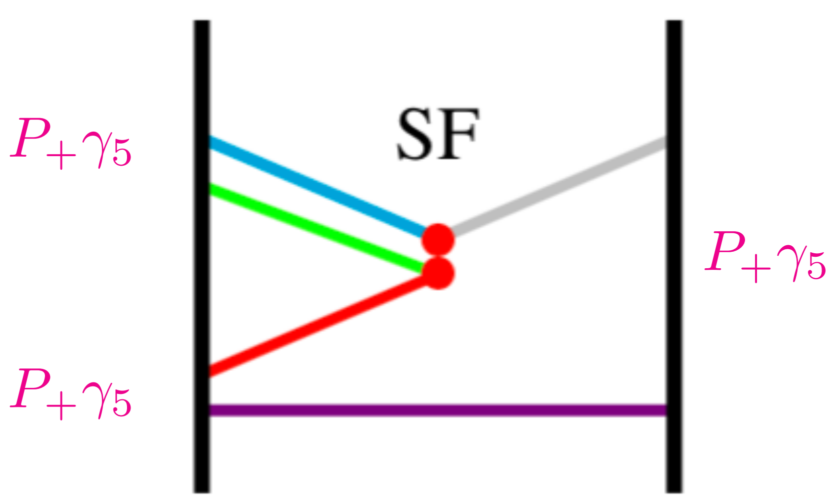
Q_5^{\pm} &\rightarrow -i\,\mathcal{Q}_5^{\pm}
\end{aligned}
$$

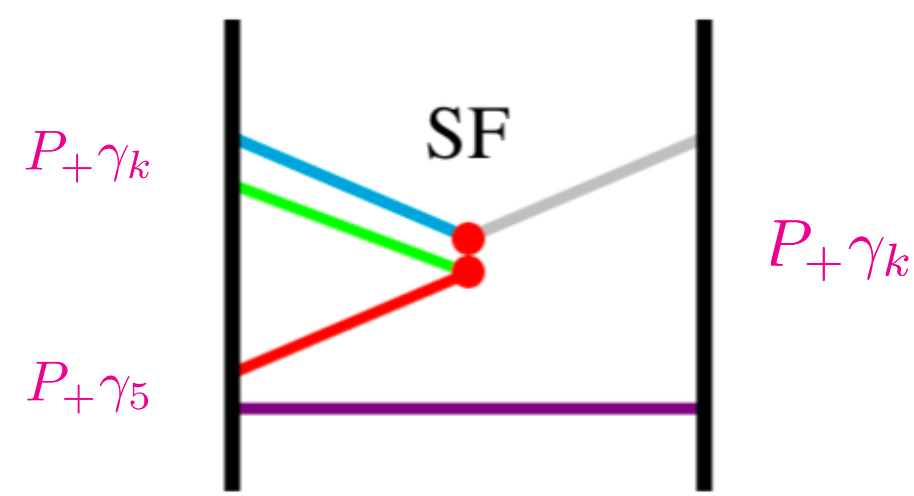
parity odd

SF renormalisation involves massless fermions, parity odd operators and SF boundaries

- We have performed some rather rudimentary cross-checks for getting the 5 RGI operators, combining: (i) our NP RG-running results in SF scheme; (ii) ETM bare matrix elements; (iii) PT switching from RI/MOM to SF at ~4-3GeV: very unstable results due to the NP RG-running in SF.



NOISY!



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$$

K^0 oscillations and operator renormalisation

M.DallaBrida, M.Papinutto, P.Vilaseca, PoS(LATTICE 2015)252

$$
\begin{aligned}
Q_1^{\pm} &\rightarrow -i\,\mathcal{Q}_1^{\pm}\\
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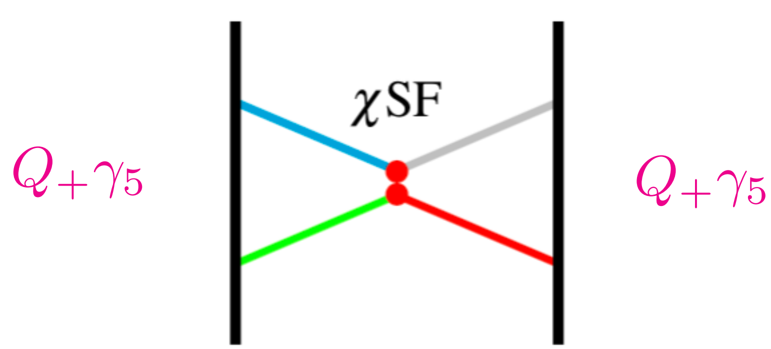
Q_4^{\pm} &\rightarrow -i\,\mathcal{Q}_4^{\pm}\\
Q_5^{\pm} &\rightarrow -i\,\mathcal{Q}_5^{\pm}
\end{aligned}
$$

parity odd

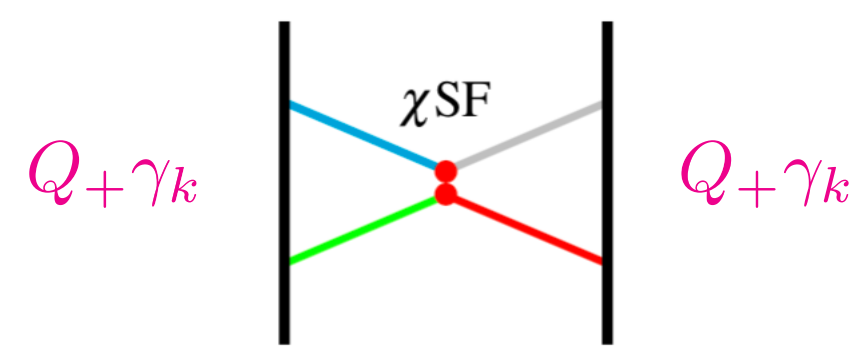
χSF renormalisation involves massless fermions, parity odd operators and χSF boundaries

- To preserve parity *P* we must introduce correlation functions with 4 distinct flavours and **two** combinations of wall sources

- The χSF renormalised correlation functions of parity-odd operators renormalise without "extra lattice subtractions" and are equal, up to $O(a^2)$, to the parity-even SF correlation functions



LESS NOISY!



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\overline{Q}_3^{+} &= -i\,[\mathcal{Z}_{32}^{-}\,\mathcal{Q}_2^{-} + \mathcal{Z}_{33}^{-}\,\mathcal{Q}_3^{-}]
\end{aligned}
$$

χSF renormalisation scheme and RG-running for dim-3 composite operators and N_f=3

(multiplicative renormalisation)

χSF renormalisation and RG-running for d=3 composite operators

- The χSF construction (S.Sint, NuclPhysB847(2011)491) has been tested:

- Theoretical expectations and perturbative tests M.DallaBrida, S.Sint, P.Vilaseca, JHEP08(2016)102

- High-precision results for the axial and vector current normalisations in $N_f = 2$ and $N_f = 3$ QCD for couplings corresponding to hadronic scales (e.g. CLS simulations) M.DallaBrida, S.Sint, T.Korsec, P.Vilaseca, Our.Phys.C79(2019)23

- Before moving on to the χSF renormalisation and RG-running of the dim-6, 4-fermion operators, we decided to do further studies on the NP RG-running ($N_f = 3$ QCD) of dim-3 operators (pseudoscalar and tensor) over energy scales ranging from $\sim\Lambda_{QCD}$ to $\sim$100 GeV

- For the pseudoscalar (quark mass) renormalisation see I.C.Plasencia, M.Dalla Brida, G.M.deDivitiis, A.Lytle, M.Papinutto, L.Pirelli, A.V. Phys.Rev.D105(2022)054506

- Mixed action logic: ensembles (sea quarks) from SF runs, measurements (valence quarks) with χSF boundaries.

- The gauge coupling is that of previous Alpha works: in a low energy region $\Lambda_{QCD} \lesssim \mu \lesssim 2\text{GeV}$ at the coupling is defined in the GF setup whilst at high energies $2\text{GeV} \lesssim \mu \lesssim 100\text{GeV}$ it is given in the SF setup

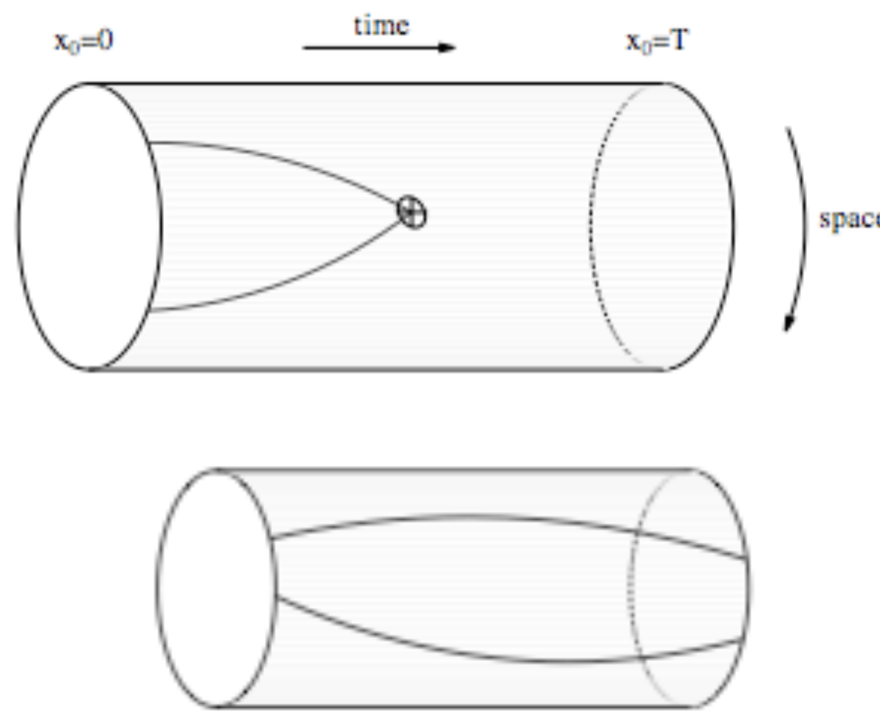
χSF renormalisation and RG-running for d=3 composite operators

- The "SF scheme" (SF boundary conditions)

$$\frac{Z_{\rm P}^{\rm SF}(g_0^2, L/a) f_{\rm P}(T/2)}{\sqrt{f_1}} = \left[\frac{f_{\rm P}(T/2)}{\sqrt{f_1}}\right]^{\rm TL}$$

cancels boundary quark field renormalization

tree level



- The χSF scheme (with χSF boundary conditions)

$$\frac{Z_{\rm P}^{\chi{\rm SF}}(g_0^2, L/a) g_{\rm P}^{ud}(T/2)}{\sqrt{g_1^{ud}}} = \left[\frac{g_{\rm P}^{ud}(T/2)}{\sqrt{g_1}^{ud}}\right]^{\rm TL} \qquad \frac{Z_{\rm S}^{\chi{\rm SF}}(g_0^2, L/a) g_{\rm S}^{uu'}(T/2)}{\sqrt{g_1^{uu'}}} = \left[\frac{g_{\rm S}^{uu'}(T/2)}{\sqrt{g_1}^{uu'}}\right]^{\rm TL}$$

- Formally:

$$f_{\rm P} = g_{\rm P}^{ud} = i\, g_{\rm S}^{uu'} \qquad f_1 = g_1^{ud} = g_1^{uu'}$$

χSF renormalisation and RG-running for d=3 composite operators

"SF scheme" I.Campos, P.Fritzsch, C.Pena, D.Preti, A.Ramos, A.V. Eur.Phys.J.C78(2018)387

χSF scheme I.C.Plasencia, M.Dalla Brida, G.M.deDivitiis, A.Lytle, M.Papinutto, L.Pirelli, A.V. Phys.Rev.D105(2022)054506

$$\frac{M^{\mathrm{RGI}}}{\bar{m}(\mu_{\mathrm{had}})} = 1.7505(89)$$

SF scheme

$$\frac{M^{\mathrm{RGI}}}{\bar{m}(\mu_{\mathrm{had}})} = 1.7519(74)$$

χSF scheme

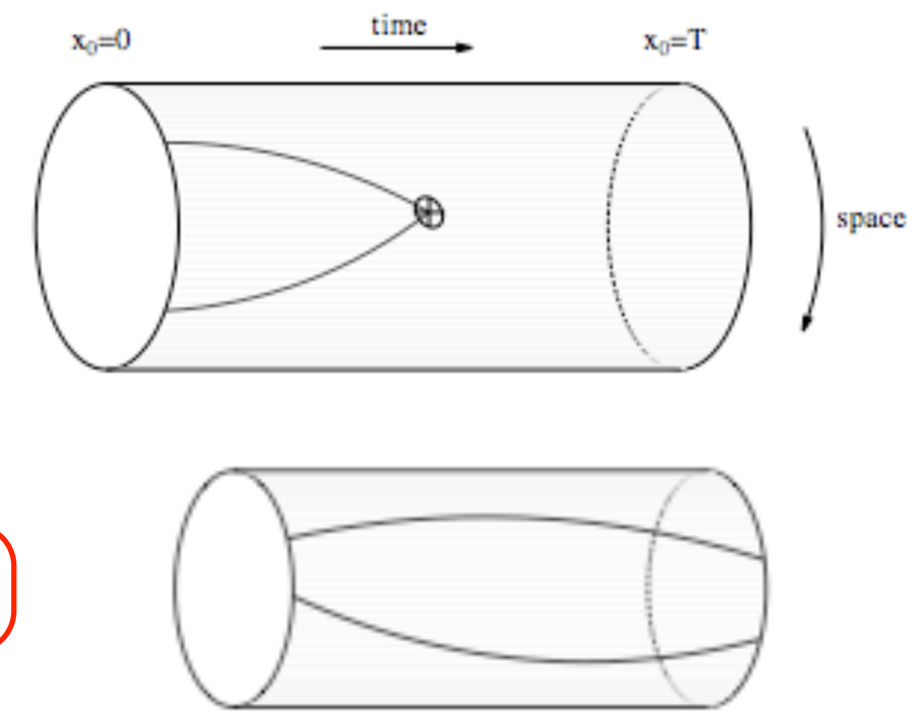
χSF renormalisation and RG-running for d=3 composite operators

- The "SF scheme" (SF boundary conditions)

$$\frac{Z_{\mathrm{T}}^{\mathrm{SF}}(g_0^2, L/a) k_{\mathrm{T}}^{\mathrm{I}}(T/2)}{\sqrt{d_1}} = \left[\frac{k_{\mathrm{T}}(T/2)}{\sqrt{d_1}}\right]^{\mathrm{TL}}$$

cancels boundary quark field renormalization

tree level



NB: d_1 can be either f_1 (with γ_5 at the boundary) or k_1 (with γ_k at the boundary) or a product of "suitable" powers of both

- The correlation function needs to be Symanzik-improved in the chiral limit: need c_{T}!

$$k_{\mathrm{T}}^{\mathrm{I}} \;=\; k_{\mathrm{T}} \,+\, a\, c_{\mathrm{T}}(g_0^2)\, \partial_0 k_{\mathrm{V}}$$

C.Pena, D.Preti, Eur.Phys.J.C78(2018)575 c_{T} from PT at N_f=2

L.Chimirri, P.Fritzsch, J.Heitger, F.Joswig,M.Panero, C.Pena, D.Preti, PoS(LATTICE 2019)212 and paper to appear shortly c_{T} from WI (NP) at N_f=3

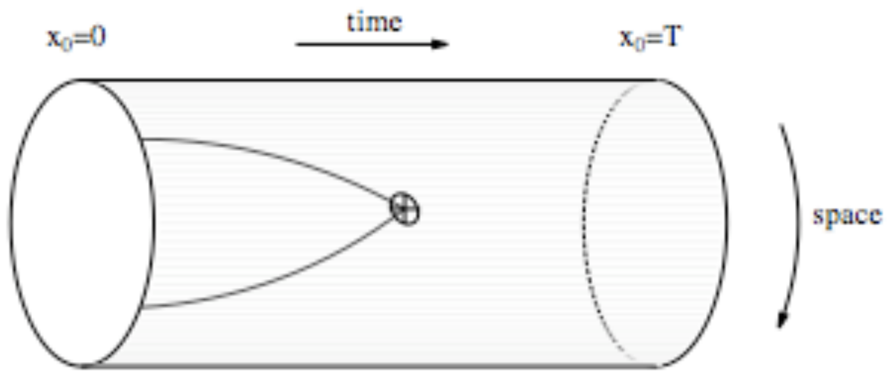
χSF renormalisation and RG-running for d=3 composite operators

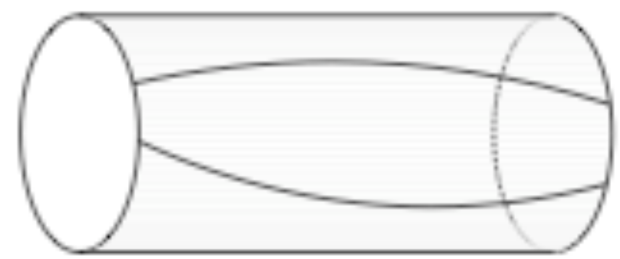
- The "χSF scheme" (χSF boundary conditions)
- M.DallaBrida, M.Papinutto, P.Vilaseca, PoS(LATTICE 2015)252
- I.C.Plasencia, M.Dalla Brida, G.M.deDivitiis, A.Lytle, M.Papinutto, L.Pirelli, A.V. PoS(LATTICE 2023)356

$$\frac{Z_{\rm T}^{\chi\rm SF}(g_0^2, L/a)\, l_{\rm T}^{ud}(T/2)}{\sqrt{d_1}} = \left[\frac{l_{\rm T}^{ud}(T/2)}{\sqrt{d_1}}\right]^{\rm TL}$$

$$d_1 = g_1^{ud} \qquad \alpha - \text{scheme}$$

$$d_1 = l_1^{ud} \qquad \beta - \text{scheme}$$

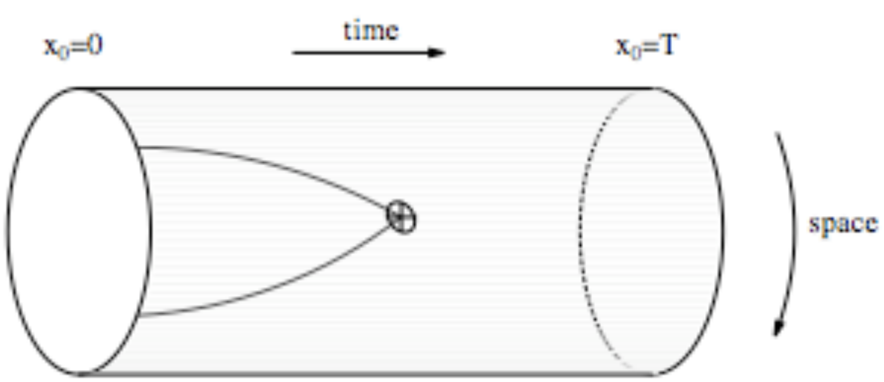


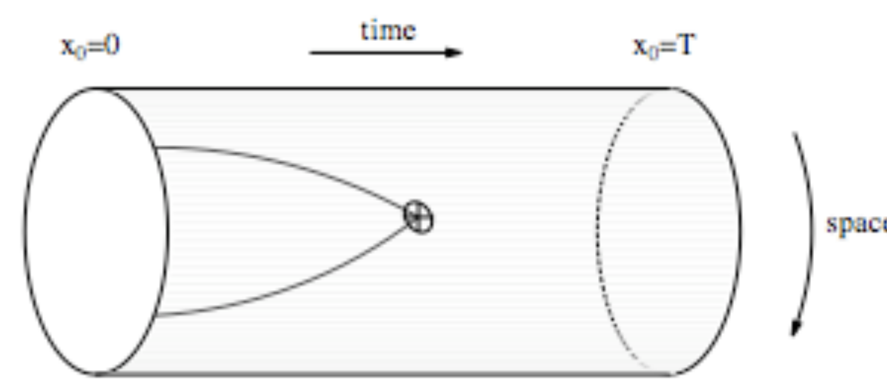


$$\frac{Z_{\rm T}^{\chi\rm SF}(g_0^2, L/a)\, l_{\rm T}^{ud}(T/2)}{d} = \left[\frac{l_{\rm T}^{ud}(T/2)}{d}\right]^{\rm TL}$$

$$d = -i\, g_{\tilde{\rm V}}^{ud} \qquad \gamma - \text{scheme}$$

$$d = l_{\tilde{\rm V}}^{uu'} \qquad \delta - \text{scheme}$$





χSF renormalisation and RG-running for d=3 composite operators

- The correlation function does NOT need to be Symanzik-improved: NO c_T!

$$\boxed{O(a^2)}$$

$$l_{\rm T}^{{\rm I},ud} = l_{\rm T}^{ud} + \cancel{c_{\rm T}(g_0^2)\, a\, \partial_0 l_{\rm V}^{ud}}$$

$$\frac{Z_{\rm T}^{\chi{\rm SF}}(g_0^2, L/a)\, l_{\rm T}^{ud}(T/2)}{\sqrt{d_1}} = \left[\frac{l_{\rm T}^{ud}(T/2)}{\sqrt{d_1}}\right]^{\rm TL}$$

$$d_1 = g_1^{ud} \qquad \alpha - \text{scheme}$$

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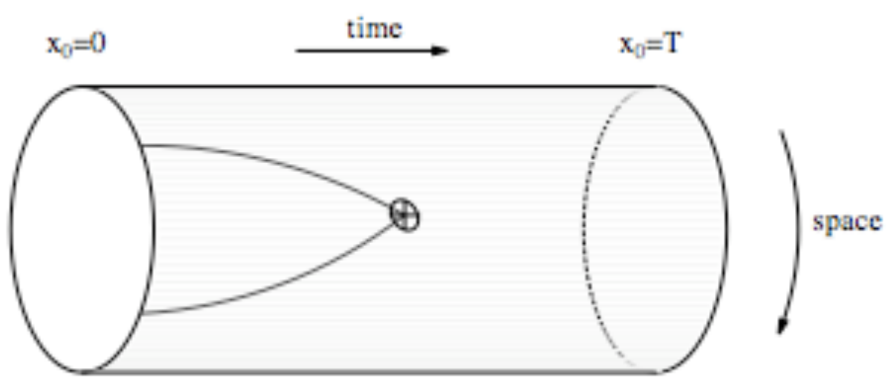

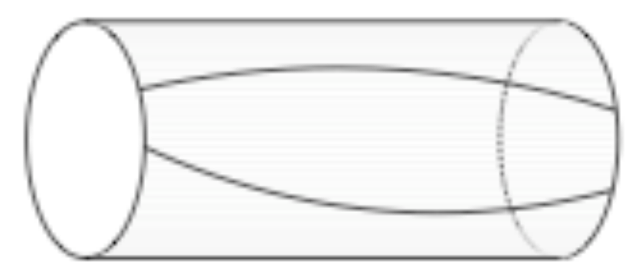

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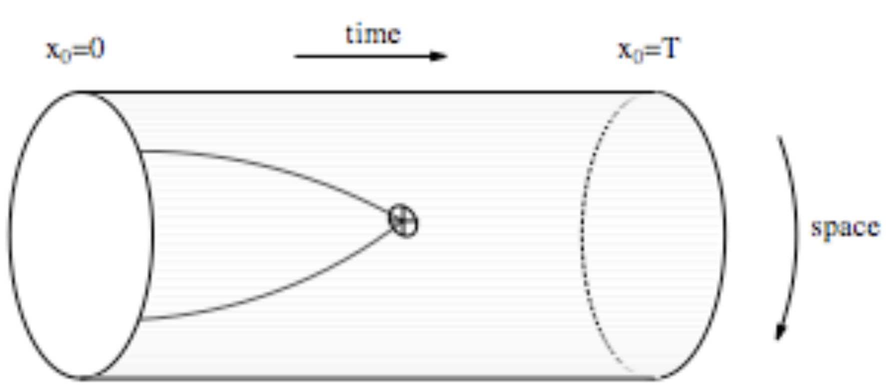
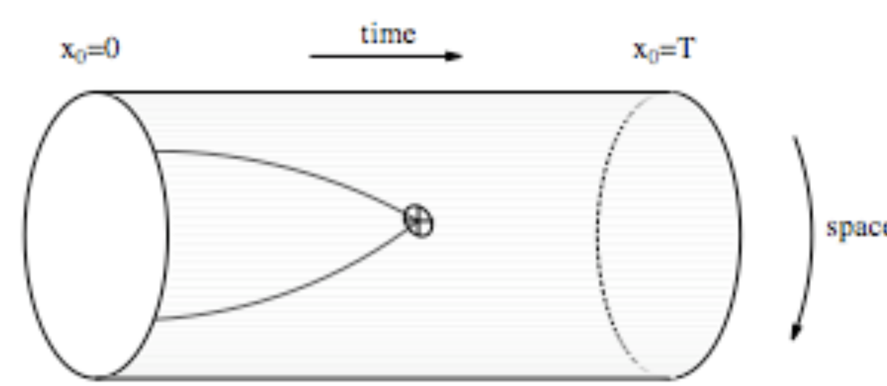
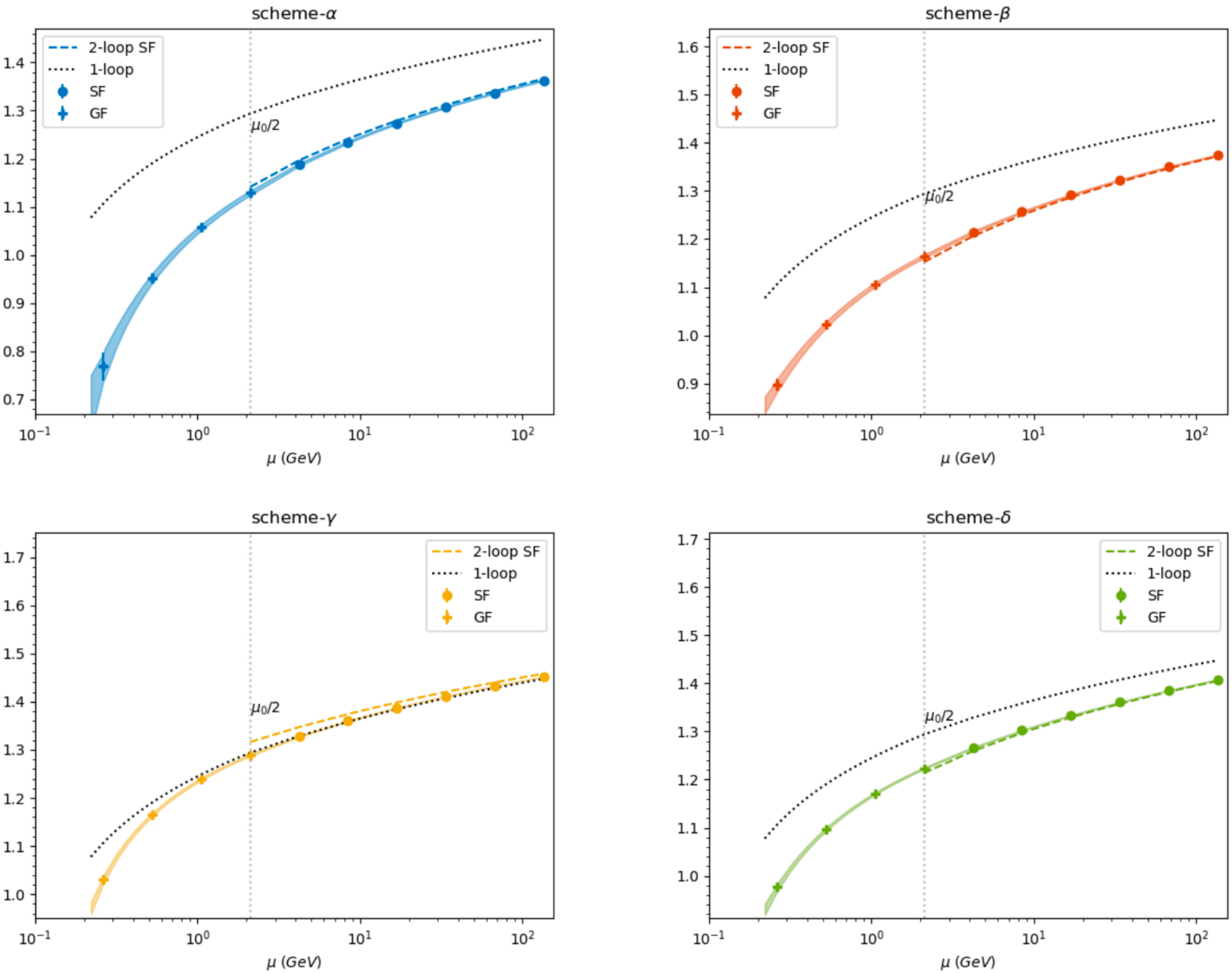




χSF renormalisation and RG-running for d=3 composite operators

$$\frac{T^{\mathrm{RGI}}}{[T(\mu)]_{\mathrm{R}}}$$



PRELIMINARY

$$\frac{T^{\mathrm{RGI}}}{[T(\mu)]_{\mathrm{R}}}$$

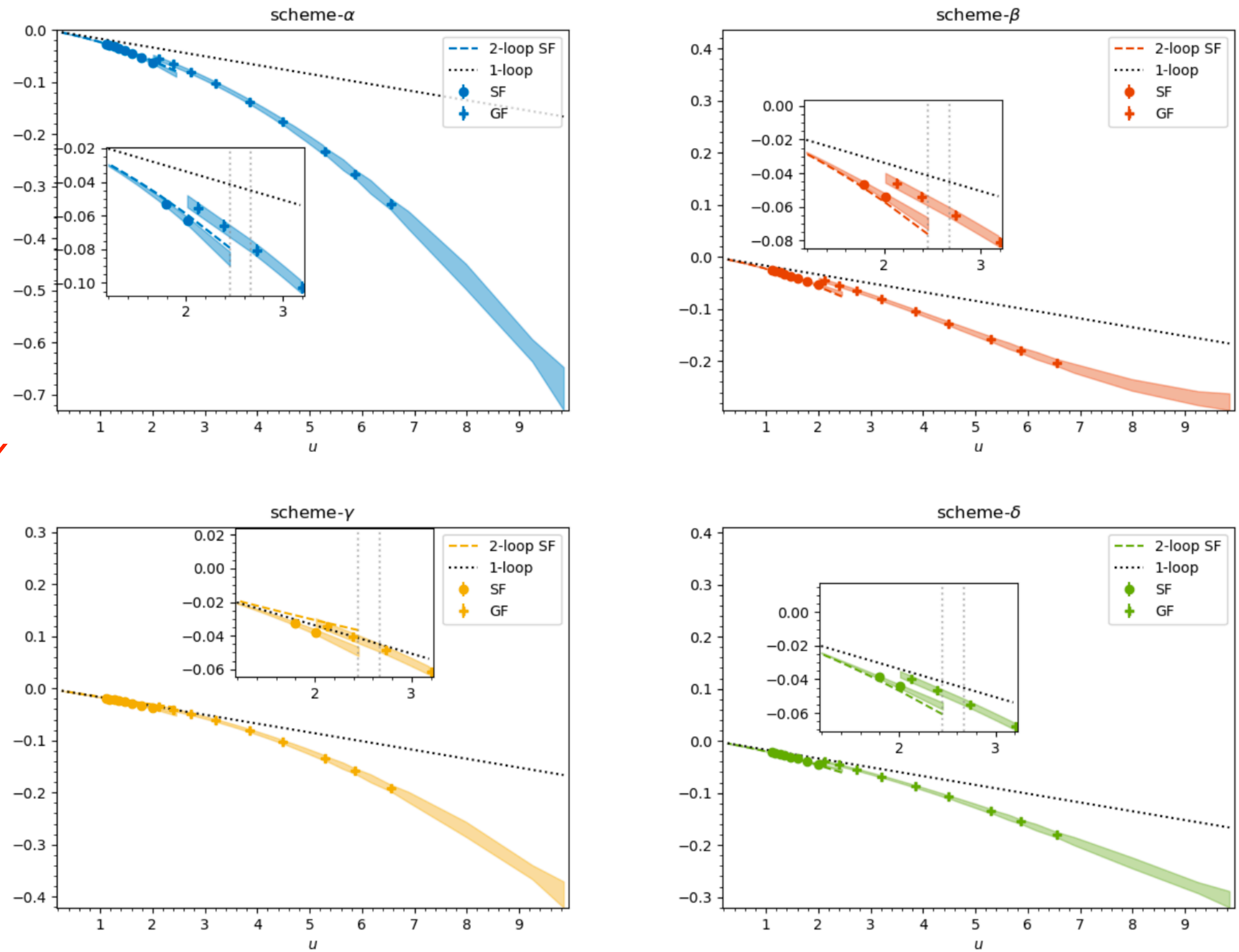
I.C.Plasencia, M.Dalla Brida, G.M.deDivitiis, A.Lytle, M.Papinutto, L.Pirelli, A.V. in preparation

χSF renormalisation and RG-running for d=3 composite operators

γ_T

PRELIMINARY

γ_T



I.C.Plasencia, M.Dalla Brida, G.M.deDivitiis, A.Lytle, M.Papinutto, L.Pirelli, A.V. in preparation

Conclusions

- Alpha proposes a "tiebreaker", in order to settle the tensions observed in B_4 and B_5
- The "new" elements would be NP χSF renormalisation and RG-running and OS bare matrix elements
- This ensures simple renormalisation properties and automatic improvement
- Preliminary results for the quark mass running in a χSF setup are perfectly compatible to those obtained in a SF setup
- Analogous results will appear soon for the tensor composite field
- The χSF renormalisation condition being simpler than the SF one, we expect significant noise reduction
- However, the quest for the $N_f = 2+1$ computation of $B_1, \ldots, B_5$ with χSF renormalisation and RG-running in $N_f = 3$ is a excruciatingly long and winding road… the end is not nigh!