Generating configurations of increasing lattice size with machine learning and the inverse renormalization group

Dimitrios Bachtis

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 813942

Outline

1. The (standard) renormalization group.

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2. The inverse renormalization group: is it possible?

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3. Example of the inverse renormalization group for the φ^4 theory.

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3. Example of the inverse renormalization group for the φ^4 theory.
4. Could we ever use the inverse renormalization group to surpass supercomputers?

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Why should a lattice field theorist be interested in the inverse renormalization group?

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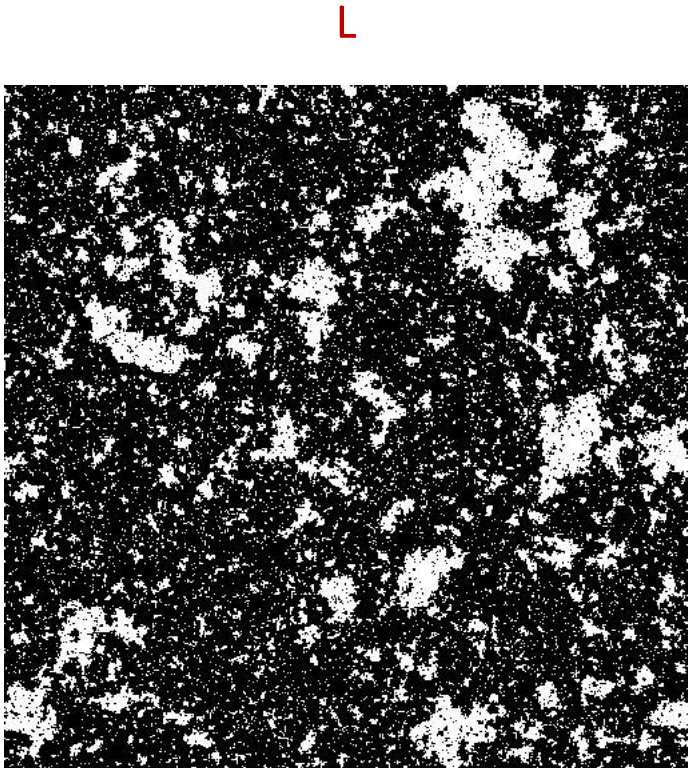
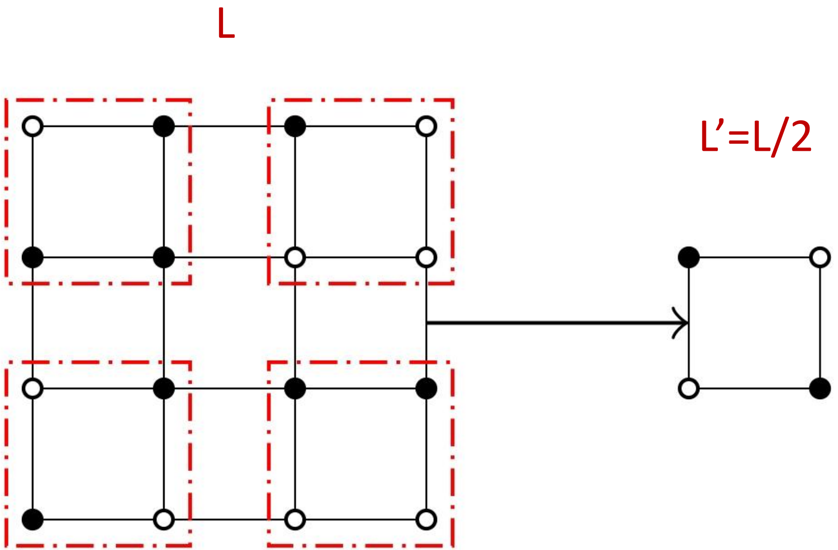
We can generate configurations of increasing lattice size without critical slowing down

Standard renormalization group



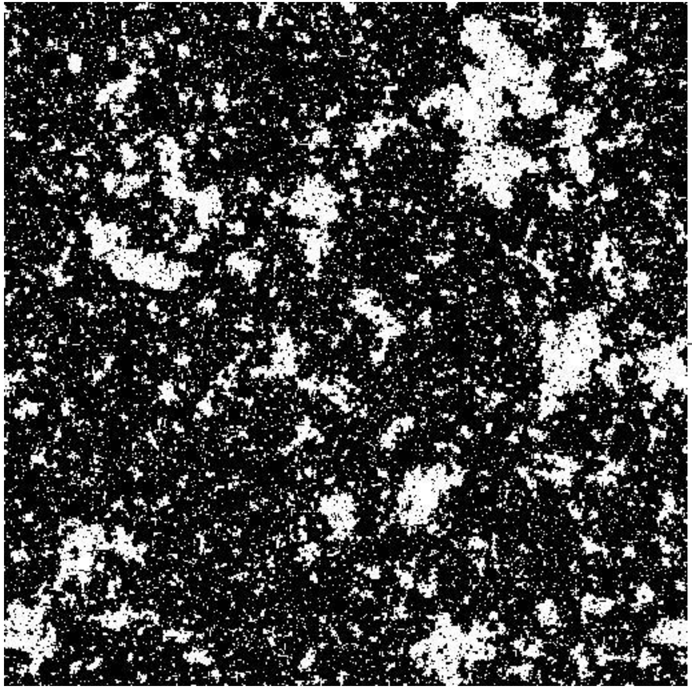
Standard renormalization group



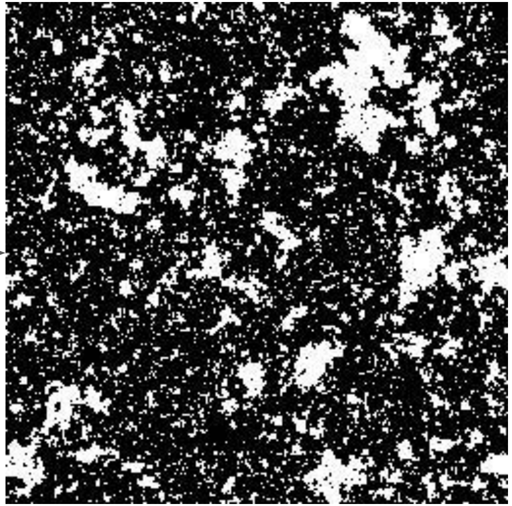
Spin blocking transformation with a rescaling factor of b=2 and the majority rule

Standard renormalization group

L



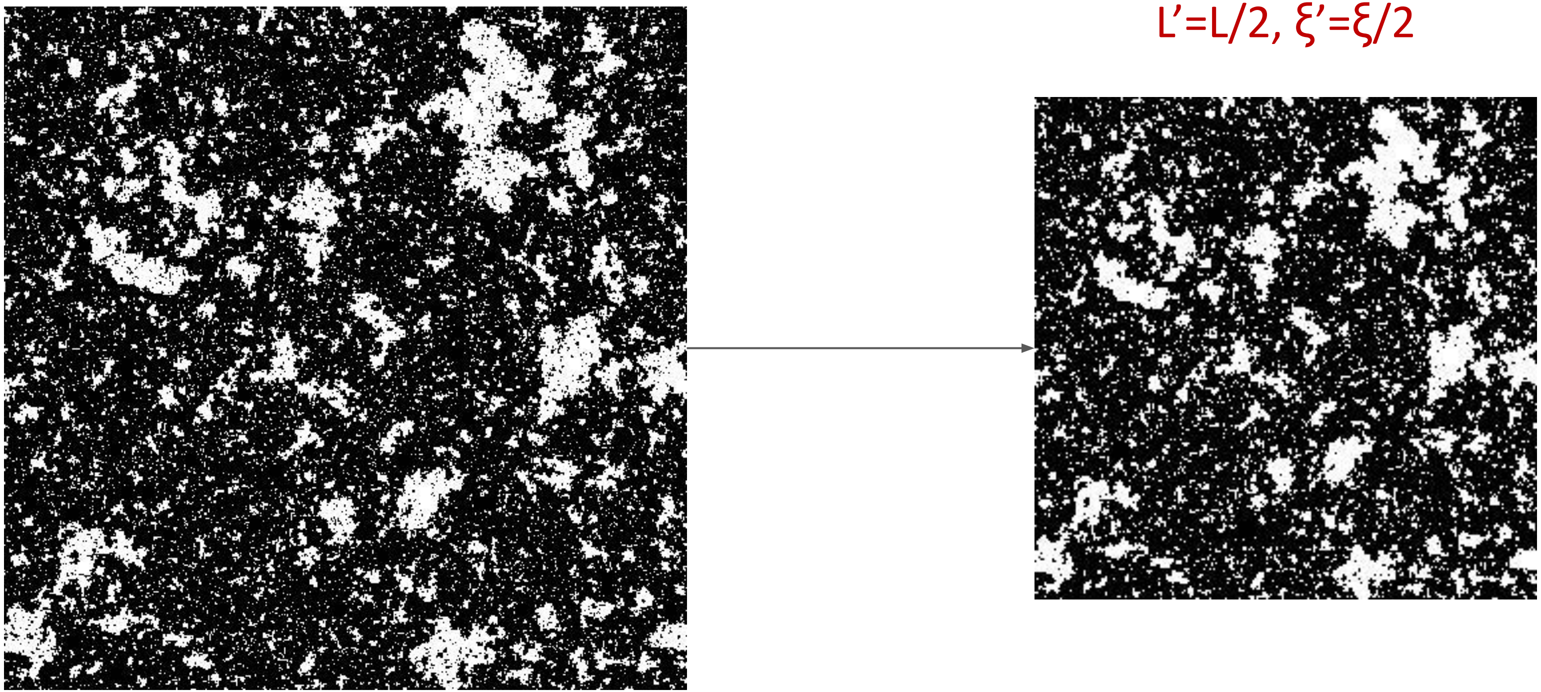
L’=L/2



Standard renormalization group

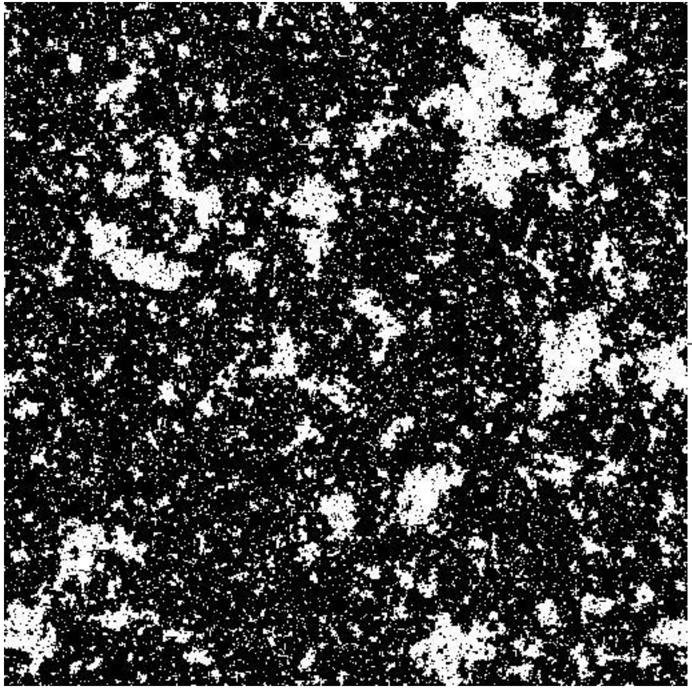
L, ξ

$L' = L/2, \ \xi' = \xi/2$

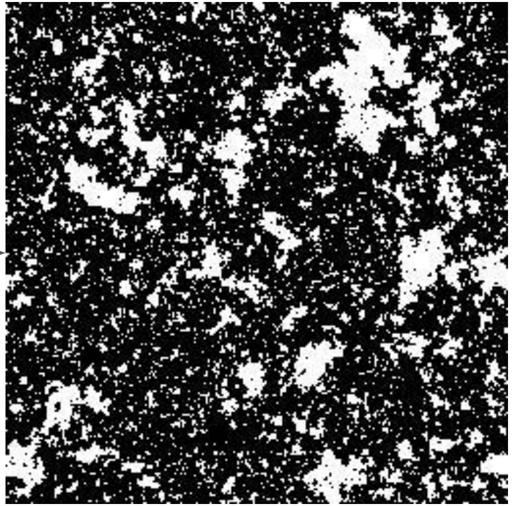


Standard renormalization group

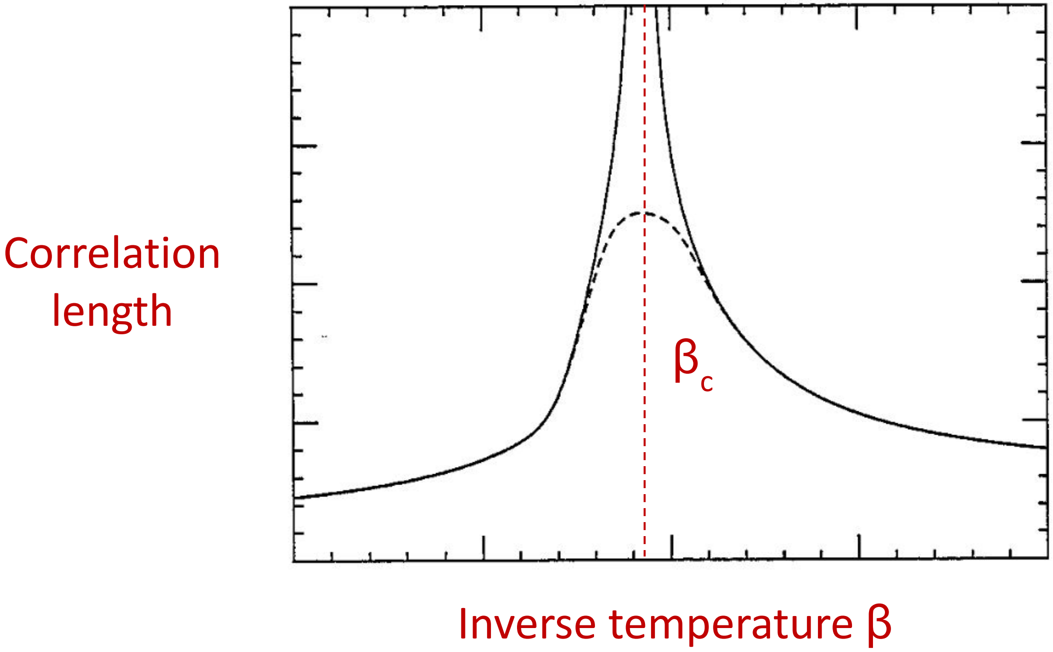
L, ξ, β



L’=L/2, ξ’=ξ/2, β’

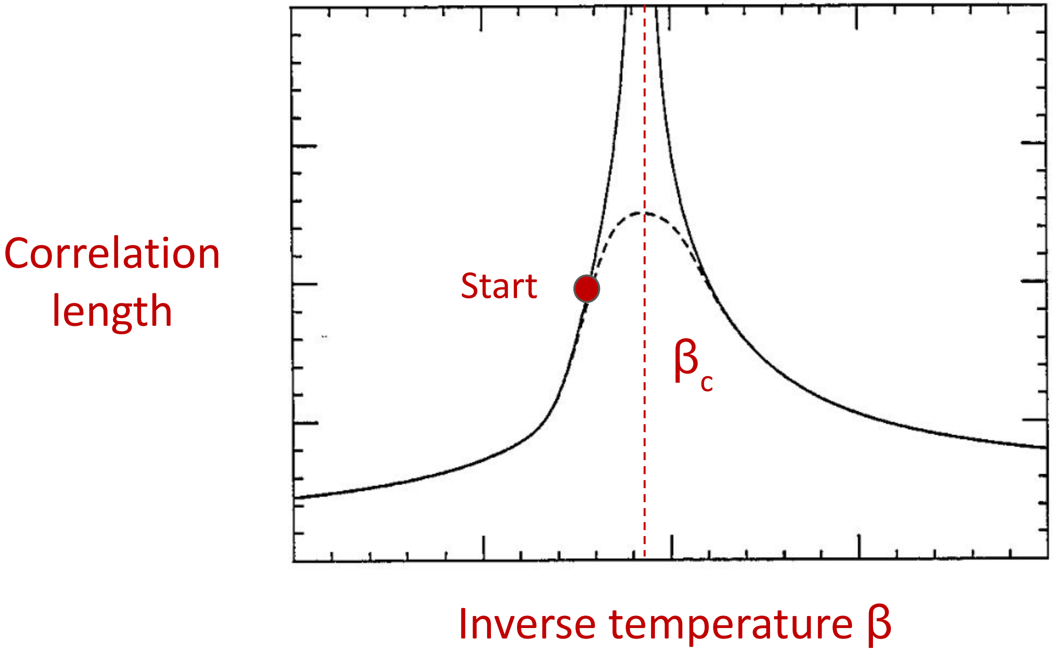


Standard renormalization group



Altered figure from (Newman, Barkema) book (Fig 4.1)

Standard renormalization group



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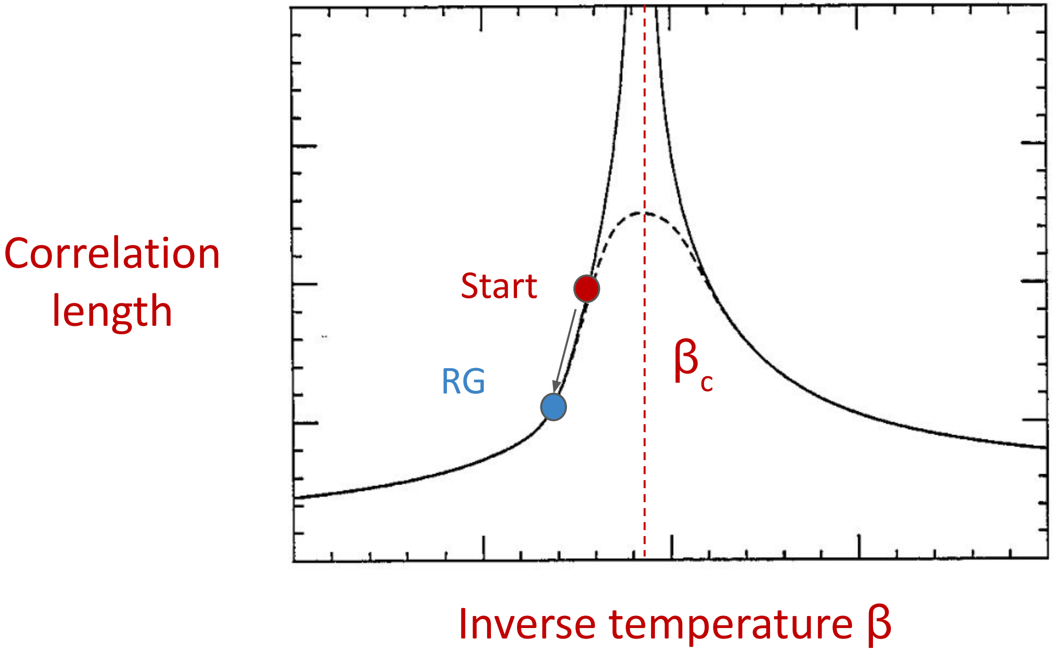
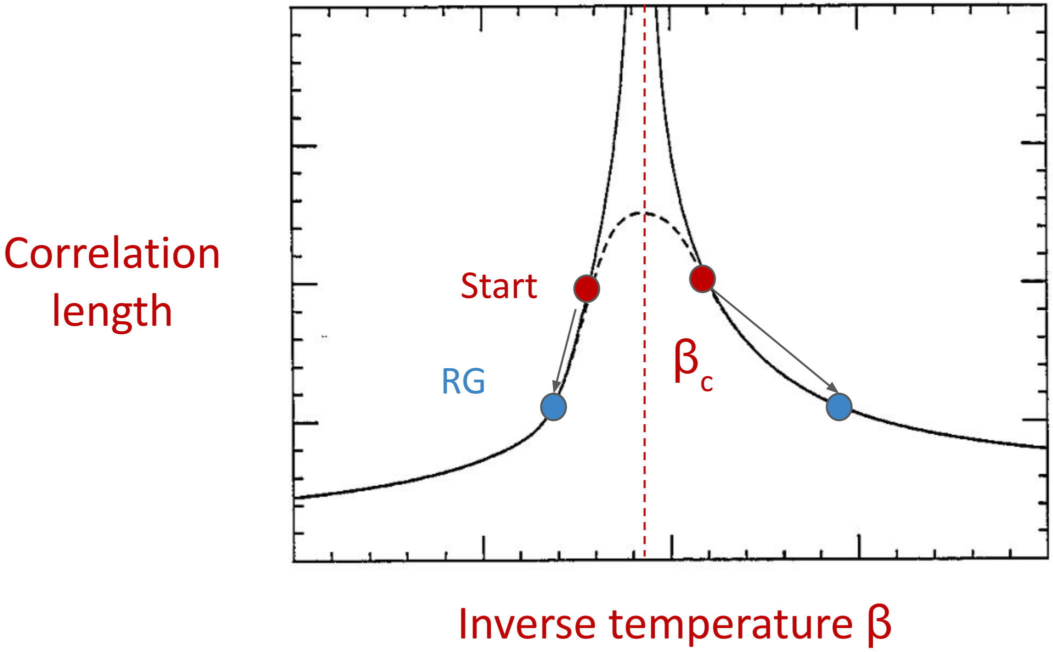


Altered figure from (Newman, Barkema) book (Fig 4.1)

Standard renormalization group



Altered figure from (Newman, Barkema) book (Fig 4.1)

Real space renormalization group

The standard (Monte Carlo) renormalization group is a powerful technique to study phase transitions:

1) Finite size effects are almost eliminated->Precision measurements can be obtained on smaller lattice sizes.
2) Locating the critical point can be achieved self-consistently (and on smaller lattices).

However, the standard renormalization group, which iteratively eliminates degrees of freedom, can only be applied for a finite number of steps before the degrees of freedom vanish.

Inverse renormalization group

Can we devise an inverse renormalization group approach that

i) retains all of the benefits of the standard RG, i.e. partial elimination of finite size effects,

ii) works for systems with continuous degrees of freedom,

iii) produces the correct fixed point structure and gives rise to inverse flows in parameter space, and

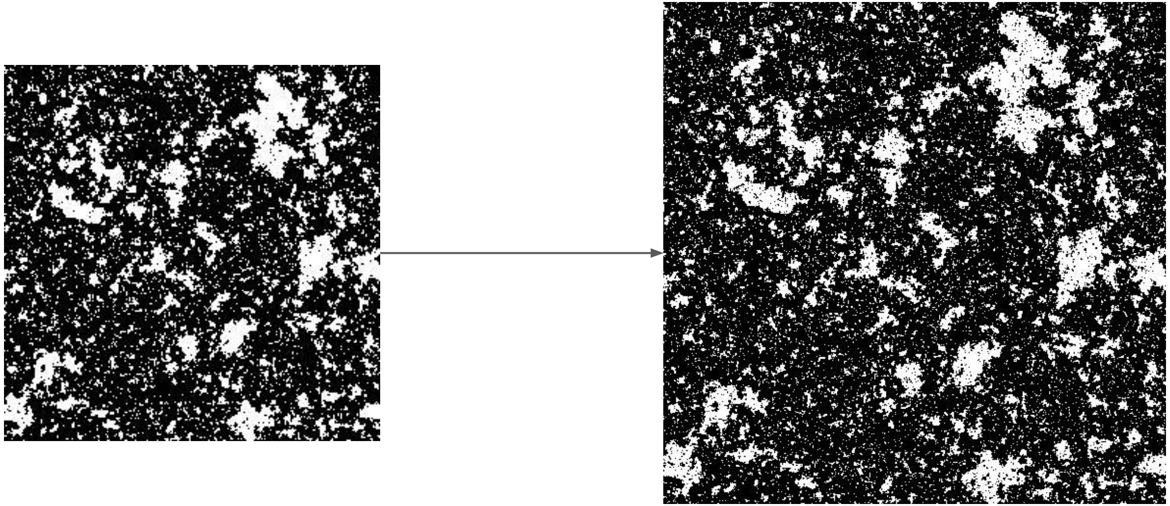
iv) can be applied in principle for an arbitrary number of steps, while evading the critical slowing down effect?

Inverse Monte Carlo Renormalization Group Transformations for Critical Phenomena, D. Ron, R. Swendsen, A. Brandt, Phys. Rev. Lett. 89, 275701 (2002)
Inverse renormalization group in quantum field theory, D. Bachtis, G. Aarts, F. Di Renzo, B. Lucini, arXiv:2107.00466

Inverse renormalization group

But how to devise an inverse transformation?

New degrees of freedom must be introduced within the system



Inverse renormalization group

Inversion of a majority rule in the **Ising** model

Original degree of freedom

+1

Possible rescaled degrees of freedom

+1	+1
+1	-1

-1	+1
+1	+1

-1	+1
+1	-1

-1	+1
+1	-1

…

For the inverse renormalization group in the Ising model, see:

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Inverse renormalization group

Inversion of a summation in the $\boldsymbol{\varphi}^4$ model

Original degree of freedom

0.40

Possible rescaled degrees of freedom

0.01	0.36
0.02	0.01

-421.1	90.1
0.5	330.9

…

Inverse Renormalization Group in Quantum Field Theory, D. Bachtis, G. Aarts, F. Di Renzo, B. Lucini, Phys. Rev. Lett. 128, 081603 (2022)

Inverse renormalization group

Inversion of a summation in the $\boldsymbol{\varphi}^4$ model

Original degree of freedom

0.40

Possible rescaled degrees of freedom

0.01	0.36
0.02	0.01

-421.1	90.1
0.5	330.9

...

Too complicated!

Inverse Renormalization Group in Quantum Field Theory, D. Bachtis, G. Aarts, F. Di Renzo, B. Lucini, Phys. Rev. Lett. 128, 081603 (2022)

The main idea:

We can learn a set of transformations, in the form of transposed convolutions, that can approximate the inversion of a standard renormalization group transformation.

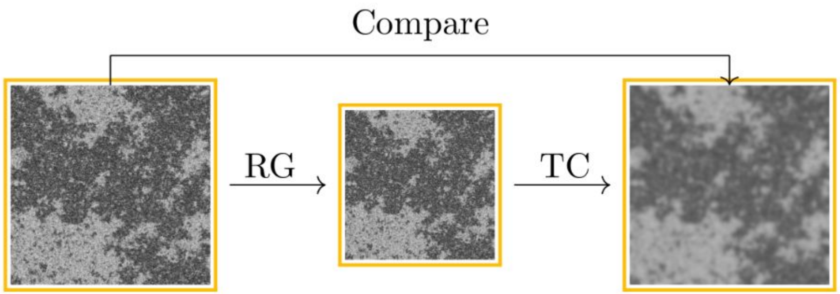


FIG. 3. Illustration of the optimization approach. Transposed convolutions (TC) are applied on configurations produced with the renormalization group (RG) to construct a set of configuration which is compared with the original.

Inverse renormalization group

$$\begin{bmatrix} 3 & 1 \\ 2 & 0 \end{bmatrix} * \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix}$$

Inverse Renormalization Group in Quantum Field Theory, D. Bachtis, G. Aarts, F. Di Renzo, B. Lucini, Phys. Rev. Lett. 128, 081603 (2022)

Inverse renormalization group

$$\begin{array}{|c|c|}\hline 3 & 1 \\ \hline 2 & 0 \\ \hline \end{array} \ast \begin{array}{|c|c|}\hline w_{11} & w_{12} \\ \hline w_{21} & w_{22} \\ \hline \end{array}$$

Example

$$\begin{array}{|c|c|}\hline 3 & 1 \\ \hline 2 & 0 \\ \hline \end{array} \ast \begin{array}{|c|c|}\hline 2 & 3 \\ \hline 0 & 1 \\ \hline \end{array} =$$

$$\begin{array}{|c|c|c|}\hline 6 & 3 & \\ \hline 0 & 3 & \\ \hline & & \\ \hline \end{array} + \begin{array}{|c|c|c|}\hline & 2 & 3 \\ \hline & 0 & 1 \\ \hline & & \\ \hline \end{array} + \begin{array}{|c|c|c|}\hline & & \\ \hline 4 & 6 & \\ \hline 0 & 1 & \\ \hline \end{array} + \begin{array}{|c|c|c|}\hline & & \\ \hline & 0 & 0 \\ \hline & 0 & 0 \\ \hline \end{array} =$$

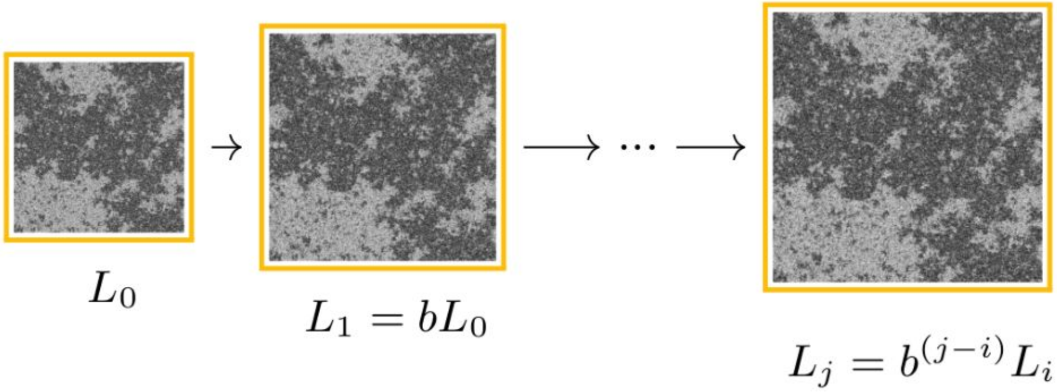
$$\begin{array}{|c|c|c|}\hline 6 & 5 & 3 \\ \hline 4 & 9 & 0 \\ \hline 0 & 1 & 0 \\ \hline \end{array}$$

Inverse Renormalization Group in Quantum Field Theory, D. Bachtis, G. Aarts, F. Di Renzo, B. Lucini, Phys. Rev. Lett. 128, 081603 (2022)

Inverse renormalization group

The benefit:

We can apply this set of inverse transformations iteratively to arbitrarily increase the size of the system, evading the critical slowing down effect.



Inverse renormalization group

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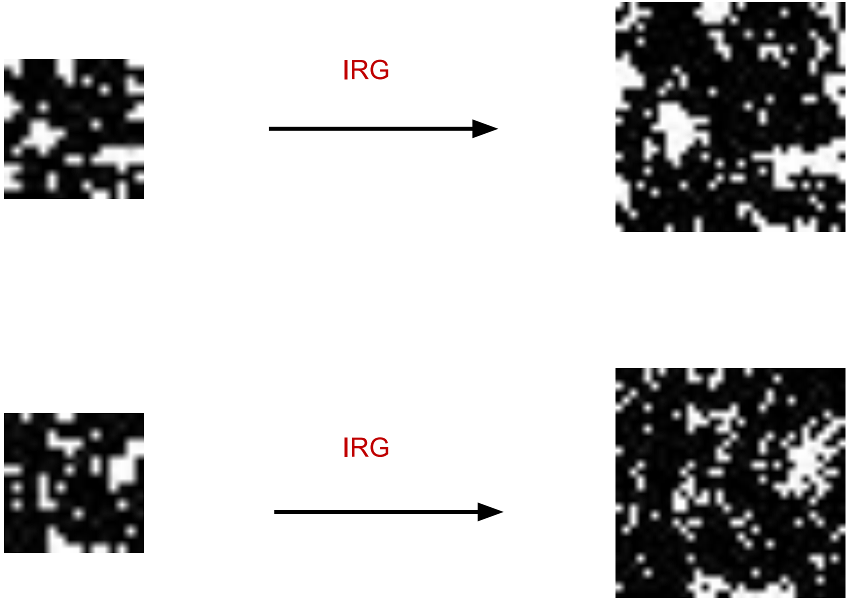
Inverse renormalization group



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Inverse renormalization group

The set of transformations can be applied iteratively to arbitrarily increase the lattice size:

$$L_j = b^{(j-i)} L_i \qquad j > i \geq 0, \text{ and } L_0 = L$$

However the increase in the lattice size will induce an analogous increase in the correlation length of the system:

$$\xi_j = b^{(j-i)} \xi_i$$

What are the implications?

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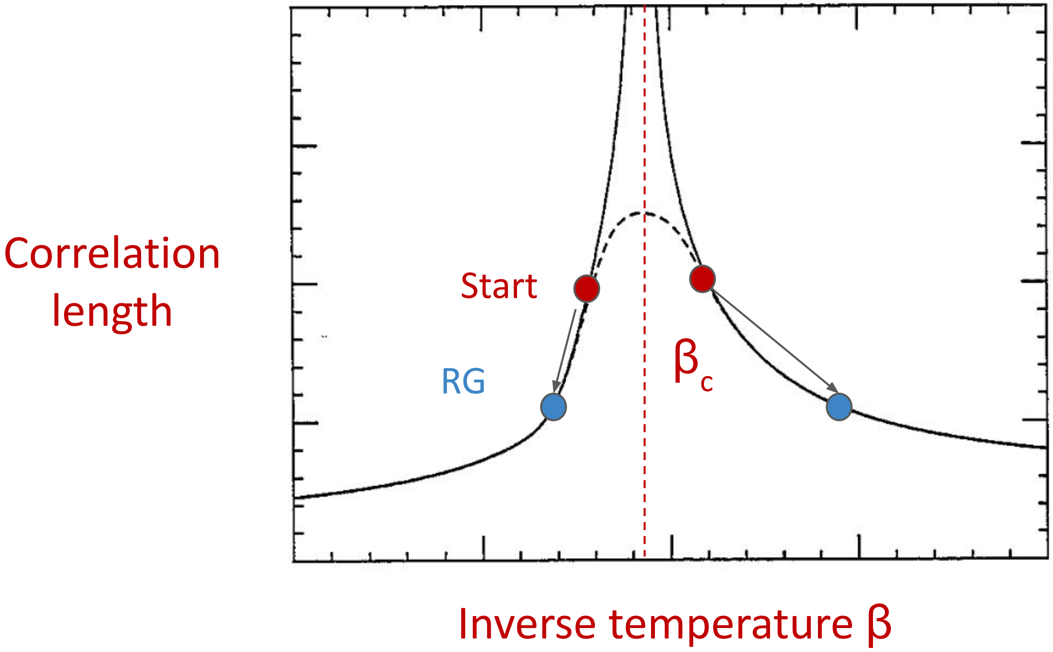
Standard renormalization group



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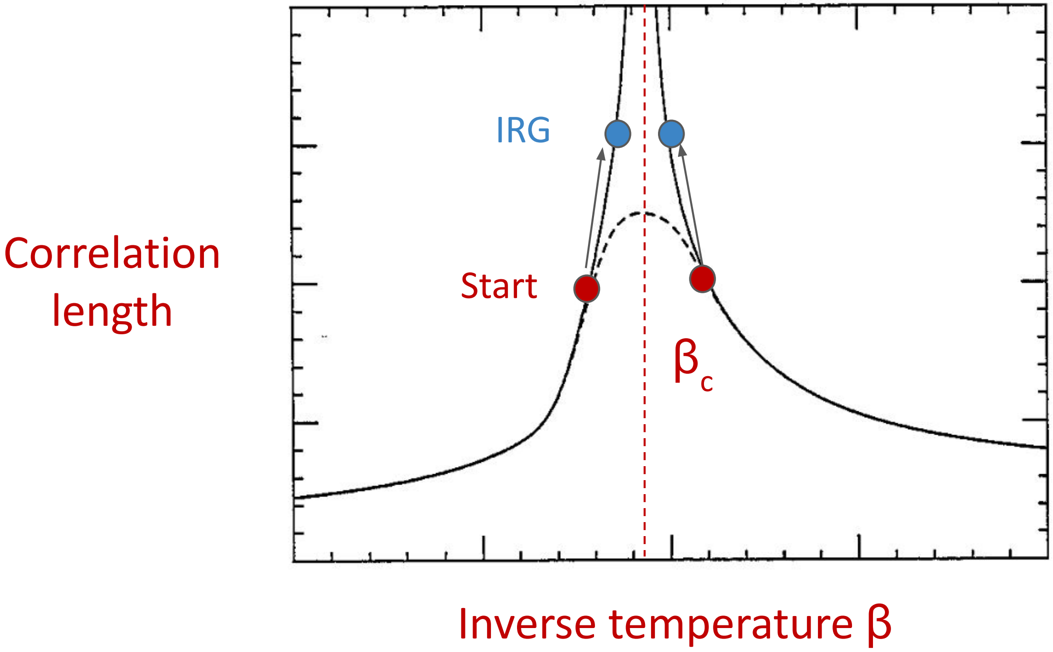
Inverse renormalization group



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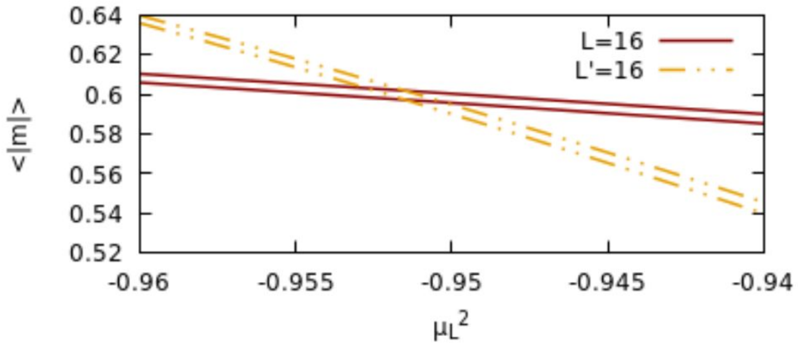
Inverse renormalization group

At the critical point the correlation length diverges, it becomes infinite, and intensive observable quantities O,O’ of original and renormalized systems become equal.

$$O'(K_c)=O(K_c)$$

Inverse renormalization group

First, we verify that the **standard MC** renormalization group method works in the φ^4 theory:

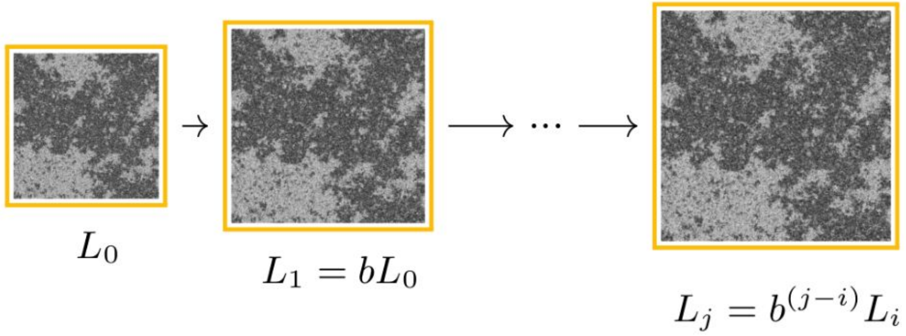


Then we invert the standard transformation that we verified as being successful.

Inverse Renormalization Group in Quantum Field Theory, D. Bachtis, G. Aarts, F. Di Renzo, B. Lucini, Phys. Rev. Lett. 128, 081603 (2022)

Inverse renormalization group

Now, we start from a lattice size L_0=32 in each dimension and apply the inverse transformations to obtain systems of lattice sizes L_1=64, L_2=128, L_3=256, L_4=512.



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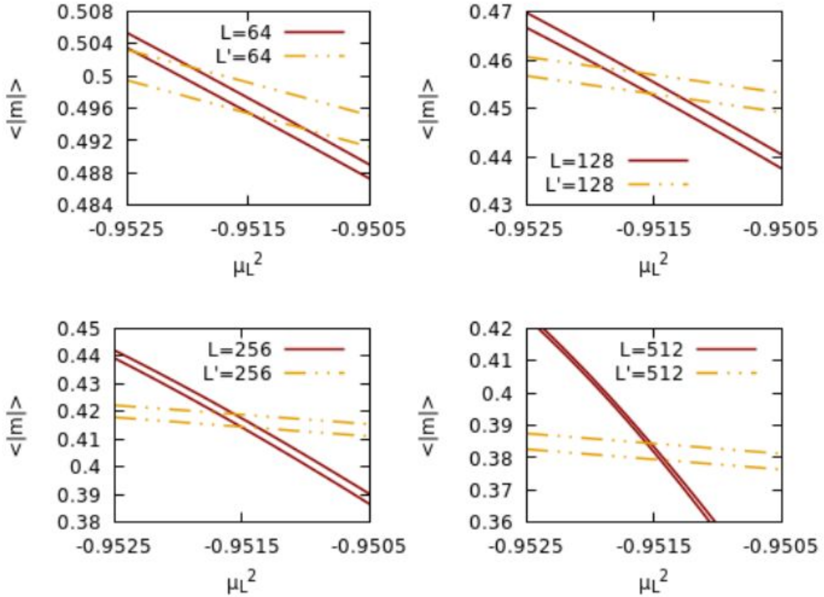

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Inverse renormalization group

Can we now use the inverse renormalization group approach to calculate critical exponents?

The relations that govern the divergence of the magnetization for an original (i) and a rescaled (j) system are

$$m_i \sim |t_i|^{\beta} \qquad\qquad m_j \sim |t_j|^{\beta}$$

They can be equivalently expressed in terms of the correlation length as

$$m_i \sim \xi_i^{-\beta/\nu} \qquad\qquad m_j \sim \xi_j^{-\beta/\nu}$$

where v is the correlation length exponent

Inverse Renormalization Group in Quantum Field Theory, D. Bachtis, G. Aarts, F. Di Renzo, B. Lucini, Phys. Rev. Lett. 128, 081603 (2022)

Inverse renormalization group

By dividing the magnetizations (or magnetic susceptibilities), taking the natural logarithm, and applying L'Hôpital's rule, we obtain

$$\frac{\beta}{\nu} = -\frac{\ln \frac{dm_j}{dm_i}\Big|_{K_c}}{\ln \frac{\xi_j}{\xi_i}} = -\frac{\ln \frac{dm_j}{dm_i}\Big|_{K_c}}{(j-i)\ln b}. \qquad \frac{\gamma}{\nu} = \frac{\ln \frac{d\chi_j}{d\chi_i}\Big|_{K_c}}{\ln \frac{\xi_j}{\xi_i}} = \frac{\ln \frac{d\chi_j}{d\chi_i}\Big|_{K_c}}{(j-i)\ln b}.$$

We can use the expressions above to calculate the critical exponents without ever experiencing a critical slowing down effect.

Inverse Renormalization Group in Quantum Field Theory, D. Bachtis, G. Aarts, F. Di Renzo, B. Lucini, Phys. Rev. Lett. 128, 081603 (2022)

Inverse renormalization group

TABLE I. Values of the critical exponents γ/ν and β/ν. The original system has lattice size $L = 32$ in each dimension and its action has coupling constants $\mu_L^2 = -0.9515$, $\lambda_L = 0.7$, $\kappa_L = 1$. The rescaled systems are obtained through inverse renormalization group transformations.

L_i/L_j	32/64	32/128	32/256	32/512	64/128	64/256	64/512	128/256	128/512	256/512
γ/ν	1.735(5)	1.738(5)	1.741(5)	1.742(5)	1.742(5)	1.744(5)	1.744(5)	1.745(5)	1.745(5)	1.746(5)
β/ν	0.132(2)	0.130(2)	0.128(2)	0.128(2)	0.128(2)	0.127(2)	0.127(2)	0.126(2)	0.126(2)	0.126(2)

TABLE II. Values of the critical exponents γ/ν and β/ν. The original system has lattice size $L = 8$ in each dimension and its action has coupling constants $\mu_L^2 = -1.2723$, $\lambda_L = 1$, $\kappa_L = 1$. The rescaled systems are obtained through inverse renormalization group transformations.

L_i/L_j	8/16	8/32	8/64	8/128	8/256	8/512	16/32	16/64	16/128	16/256	16/512
γ/ν	1.694(6)	1.708(6)	1.717(6)	1.723(6)	1.727(6)	1.730(6)	1.721(6)	1.728(6)	1.732(6)	1.735(6)	1.737(6)
β/ν	0.154(2)	0.147(2)	0.142(2)	0.139(2)	0.137(2)	0.135(2)	0.140(2)	0.136(2)	0.134(2)	0.132(2)	0.131(2)

L_i/L_j	32/64	32/128	32/256	32/512	64/128	64/256	64/512	128/256	128/512	256/512
γ/ν	1.735(6)	1.738(6)	1.740(6)	1.740(6)	1.741(6)	1.742(6)	1.742(7)	1.743(6)	1.743(7)	1.743(7)
β/ν	0.133(2)	0.131(2)	0.130(2)	0.129(2)	0.129(2)	0.129(2)	0.128(2)	0.128(2)	0.127(2)	0.127(2)

Ising universality class: $\gamma/v=1.75$, $\beta/v=0.125$.

Inverse Renormalization Group in Quantum Field Theory, D. Bachtis, G. Aarts, F. Di Renzo, B. Lucini, Phys. Rev. Lett. 128, 081603 (2022)

Inverse renormalization group

Could we ever use the inverse renormalization group to surpass supercomputers?

Inverse renormalization group

3D Edwards Anderson model

Paramagnetic phase
β->0

Spin Glass phase
β->∞

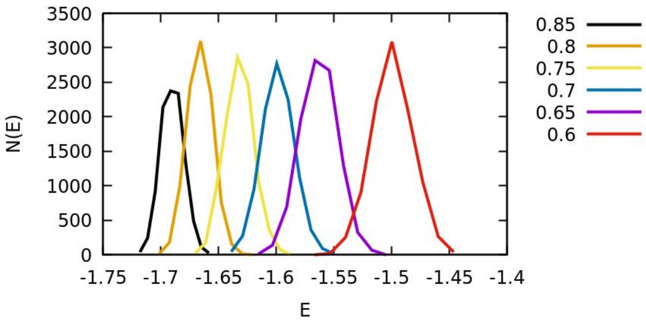
System freezes to a configuration

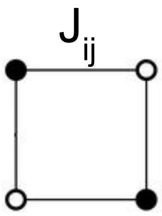
Inverse renormalization group

3D Edwards Anderson model

The problem is computationally hard:

i) Metropolis algorithm does not even thermalize in sufficient time as we approach the spin glass phase.
ii) Cluster algorithms are not efficient in 3D. We need to resort to parallel tempering/replica exchange techniques in >=3D.
iii) System has probabilistic coupling constants, and we need to simulate a large number of realizations of disorder to calculate expectation values.





Janus Collaboration:

Has access to special-purpose supercomputers which are constructed exclusively to study spin glasses.

Has thermalized lattices up to $L=40^3$

Critical parameters of the three-dimensional Ising spin glass

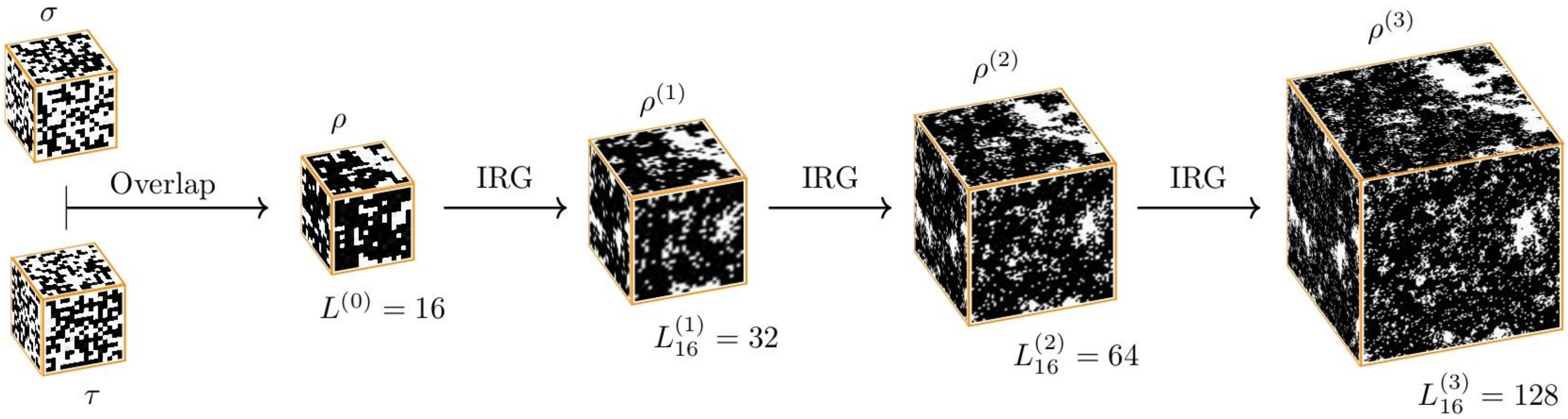
Janus Collaboration: M. Baity-Jesi, R. A. Baños, A. Cruz, L. A. Fernandez, J. M. Gil-Narvion, A. Gordillo-Guerrero, D. Iñiguez, A. Maiorano, F. Mantovani, E. Marinari, V. Martin-Mayor, J. Monforte-Garcia, A. Muñoz Sudupe, D. Navarro, G. Parisi, S. Perez-Gaviro, M. Pivanti, F. Ricci-Tersenghi, J. J. Ruiz-Lorenzo, S. F. Schifano, B. Seoane, A. Tarancon, R. Tripiccione, D. Yllanes

We report a high-precision finite-size scaling study of the critical behavior of the three-dimensional Ising Edwards-Anderson model (the Ising spin glass). We have thermalized lattices up to L=40 using the Janus dedicated computer. Our analysis takes into account leading-order corrections to scaling. We obtain Tc = 1.1019(29) for the critical temperature, \nu = 2.562(42) for the thermal exponent, \eta = -0.3900(36) for the anomalous dimension and \omega = 1.12(10) for the exponent of the leading corrections to scaling. Standard (hyper)scaling relations yield \alpha = -5.69(13), \beta = 0.782(10) and \gamma = 6.13(11). We also compute several universal quantities at Tc.

Inverse renormalization group

Inverse renormalization group:

We have constructed configurations for lattices up to L=128^3



Inverse renormalization group

We have answered these questions:

- ➔ How to generate configurations of systems with larger lattice size without critical slowing down effect.
- ➔ How inverse renormalization group flows emerge.
- ➔ How to calculate multiple critical exponents with the inverse renormalization group.

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 813942

- ➔ We have discussed applications of the inverse renormalization group to computationally hard problems.