PRECISION DRELL-YAN PHENOMENOLOGY AT N3LO QCD

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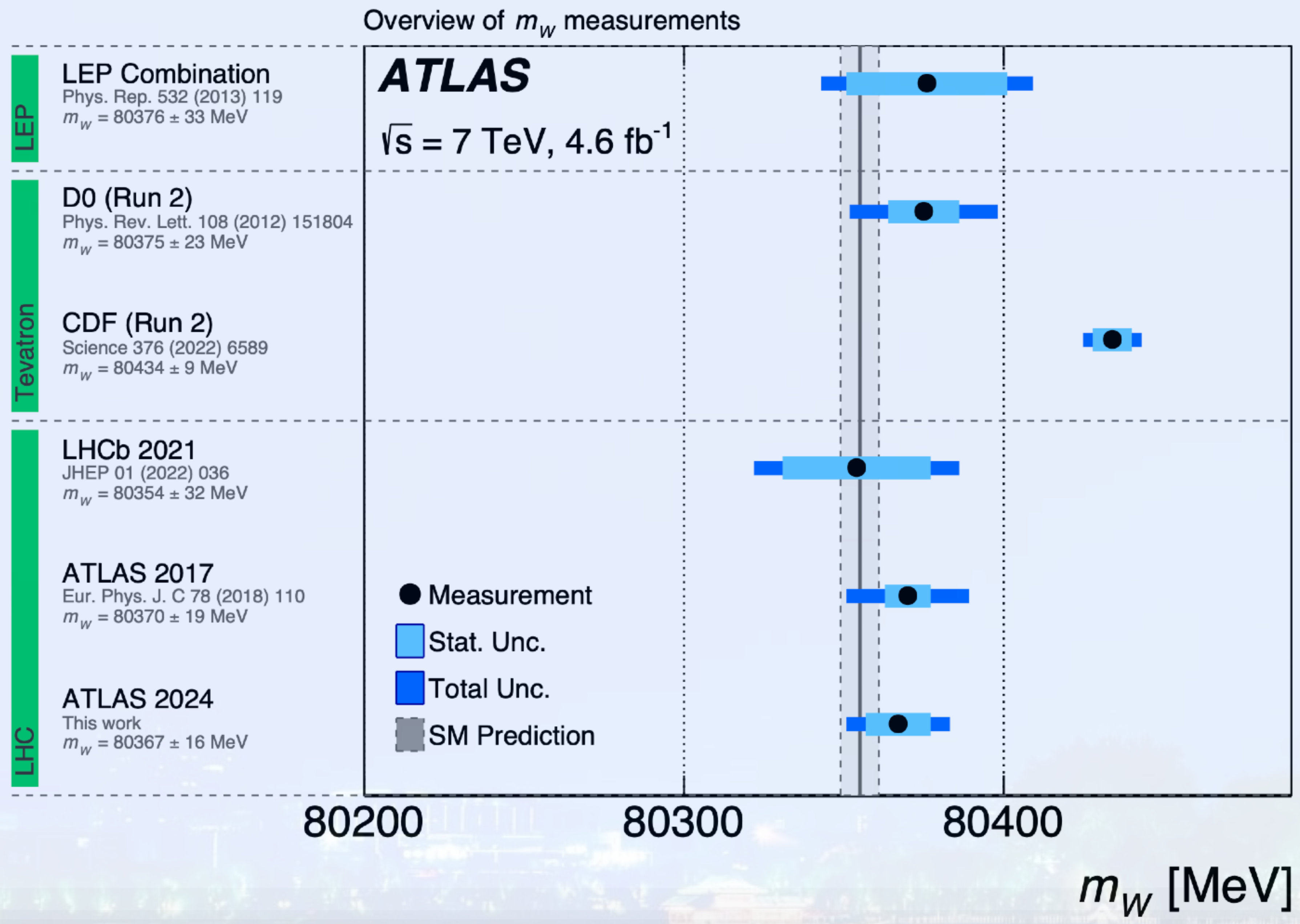
Loops and Legs in Quantum Field Theory

Wittenberg, 16 April, 2024

SHANDONG UNIVERSITY 1901

W MASS MEASUREMENTS



(MeV)	mW	Total Unc.	Stat. Unc.	Exp. Unc.	Th. Unc.
SM	80355	6	–	–	6
CDFII	80433.5	9.4	6.4	5.3	5.2
ATLAS	80366.5	15.9	9.8	8.7	9.0
LHCb	80354	32	23	10	19

- Indirect measurement of m_T^W, p_T^l, p_T^ν

$$p_T^{l(\nu)} = \sqrt{(p_x^{l(\nu)})^2 + (p_y^{l(\nu)})^2}$$

$$E_T^{l(\nu)} = \sqrt{m^2 + (p_x^{l(\nu)})^2 + (p_y^{l(\nu)})^2} \approx p_T^{l(\nu)}$$

$$m_T^W = \sqrt{2E_T^l E_T^\nu(1 - \cos\Delta\phi)}$$

- Full error = Experiment + Theory model

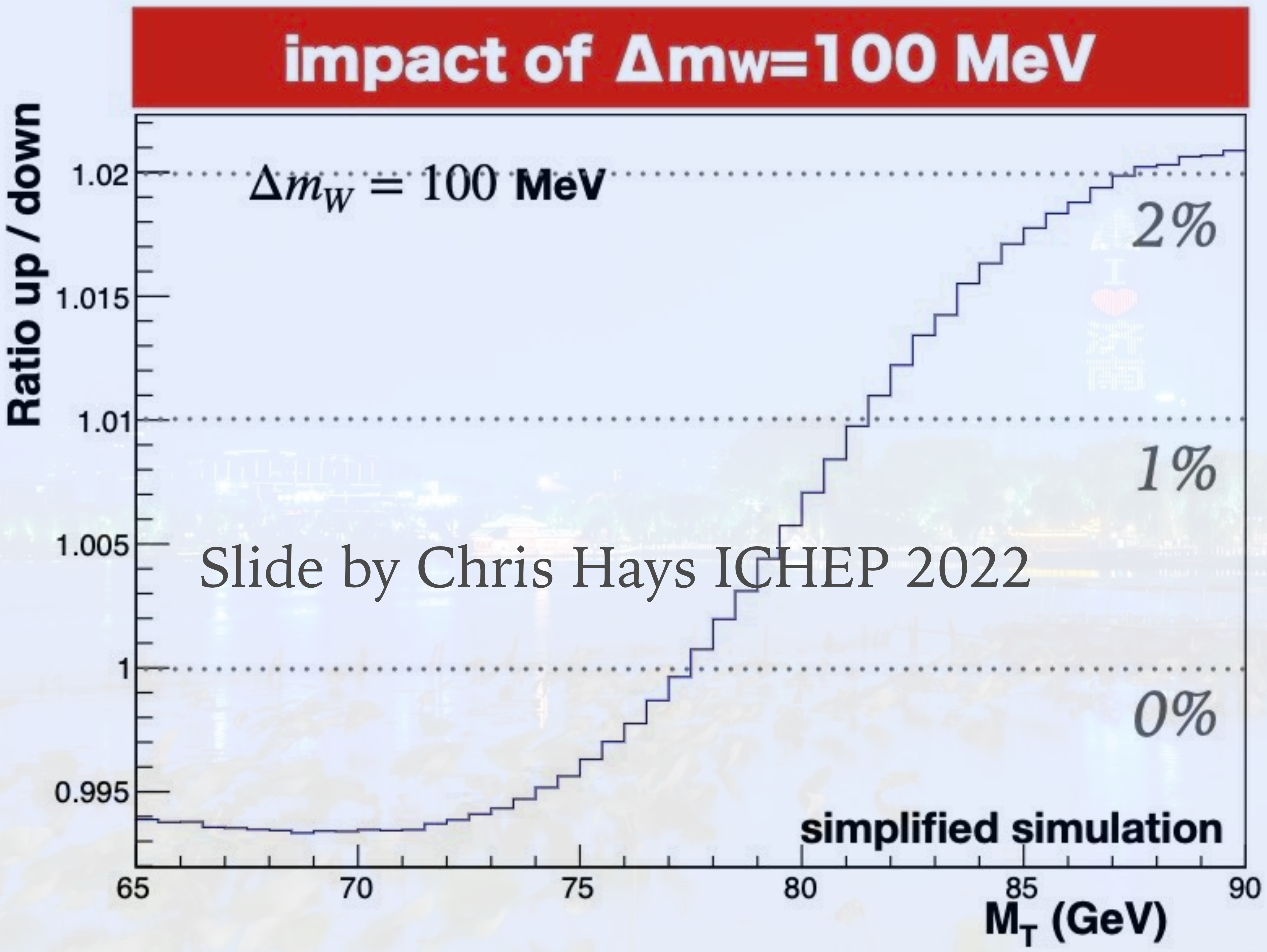
- Likelihood fit of m_W, Γ_W, nuisance parameters, templates
- Robust theory predictions for EW, QCD, PDF, PS etc.
- Parameter tune based on p_T^Z, A_i, none perturbative parameters

Table 2. Uncertainties on the combined M_W result. CDFII uncertainties

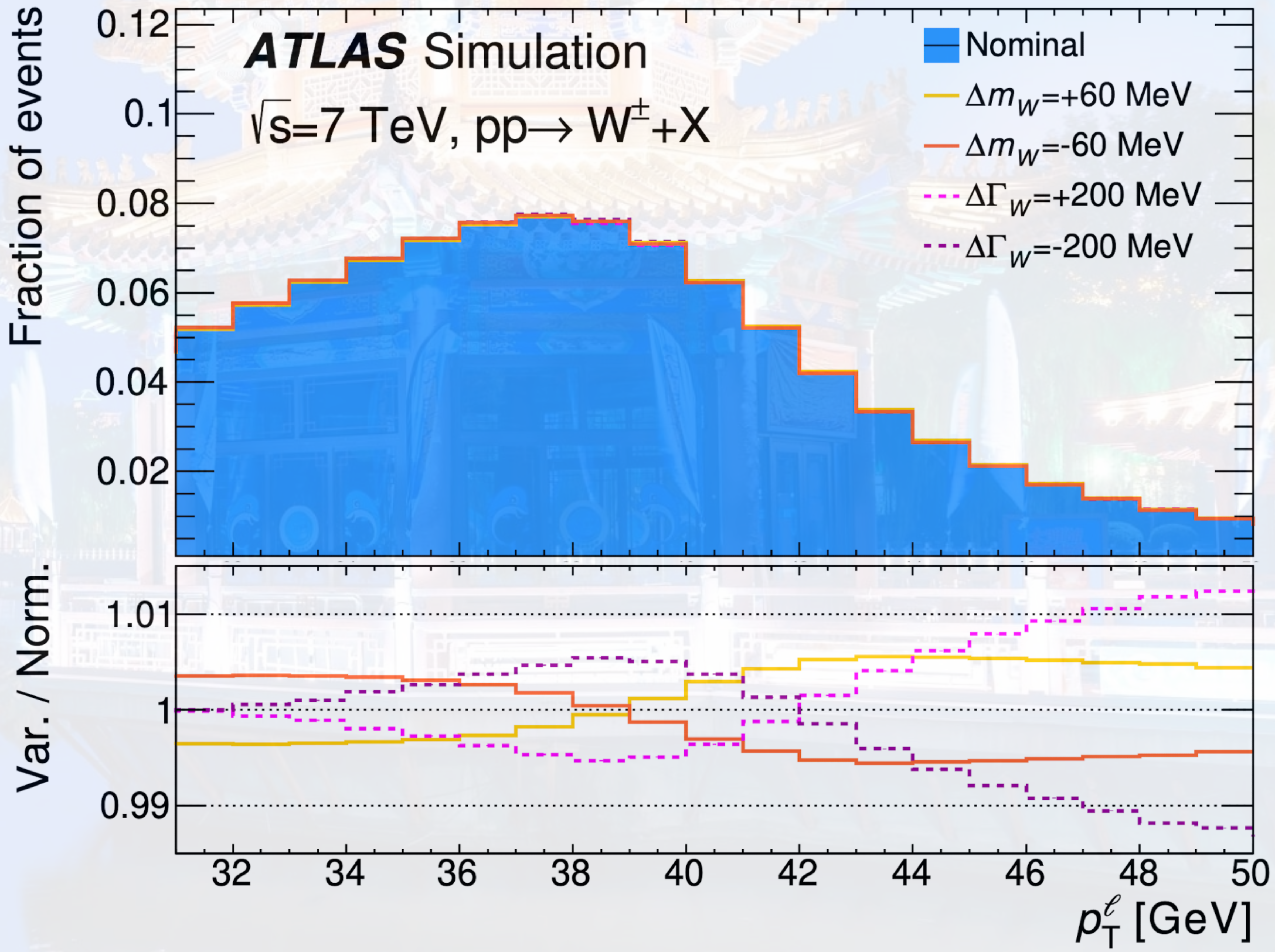
Source	Uncertainty (MeV)
Lepton energy scale	3.0
Lepton energy resolution	1.2
Recoil energy scale	1.2
Recoil energy resolution	1.8
Lepton efficiency	0.4
Lepton removal	1.2
Backgrounds	3.3
p_T^Z model	1.8
p_T^W/p_T^Z model	1.3
Parton distributions	3.9
QED radiation	2.7
W boson statistics	6.4
Total	9.4

W MASS MEASUREMENTS

- $d\sigma/dm_T^W$ templates with $\Delta m_W = 100$ MeV



- $d\sigma/dp_T^l$ templates with $\Delta m_W = 60$ MeV



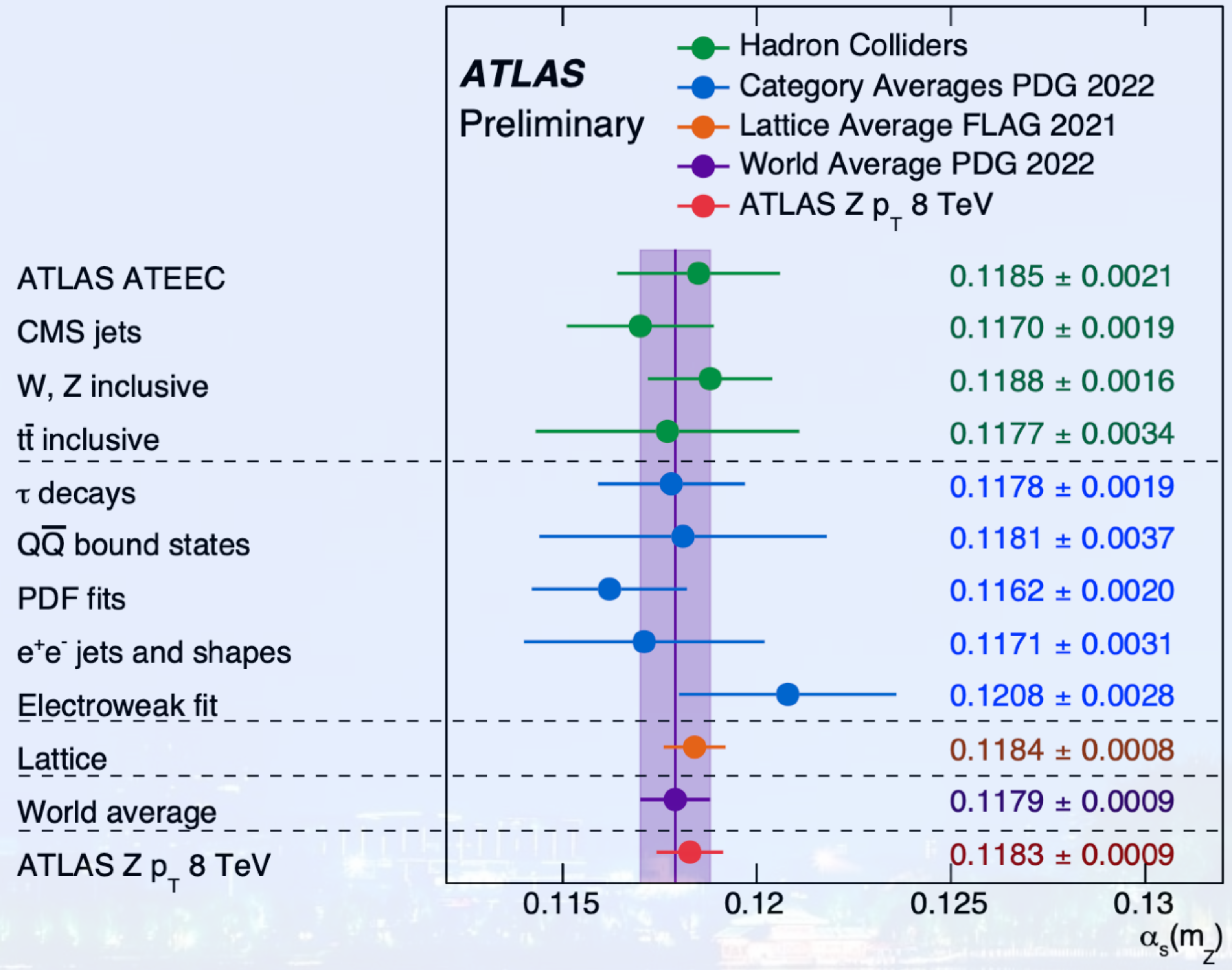
$\Delta m_W =$ 100 MeV ~ 0.5-2% change in $d\sigma/dm_T^W$

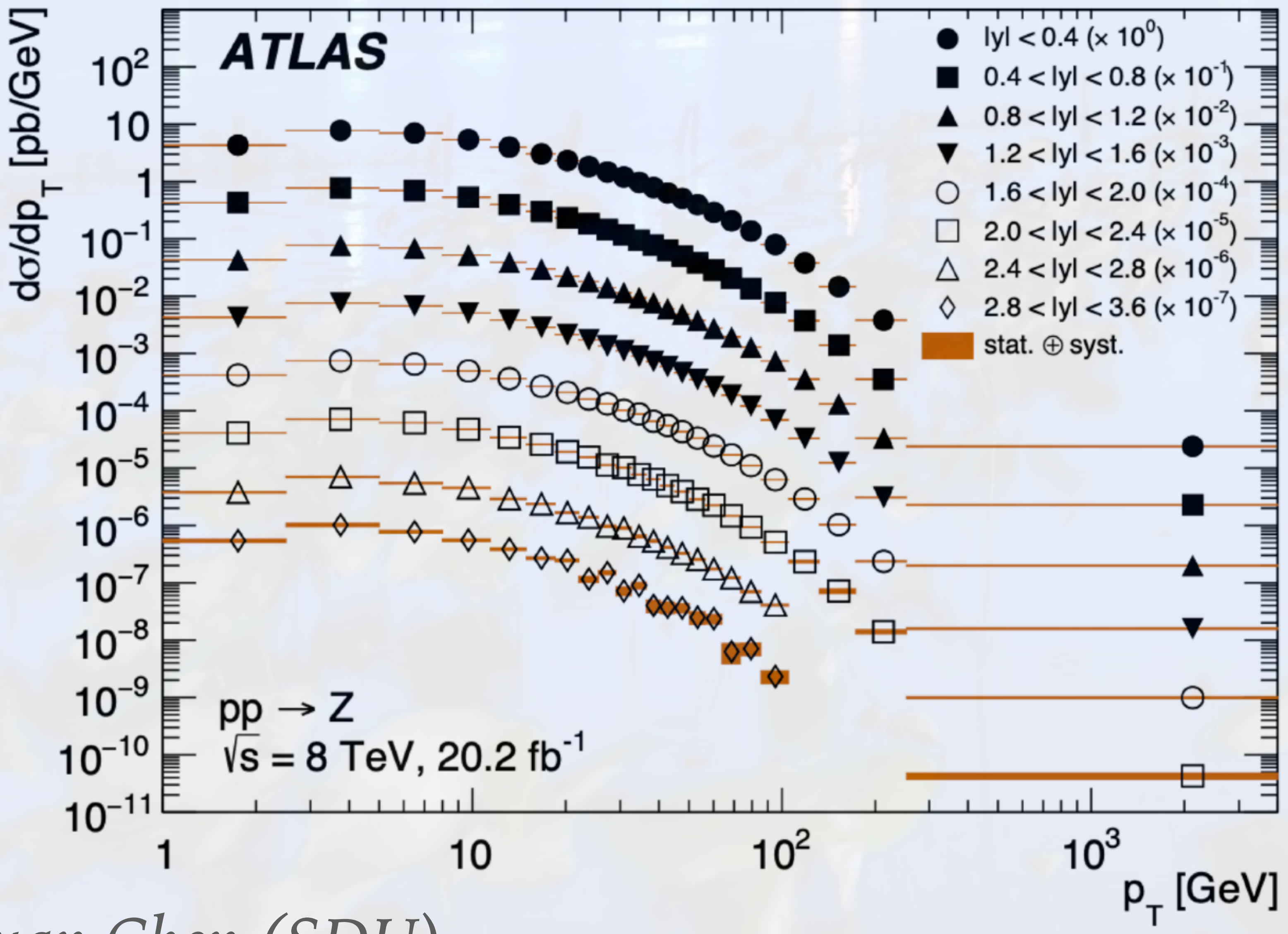
→ $\Delta m_W =$ 10 MeV ~ 0.1% precision in $d\sigma/dm_T^W$

$\Delta m_W =$ 60 MeV ~ 0.5% change in $d\sigma/dp_T^l$

$\Delta \Gamma_W =$ 200 MeV ~ 0.5-1% change in $d\sigma/dp_T^l$

α_s MEASUREMENT BY ATLAS





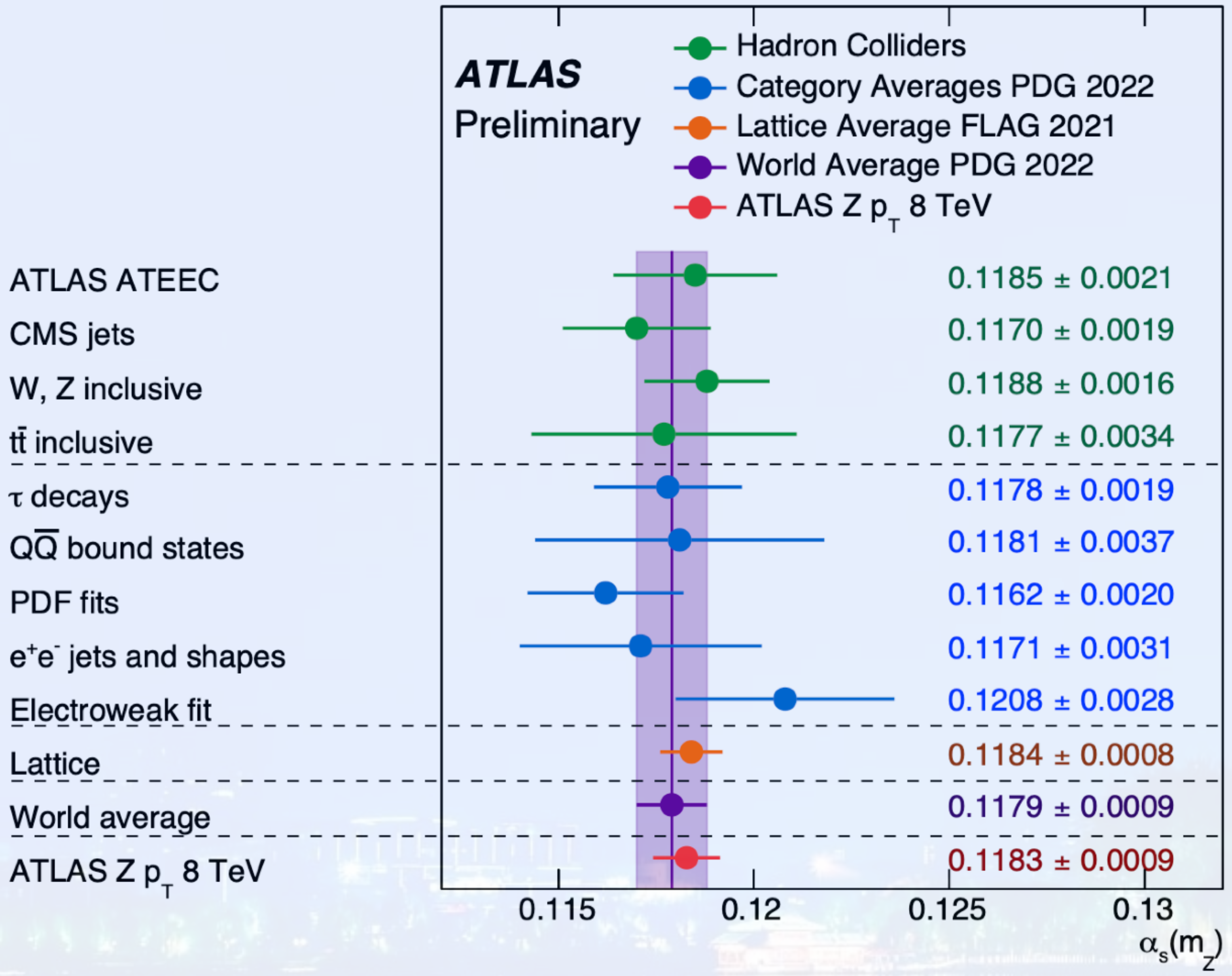
- World average: $\alpha_s(m_Z) = 0.1179 \pm 0.0009$
- ATLAS p_T^Z @ 8 TeV: $\alpha_s(m_Z) = 0.1183 \pm 0.0009$

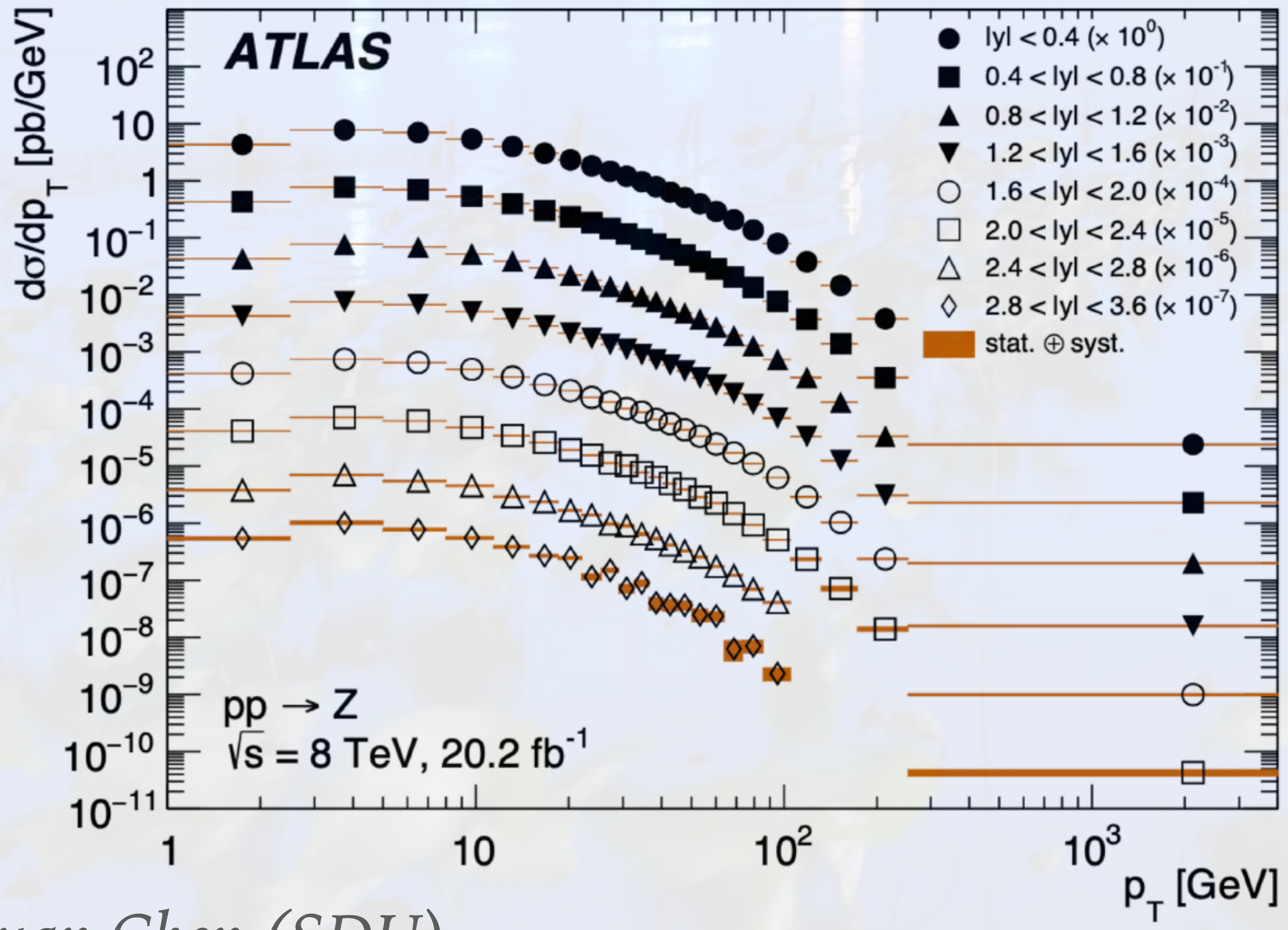
ATLAS 2309.12986

See also ATLAS JHEP 07 (2023) 085

ATLAS 2309.09318

α_s MEASUREMENT BY ATLAS





- World average: $\alpha_s(m_Z) = 0.1179 \pm 0.0009$ *ATLAS 2309.12986*
- ATLAS p_T^Z @ 8 TeV: $\alpha_s(m_Z) = 0.1183 \pm 0.0009$ *See also ATLAS JHEP 07 (2023) 085*
  - Indirect measurement of $d\sigma/dp_T^Z/dy^Z$ distributions *ATLAS 2309.09318*
    - $80 < m_{ee(\mu\mu)} < 100$ GeV
    - $p_T^Z < 29$ GeV in 8 slices of $|y^Z| < 3.6$
    - $|\eta_{e_1}| < 2.4$, $2.5 < |\eta_{e_2}| < 4.9$ with $p_T^{e_1(e_2)} > 25\ (20)$ GeV
    - $|\eta_{e(\mu)}| < 2.4$ with $p_T^{e(\mu)} > 20$ GeV
  - DYTurbo with xFitter to find the best α_s that describe the data
  - Experiment unc. : ± 0.00044
  - Theory model unc. : $^{+0.00072}_{-0.00076}$ $\pm$??

8 TeV
20.2 fb^{-1}

Error budget of $\alpha_s(m_Z)$

Experimental uncertainty	+0.00044	-0.00044
PDF uncertainty	+0.00051	-0.00051
Scale variations uncertainties	+0.00042	-0.00042
Matching to fixed order	0	-0.00008
Non-perturbative model	+0.00012	-0.00020
Flavour model	+0.00021	-0.00029
QED ISR	+0.00014	-0.00014
N4LL approximation	+0.00004	-0.00004
Total	+0.00084	-0.00088

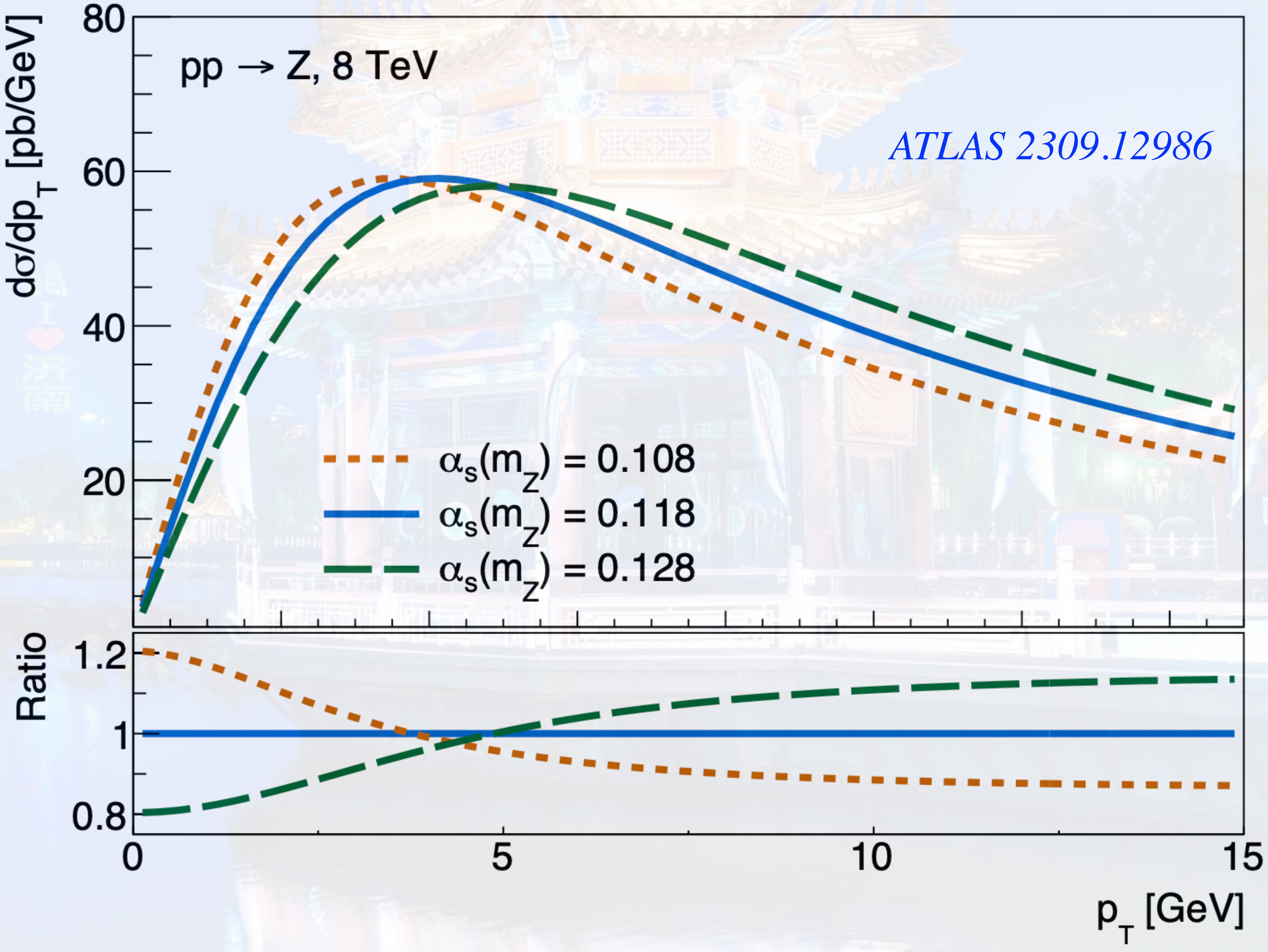
α_s MEASUREMENT BY ATLAS

- χ^2 fit in xFitter framework:

Eur. Phys. J. C **75** *(2015) 304*

$$\chi^2(\beta_{\text{exp}}, \beta_{\text{th}}) = \sum_{i=1}^{N_{\text{data}}} \frac{\left(\sigma_i^{\text{exp}} + \sum_j \Gamma_{ij}^{\text{exp}} \beta_{j,\text{exp}} - \sigma_i^{\text{th}} - \sum_k \Gamma_{ik}^{\text{th}} \beta_{k,\text{th}}\right)^2}{\Delta_i^2} + \sum_j \beta_{j,\text{exp}}^2 + \sum_k \beta_{k,\text{th}}^2 .$$

- Δ_i experimental uncertainties
- $\beta_{exp\ (th)}$ nuisance parameters
- Γ_{ik}^{th} covariant matrix covers:
  - PDF Hessian uncertainties
  - Non-perturbative form factor



$\Delta\alpha_s =$ 0.01~ 10-20% change in $d\sigma/dp_T^Z$ → $\Delta\alpha_s =$ 0.001 ~ 1-2% precision in $d\sigma/dp_T^Z$

Precision Theory Tools Inside Measurements

PRECISION PREDICTIONS IN m_W MEASUREMENT

➤CDF II use ResBos to generate theory templates

➤NLO+NNLL QCD accuracy for W/Z production

Balazs, Brock, Landry, Nadolsky and Yuan`97 to`03

➤CSS factorisation and resummation of p_T in b space:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2\,\mathrm{d}^2\vec{p}_T\,\mathrm{d}y\,\mathrm{d}\cos\theta\,\mathrm{d}\phi} = \sigma_0 \int \frac{\mathrm{d}^2 b}{(2\pi)^2} e^{i\vec{p}_T\cdot\vec{b}} e^{-S(b)}$$
$$\times\, C\otimes f(x_1,\mu)\, C\otimes f(x_2,\mu) + Y(Q,\vec{p}_T,x_1,x_2,\mu_R,\mu_F)$$

Collins, Soper and Sterman`85

➤Non-perturbative effects at $\alpha_s(\Lambda)$ and large b:

$$S(b) = S_{\mathrm{NP}} S_{\mathrm{Pert}}\,,$$

Collins and Soper `77

$$S_{\mathrm{Pert}}(b) = \int_{C_1^2/(b^*)^2}^{C_2^2Q^2} \frac{\mathrm{d}\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{C_2^2Q^2}{\bar{\mu}^2}\right) A\,(\bar{\mu},C_1) + B\,(\bar{\mu},C_1,C_2)\right]$$

$$S_{\mathrm{NP}} = \left[-g_1 - g_2 \ln\left(\frac{Q}{2Q_0}\right) - g_1 g_3 \ln\left(100 x_1 x_2\right)\right] b^2$$

S_{NP} assumes the BLNY functional form

Brock, Landry, Nadolsky and Yuan `02

➤LO+LL lepton EM radiation with PHOTOS and HORACE Golonka and Was `06

Carloni Calame, Montagna et. al. `07

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  Carloni Calame, Montagna et. al. `07
- ➤Use data driven method:

Fix	g1	g2	g3	α_s
p_T^Z	Global fit `03	CDFII fit	Global fit `03	CDFII fit
p_T^Z/p_T^W			Global fit `03	

Global fit by Brock, Landry, Nadolsky and Yuan `03

$m_T^W \sim 0.7$ MeV, $p_T^l \sim 2.3$ MeV, $p_T^\nu \sim 0.9$ MeV

CDF supplementary materials `22

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p_T^Z/p_T^W			Global fit `03	

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$m_T^W \sim 0.7$ MeV, $p_T^l \sim 2.3$ MeV, $p_T^\nu \sim 0.9$ MeV

CDF supplementary materials `22

➤Scale uncertainty of p_T^Z/p_T^W by DYQT

Bozzi, Catani, Ferrera, de Florian, Grazzini `09 `11

$m_T^W \sim 3.5$ MeV, $p_T^l \sim 10.1$ MeV, $p_T^\nu \sim 3.9$ MeV

Not included in final result CDF sm`22

PRECISION PREDICTIONS IN m_W MEASUREMENT

- ResBos → ResBos2
  - NNLO+N3LL accuracy for W/Z production

    Isaacson, Fu, Yuan `23
  - Upgrade CSS formalism to N3LL
  - Rescale NLO to NNLO from MCFM:

    Campbell, Ellis and Giele `15

$$\frac{d\sigma^{A_i}_{NLO}}{dp_T dy dQ} \to K^{A_i}_{\frac{NNLO}{NLO}}(p_T, y, Q)\frac{d\sigma^{A_i}_{NLO}}{dp_T dy dQ}$$

  - Dependence of angular coefficients recently included with more rescaling: $\frac{d\sigma}{d\cos\theta d\phi}$

$$= L_0(1+\cos^2\theta) + A_0(1-3\cos^2\theta) + A_1\sin 2\theta\cos\phi$$
$$+A_2\sin^2\theta\cos 2\phi + A_3\sin\theta\cos\phi + A_4\cos\theta$$
$$+A_5\sin^2\theta\sin 2\phi + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi$$

Isaacson, Fu and Yuan`22 `23

- A_i at each fixed order:
  - LO: L_0, A_4
  - NLO: L_0, $A_0 = A_2$, A_1, A_3, A_4
  - NNLO: L_0, $A_0 \neq A_2$, A_1, A_3, A_4, A_5, A_6, A_7
- Resummation choices for only L_0, A_4 or all A_i
- The AZ-tune is also used in ATLAS analysis:
  - PYTHIA 8 + PS for modelling
  - Tune $d\sigma^{A_i}_{NNLO}/dp_T/dy/dQ$ to best fit p_T^Z
  - Test the tuned model on p_T^W then apply to p_T^l, m_T

PRECISION PREDICTIONS IN m_W MEASUREMENT

- ResBos → ResBos2
  - NNLO+N3LL accuracy for W/Z production
    Isaacson, Fu, Yuan `23
  - Upgrade CSS formalism to N3LL
  - Rescale NLO to NNLO from MCFM:
    Campbell, Ellis and Giele `15

$$\frac{d\sigma_{NLO}^{A_i}}{dp_T dy dQ} \to K_{\frac{NNLO}{NLO}}^{A_i}(p_T, y, Q)\frac{d\sigma_{NLO}^{A_i}}{dp_T dy dQ}$$

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Theoretical systematics in LHC precision measurements

PRECISION PREDICTIONS IN ATLAS α_s DETERMINATION

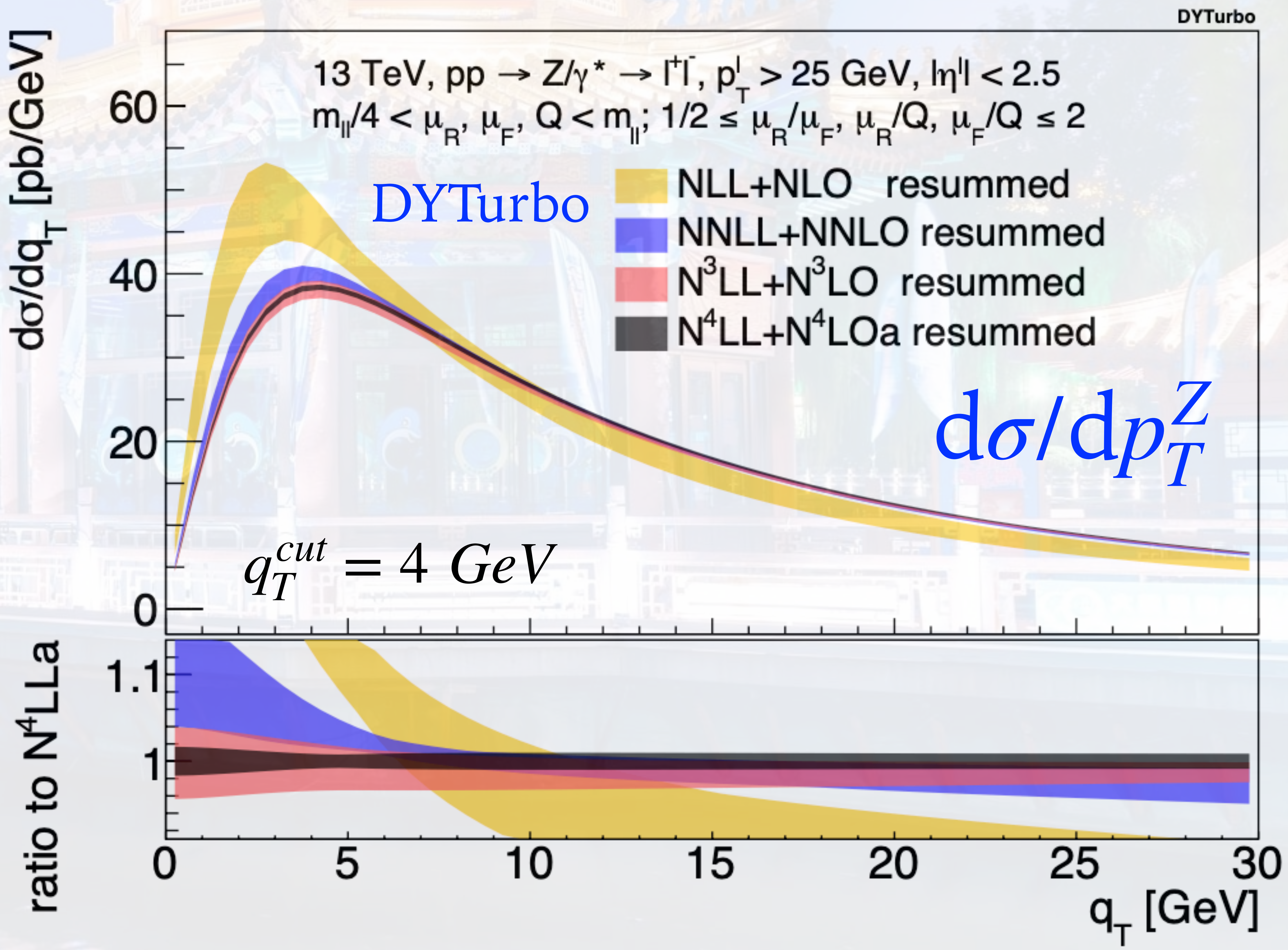
- ATLAS use DYTurbo as theory input
  Camarda, Boonekamp et. al. \`20
  - aN4LO + aN4LL accuracy for DY production
    Camarda, Cieri, Ferrera \`23
  - FO: NNLO qT slicing from DYqT + $\mathcal{O}(\alpha_S^3)$ for $\delta(qT)$ + MCFM @ $\mathcal{O}(\alpha_S^3)$ for qT > 5 GeV. Neumann, Campbell \`22
  - CSS resummation of p_T in b space:
    - Expansion up to $\mathcal{O}(\alpha_S^4)$ for small qT (approx.)
    - Exact B4 coefficient with all other N4LL components approx. (A5, H4, DGLAP etc.) Moult, Zhu, Zhu \`22
  - aN3LO PDF MSHT20: approx. in DGLAP, TH input
    McGowan, Cridge, Larland-Lang, Thorne \`22
  - Non-perturbative effects at $\alpha_s(\Lambda)$ and large b:

$$S_{NP}(b) = \exp\{-g_1 b^2 - g_K(b)\,\ln(M^2/Q_0^2)\}$$

$$g_K(b) = g_0\left(1 - \exp\left[-\frac{C_F\alpha_S((b_0/b_\star)^2)b^2}{\pi g_0 b_{\rm lim}^2}\right]\right)$$

Collins, Rogers \`14

- LL ISR photons radiation + normalisation to NLO QED and virtual EW cor. in ReneSANCe
  Bondarenko, Dydyshka et. al. \`22



S. Camarda, L. Cieri, G. Ferrera Phys. Lett. B 845 (2023)

- See also DYTurbo in α_s fitting with CDF data
  Camarda , Ferrera, Schott \`23

RESUMMATION FRAMEWORKS (QT FACTORISATION)

- Resummation kernels: $d\sigma = \sigma_{LO} \otimes H \otimes B \otimes B \otimes S$
  - In SCET: $$\frac{d\sigma}{dp_T^2} = \pi\sigma_{LO}^Z \int dx_a dx_b \delta\left(x_a x_b - \frac{m_{ll}^2}{E_{CM}^2}\right) \int \frac{d^2\vec{b}}{(2\pi)^2} e^{i\vec{p}_T\cdot\vec{b}} W(x_a, x_b, m_{ll}, \vec{b}),$$

$$W(x_a, x_b, m_{ll}, \vec{b}) = H(m_{ll}, \mu_h) U_h(m_{ll}, \mu_B, \mu_h) S_\perp(\vec{b}, \mu_s, \nu_s) U_s(b, \mu_B, \mu_s; \nu_B, \nu_s) \prod_{\gamma=a,b} B_{g/N_\gamma}^{\alpha\beta}(x_\gamma, \vec{b}, m_{ll}, \mu_B, \nu_B),$$

$$U_s(b, \mu, \mu_s; \nu, \nu_s) = \exp\left[2\int_{\mu_s}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \left(\Gamma_{\text{cusp}}(\alpha_s(\bar{\mu})) \ln\frac{b^2\bar{\mu}^2}{b_0^2} - \gamma_s(\alpha_s(\bar{\mu}))\right)\right] \left(\frac{\nu^2}{\nu_s^2}\right)^{\int_{\mu}^{b_0/b} \frac{d\bar{\mu}}{\bar{\mu}} 2\Gamma_{\text{cusp}}\left[\alpha_s(\bar{\mu})\right] + \gamma_r\left[\alpha_s(b_0/b)\right]}.$$

  - In qT (CSS): $$S_c(M, b) = \exp\left[-\int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2}\left(A_c(\alpha_s(q^2)) \ln\frac{M^2}{q^2} + B_c(\alpha_s(q^2))\right)\right]$$

$$\frac{d\sigma}{dp_T^2 dy} = \frac{m_{ll}^2}{s} \sigma_{LO}^Z \int_0^{+\infty} db \frac{b}{2} J_0(bp_T) S_c(m_{ll}, b) \sum_{a_1,a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} [HC_1C_2]_{gg:a_1a_2} \prod_{i=1,2} f_{a_i/h_i}(x_i/z_i, b_0^2/b^2)$$

  - In momentum space (RadISH):

$$\sum(p_T) = \int_0^{p_T} dk_T \frac{d\sigma(k_T)}{dk_t} = \sigma_{LO}^H \int_0^\infty [dk_1] R'(m_H, k_{t,1}) \exp(-R(m_H, \epsilon k_{t,1})) \sum_{n=0}^{\infty} \frac{1}{n!} \prod_{i=2}^{n+1} \int_{\epsilon k_{t,1}}^{k_{t,1}} [dk_i] R'(m_H, k_{t,i}) \Theta\left(p_T - \left|\sum_{j=1}^{n+1} \vec{k}_{t,j}\right|\right)$$

COMPONENTS OF QT FACTORISATION (SCET)

FO	α_s^n	$H(m_V,\mu)$	$I_{i/j}^{(n)}(x,b)$	$\ln W(x_a, x_b, m_V, \vec{b}, \mu = b_0/b) \sim \int_{\mu_h}^{\mu} d\bar{\mu}/\bar{\mu}\left(A(\alpha_s(\bar{\mu}))\ln\frac{m_V^2}{\bar{\mu}^2} + B(\alpha_s(\bar{\mu}))\right)$						
$\frac{d\hat{\sigma}^V_{NLO}}{dq_T}$	NLO	✓	✓	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1				
$\frac{d\hat{\sigma}^V_{NNLO}}{dq_T}$	N2LO	✓	✓	$\ln^3(b^2m_V^2)$	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1			
$\frac{d\hat{\sigma}^V_{N^3LO}}{dq_T}$	N3LO	✓	✓	$\ln^4(b^2m_V^2)$	$\ln^3(b^2m_V^2)$	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1		
$\frac{d\hat{\sigma}^V_{N^4LO}}{dq_T}$	N4LO	✓	✗	$\ln^5(b^2m_V^2)$	$\ln^4(b^2m_V^2)$	$\ln^3(b^2m_V^2)$	$\ln^2(b^2m_V^2)$	$\ln(b^2m_V^2)$	1	
…	…			…	…	…	…		…	…
$\frac{d\hat{\sigma}^V_{N^kLO}}{dq_T}$	NKLO			$\ln^{k+1}(b^2m_V^2)$	$\ln^k(b^2m_V^2)$	$\ln^{k-1}(b^2m_V^2)$	$\ln^{k-2}(b^2m_V^2)$	$\ln^{k-3}(b^2m_V^2)$	…	…
…	…			…	…	…	…		…	…
Resum				LL	NLL	NNLL	N3LL	N4LL	…	$N^{k+1}LL$
A				A1 ✓	A2 ✓	A3 ✓	A4 ✓	A5 ✗	…	A_{k+2}
B					B1 ✓	B2 ✓	B3 ✓	B4 ✓	…	B_{k+1}

N3LO Phenomenology Progress

Parton Distributions @ N3LO

State-of-the-art Parton Distribution Functions

- Option A: solve proton wave function with Lattice QCD *Recent progress in D. Chakrabarti, P. Choudhary et. al. 2304.09908*
- Option B: collinear factorisation $f_a \to f_a(x, \mu)$ with p-QCD evolution of factorisation scale

$$\frac{d}{d\ln\mu^2}\begin{pmatrix} f_q \\ f_g \end{pmatrix} = \begin{pmatrix} P_{q\leftarrow q} & P_{q\leftarrow g} \\ P_{g\leftarrow q} & P_{g\leftarrow g} \end{pmatrix} \otimes \begin{pmatrix} f_q \\ f_g \end{pmatrix}$$

DGLAP evolution with

$$p_{a\leftarrow b} = \frac{\alpha_s}{\pi} P^{(0)}_{a\leftarrow b} + \frac{\alpha_s^2}{\pi^2} P^{(1)}_{a\leftarrow b} + \frac{\alpha_s^3}{\pi^3} P^{(2)}_{a\leftarrow b} + \cdots$$

1970's 1980 2004

More details on Wednesday

$$\gamma^{(3)}_{j\leftarrow i}(N) = -\int_0^1 dx\, x^{N-1} P^{(3)}_{j\leftarrow i}(x) \qquad \gamma^{(3)}_{q\leftarrow g}(N),\ \gamma^{(3)}_{q\leftarrow q}(N),\ \gamma^{(3)}_{g\leftarrow q}(N)$$

For $N = 2,4,\cdots 20$

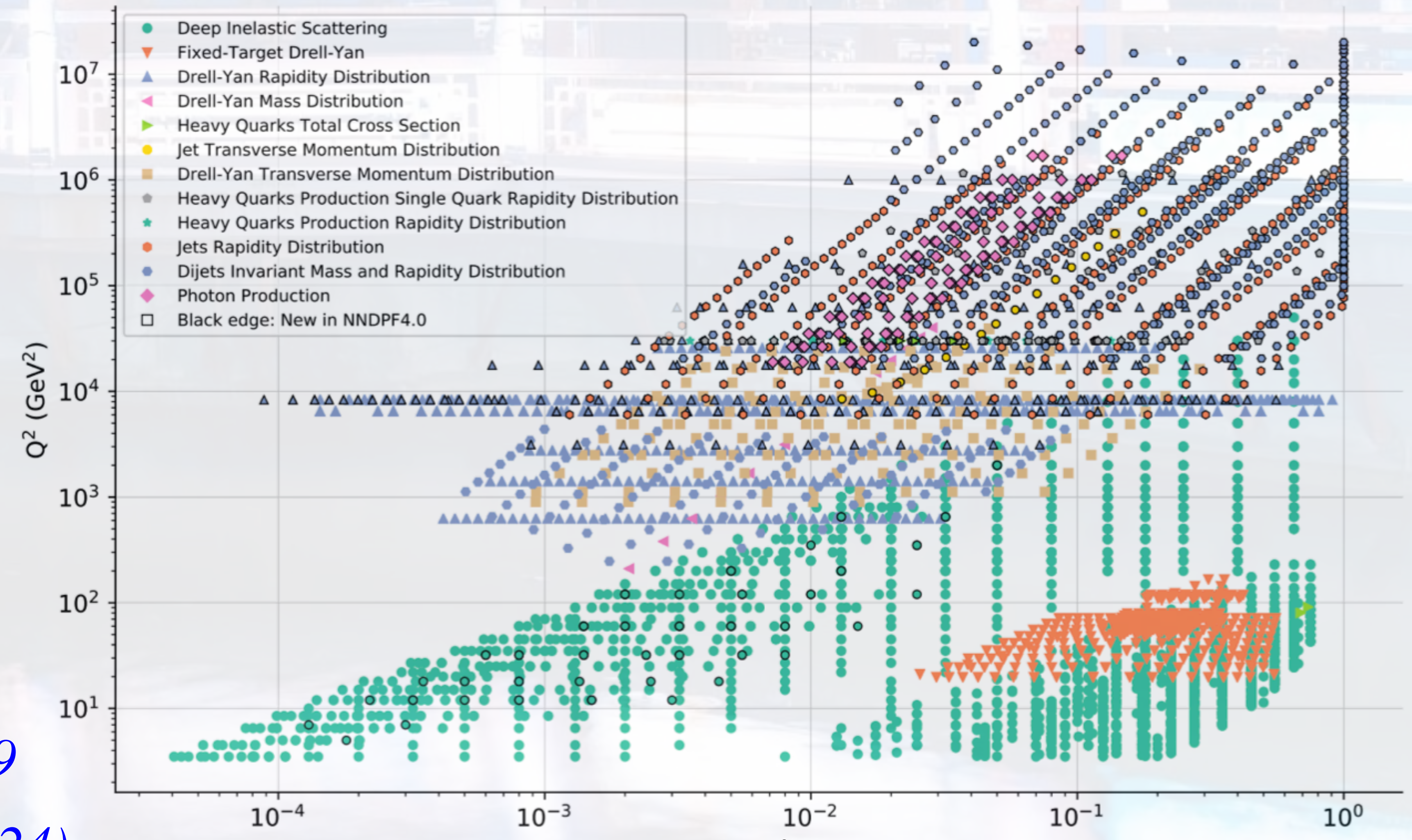
G. Falcioni, F. Herzog et. al. Phys.Lett.B 842 (2023)
G. Falcioni, F. Herzog, S. Moch, A. Vogt Phys.Lett.B 846 (2023)
G. Falcioni, F. Herzog, S. Moch, A. Vogt 2404.09959

See also full result of N_f^2, $N_f C_f^2$ contribution in *Gehrmann, von Manteuffel et. al. JHEP 01 (2024) 029*
Gehrmann, von Manteuffel et. al. Phys.Lett.B 849 (2024)

- Experiment input
  - All past and current measurements of DIS, DY, jets etc. provide fitting targets of $f_a(x, Q)$
  - Differential and total cross sections provide sensitivity in different regions of $x \in [0,1]$
  - Various technology for fitting: functional form, neural network, fast evaluation grids etc.

NNPDF4.0 Coverage

Q^2



x

Parton Distributions @ N3LO

State-of-the-art Parton Distribution Functions

NNPDF 2402.18635

- Approximated N3LO PDF available:

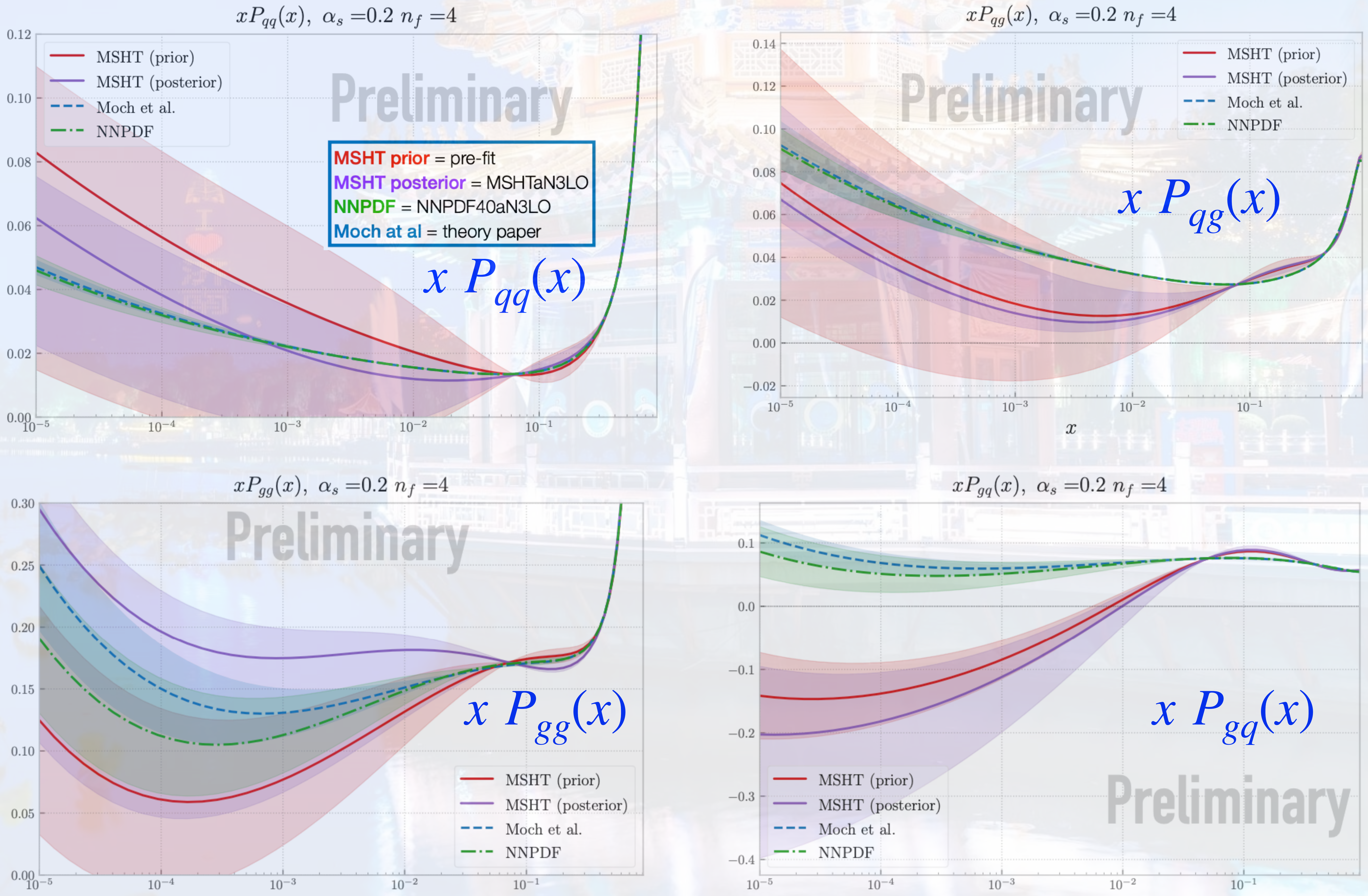
  MSHT20aN3LO *Eur.Phys.J.C 83 (2023) 4*

  NNPDFaN3LO NNPDF 2402.18635

- More precise 4-loop splitting functions affect small x region:

  4 → 10 Mellin Moments

- Large correction at aN3LO at small x region outside 68% c.l. region.
- Missing Higher Order Uncertainty (MHOU) not included in standard NNLO PDF.
- Crucial to consider MHOU and IHOU to understand consistency between NNLO and N3LO PDF.



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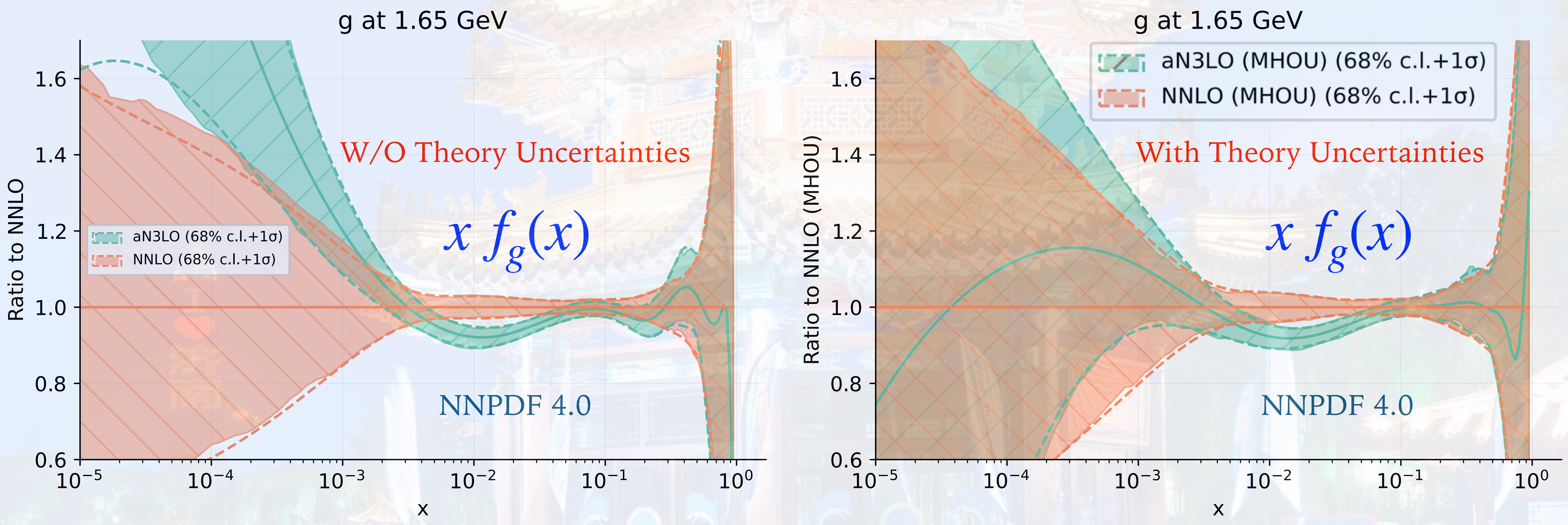
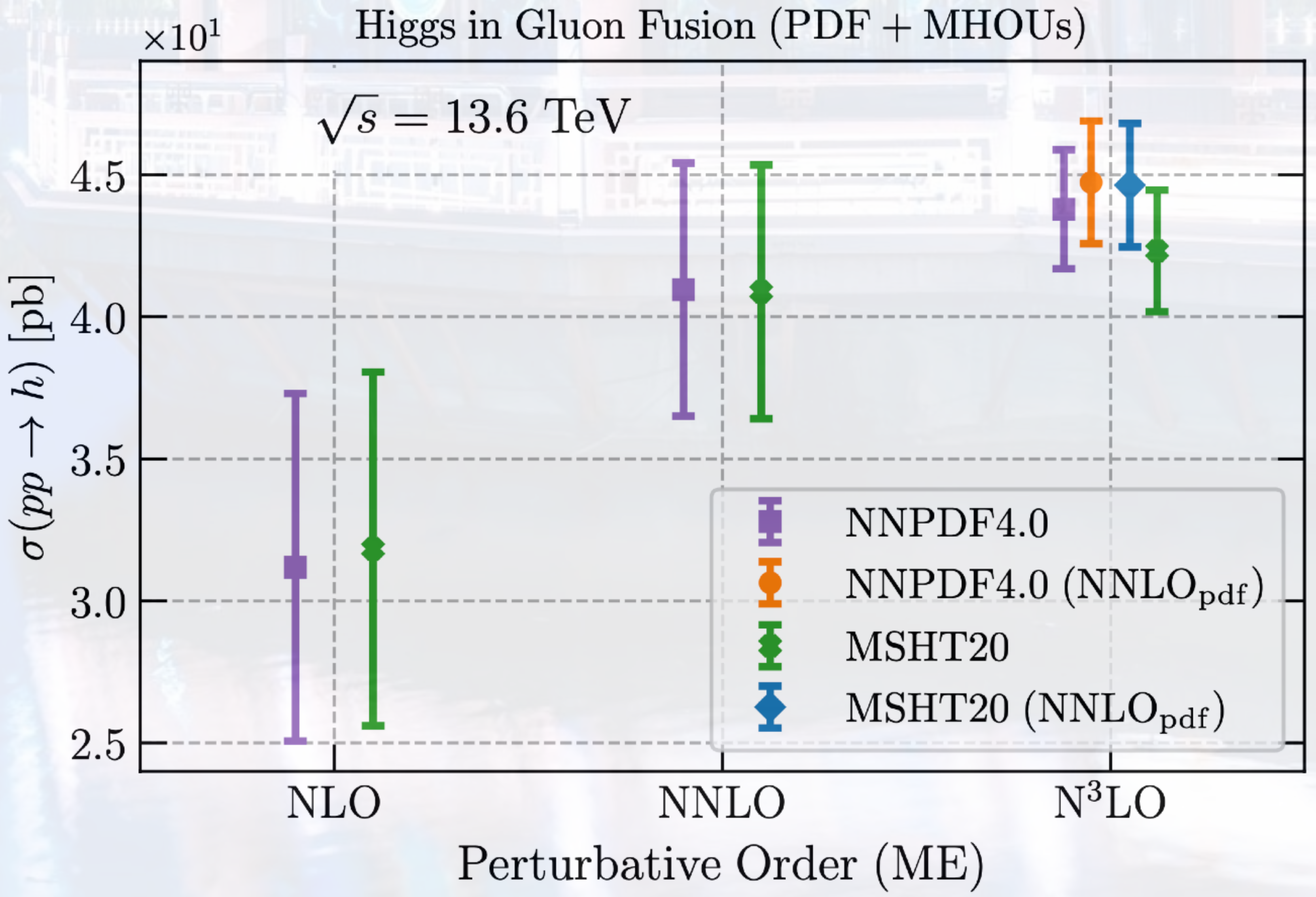




NNPDF 2402.18635

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

- Differential N3LO accuracy
  - Projection to Born

$$\frac{d\sigma^F_{N^kLO}}{d\mathcal{O}} = \left(\frac{d\sigma^{F+jet}_{N^{k-1}LO}}{d\mathcal{O}} - \frac{d\sigma^{F+jet}_{N^{k-1}LO}}{d\tilde{\mathcal{O}}}\right) + \frac{d\sigma^F_{N^kLO}}{d\tilde{\mathcal{O}}}$$

    - Jet production in DIS (NNLOJET) Currie, Gehrmann, Glover, Huss, Niehues `18
    - Higgs decay to $b\bar{b}$ (MCFM) Mondini, Schiavi, Williams `19
    - Higgs production via ggF (RapidiX+NNLOJET) XC, Gehrmann, Glover, Huss, Mistlberger, Pelloni `21
  - qT slicing

$$d\sigma^F_{N^kLO} = \mathcal{H}^F_{N^kLO} \otimes d\sigma^F_{LO}\Big|_{\delta(\tau)} + \left[d\sigma^{F+jet}_{N^{k-1}LO} - d\sigma^{F\ CT}_{N^kLO}\right]_{\tau>\tau_{cut}} + \mathcal{O}(\tau^2_{cut}/Q^2)$$

    - Higgs production via ggF (HN3LO+NNLOJET) Cieri, XC, Gehrmann, Glover, Huss `18
    - Higgs pair production via ggF (with modified iHixs2) Chen, Li, Shuo, Wang `19
    - Drell-Yan production (NNLOJET) XC, Gehrmann, Glover, Huss, Yang, Zhu `21 `22 (MCFM) Neumann and Campbell `22 `23
- Combined with resummation (N3LL/aN4LL at small qT)
  - Drell-Yan production Ju and Schönherr `21 (DYTurbo) Camarda, Cieri, Ferrera `21 `23 (RadISH(N3LL)+NNLOJET) XC, Gehrmann, Glover, Huss, Monni, Re, et. al. `18 `19 `22 (CuTe-MCFM) Neumann and Campbell `22 `23
  - Higgs production via ggF (SCET+NNLOJET) XC, Gehrmann et. al.`18 (SCETlib) Billis, Dehnadi, et. al.`21

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

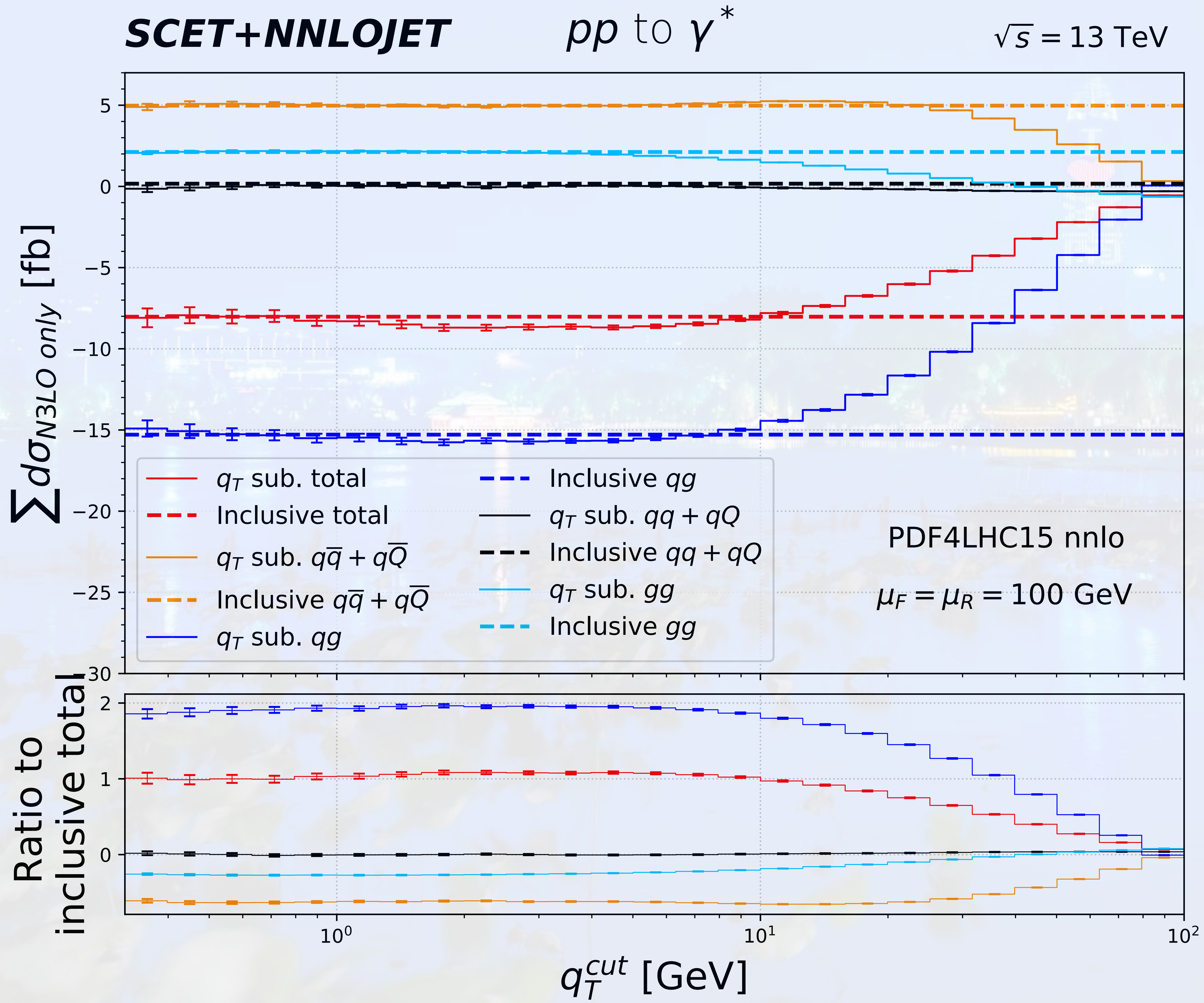
➤qT slicing at N3LO for neutral and charged current production (NNLOJET)

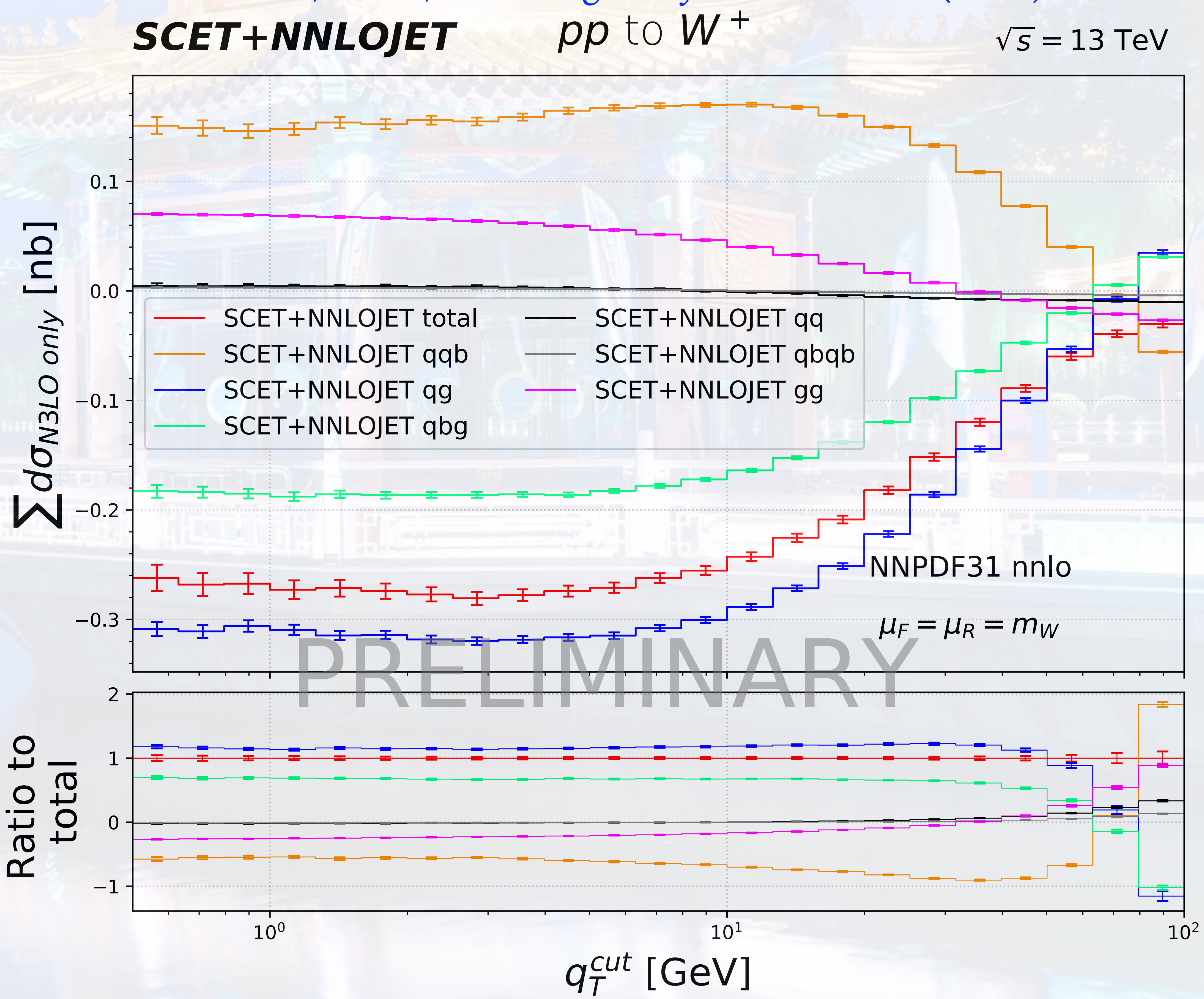
$$\sum \mathrm{d}\sigma^{V}_{N3LO} \equiv \sum_{\mathrm{d}p_{T,V}} \mathrm{d}\sigma^{V+jet}_{NNLO}/\mathrm{d}p_{T,V}|_{p_{T,V}>q_T^{cut}} + \sum_{\mathrm{d}p_{T,V}} \mathrm{d}\sigma^{V\,SCET}_{N^3LO}/\mathrm{d}p_{T,V}|_{p_{T,V}\in[0,q_T^{cut}]}$$

NC and CC Validated against inclusive XS within ± 5% uncertainty

$$\Delta\sigma^{\gamma^*}_{N^3LO} = -7.98 \pm 0.36\, fb \quad vs. \quad -8.03\, fb$$

Duhr, Dulat, Mistlberger *Phys.Rev.Lett.* 125 (2020)





XC, Gehrmann, Glover, Huss, Yang, Zhu *Phys.Rev.Lett.* 128 (2022) 5

XC, Gehrmann, Glover, Huss, Yang, Zhu *Phys.Lett.B 840 (2023)*

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

More details in Tobias' talk

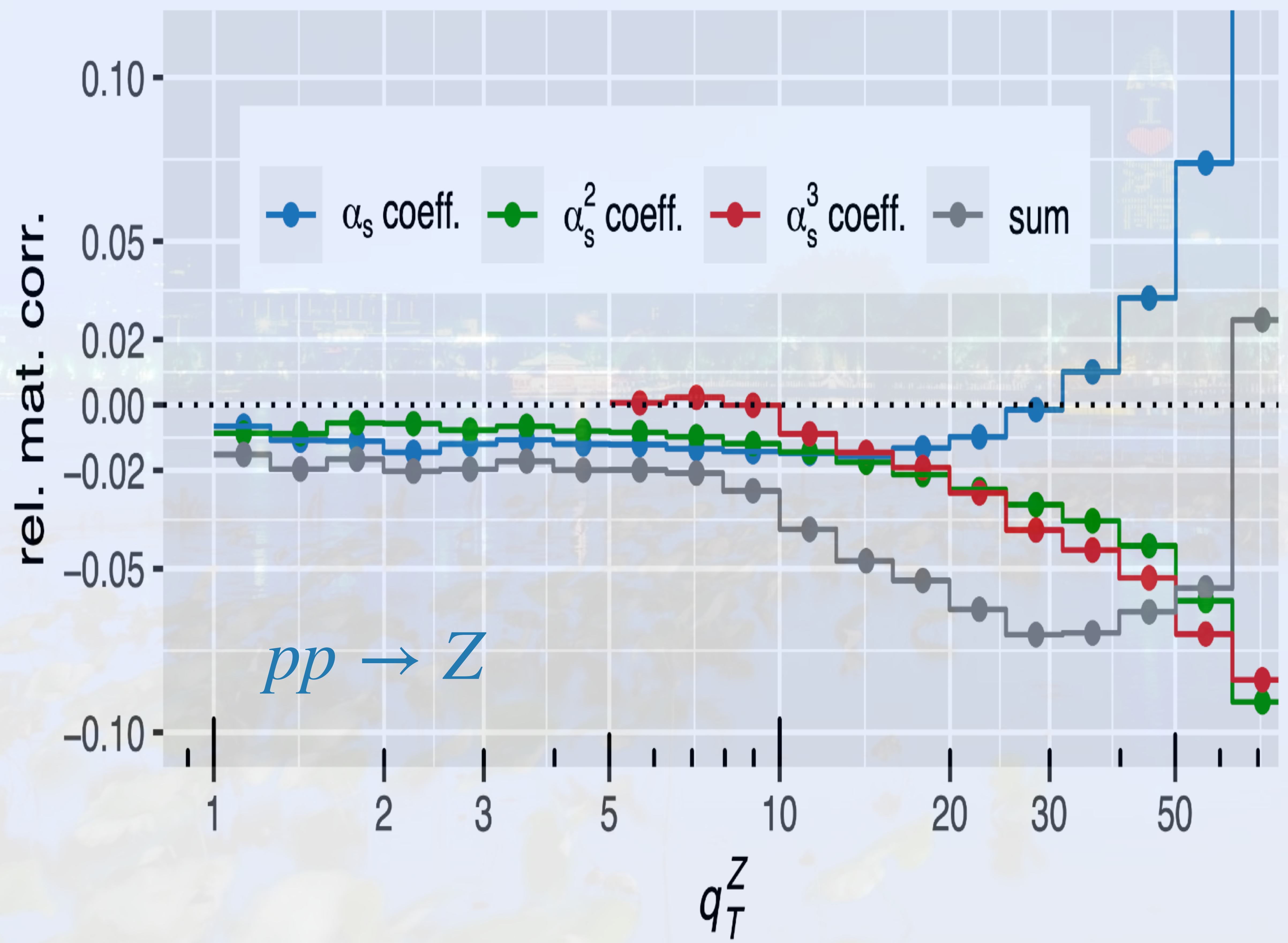
➤qT slicing at N3LO for neutral and charged current production (MCFM)

$$\sum d\sigma^{V}_{N3LO} \equiv \sum_{dp_{T,V}} d\sigma^{V+jet}_{NNLO}/dp_{T,V}|_{p_{T,V}>q_T^{cut}} + \sum_{dp_{T,V}} d\sigma^{V\,SCET}_{N^3LO}/dp_{T,V}|_{p_{T,V}\in[0,q_T^{cut}]}$$

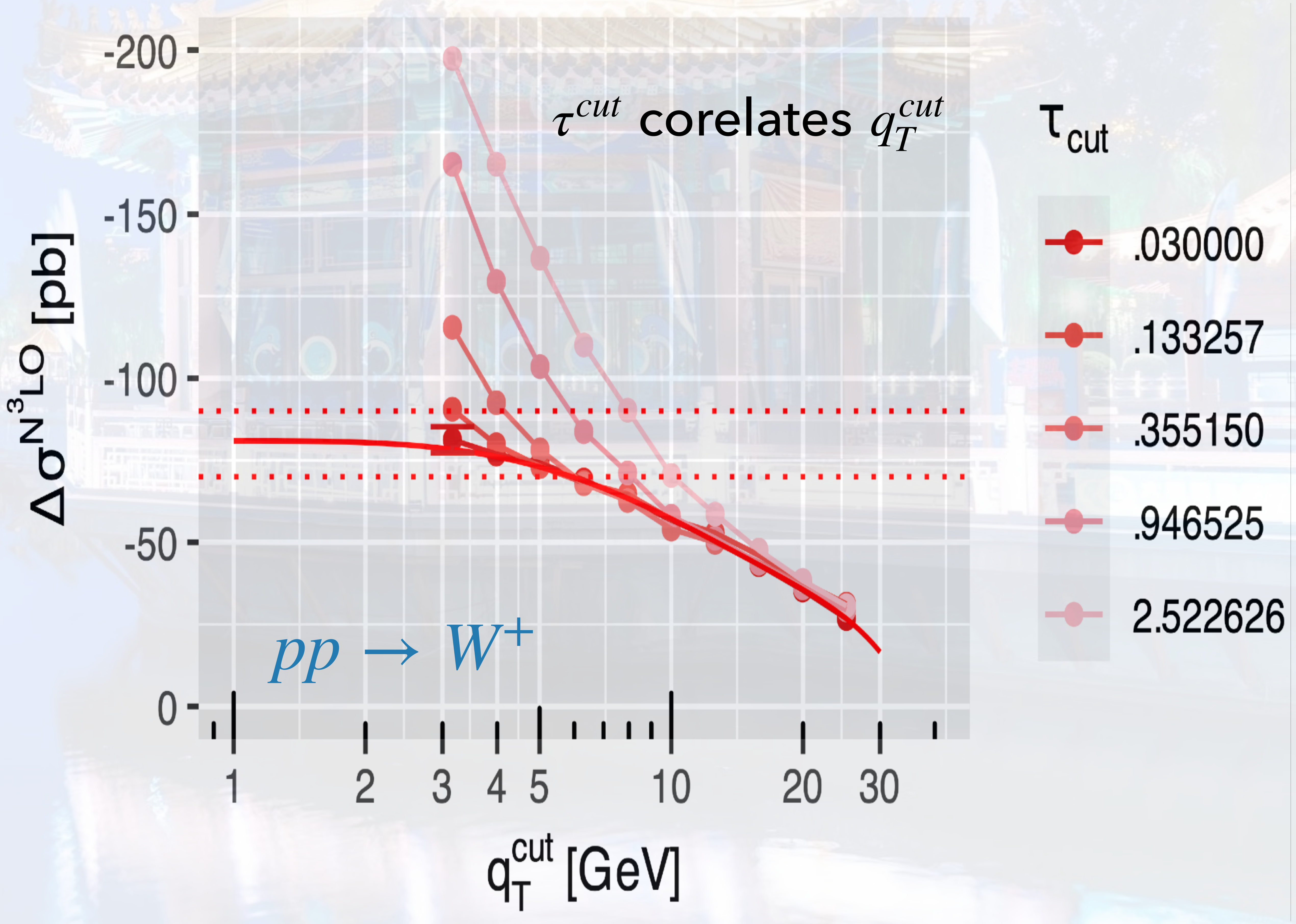
NC MCFM: $-22.6\,\text{pb} \pm 1.4\,\text{pb}\,(\text{num.}) \pm 1\,\text{pb}\,(\text{slicing})$

NC NNLOJET: $-18.7\,\text{pb} \pm 1.1\,\text{pb}\,(\text{num.}) \pm 0.9\,\text{pb}\,(\text{slicing})$

CC agree to inclusive XS within ± 60% uncertainty of $\Delta(\alpha_s^3)$



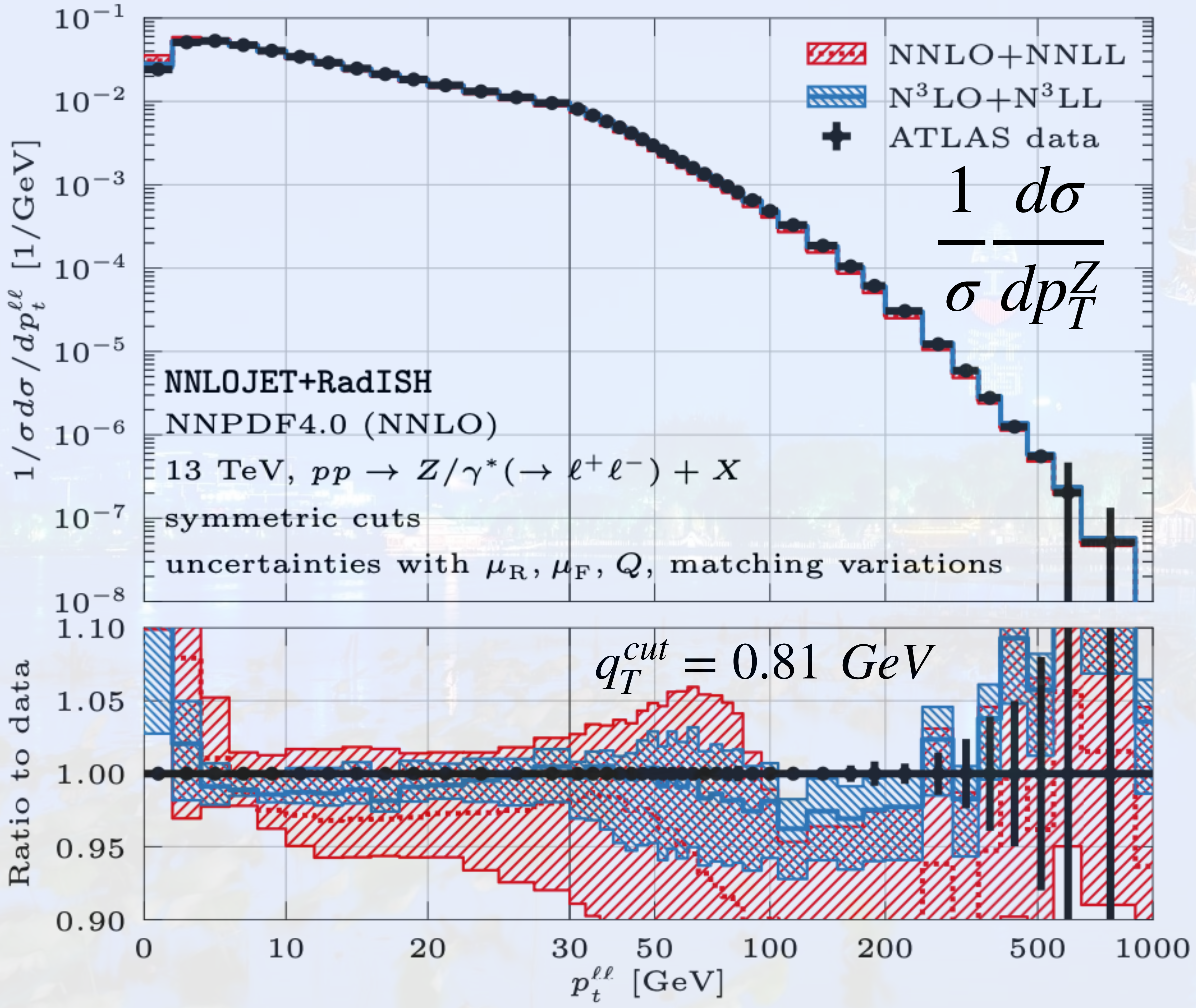
Neumann and Campbell *Phys.Rev.D* 107 (2023) 1



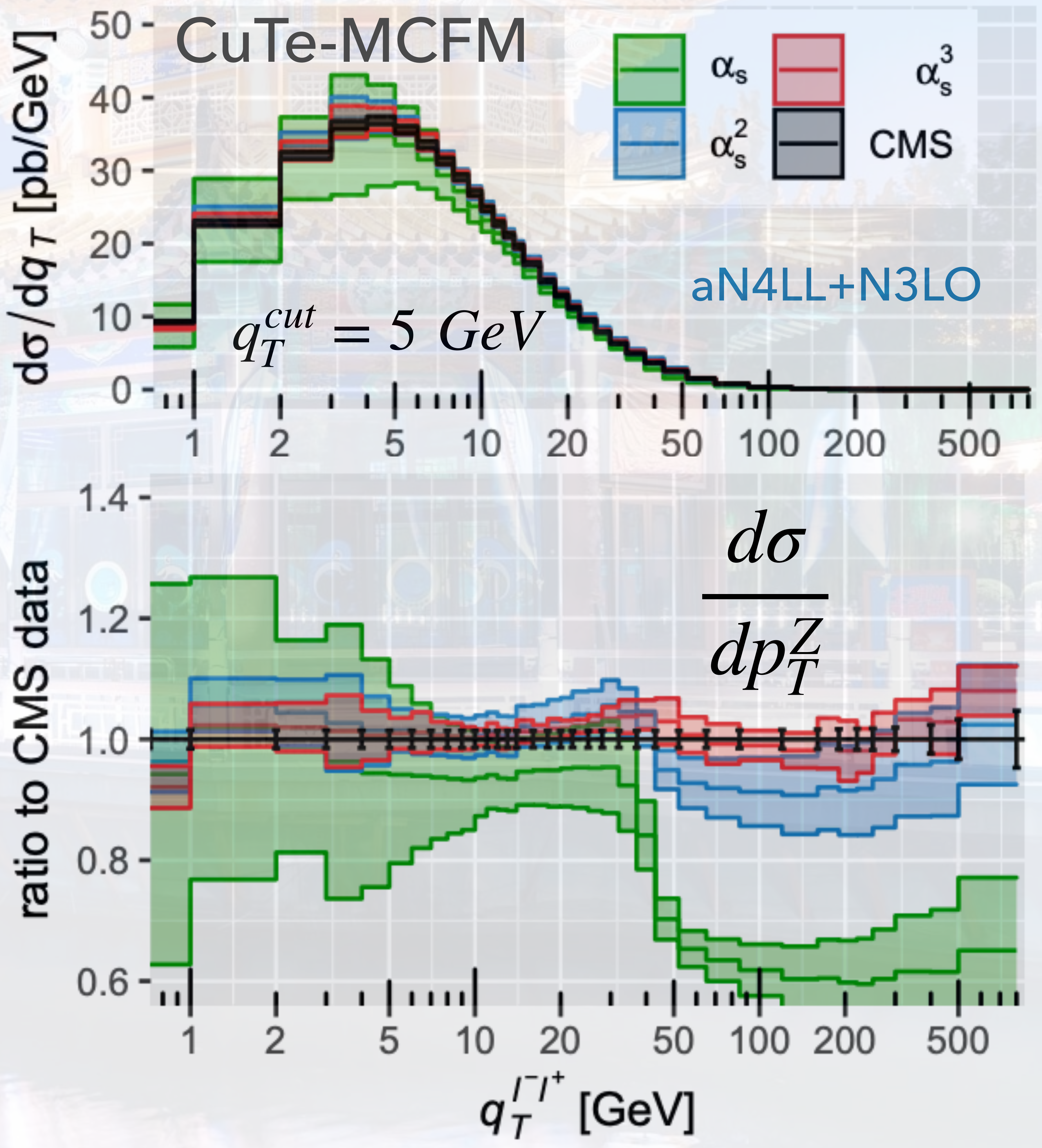
Neumann and Campbell *JHEP 11 (2023) 127*

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

➤Differential N3LO predictions for neutral current production



XC, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli
Phys.Rev.Lett. 128 (2022) 25



Neumann and Campbell *Phys.Rev.D* 107 (2023) 1

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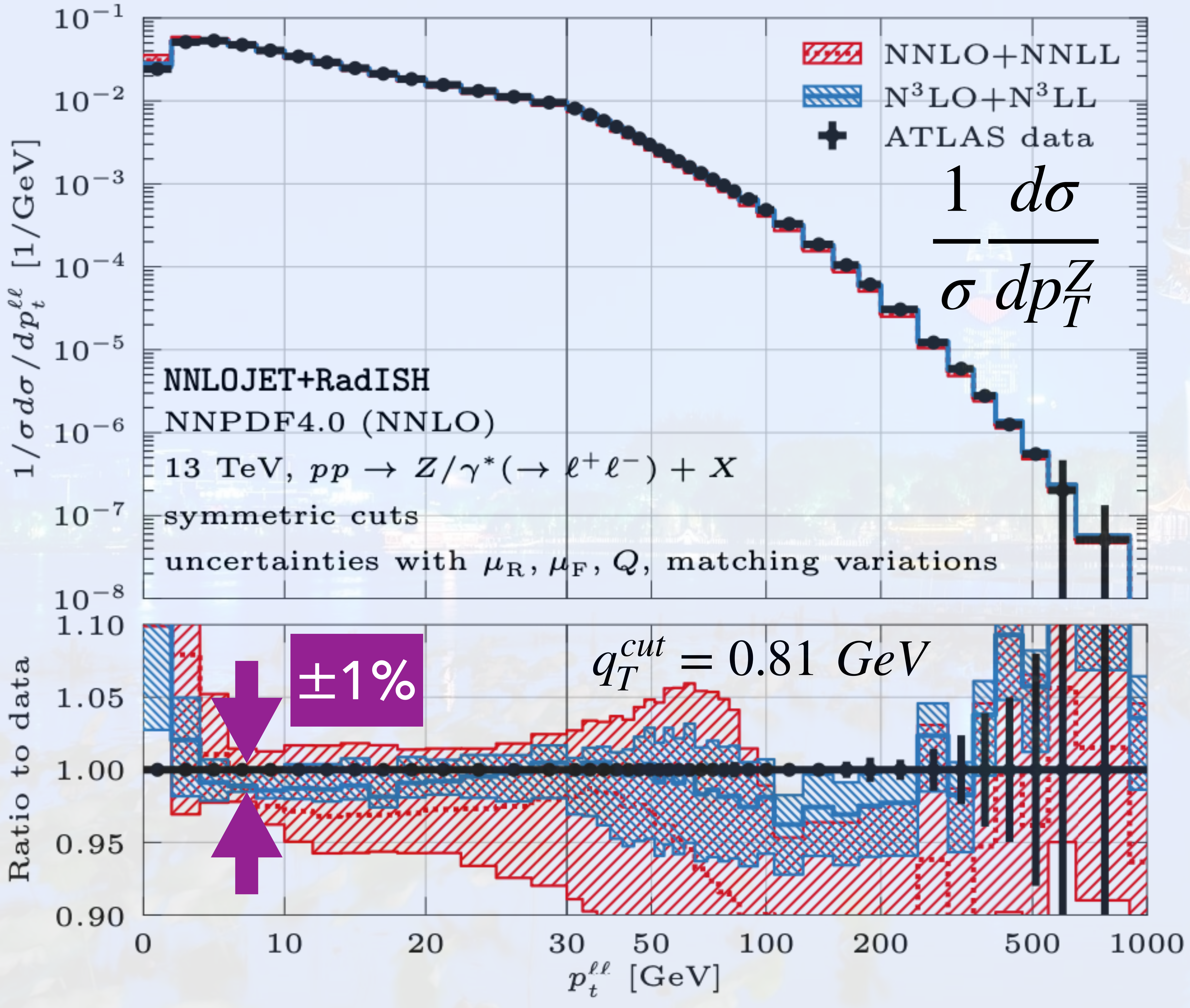
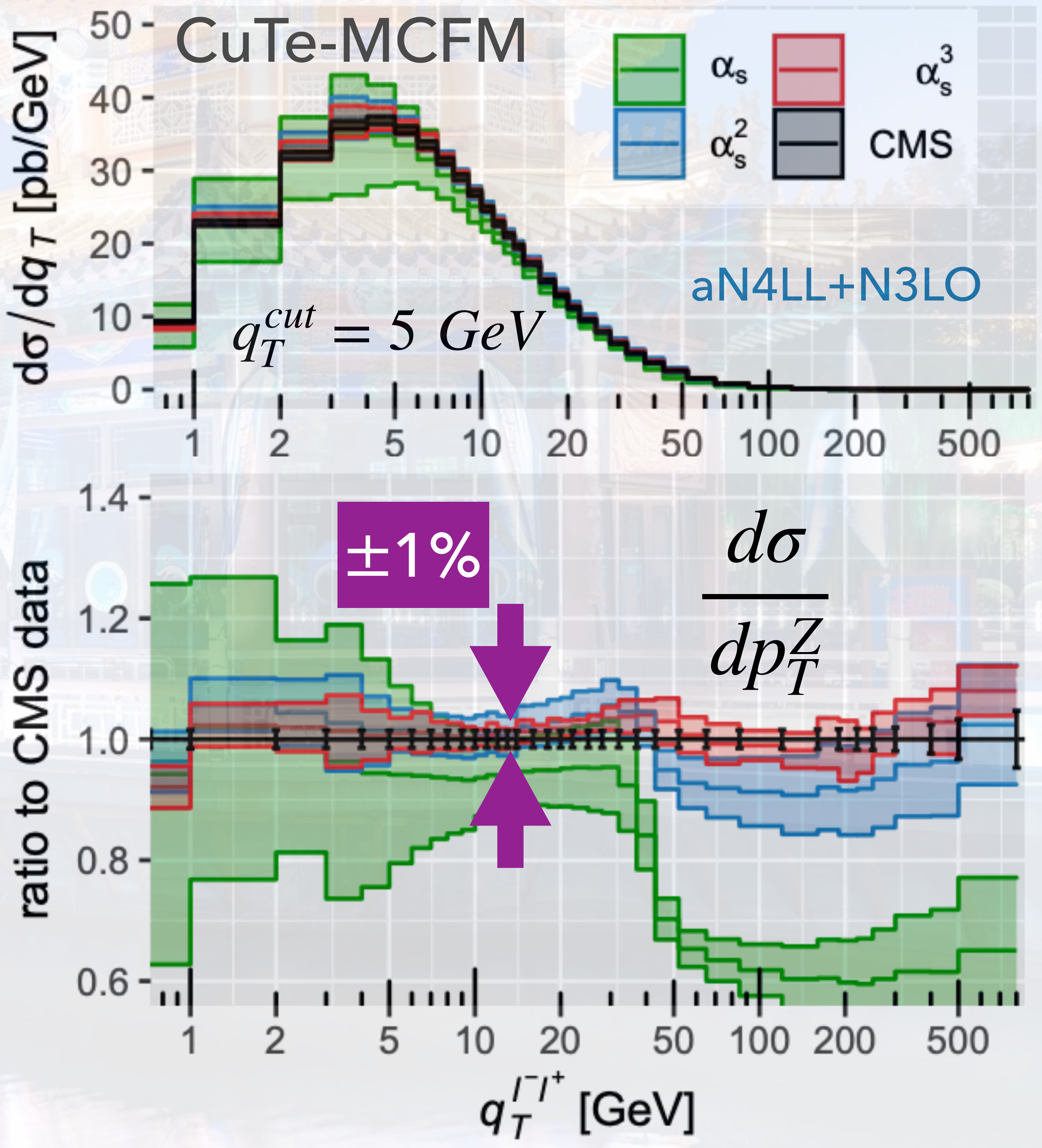


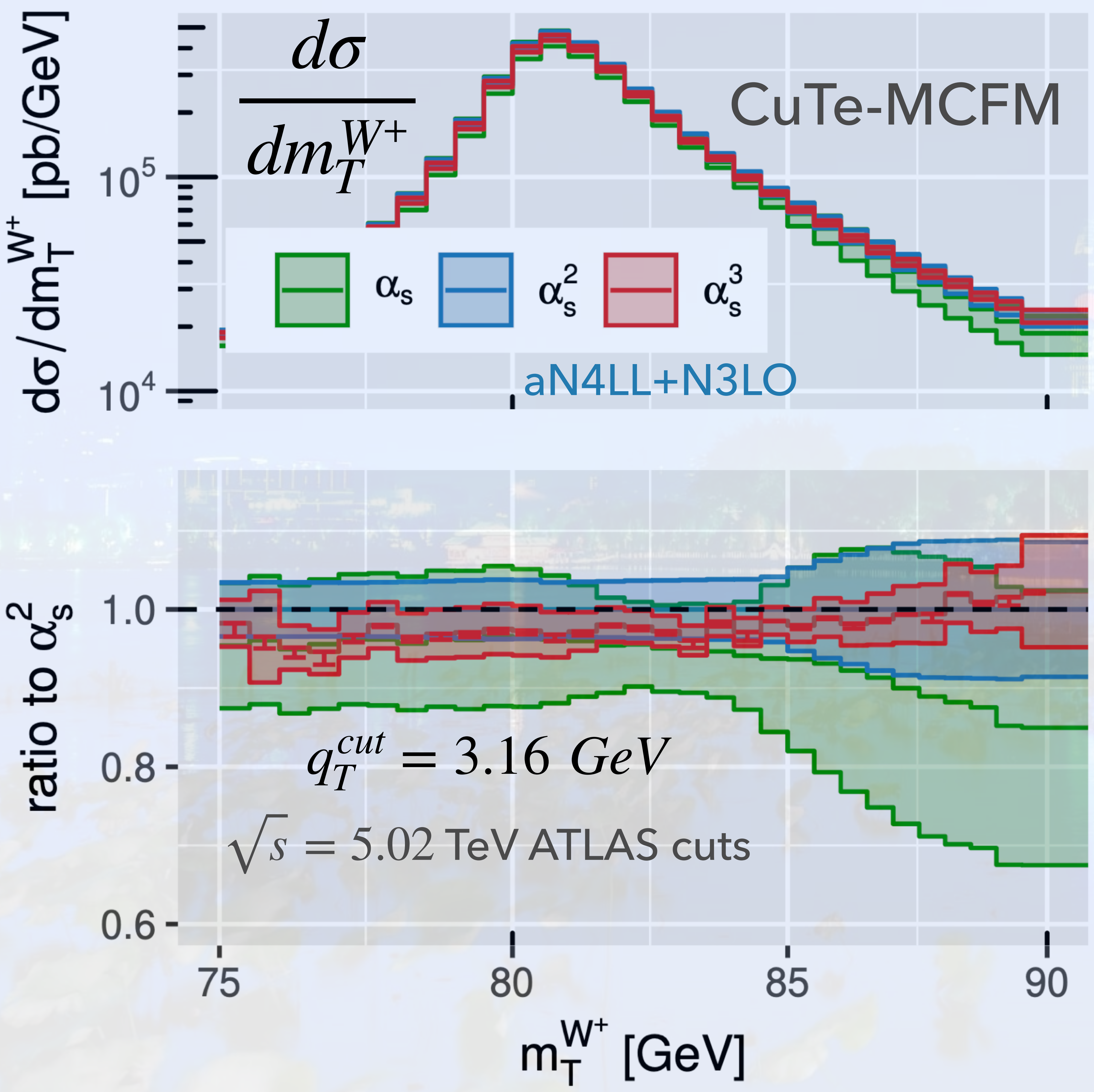
XC, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Torrielli *Phys.Rev.Lett.* 128 (2022) 25



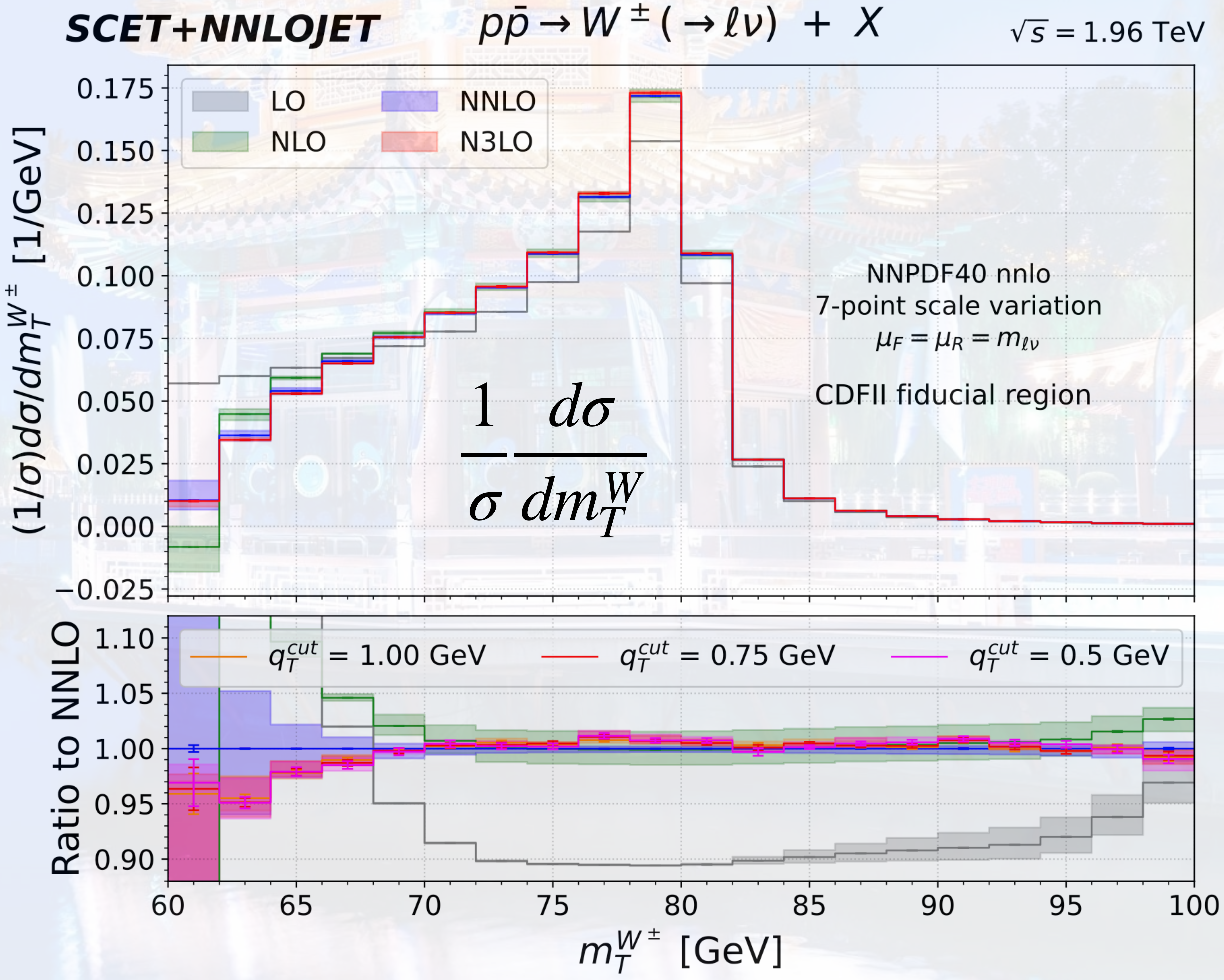
Neumann and Campbell *Phys.Rev.D* 107 (2023) 1

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

➤Differential N3LO predictions for charged current production



Neumann and Campbell *JHEP 11 (2023) 127*



XC, Gehrmann, Glover, Huss, Yang, Zhu *Phys.Lett.B 840 (2023)*

STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

➤ Precise AZ tune at N3LO:

Require DY grids with all D.O.F.

$$\frac{d\sigma}{dm_{l\nu}dp_T dy}\left[(1+\cos^2\theta)+\sum_{i=0}^{7}A_i f_i(\theta,\phi)\right]$$

ResBos2

ATLAS with PYTHIA 8

- ➤ Numerically challenging for D.O.F = 11 (may drop $A_{5,6,7}$ for being very small)
- ➤ MC error of each grid bin + interpolation error cross bins (prefer fine granularity)
- ➤ Once $A_i(p_T, y, m_{l\nu})$ available, no new calculation is needed for different fiducial cuts

Z+J @ NNLO

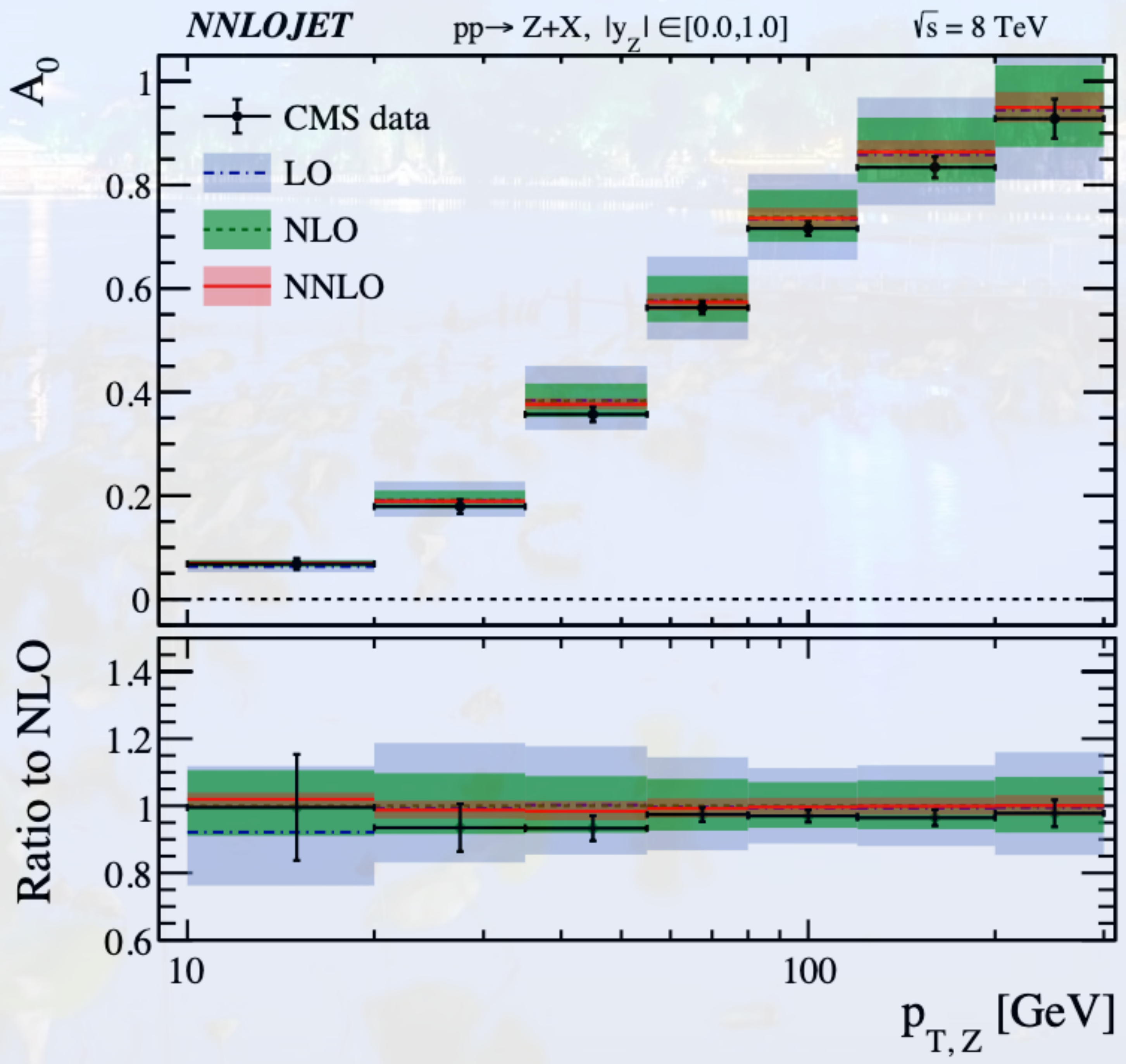
$A_0(p_T, y)$

$|y| < 1$

Inclusive in $m_{l\nu}$

Smallest bin @ 10 GeV

Gauld, Gehrmann-De Ridder, Gehrmann, Glover, Huss `17



W+J @ NNLO

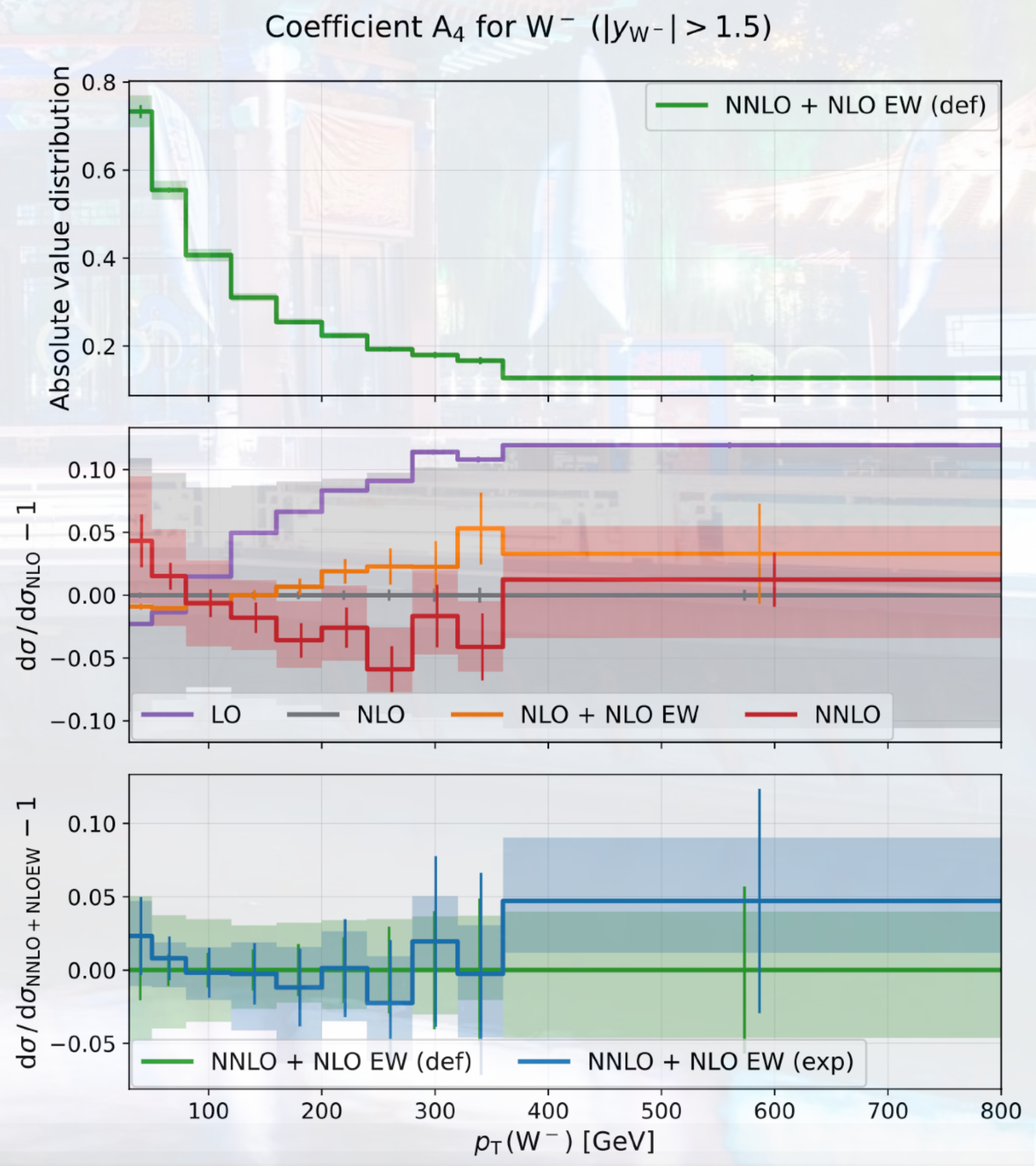
$A_4(p_T, y)$

$|y| > 1.5$

Inclusive in $m_{l\nu}$

Smallest bin ~ 20 GeV

Pellen, Poncelet, Popescu, Vitos `22

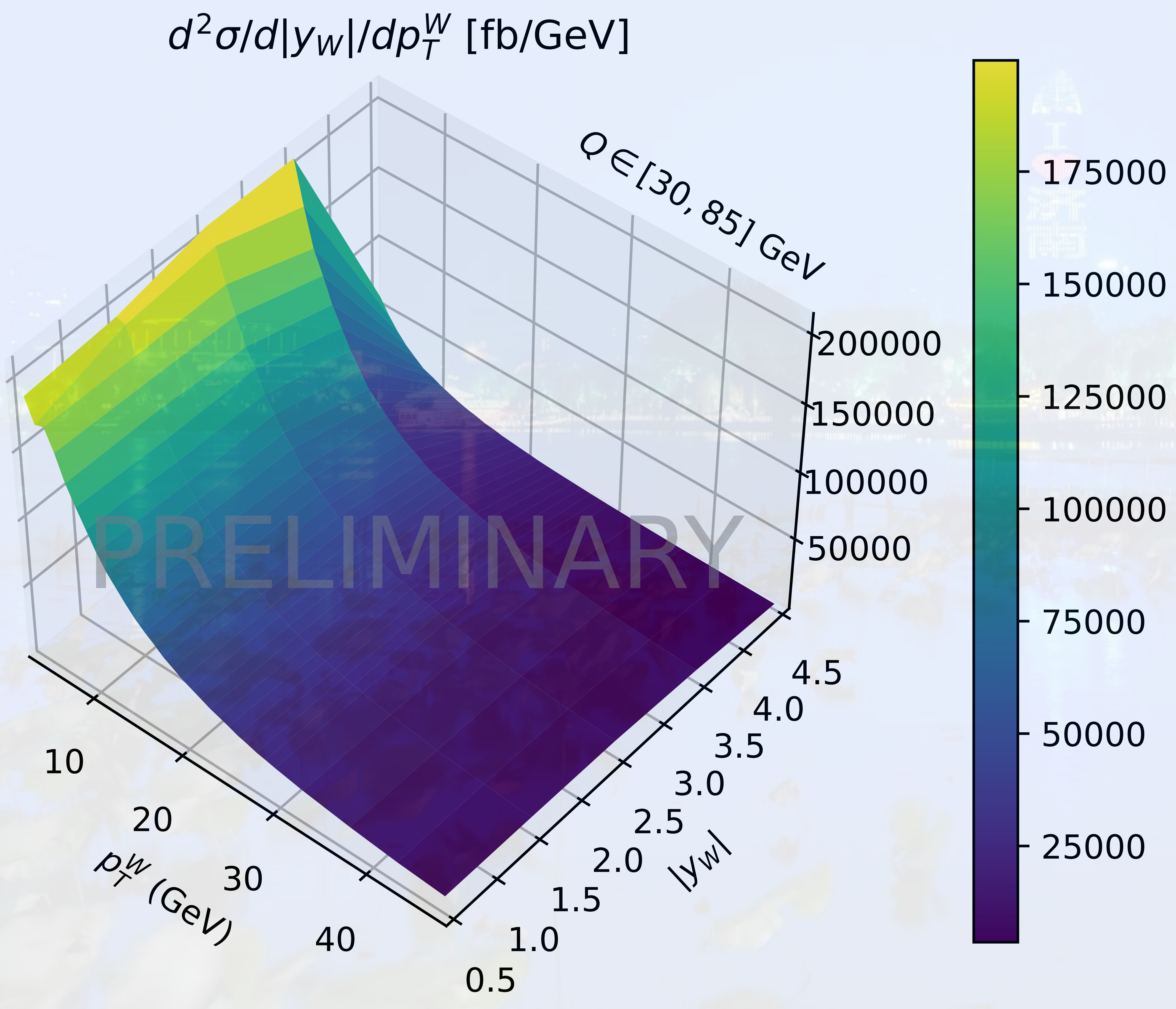


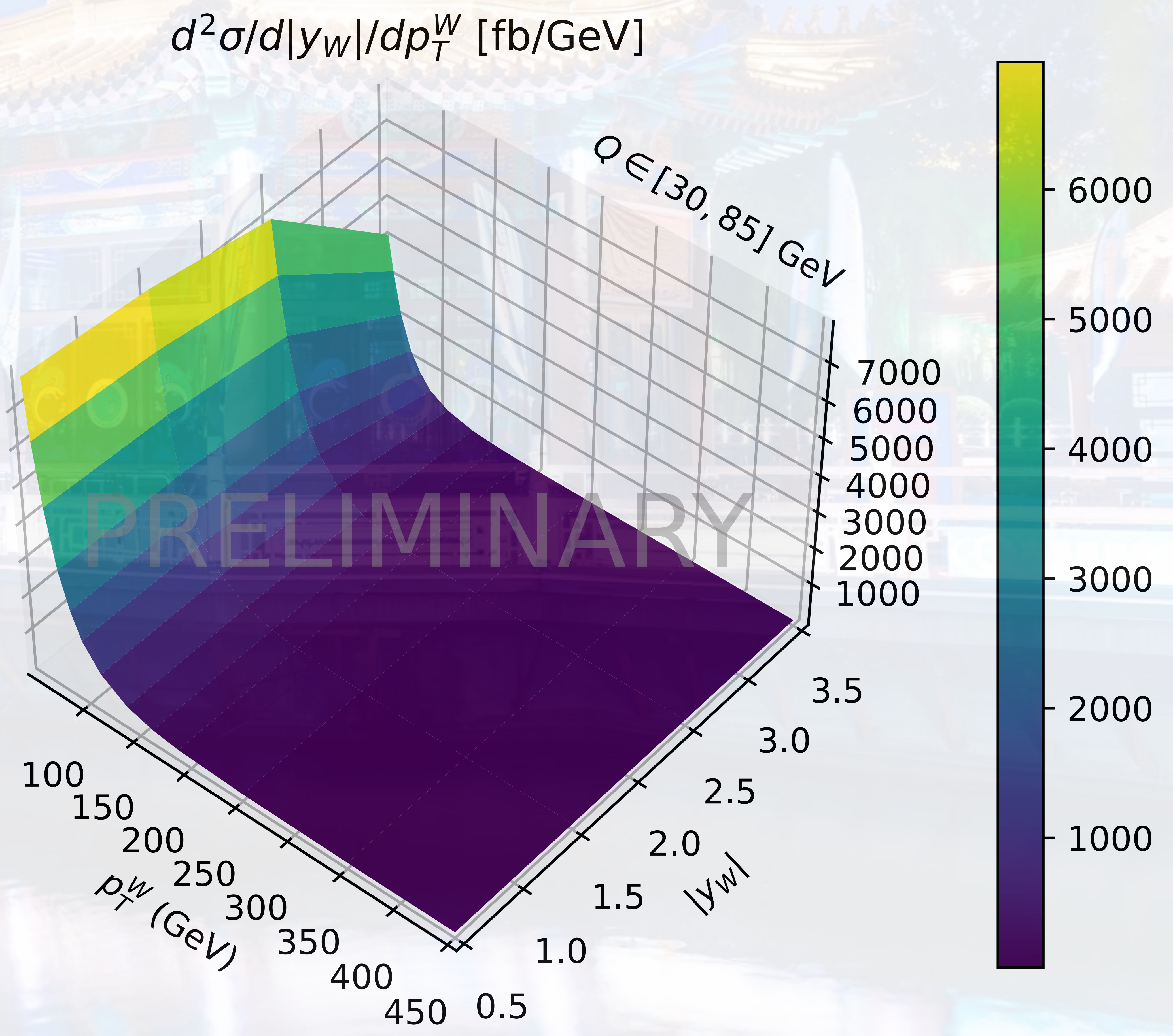
STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

➤ Precise AZ tune at N3LO (fully differential):

In collaboration with T. Gehrmann, A. Huss

$d^3\sigma$ for $p_T^{W^+} \in [2,\ 500]$ GeV, $|y_{W^+}| \in [0,\ 4]$ and $Q \in [30,\ 85]$ GeV



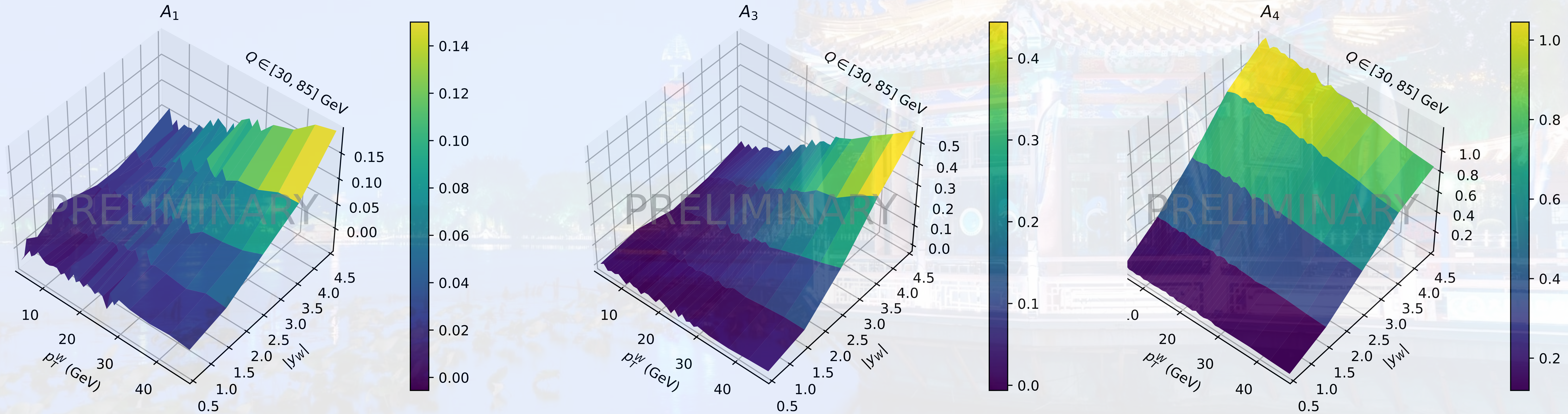


STATE-OF-THE-ART PREDICTIONS: $d\sigma_{N^3LO}$

➤Precise AZ tune at N3LO (fully differential):

In collaboration with T. Gehrmann, A. Huss

A_i for $p_T^{W^+} \in [2,\ 50]$ GeV, $|y_{W^+}| \in [0,\ 5]$ and $Q \in [30,\ 85]$ GeV



➤Numerically more challenging than the unpolarised contribution, different challenges for A_i

➤Different shape for low and high p_T^W, both regions have smooth distributions

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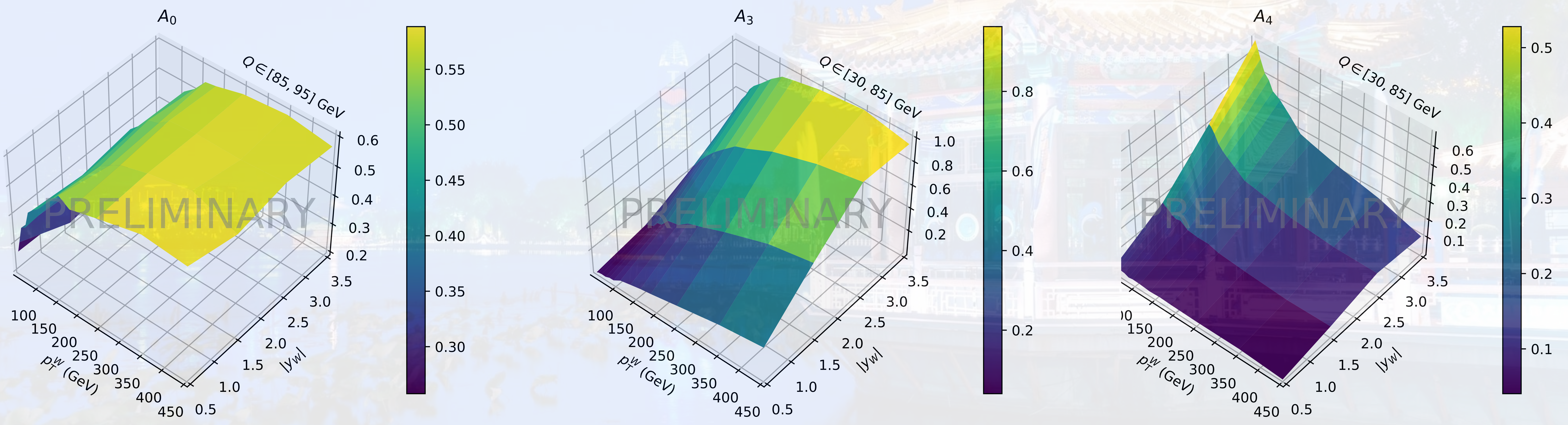

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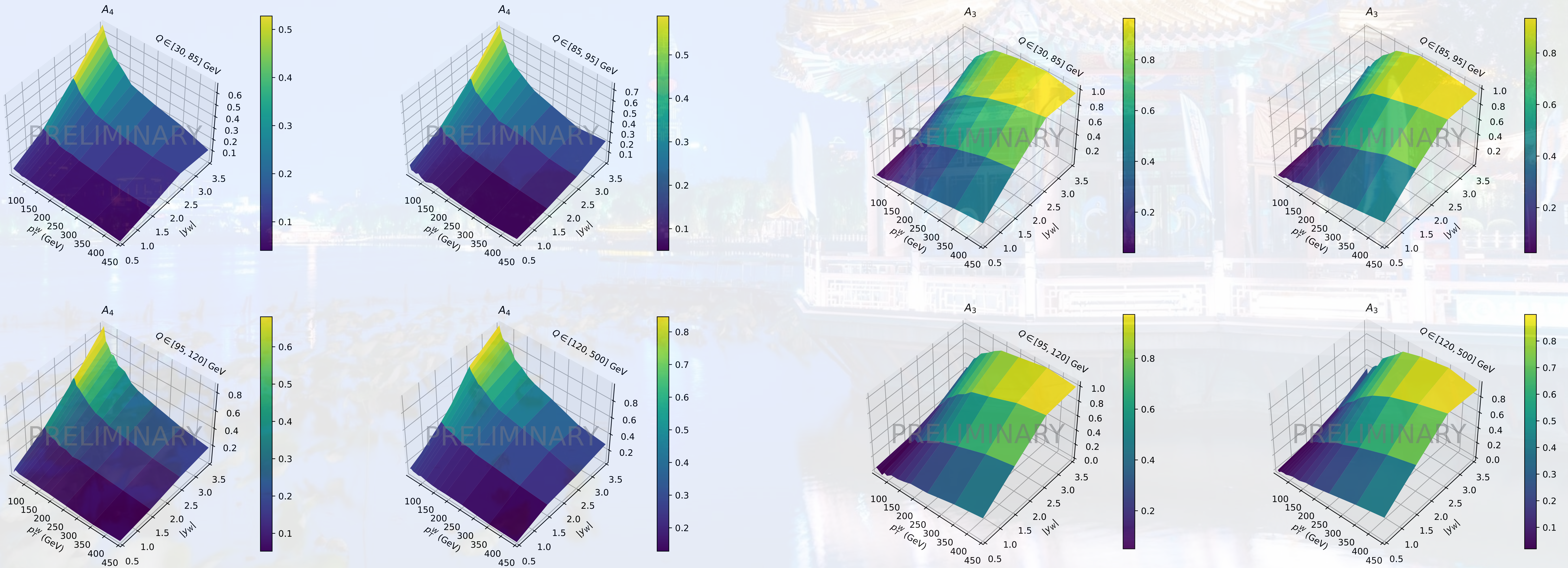

CONCLUSION AND OUTLOOK

- The Standard Candle of the Standard Model requires precision phenomenology
- The determination of m_W and α_s need delicate treatment and thorough understanding of experiment and theory uncertainties.
- Best predictions for NC and CC DY production at N3LO QCD achieves 1% accuracy.
- Thorough study of resummation uncertainties (matching and scheme choice), mixed QCD-EW, approximated PDFs, indicate corrections and irreducible errors at % level.
- Require collective efforts to turn controversial results to convincing results: most of the approximations are expected to be replaced during LHC Run 3.
- The AZ tune method is a powerful tool to bridge MC samples and EXP data. New numerical challenges but seem to be manageable.

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Thank You for Your Attention

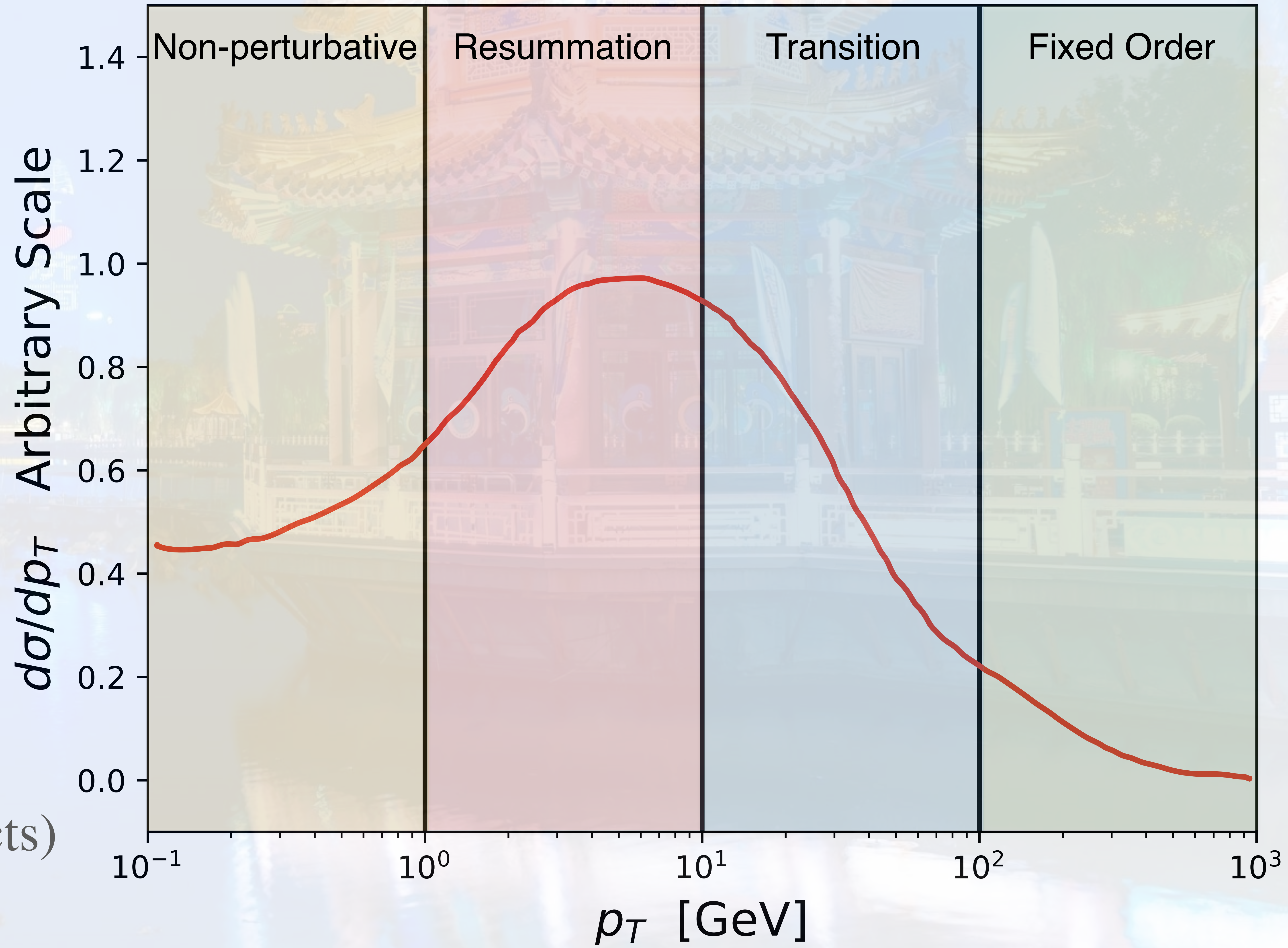
PRECISION PREDICTIONS AT HADRON COLLIDER

- Beyond QCD improved parton model
  - pQCD describes the tail of spectrum
  - Large logarithmic divergence

$$\ln\frac{p_T}{Q} \text{ as } p_T \to 1 \text{ GeV}$$

  - Various LP resummation schemes
  - Multiple solutions in transition region
  - Non-perturbative effects ~ 1 GeV (Short distance and long distance effects)

p_T Spectrum = multi-scale problem



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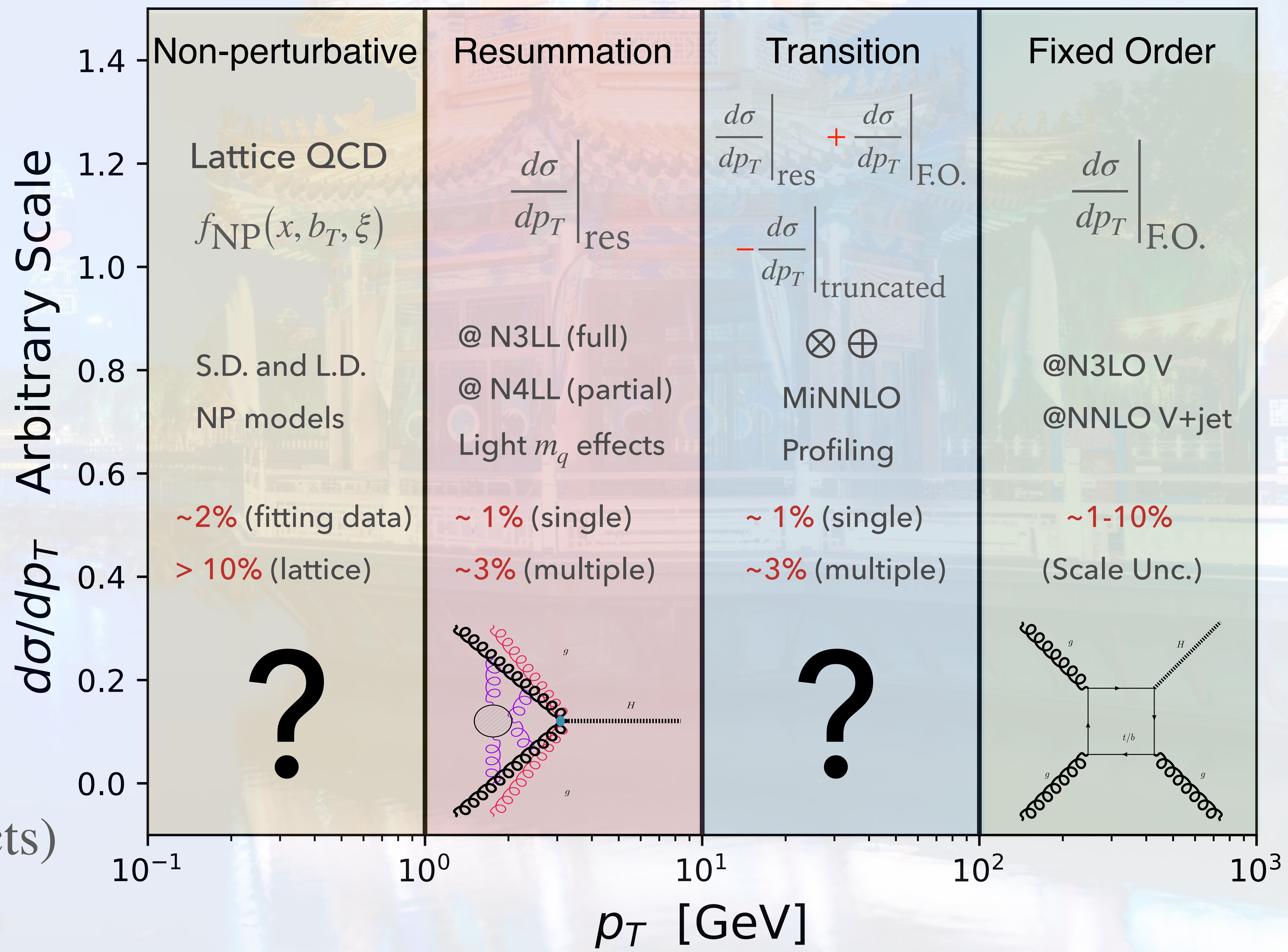


LIGHT QUARK MASS EFFECT AT SMALL TRANSVERSE MOMENTUM

➤At few GeV b and c quark mass are comparable to the resummation and factorisation scales

➤Retain full quark mass dependence in FO, PDF and resummation: GM-VFN scheme
Collins `98

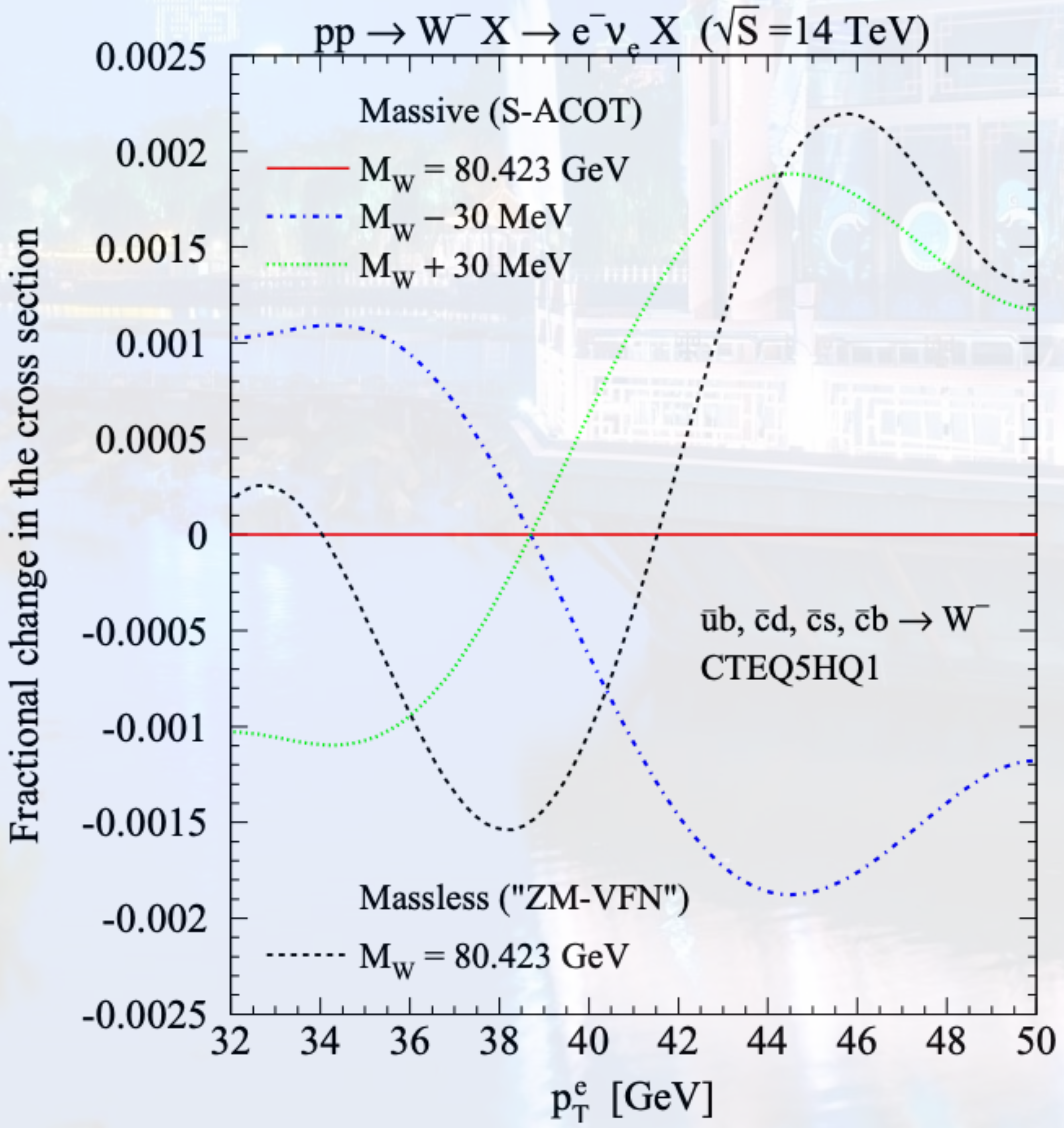
➤Reasonably good approximation in S-ACOT scheme (ignore quark mass from initial states)
Kramer, Olness and Soper `00
Nadolsky, Kidonakis, Olness, Yuan `03

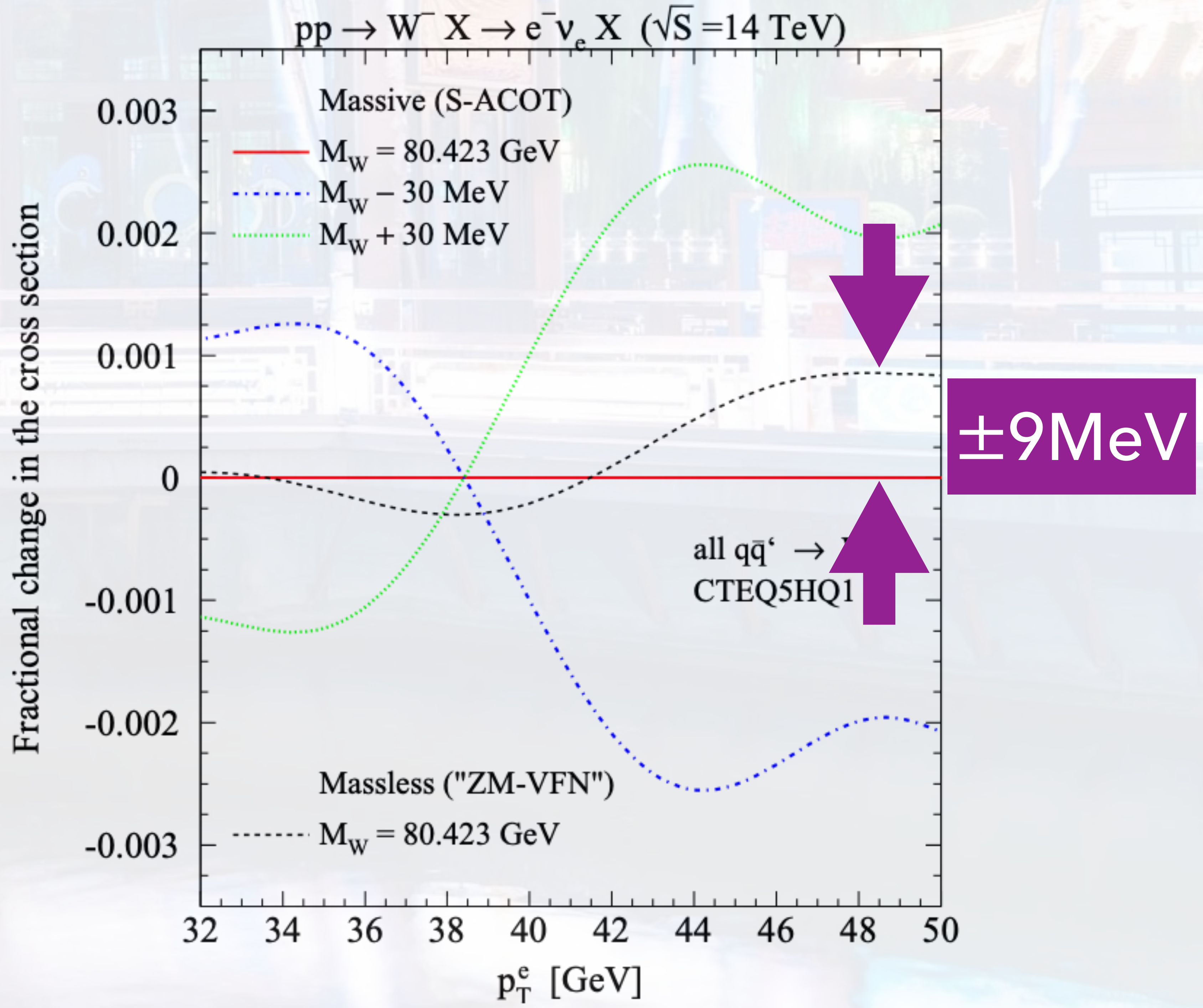
➤NLO+NLL indicate 9 MeV (LHC) and 3 MeV (Tevatron) shift of mW.
Berge, Nadolsky, Olness `05

➤Extension to NNLL' is available
Pietrulewicz, Samitz, Spiering, Tackmann `17

➤Revisit m_q uncertainty in m_W is needed with modern tools! (FO, resummation scheme, PDF)

	W^+					W^-					Z^0				
Subprocesses	$u\bar{d}$	$u\bar{s}$	$c\bar{d}$	$c\bar{s}$	$c\bar{b}$	$d\bar{u}$	$s\bar{u}$	$d\bar{c}$	$s\bar{c}$	$b\bar{c}$	$u\bar{u}$	$d\bar{d}$	$s\bar{s}$	$c\bar{c}$	$b\bar{b}$
Tevatron Run-2	90	2	1	7	0	90	2	1	7	0	57	35	5	2	1
LHC	74	4	1	21	0	67	2	3	28	0	36	34	15	9	6





Berge, Nadolsky, Olness `05

W MASS IN CDFII MEASUREMENT

- $d\sigma/dm_T^W$ Template fit to best best parameter values:
  - Relativistic Breit-Wigner form: $(s^2 - m_W^2 + is^2\Gamma_W/m_W)^{-1}$ with fixed Γ_W
  - Binned maximum-likelihood fit: (Poisson distribution cross bins)

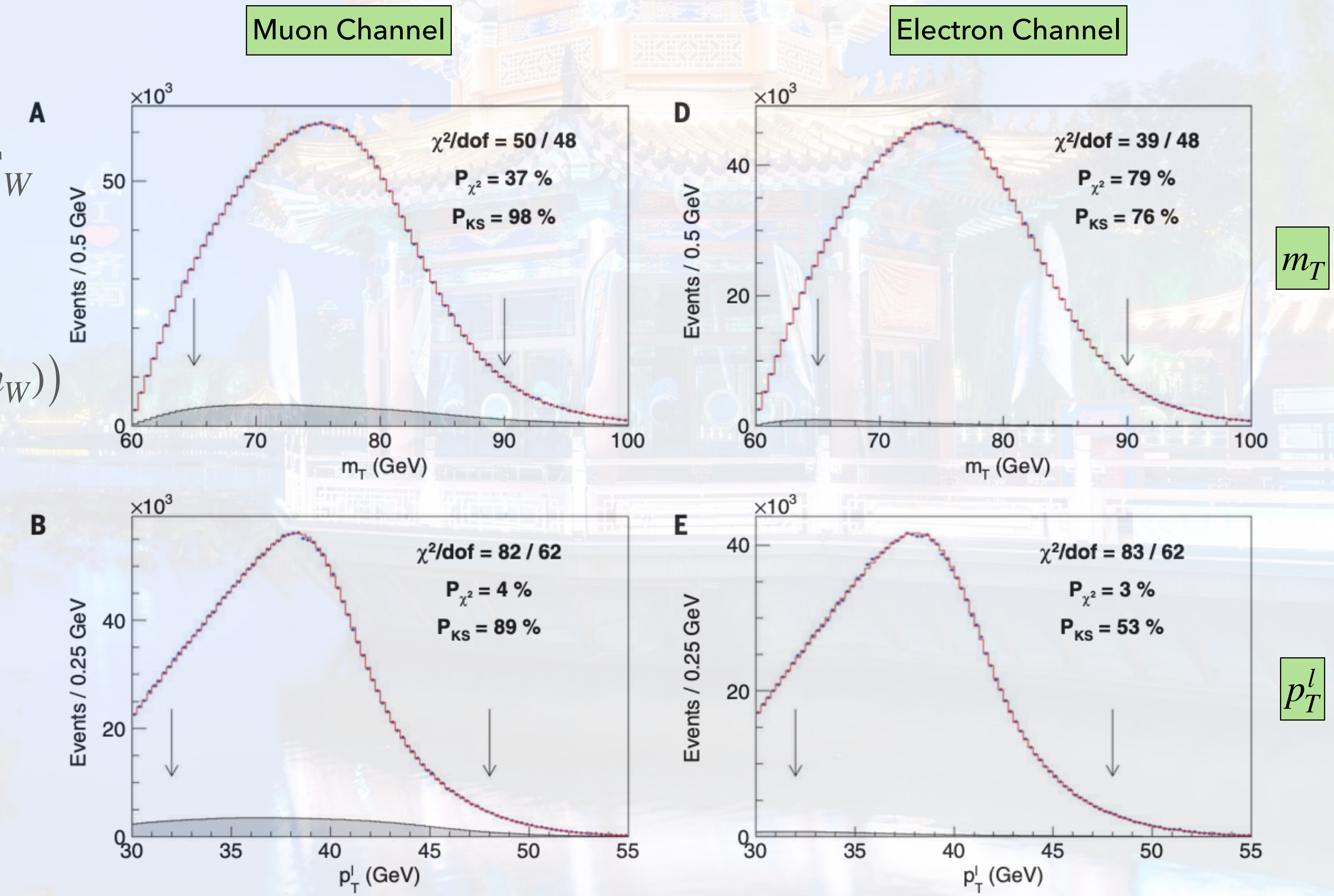
$$-\ln\mathscr{L}_b(m_W) = -\sum_b \left(n_b \ln\left(\Delta\sigma_b(m_W)\right) - \Delta\sigma_b(m_W)\right)$$

  n_b: observed event, $\Delta\sigma_b(m_W)$: predicted

  - The best linear unbiased estimator to combine each observable:
    - $\chi^2/\text{dof} = 7.4/5 \rightarrow$ p-value $= 20\%$
    - Weight distribution:

$$m_T^W \sim 64.2\,\%\,,\ p_T^l \sim 25.4\,\%\,,\ p_T^\nu \sim 10.4\,\%$$

CDFII: Best fitted results for m_T^W, p_T^l



PRECISION PREDICTIONS IN RESBOS2

- ResBos → ResBos2
  - NNLO+N3LL accuracy for W/Z production

    Isaacson, Fu, Yuan `23
  - Upgrade CSS formalism to N3LL
  - Rescale NLO to NNLO from MCFM:

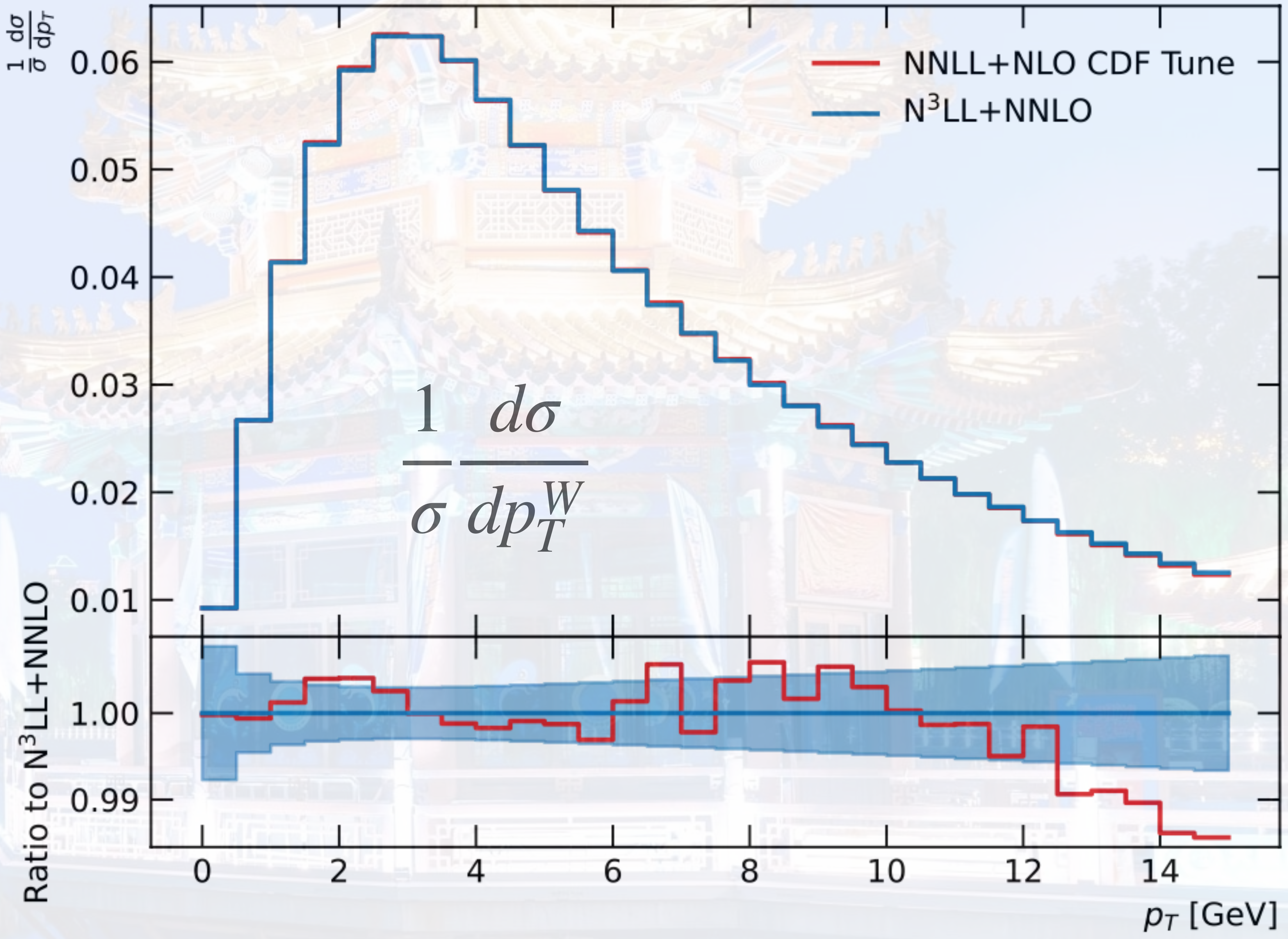
    Campbell, Ellis and Giele `15

$$\frac{d\sigma^{A_i}_{NLO}}{dp_T dy dQ} \to K^{A_i}_{\frac{NNLO}{NLO}}(p_T, y, Q)\frac{d\sigma^{A_i}_{NLO}}{dp_T dy dQ}$$

  - Dependence of angular coefficients recently included with more rescaling: $\frac{d\sigma}{d\cos\theta d\phi}$

$$= L_0(1+\cos^2\theta) + A_0(1-3\cos^2\theta) + A_1\sin 2\theta\cos\phi$$
$$+A_2\sin^2\theta\cos 2\phi + A_3\sin\theta\cos\phi + A_4\cos\theta$$
$$+A_5\sin^2\theta\sin 2\phi + A_6\sin 2\theta\sin\phi + A_7\sin\theta\sin\phi$$

Isaacson, Fu and Yuan`22 `23



- Pseudo data: NNLO+N3LL p_T^Z with global fit
- Fit g2, α_s in NLO+NNLL p_T^Z to pseudo data
- Use fitted g2, α_s in NLO+NNLL W templates

PRECISION PREDICTIONS IN RESBOS2

➤W mass details by ResBos2 Isaacson, Fu and Yuan`22 `23

Width	Mass Shift [MeV]
2.0475 GeV	2.0 ± 0.5
2.1315 GeV	0.3 ± 0.5
NLO	1.2 ± 0.5

	m_T		$p_T(\ell)$		$p_T(\nu)$	
PDF Set	NNLO	NLO	NNLO	NLO	NNLO	NLO
CT18	0.0 ± 1.3	1.8 ± 1.2	0.0 ± 15.9	2.0 ± 14.3	0.0 ± 15.5	2.9 ± 14.2
MMHT2014	1.0 ± 0.6	2.6 ± 0.6	6.2 ± 7.8	36.7 ± 7.0	3.9 ± 7.5	36.0 ± 6.7
NNPDF3.1	1.1 ± 0.3	2.1 ± 0.4	2.1 ± 3.8	13.5 ± 4.9	5.4 ± 3.7	10.0 ± 4.9
CTEQ6M	N/A	2.8 ± 0.9	N/A	19.0 ± 10.4	N/A	20.9 ± 10.2

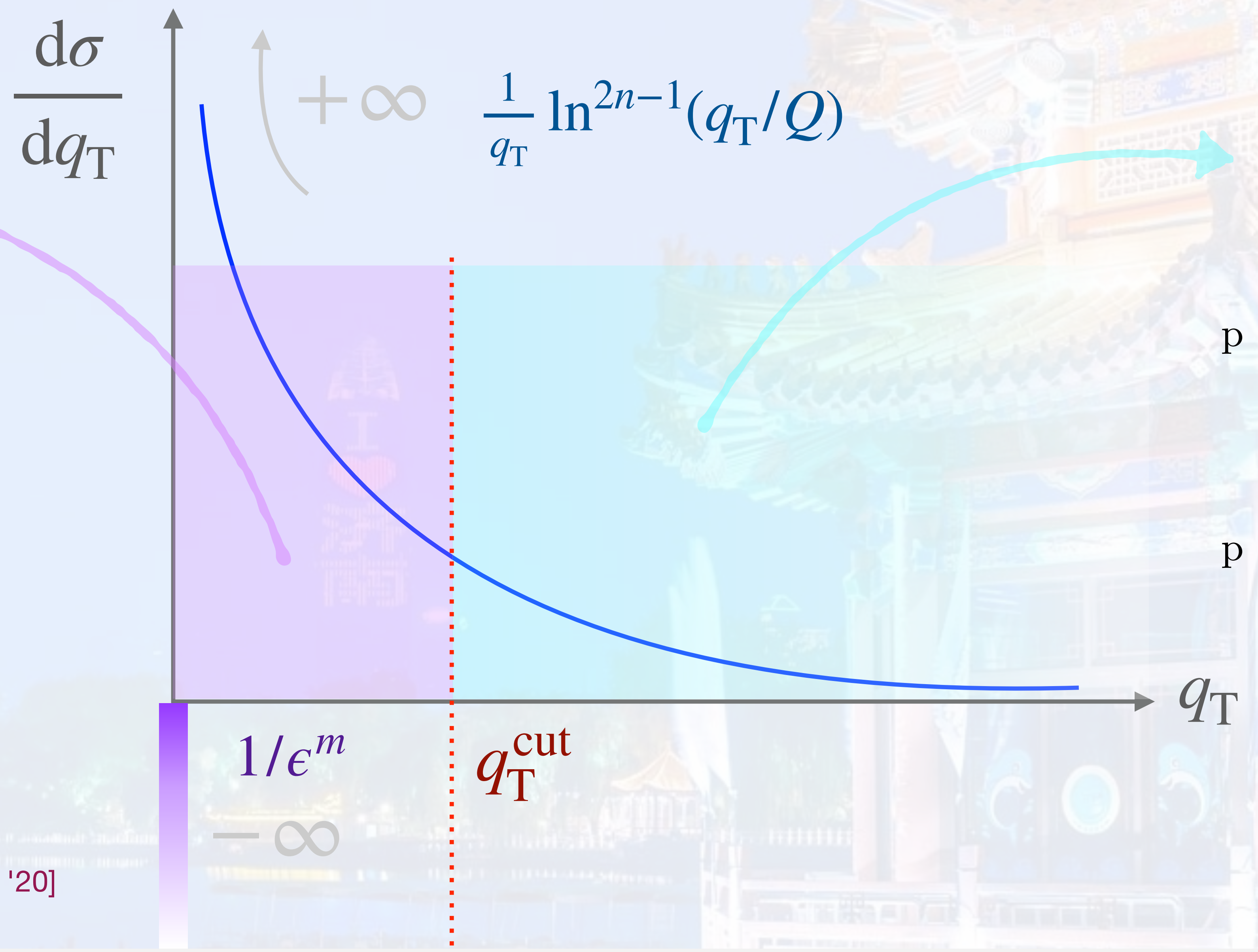
	Mass Shift [MeV]	
Observable	Smearing 1	Smearing 2
m_T	0.2 ± 1.8 ± 1.0	1.0 ± 2.1 ± 1.3
$p_T(\ell)$	4.3 ± 2.7 ± 1.3	4.5 ± 2.6 ± 1.4
$p_T(\nu)$	3.0 ± 3.4 ± 2.2	3.8 ± 4 ± 2.7

	Mass Shift [MeV]					
	m_T		$p_T(\ell)$		$p_T(\nu)$	
Scale	ResBos2	+Detector Effect+FSR	ResBos2	+Detector Effect+FSR	ResBos2	+Detector Effect+FSR
Upper	1.2 ± 0.5	0.8 ± 1.8 ± 1.1	3.1 ± 2.1	-6.5 ± 2.7 ± 1.3	1.4 ± 2.1	-4.9 ± 3.4 ± 2.0
Lower	1.2 ± 0.5	-0.7 ± 1.8 ± 01.	1.8 ± 2.1	9.4 ± 2.6 ± 1.2	0.0 ± 2.1	4.8 ± 3.4 ± 1.9

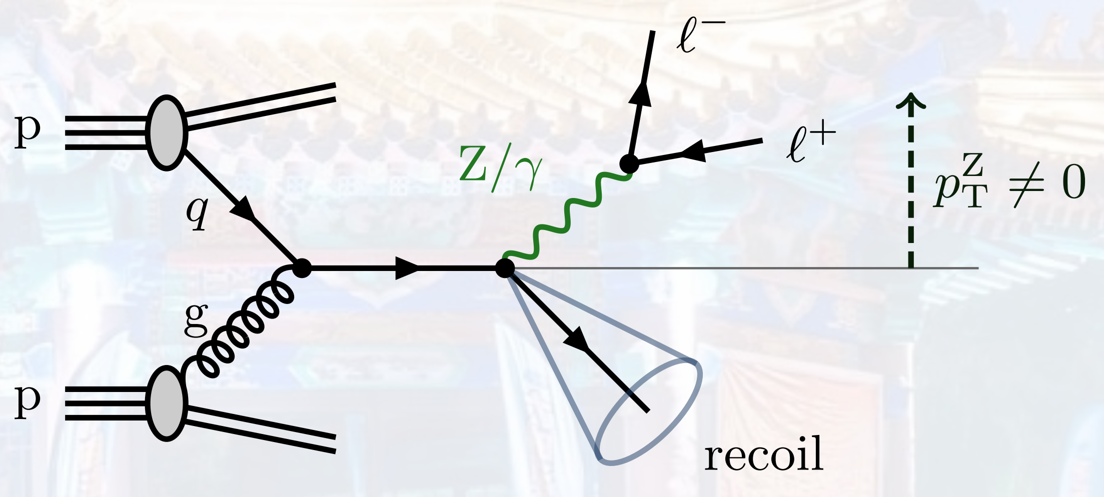
q_T SUBTRACTION @ N³LO

q_T resummation

- expand to fixed order
- $\mathcal{O}(\alpha_s^3)$ ingredients:
  - hard function $H_{q\bar{q}}$ [Gehrmann, Glover, Huber, Ikizlerli, Studerus '10]
  - soft function $S(\mathbf{b}_\perp)$ [Li, Zhu '16]
  - beam function $B_q(\mathbf{b}_\perp)$ [Luo, Yang, Zhu, Zhu '19] [Ebert, Mistlberger, Vita '20]



V+jet @ NNLO



[Catani, Grazzini '07]

$$d\sigma^V_{\rm N^3LO} = d\sigma^V_{\rm N^3LO}\Big|_{q_T<q_T^{\rm cut}} + d\sigma^V_{\rm N^3LO}\Big|_{q_T>q_T^{\rm cut}}$$

$$= \mathcal{H}^V_{\rm N^3LO} \otimes d\sigma^V_{\rm LO} + \left[d\sigma^{V+\rm jet}_{\rm NNLO} - d\sigma^{V,\rm CT}_{\rm N^3LO}\right]_{q_T>q_T^{\rm cut}} + \mathcal{O}\left((q_T^{\rm cut}/Q)^n\right)$$

Competing interests: $q_T^{\rm cut}$ as small as possible ⇜⇝ $q_T^{\rm cut}$ as large as possible

↪ suppress power corrections ↪ numerical stability & efficiency

BACKUP SLIDES

- ➤Differential N3LO predictions for neutral current production with fiducial cuts
  - ➤ Resum all order contributions at N3LL using RadISH and matched to N3LO

Order	σ [pb] Symmetric cuts		σ [pb] Product cuts	
k	N^{k}LO	N^{k}LO+N^{k}LL	N^{k}LO	N^{k}LO+N^{k}LL
0	$721.16^{+12.2\%}_{-13.2\%}$	—	$721.16^{+12.2\%}_{-13.2\%}$	—
1	$742.80(1)^{+2.7\%}_{-3.9\%}$	$748.58(3)^{+3.1\%}_{-10.2\%}$	$832.22(1)^{+2.7\%}_{-4.5\%}$	$831.91(2)^{+2.7\%}_{-10.4\%}$
2	$741.59(8)^{+0.42\%}_{-0.71\%}$	$740.75(5)^{+1.15\%}_{-2.66\%}$	$831.32(3)^{+0.59\%}_{-0.96\%}$	$830.98(4)^{+0.74\%}_{-2.73\%}$
3	$722.9(1.1)^{+0.68\%}_{-1.09\%} \pm 0.9$	$726.2(1.1)^{+1.07\%}_{-0.77\%}$	$816.8(1.1)^{+0.45\%}_{-0.73\%} \pm 0.8$	$816.6(1.1)^{+0.87\%}_{-0.69\%}$

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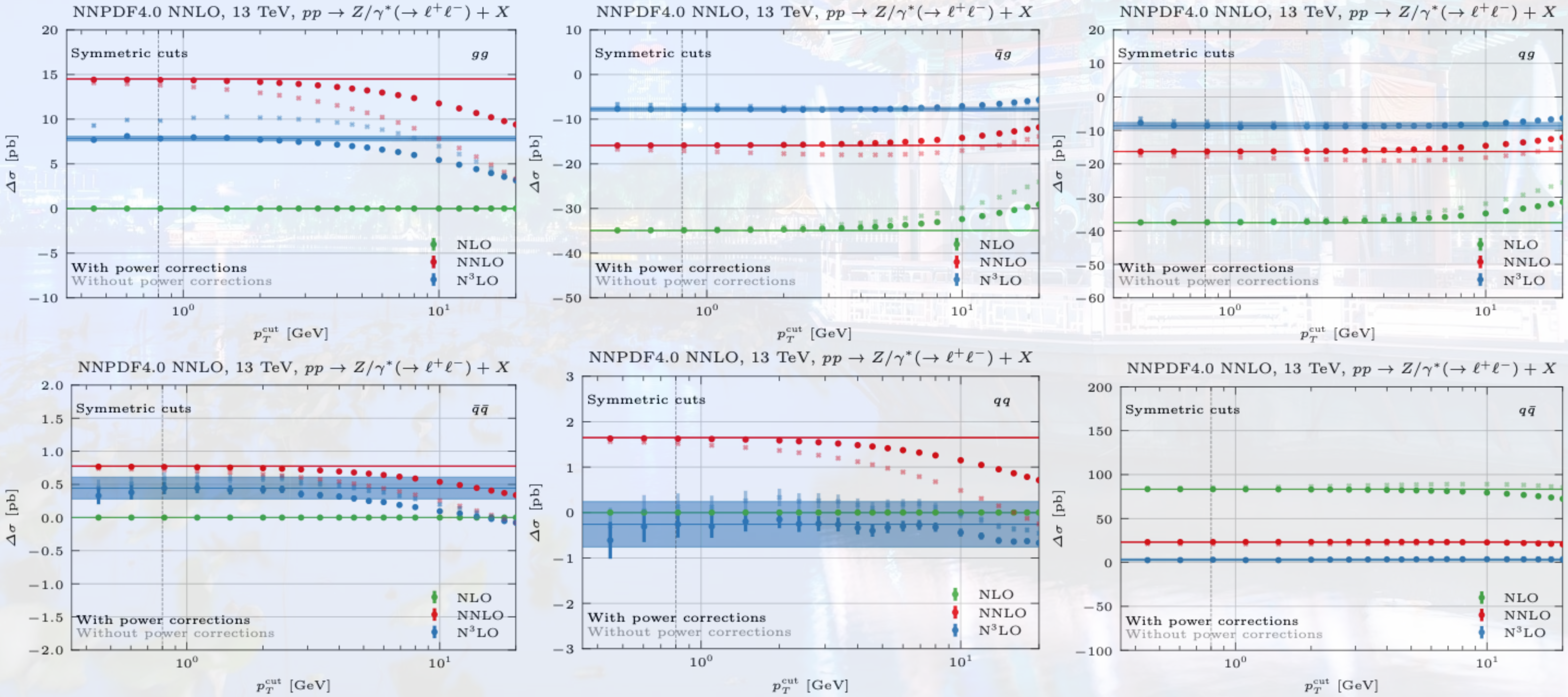
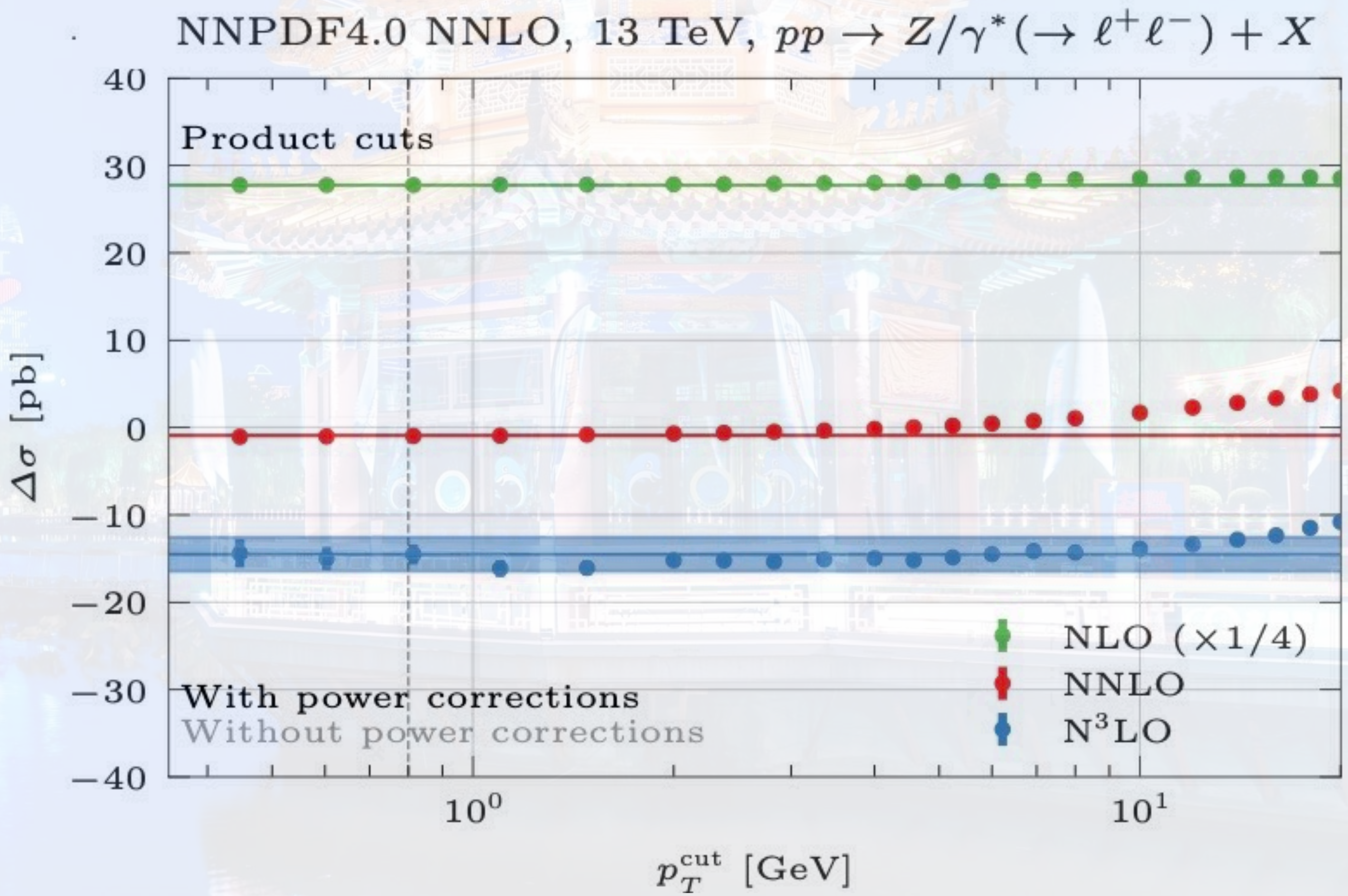


BACKUP SLIDES

- Differential N3LO predictions for neutral current production with fiducial cuts
  - Apply ATLAS fiducial cuts at 13 TeV
    - Dynamical scale $\mu_F = \mu_R = \sqrt{m_{ll}^2 + p_T^{ll2}}$
    - $m_{ll} \in [66,116]$ GeV, $|\eta^{l^\pm}| < 2.5$
    - Symmetric cuts: $|p_T^{l^\pm}| > 27$ GeV
      Introduce power correction at $\mathcal{O}(q_T^{cut}/m_{ll})$
    - Solution:
      - Apply Lorentz Boost below q_T^{cut}
        Buonocore, Rottoli, Kallweit, Wiesemann `21
        Camarda, Cieri, Ferrera `21
      - Product cuts: $\sqrt{p_T^{l^+} p_T^{l^-}} > 27$ GeV
        Salam, Slade `21
        $\min\{p_T^{l^+}, p_T^{l^-}\} > 20$ GeV
  - Typical fiducial cuts for m_T^V, p_T^V in DY production
    - Large log terms appear in $p_T^l \sim m_V/2$, $m_T^V \sim 2 \times \min[p_T^l]$, $p_T^V \sim 0$



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