Top-Quark Loops in $gg \to ZZ$ at NLO QCD

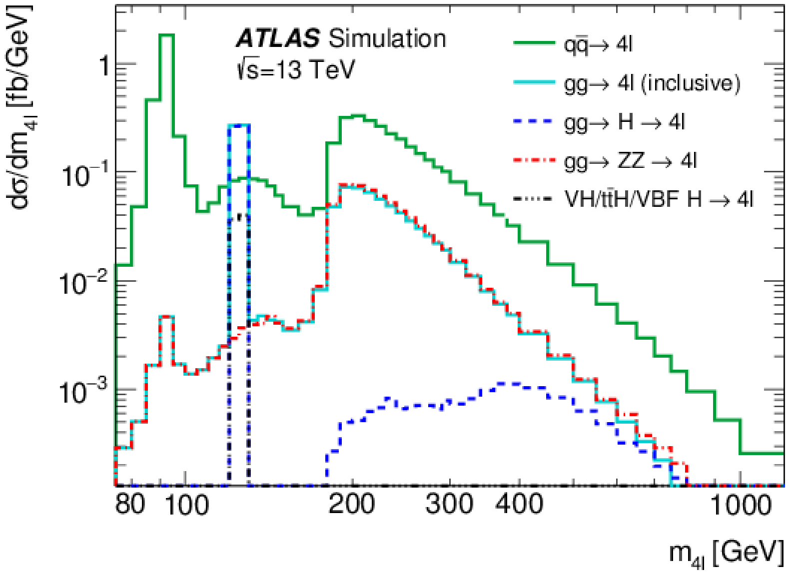
Marco Vitti (Karlsruhe Institute of Technology, TTP and IAP)

In collaboration with **G. Degrassi** and **R. Gröber** [2404.WXYZ]

Loops and Legs in Quantum Field Theory, Wittenberg, 16 Apr 2024

$pp \to ZZ$ at the LHC

- Probe of EW theory: polarisation measurements, Higgs production
- Indirect access to Higgs width [Kauer, Passarino – 1206.4803] [Caola, Melnikov – 1307.4935] [Campbell, Ellis, Williams - 1311.3589]



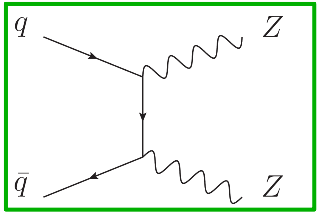
[ATLAS – 1902.05892]

Comparing on-shell and off-shell signal strengths

$$\frac{\mu_{\rm on}}{\mu_{\rm off}} \propto \frac{\kappa^2_{ggh}(m_h)\kappa^2_{hZZ}(m_h)}{\Gamma_h/\Gamma_h^{\rm SM}} \frac{1}{\kappa^2_{ggh}(m_{ZZ})\kappa^2_{hZZ}(m_{ZZ})},$$

Accurate theoretical predictions needed in both regions

Theoretical Predictions



Dominant contribution

NNLO QCD

[Brown, Mikaelian – (‘79); Ohnemus, Owens - (‘91); Mele, Nason, Ridolfi - (‘91); Cascioli et al. - 1405.2219; Heinrich et al. - 1710.06294; Gehrmann et al. - 1404.4853; Caola et al. - 1408.6409; Gehrmann et al. - 1503.04812; Grazzini et al. - 1507.06257; Kallweit, Wiesemann - 1806.05941]

NLO EW

[Bierweiler et al. – 1305.5402; Baglio, Ninh, Weber – 1307.4331; Chiesa et al. - 2005.12146]

$\sqrt{s}$	8 TeV	13 TeV	8 TeV	13 TeV
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LO	$8.1881(8)^{+2.4\%}_{-3.2\%}$	$13.933(1)^{+5.5\%}_{-6.4\%}$	-27.5%	-29.8%
NLO	$11.2958(4)^{+2.5\%}_{-2.0\%}$	$19.8454(7)^{+2.5\%}_{-2.1\%}$	0%	0%
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ggNLO$_{gg}$	$1.4787(4)^{+15.9\%}_{-13.1\%}$	$3.626(1)^{+15.2\%}_{-12.7\%}$	$+86.3\%$	$+80.8\%$
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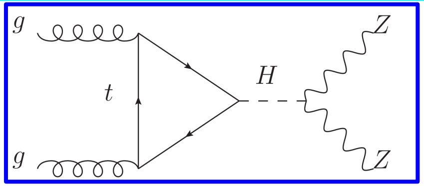
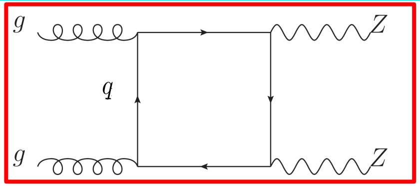
[Grazzini, Kallweit, Wiesemann, Yook - 1811.09593]

Theoretical Predictions





LO loop-induced (α_s^2 correction)

[Dicus, Kao, Repko – (‘87); Glover, Van der Bij – (‘89)]

Contributes to ~10% of hadronic xsec

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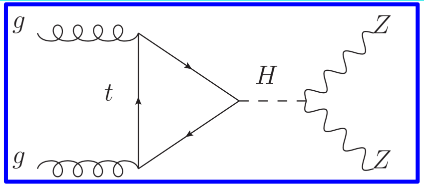

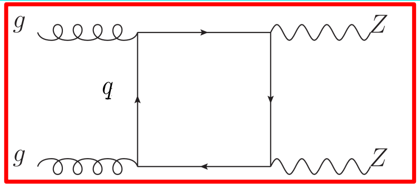

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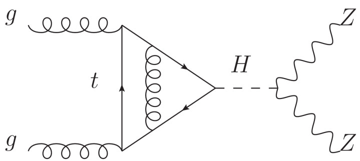
Virtual NLO QCD

Higgs-mediated

[Spira et al. - 9504378 ; Aglietti et al. - 0611266 ;
Harlander, Kant - 0509189; Anastasiou et al. - 0611236]



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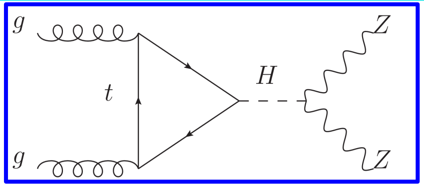
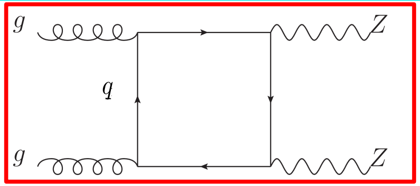




LO loop-induced (α_s^2 correction)

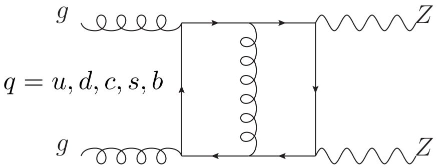
[Dicus, Kao, Repko – ('87); Glover, Van der Bij – ('89)]

Contributes to ~10% of hadronic xsec

Virtual NLO QCD

Non-resonant (light quarks)

[von Manteuffel, Tancredi – 1503.08835;
Caola et al. - 1509.06734]



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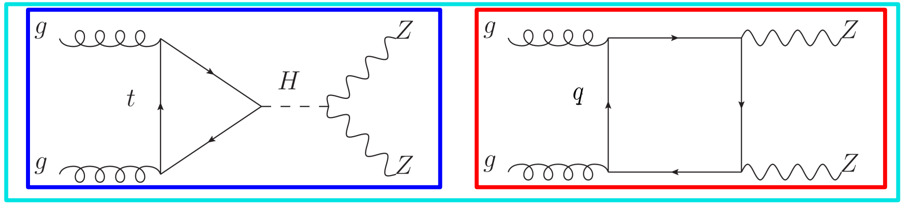

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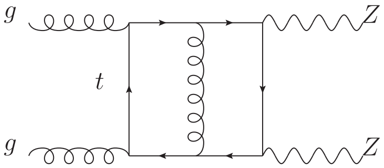
Contributes to ~10% of hadronic xsec

Virtual NLO QCD

Non-resonant (top quark)

No exact results in full analytic form



$$\Rightarrow I(\hat{s}, \hat{t}, m_Z^2, m_t^2)$$

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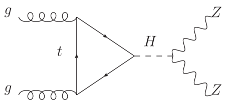
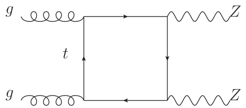
[Grazzini, Kallweit, Wiesemann, Yook - 1811.09593]

Importance of top-quark effects

$2\,Re\,($  $*$  $)$

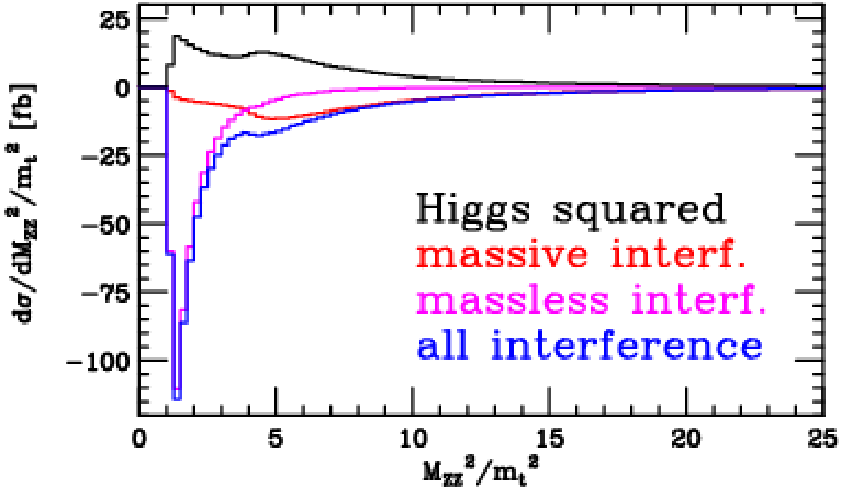
- Dominant contribution to interference for large invariant masses
- Exact numerical results available

  [Agarwal, Jones, von Manteuffel - 2011.15113 ; Brønnum-Hansen, Wang – 2101.12095]

  (see Chen Yu's Talk)

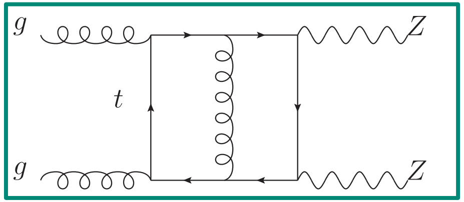
- Large effects found also at NLO QCD

  [Agarwal, Jones, Kerner, von Manteuffel - 2404.05684]



[Campbell et al. - 1605.01380]

Analytic Approximations



Exploit **hierarchies** of masses/kinematic invariants

Pros: simplified integral structures; can change parameters easily

Cons: proliferation of integrals; restricted to specific phase-space regions

- Limit $m_t \to \infty$
  [Dowling, Melnikov – 1503.01274; Caola, et al. – 1605.04610]
- Large mass expansion (LME)
  [Campbell et al. - 1605.01380; Gröber, Maier, Rauh – 1908.04061]
- High-energy expansion: $m_Z^2 \ll m_t^2 \ll \hat{s}, \hat{t}$
  [Davies, Mishima, Steinhauser, Wellmann - 2002.05558]

This talk: pT expansion $m_Z^2, p_T^2 \ll m_t^2, \hat{s}$

Previously applied to

$gg \to HH$ [Bonciani, Degrassi, Giardino, Gröber - 1806.11564]

$gg \to ZH$ [Alasfar, Degrassi, Giardino, Gröber, MV - 2103.06225]

pT Expansion - Calculation Overview

Karlsruhe Institute of Technology

1. Generation of Feynman diagrams - O(100 diags) (FeynArts [Hahn - 0012260])

2. Lorentz decomposition of the amplitude: contractions, Dirac traces... (FeynCalc [Shtabovenko et al. - 2001.04407])

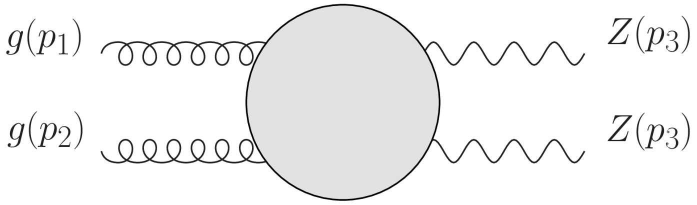
$$\mathcal{A}_{\mu\nu\rho\sigma} = \sum_{i=1}^{16} \mathcal{P}^{(i)}_{\mu\nu\rho\sigma} \, A^{(i)} \qquad A^{(i)} = \sum_{i=1}^{n} C^{(i)} \, I^{(i)}(\hat{s}, \hat{t}, m_Z^2, m_t^2)$$

3. Expansion of form factors in the limit of small p_T, m_Z (Mathematica)

4. Decomposition of scalar integrals using IBP identities (LiteRed [Lee - 1310.1145])

5. Evaluation of master integrals

pT Expansion - Details



- We assume the limit of a **forward kinematics**

$$(p_1 + p_3)^2 \to 0 \Leftrightarrow \hat{t} \to 0 \Rightarrow p_T \to 0$$

- Then Taylor-expand the form factors in the ratios

$$\boxed{\frac{m_Z^2}{\hat{s}}, \frac{p_T^2}{\hat{s}} \ll 1} \qquad \boxed{\frac{p_T^2}{4m_t^2} \ll 1}$$

Expansion at integrand level

- One scale removed

$$I(\hat{s}, p_T^2, m_Z^2, m_t^2) \to I'(\hat{s}, p_T^2, m_t^2)$$

- IBP Reduction of $I'(\hat{s}, p_T^2, m_t^2)$
- The MIs depend on the ratio $\hat{s}/m_t^2 \Rightarrow$ **single-scale integrals**

$$I(\hat{s}, p_T^2, m_Z^2, m_t^2) \to I'(\hat{s}, p_T^2, m_t^2) \to \mathrm{MI}(\hat{s}/mt^2)$$

pT Expansion - Master Integrals

- 52 MIs: same for $gg \to HH$, $gg \to ZH$

- 50 MIs expressed in terms of Generalized Polylogarithms

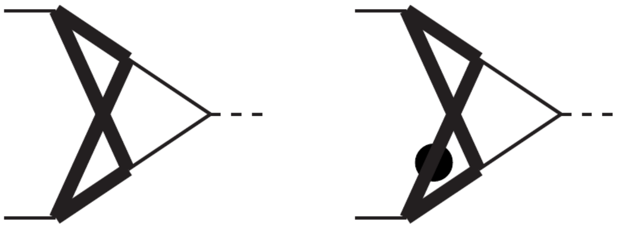
  [Bonciani, Mastrolia, Remiddi ('03) - Aglietti et al. ('06) - Anastasiou et al. ('06) - Caron-Huot, Henn ('14) - Becchetti, Bonciani ('17) - Bonciani, Degrassi, Vicini ('10)]

  Evaluated using handyG [Naterop, Signer, Ulrich - 1909.01656]

- Two elliptic integrals [von Manteuffel, Tancredi ('17)]

  Semi-analytical evaluation implemented in FORTRAN routine

  [Bonciani, Degrassi, Giardino, Gröber - 1812.02698]



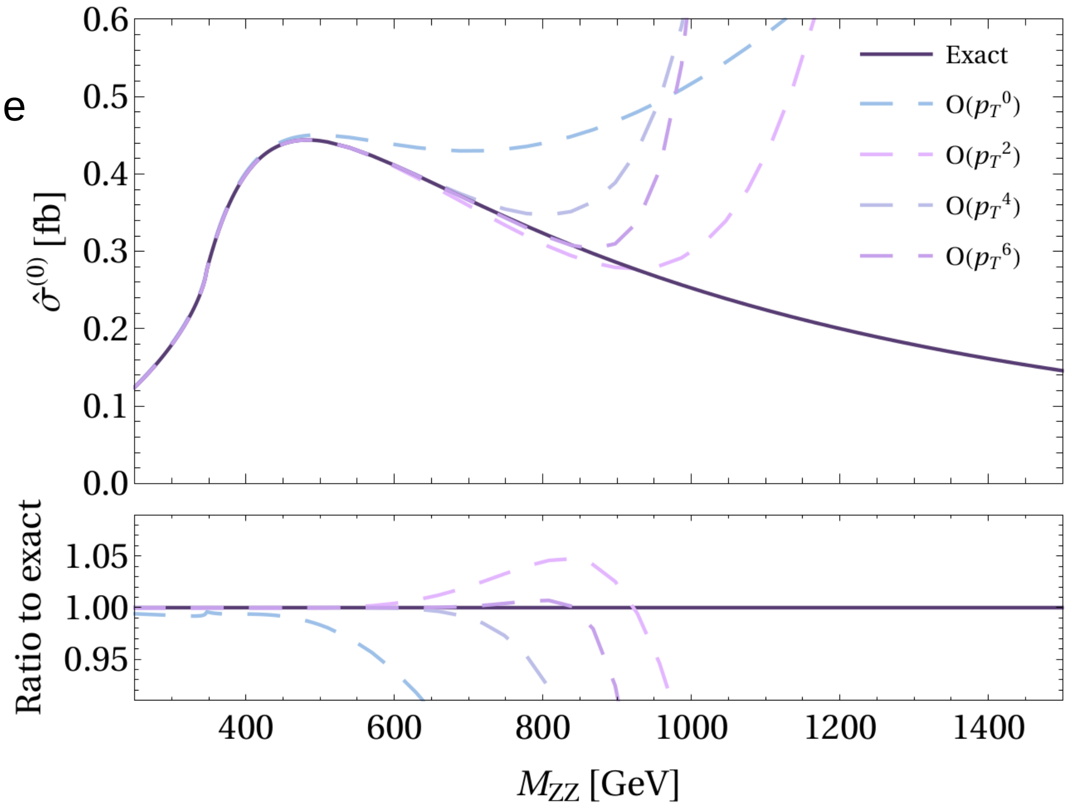
Validation at LO

- Three orders sufficient for permille accuracy

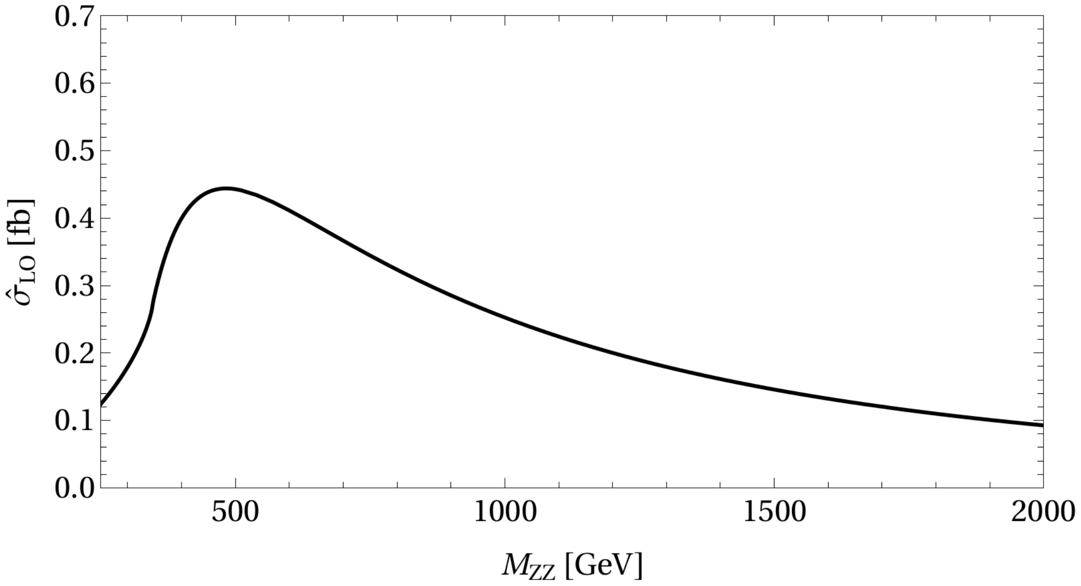
- For $M_{ZZ} \gtrsim 700\ \mathrm{GeV}$ the assumption

$$p_T^2 \ll 4m_t^2$$

can be violated



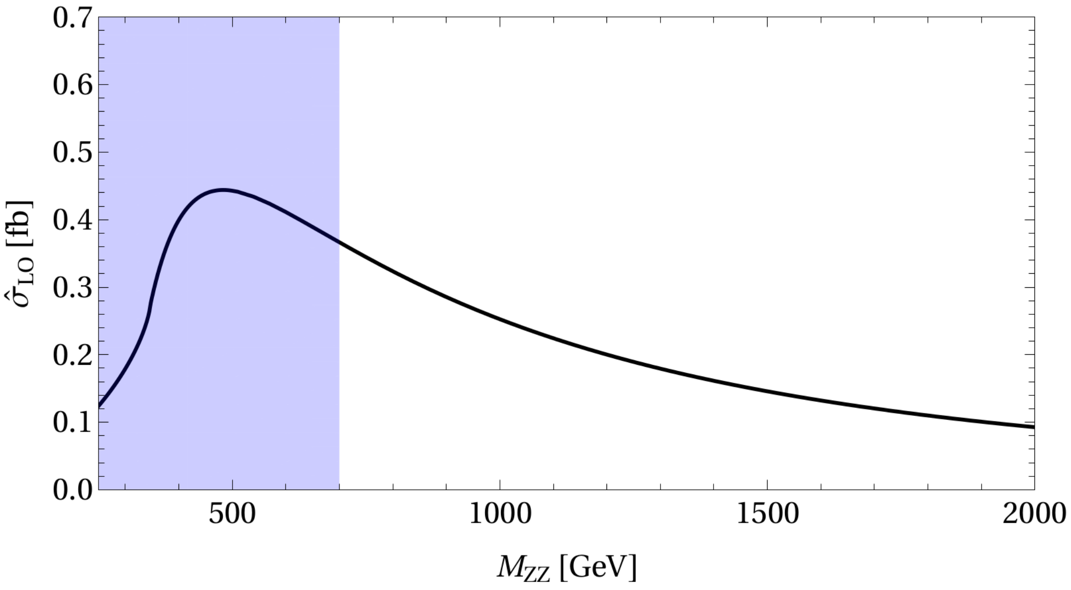
Complementing Phase-Space Coverage



Complementing Phase-Space Coverage

- p_T exp

$p_T^2 \lesssim 4m_t^2$
or
$|\hat{t}| \lesssim 4m_t^2$



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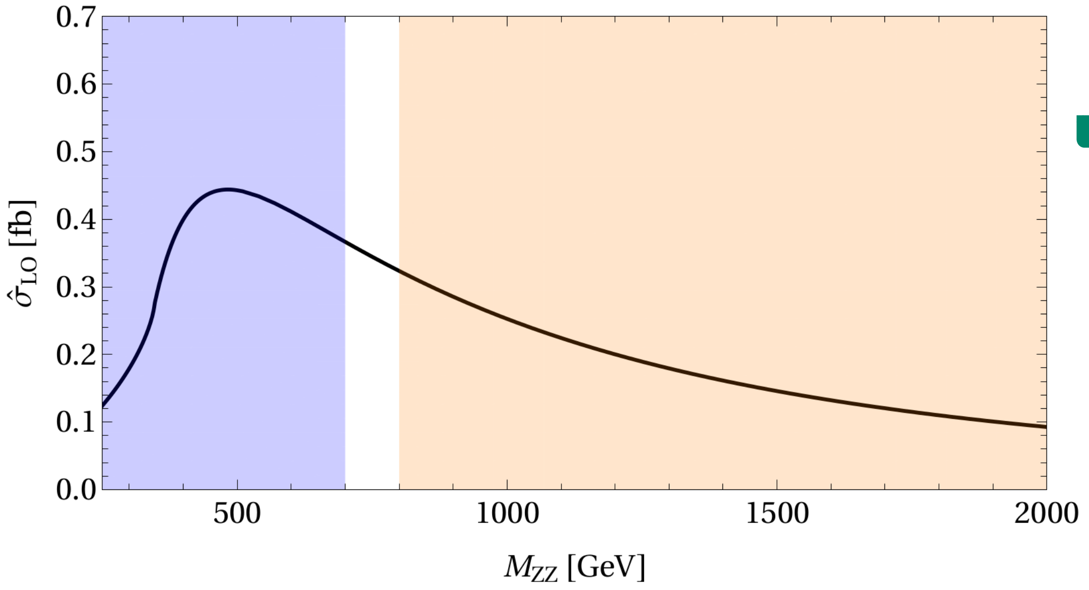


- High-Energy exp:

$|\hat{t}| \gtrsim 4m_t^2$

[Davies, Mishima, Steinhauser, Wellmann - 2002.05558]

The two expansions can be combined

Needed refinement using Padé approximants

[Bellafronte, Degrassi, Giardino, Gröber, MV -2103.06225]

Merging pT and HE Expansions at NLO

Improve the convergence of a series expansion by matching the coefficients of the **Padé approximant** [m/n] [e.g. Fleisher, Tarasov ('94) - Campbell et al. - 1605.01380]

$$f(x) \overset{x\to 0}{\simeq} c_0 + c_1 x + \cdots + c_q x^q \qquad f(x) \simeq [m/n](x) = \frac{a_0 + a_1 x + \cdots + a_m x^m}{1 + b_1 x + \cdots + b_n x^n} \quad (q = m + n)$$

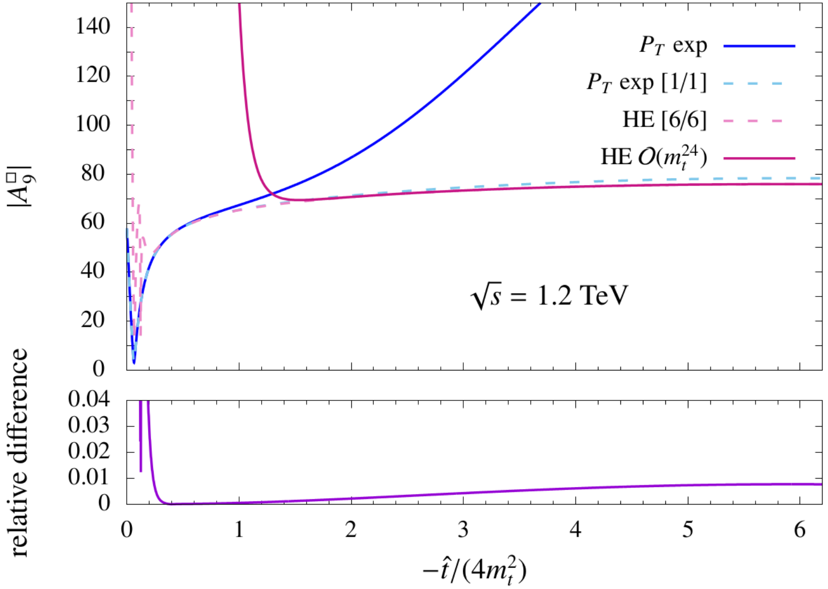
- For each FF we merged the following results

  - pT exp improved by [1/1] Padé

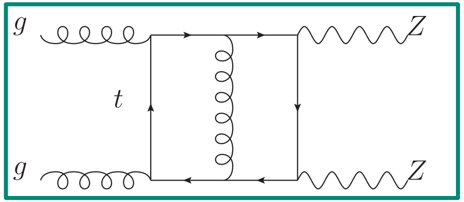
  - HE exp improved by [6/6] Padé
    [Davies, Mishima, Steinhauser, Wellmann - 2002.05558]

- Padé results are stable and comparable in the region $|\hat{t}| \sim 4 m_t^2$ $\rightarrow$ can switch without loss of accuracy

- Evaluation time for a phase-space point below 0.1 *s* $\Rightarrow$ suitable for Monte Carlo

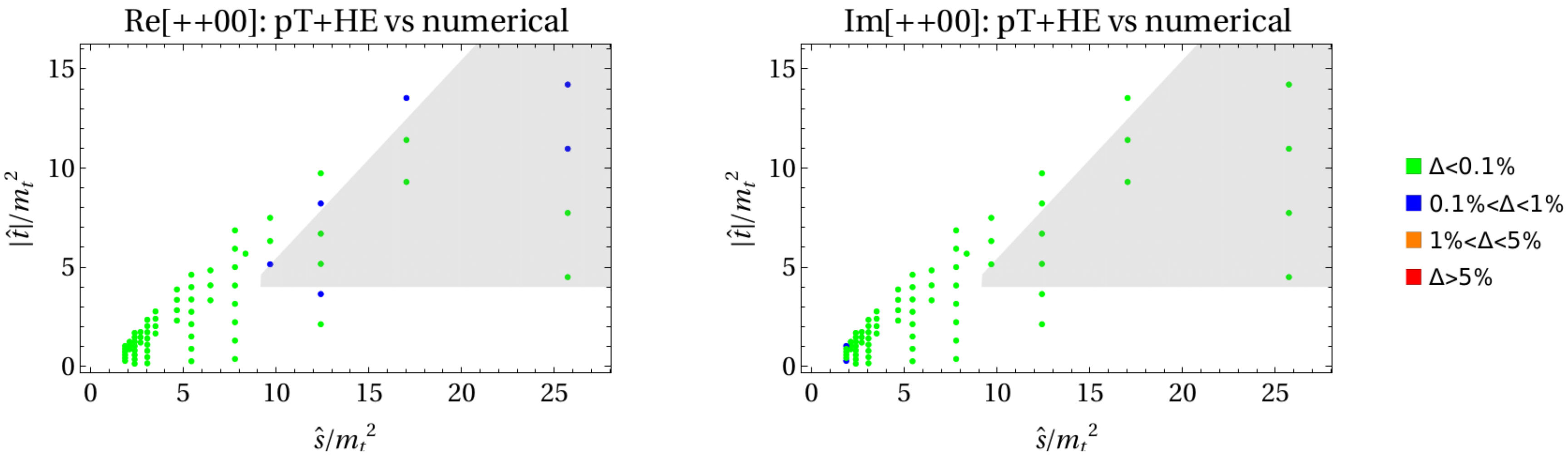


Comparing with Numerical Results



Comparison with helicity amplitudes of [Agarwal, Jones, von Manteuffel - 2011.15113]

$$\mathcal{M}^{\text{fin}}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} = \left(\frac{\alpha_s}{2\pi}\right)\mathcal{M}^{(1)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} + \left(\frac{\alpha_s}{2\pi}\right)^2 \boxed{\mathcal{M}^{(2)}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}} + \mathcal{O}(\alpha_s^3)$$



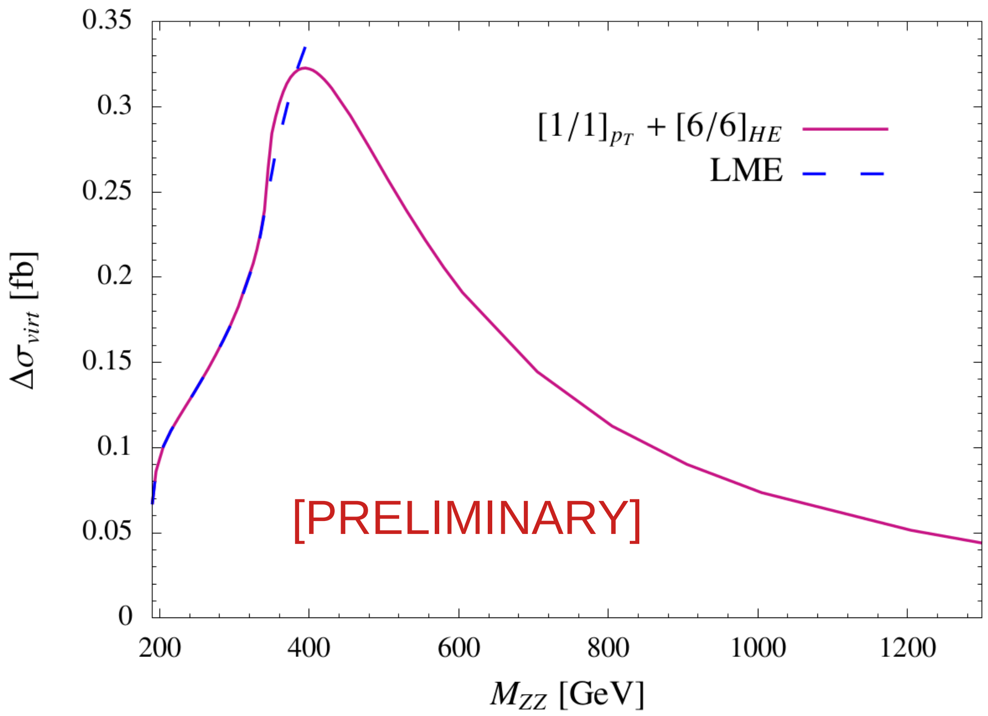
Results

$$\mathcal{V}_{\rm fin} = \frac{G_F^2 m_Z^4}{16}\left(\frac{\alpha_s}{\pi}\right)^2 \left\{ \sum_i \left|\mathcal{A}_i^{(0)}\right|^2 \frac{C_A}{2}\left(\pi^2 - \log^2\left(\frac{\mu_R^2}{\hat{s}}\right)\right) + 2\sum_i \mathrm{Re}\left[\mathcal{A}_i^{(0)}\left(\mathcal{A}_i^{(1)}\right)^*\right]\right\}$$

$$\Delta\sigma_{\rm virt} = \int_{\hat{t}^-}^{\hat{t}^+} d\hat{t}\,\frac{1}{2}\,\frac{1}{16\pi\hat{s}^2}\left(\frac{\alpha_s}{\pi}\right)\mathcal{V}_{\rm fin}(\hat{t})$$

LME from

[Davies, Mishima, Steinhauser, Wellmann - 2002.05558]

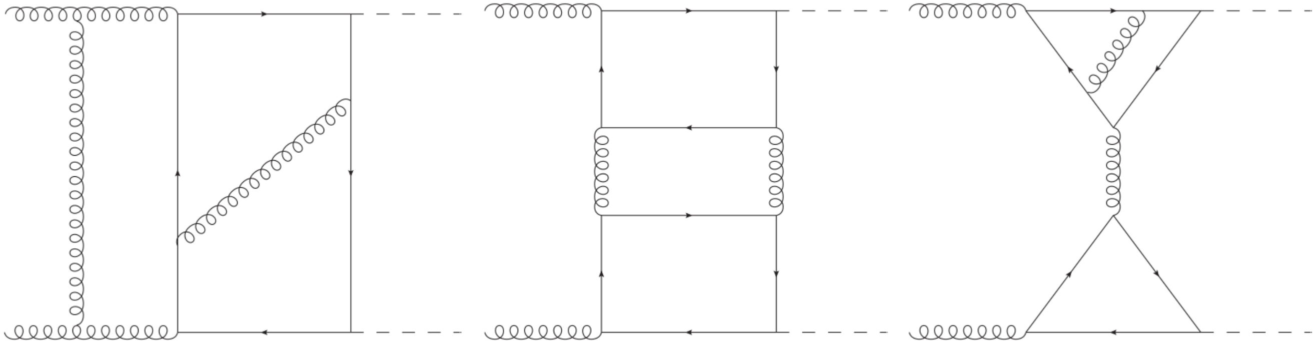


Conclusions

- We obtained an analytic approximation of the top-mediated virtual corrections to $gg \to ZZ$
- Combination of pT and HE expansion covers the complete phase space
- Accuracy suitable for pheno applications
- Implementation in Monte Carlo feasible
- Allows to study top-mass scheme uncertainty

Going to NNLO QCD...

- Can we use the forward expansion for higher orders?
- Consider a “simpler” case: $gg \to HH$



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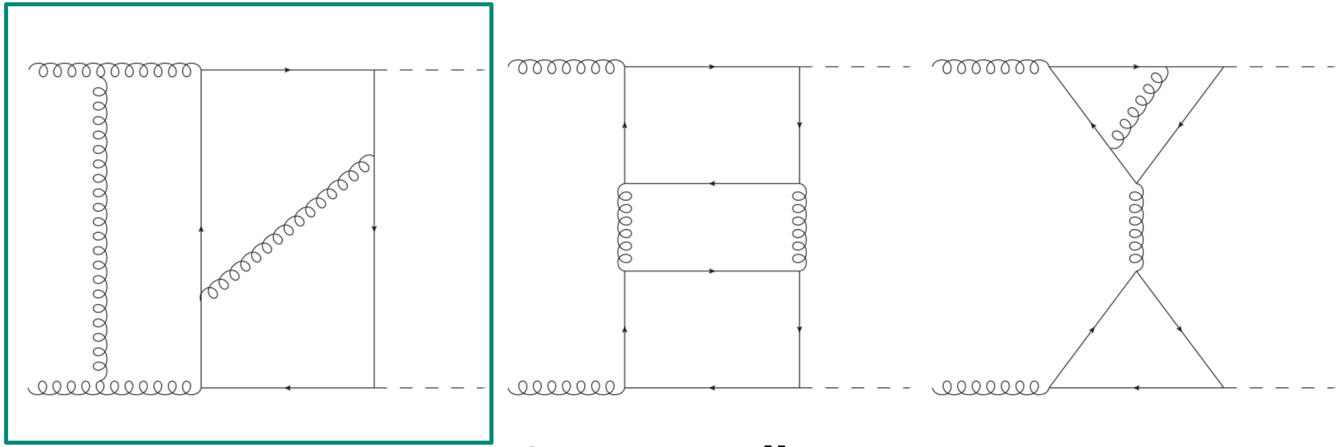

Conceptually yes

Practical implementation promising
for the $t \to 0$ expansion $\{t^0, m_H^0\}$
[Davies, Schönwald, Steinhauser 2307.04796]

(see also Josh’s talk)

Going to NNLO QCD...

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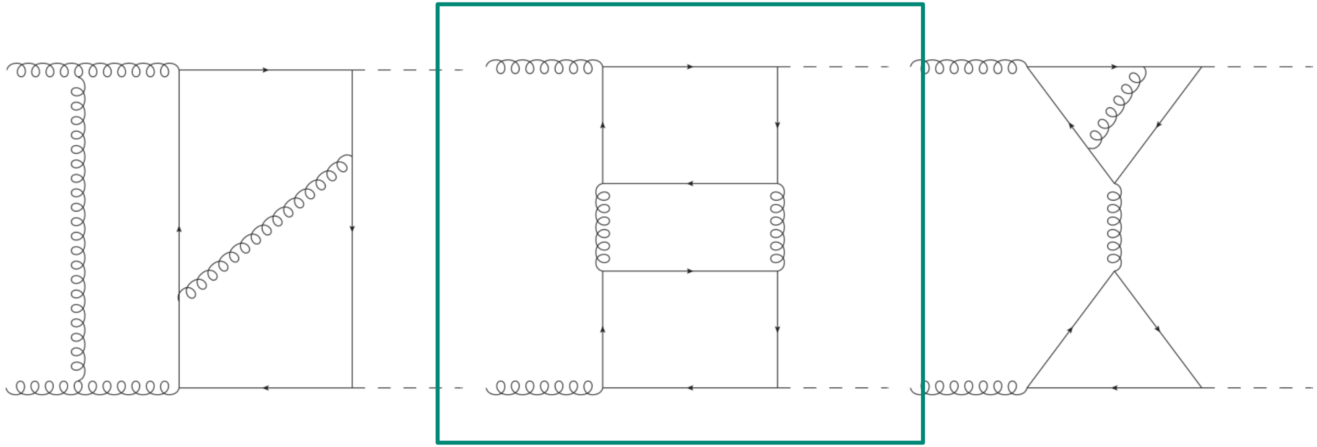

More involved due to t-channel
cuts through massless lines
$\Rightarrow$ A Taylor expansion is not suitable

Going to NNLO QCD...

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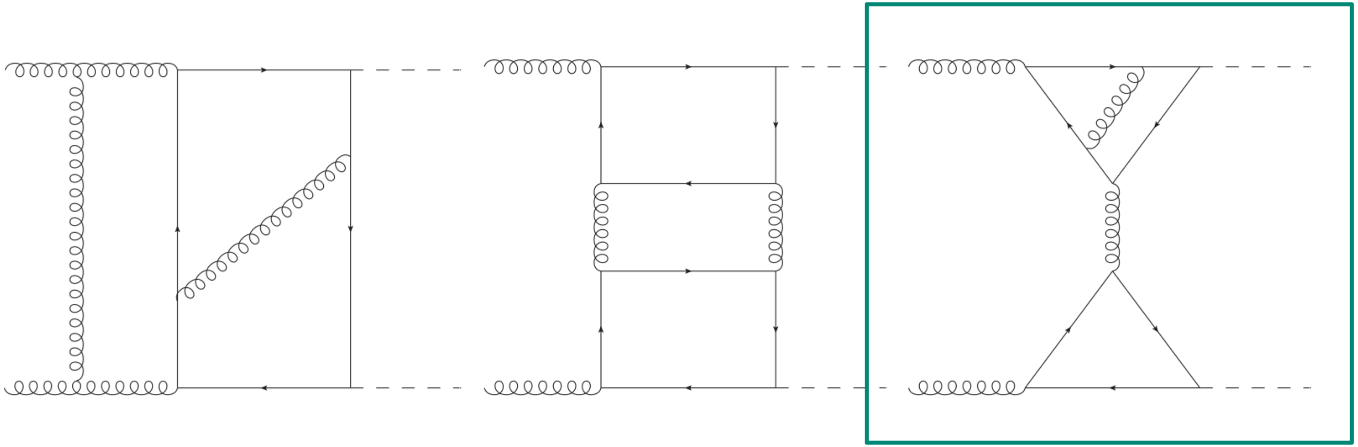
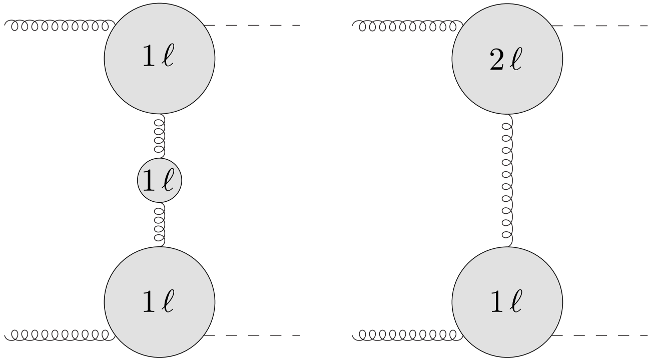


Start by studying the 1PR piece

1PR Contribution to *gg*→*HH* @ 3 Loops

(In collaboration with J. Davies, K. Schönwald, M. Steinhauser)

$$\mathcal{M}^{ab} = \varepsilon_{1,\mu}\varepsilon_{2,\nu}\mathcal{M}^{\mu\nu,ab} = \varepsilon_{1,\mu}\varepsilon_{2,\nu}\delta^{ab}X_0 s\left(F_1 A_1^{\mu\nu} + F_2 A_2^{\mu\nu}\right)$$

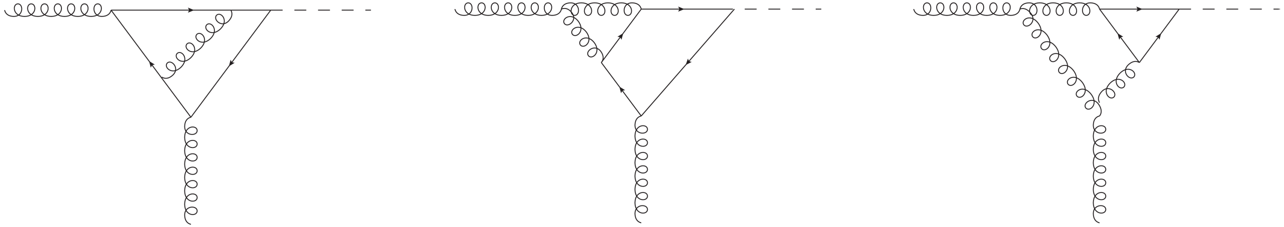


Goal: compute $F_1^{(3\ell,\ 1\mathrm{PR})}$ $F_2^{(3\ell,\ 1\mathrm{PR})}$

Approach: construct the *gg*→*HH* form factors from the 1PI ggH subamplitudes

$$\mathcal{V}^{\alpha\beta}(q_s, q_2) = F_a\ g^{\alpha\beta}(q_s \cdot q_2) + F_b\ q_s^\alpha q_2^\beta + F_c\ q_2^\alpha q_s^\beta + F_d\ q_s^\alpha q_s^\beta + F_e\ q_2^\alpha q_2^\beta$$

$$q_2^2 = 0, q_s^2 \neq 0$$
$$m_H \neq 0$$



Strategy

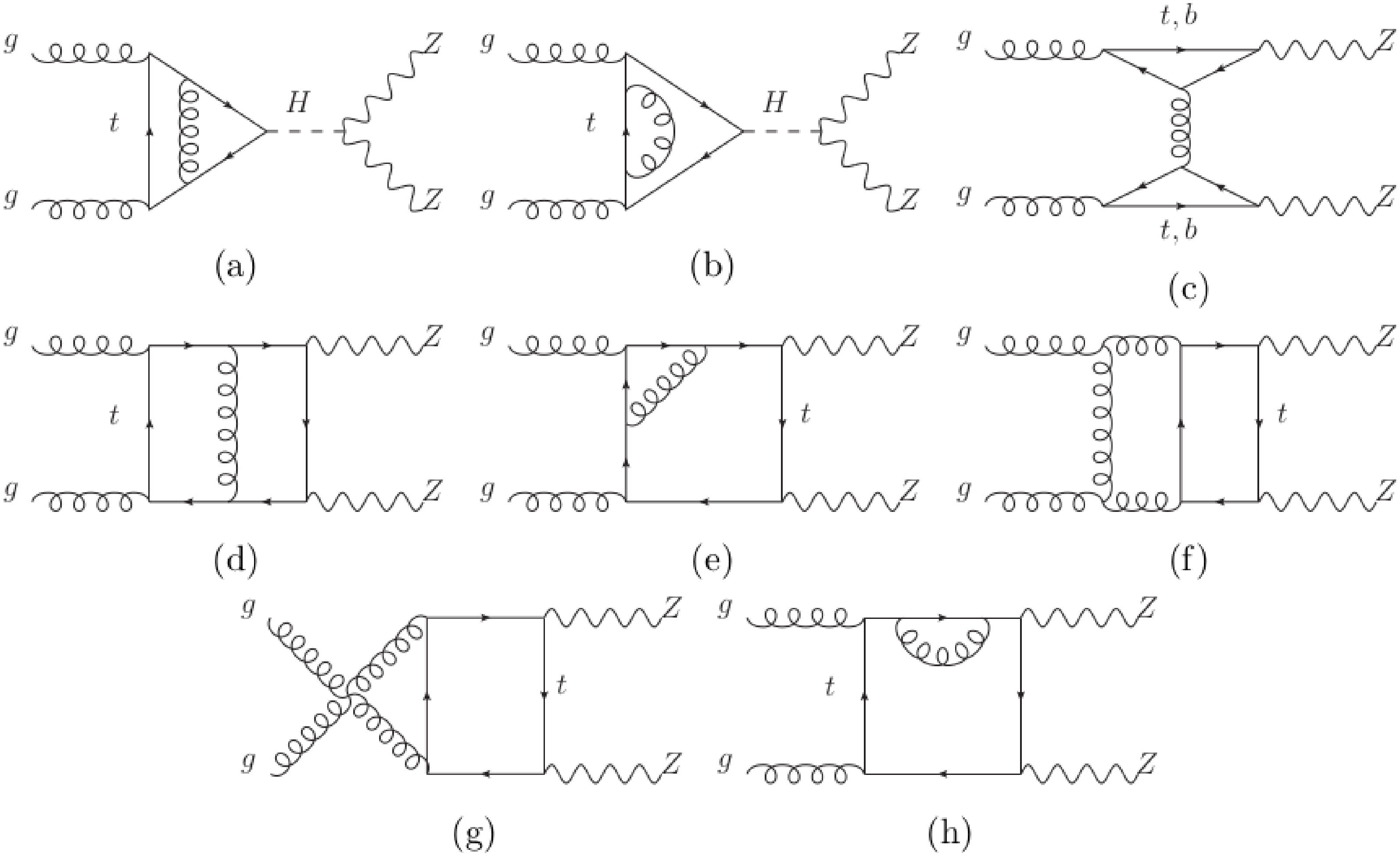
1. Generation of diagrams with qgraf [Nogueira, ‘93]
2. Manipulation with Tapir [Gerlach, Herren, Lang - 2201.05618], q2e/exp [Harlander, Seidensticker Steinhauser – ‘97], FORM [Ruijl, Ueda, Vermaseren - 1707.06453]
3. IBP reduction (KIRA [Klappert, Lange, Maierhöfer, Usovitsch - 2008.06494])
4. The MIs can be expanded for $m_H \to 0$ (LiteRed [Lee - 1310.1145])
5. Results mapped onto single-scale “forward” topologies [Davies, Mishima, Schönwald, Steinhauser - 2302.01356]
6. Evaluated semi-analytically using “expand-and-match” approach [Fael, Lange, Schönwald, Steinhauser – 2106.05296; 2202.05276]

Status

- Two-loop: results in agreement with [Degrassi, Giardino, Gröber – 1603.00385]
- Three-loop: agreement with LME result of [Davies, Steinhauser - 1909.01361]
- Numerical implementation of the results: in progress...

Thank you for your attention

Backup



g
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(g)
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(h)

pT expansion: example

Karlsruhe Institute of Technology

1) Consider a **one-loop** box integral

$$\int d^D q \; \frac{(q^2)^{n_1}(q\cdot p_1)^{n_2}(q\cdot p_2)^{n_3}(q\cdot p_3)^{n_4}}{(q^2-m_t^2)[(q+p_2)^2-m_t^2][(q-p_1-p_3)^2-m_t^2][(q-p_1)^2-m_t^2]}$$

2) Focus on the p3-dependent part; make transverse momentum explicit

$$\frac{(q\cdot p_3)^{n_4}}{[(q-p_1-p_3)^2-m_t^2]} \qquad\qquad p_3^\mu = -p_1^\mu - \frac{t'}{s'}(p_1-p_2)^\mu + r_\perp^\mu$$

3) In the forward limit $p_3^\mu \simeq -p_1^\mu$

$$\int d^D q \; \frac{(q^2)^{n_1}(q\cdot p_1)^{n_2'}(q\cdot p_2)^{n_3'}(q\cdot r_\perp)^{n_4'}}{(q^2-m_t^2)^{l_1}[(q+p_2)^2-m_t^2][(q-p_1)^2-m_t^2]}$$

4) IBP reduction $\rightarrow$ the MIs do not depend on $r_\perp$