Linear power corrections to top quark production processes

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based on JHEP05(2023)153, JHEP01(2024)074
with **S. Makarov**, **K. Melnikov**, **P. Nason**

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16 April 2024

Part I

Introduction: Renormalons & Power Corrections

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- *master equation* for hadron colliders:

$$d\sigma = \sum_{i,j} \int dx_1 dx_2\, f_{i/p}(x_1)\, f_{j/p}(x_2)\, d\hat{\sigma}_{ij}(x_1 P_1, x_2 P_2) \tag{1}$$

  - $f_{i/p}$: parton distribution function (PDF)
  - $d\hat{\sigma}_{ij}$: partonic cross-section

cross-sections: QCD corrections

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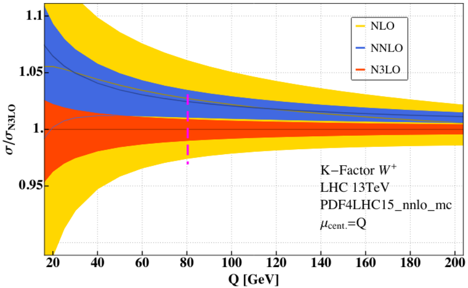

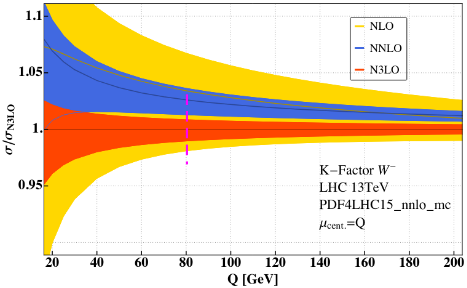

Drell-Yan N^{3}LO $\sim$ 1% correction

[Duhr, Dulat, Mistlberger; JHEP 11 (2020) 143]

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$\rightarrow$ **non-perturbative corrections may become relevant!**

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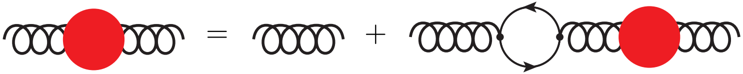

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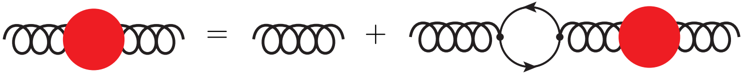
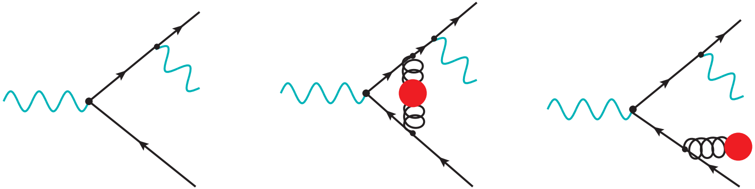


- example: 3-jet event $Z^*/\gamma^* \to q\bar{q}\gamma$



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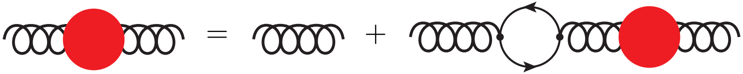

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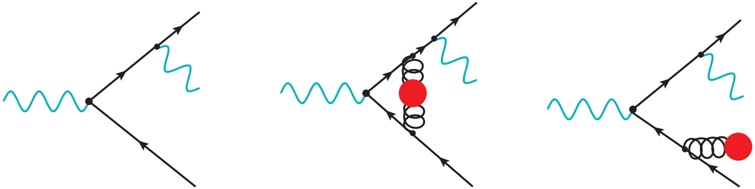


- each diagram can be computed perturbatively,

$$d\sigma = d\sigma^{(0)} + \left(\frac{\alpha_s}{\pi}\right) d\sigma^{(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 n_f\, d\sigma^{(2)} + \left(\frac{\alpha_s}{\pi}\right)^3 n_f^2\, d\sigma^{(3)} + \dots \quad (3)$$

non-perturbative physics: Renormalons

- can resum leading-n_f contributions via integral

$$\int_0^Q dk\ k^{p-1}\alpha_s(k) = \alpha_s(Q)\, Q^p \sum_{n=0}^{\infty} \underbrace{\left(\frac{\beta_0}{2\pi}\alpha_s(Q)\right)^n \frac{1}{p^{n+1}} n!}_{\text{factorial growth}}, \tag{4}$$

$$\text{with} \quad \alpha_s(\mu) = \frac{1}{\frac{\beta_0}{2\pi}\log\frac{\mu}{\Lambda_{\text{QCD}}}}, \quad \beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f\,. \tag{5}$$

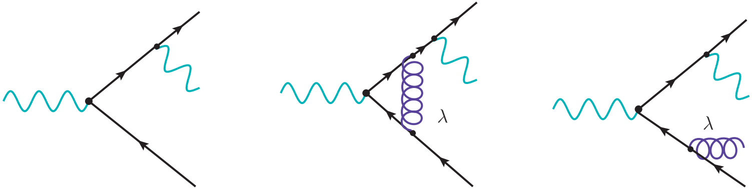
- series is not Borel summable, ambiguity given by

$$\int dk\ k^{p-1}\,\frac{2\pi}{\beta_0}\,\frac{\Lambda_{\text{QCD}}}{k-\Lambda_{\text{QCD}}} = \pm 2\pi i \frac{2\pi}{\beta_0}\Lambda_{\text{QCD}}^p$$

$\rightarrow$ ambiguity removed by non-perturbative power corrections $\Lambda_{\text{QCD}}^p/Q^p$

linear power corrections

- power corrections can be computed by considering perturbative corrections with massive gluon of mass λ



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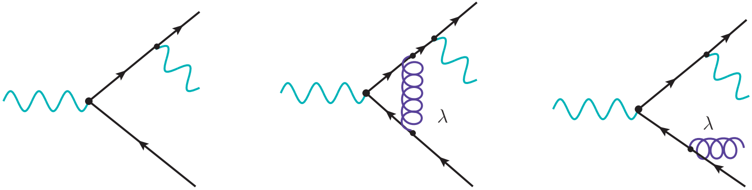

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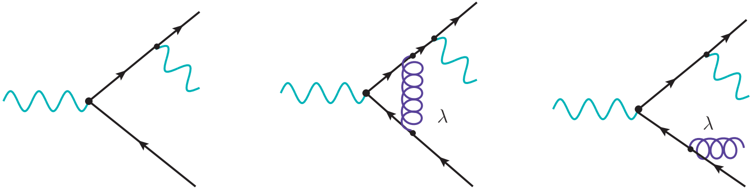


- direct relation between $\lambda^p \to \Lambda_{\mathrm{QCD}}^p$
- for phenomenological applications only linear terms λ/Q are relevant, higher orders in λ are surpressed by $\mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2/Q^2\right)$

non-perturbative physics

In order to compute linear power corrections with renormalon calculus,

- compute NLO corrections with a massive gluon of mass λ
- can be applied only to processes with no gluons in the Born process

Recent progresses with renormalon calculus,

- linear power corrections to event shape variables $\rightarrow$ can be used for α_s extraction

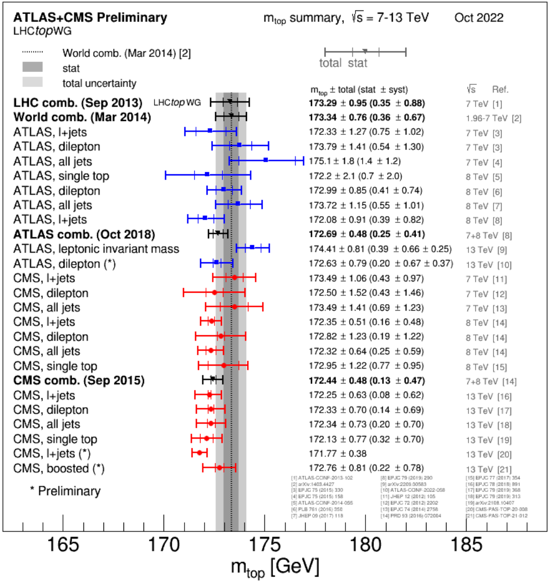
  [F. Caola, S. Ferrario Ravasio, G. Limatola, K. Melnikov, P. Nason, M.A. Ozcelik, JHEP 12 (2022) 062]

- linear power corrections to top quark processes at the LHC $\rightarrow$ may be used for m_t extract.

  [S. Makarov, K. Melnikov, P. Nason, M.A. Ozcelik, JHEP 05 (2023) 153 & JHEP 01 (2024) 074]

Top mass determination

- most precise top mass measurements at LHC come from "direct measurements"
  - → kinematic reconstruction of measured top quark decay products
- currently, typical error quoted to be around $\Delta m_t \sim 500$ MeV
  - → future (HL-LHC), envisaged uncertainty $\Delta m_t \sim 200$ MeV
- direct measurements → measured top quark mass is the value of the top mass parameter in the Monte Carlo generator



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$\rightarrow$ investigate mass scheme-dependence on linear power corrections to top quark production cross-sections and observables

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    $\to$ ideal process to study with renormalon calculus

Part II

Linear power corrections to top quark processes

[S. Makarov, K. Melnikov, P. Nason, M.A. Ozcelik, JHEP 05 (2023) 153 & JHEP 01 (2024) 074]

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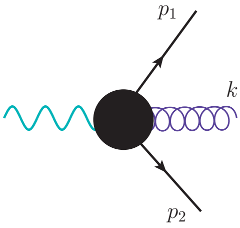

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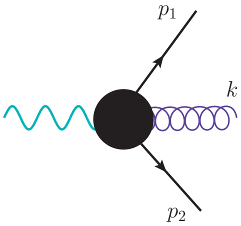

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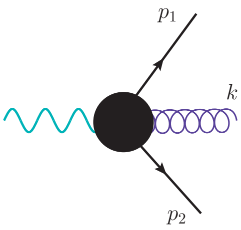

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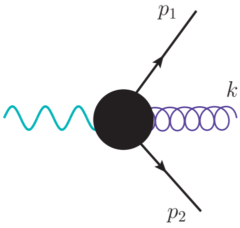

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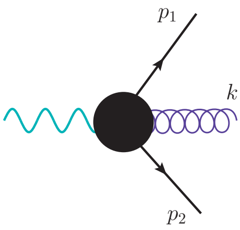

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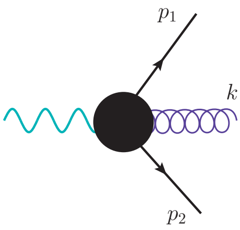

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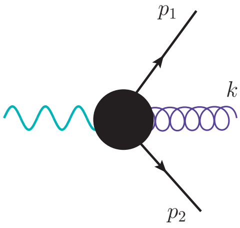

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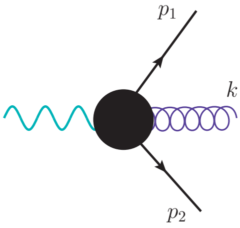

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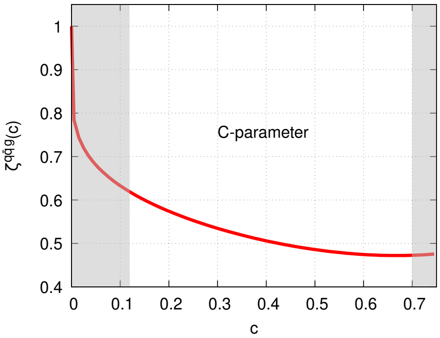
    $\rightarrow$ *presence* of linear power corrections

C-parameter: linear power corrections

linear power corrections to C-parameter: **[F. Caola et al, JHEP 12 (2022) 062]**

$$\mathcal{T}_\lambda[I_C] = \frac{15}{128\pi}\frac{s_{12}^3}{1-z_3}\left(\frac{\lambda}{q}\right)\left[\frac{(1+z_3)}{2}K\left(c_{12}^2\right) - (1-z_1 z_2)\,E\left(c_{12}^2\right)\right] \quad (7)$$



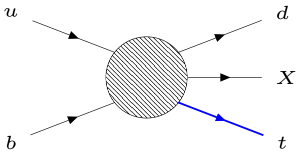
→ find agreement previous with results (2-jet limit, ...)

→ have **analytic** results for entire 3-jet region and these are superior to numerical methods

→ could be used for α_s determinations

[P. Nason, G. Zanderighi, JHEP 06 (2023) 058]

Massive case: presence or absence of linear power correction



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- single top production,
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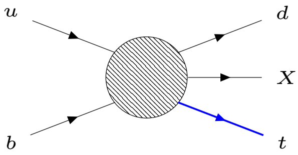

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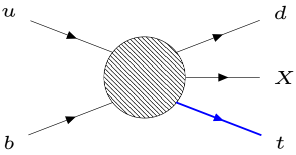

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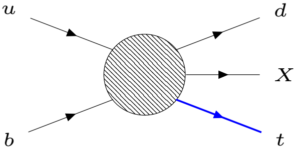

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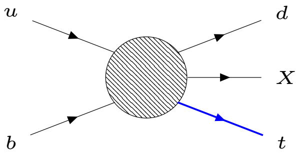

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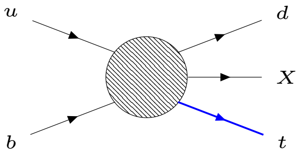

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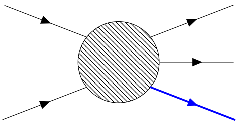

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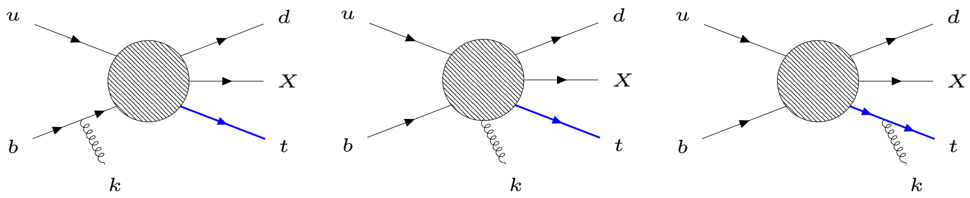
  $\rightarrow$ can use *Low-Burnett-Kroll* theorem

Real radiation: Low-Burnett-Kroll theorem



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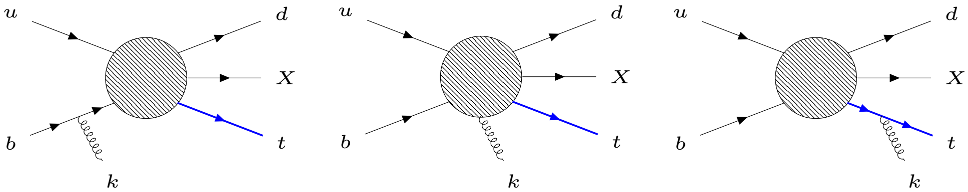

- the reduced real radiation amplitude $\mathcal{M}^\mu$ reads:

$$\begin{aligned}\mathcal{M}^\mu =& \bar{u}(q_t)\gamma^\mu \frac{\not{q}_t + \not{k} + m_t}{d_t} \mathrm{N}(q_t + k, p_b, q_d, \ldots) u(p_b) \\ &+ \bar{u}(q_t)\mathrm{N}(q_t, p_b - k, q_d, \ldots)\frac{\not{p}_b - \not{k}}{d_b}\gamma^\mu u(p_b) + \mathcal{M}^\mu_{\mathrm{reg}}(q_t, p_b, q_d, ..|k),\end{aligned} \quad (8)$$

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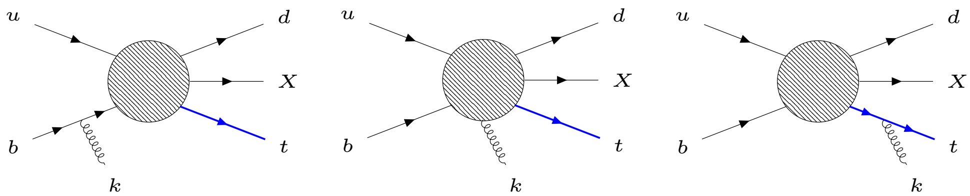

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- need $\mathcal{M}^\mu_{\mathrm{reg}}$ up to sub-leading term within $\mathcal{M}$

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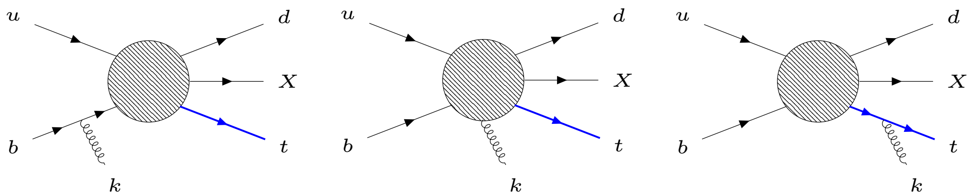

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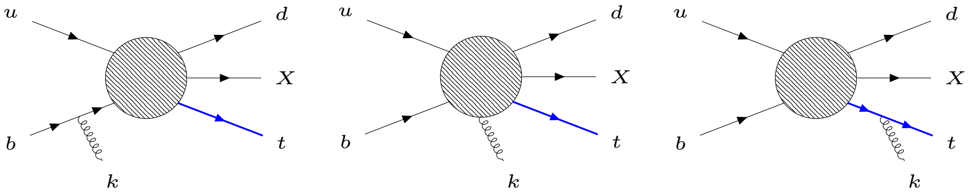

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$$\mathcal{M}^\mu_{\mathrm{reg}}(q_t, p_b, q_d, \ldots | k = 0) = -\bar{u}_t \left[\frac{\partial \mathrm{N}(q_t, p_b, q_d, \ldots)}{\partial q_{t,\mu}} + \frac{\partial \mathrm{N}(q_t, p_b, q_d, \ldots)}{\partial p_{b,\mu}} \right] u_b. \quad (8)$$

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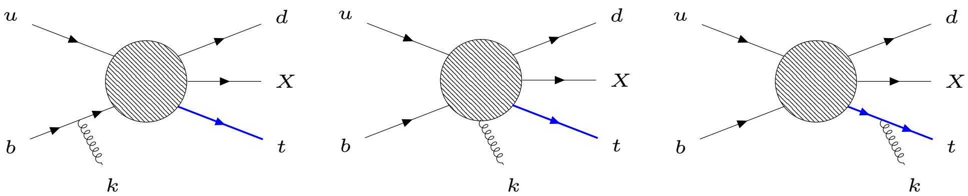

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- we now have all the ingredients
  $\rightarrow$ construct $|\mathcal{M}|^2$ up to *next-to-soft* approximation

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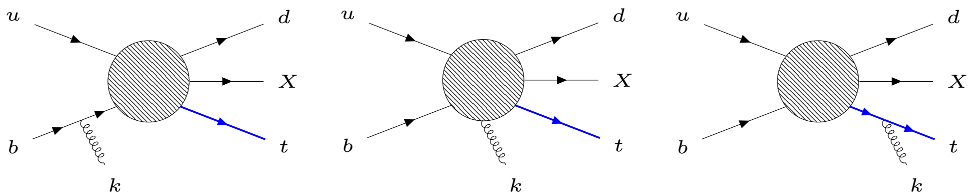

- we obtain

$$|\mathcal{M}|^2 = -J^\mu J_\mu F_{\rm LO}(q_t, p_b, q_d, ...) - J_\mu L^\mu F_{\rm LO}(q_t, p_b, q_d, ...). \quad (8)$$

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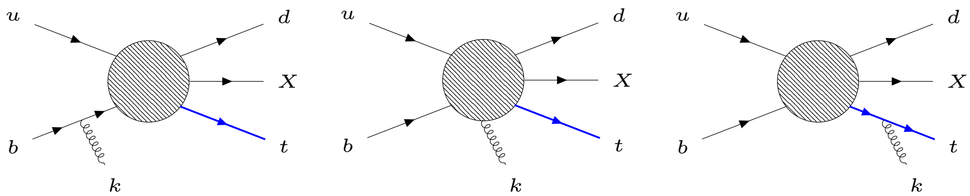

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$$J^\mu = J_t^\mu + J_b^\mu, \quad L^\mu = L_t^\mu - L_b^\mu, \quad J_t^\mu = \frac{2q_t + k^\mu}{d_t}, \quad J_b^\mu = \frac{2p_b^\mu - k^\mu}{d_b},$$

$$L_t^\mu = J_t^\mu k^\nu \frac{\partial}{\partial q_t^\nu} - \frac{\partial}{\partial q_{t,\mu}}, \qquad L_b^\mu = J_b^\mu k^\nu \frac{\partial}{\partial p_b^\nu} + \frac{\partial}{\partial p_{b,\mu}}$$

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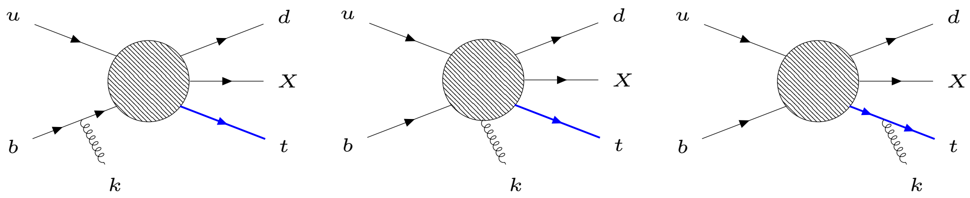

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- $|\mathcal{M}|^2$ in the next-to-soft approximation can be related to the leading-order $F_{\rm LO}$ in a *process-independent* manner

Real radiation: phase-space integration

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- perform momentum mapping:

$$q_t = p_t - k + \frac{p_t k}{p_t p_d} p_d, \qquad q_d = p_d - \frac{p_t k}{p_t p_d} p_d \tag{9}$$

$$q_t + q_d + k + p_X = p_t + p_d + p_X \tag{10}$$

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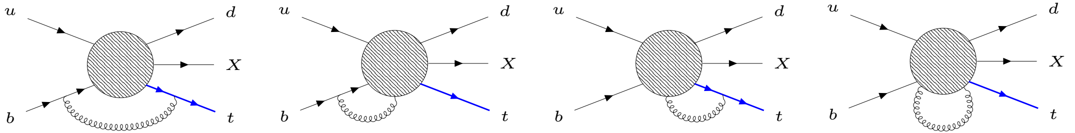
- can now write

$$\begin{aligned} \text{dLips}(p_u, p_b; q_d, q_t, p_X, k) = &\text{dLips}(p_u, p_b; p_d, p_t, p_X) \frac{d^4 k}{(2\pi)^3} \delta_+(k^2 - \lambda^2) \times \\ &\times \left[1 + \frac{p_d k}{p_t p_d} - \frac{p_t k}{p_t p_d}\right] + \mathcal{O}(\lambda^2) \end{aligned} \tag{11}$$

Virtual corrections: Low-Burnett-Kroll theorem



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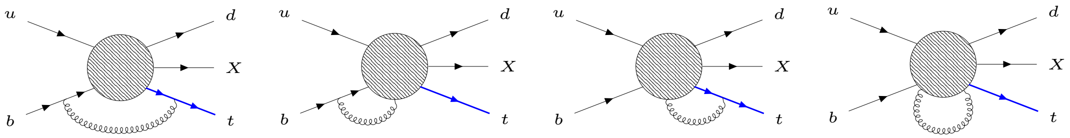

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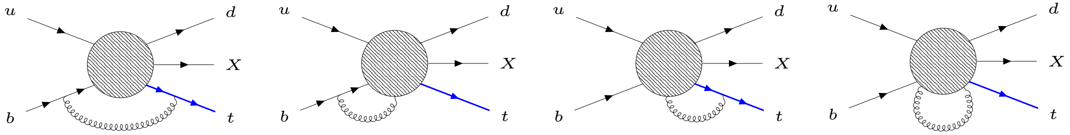

- can proceed in similar fashion as before and obtain

$$\delta[\mathcal{M}\mathcal{M}^+]_{\mathrm{virt}} = \int \frac{\mathrm{d}^4 k}{(2\pi)^4}\frac{-i}{k^2-\lambda^2}\Big[2J_t^\alpha J_{b,\alpha}F_{\mathrm{LO}} + J_t^\alpha J_{b,\alpha}k^\mu\, D_{p,\mu}F_{\mathrm{LO}} - (J_t^\alpha + J_b^\alpha)D_{p,\alpha}F_{\mathrm{LO}}$$

$$+ J_t^\alpha \mathrm{Tr}\left[(D_{p,\alpha}\not{p}_t)\mathrm{N}\not{p}_b\bar{\mathrm{N}}\right] + J_b^\alpha \mathrm{Tr}\left[(\not{p}_t + m_t)\mathrm{N}(D_{p,\alpha}\not{p}_b)\bar{\mathrm{N}}\right]\Big]. \quad (12)$$

$$D_p^\mu = \frac{\partial}{\partial p_{t,\mu}} + \frac{\partial}{\partial p_{b,\mu}}$$

Real & virtual contributions

- we can now integrate out gluon momentum and obtain for the real and virtual contributions:

$$\mathcal{T}_\lambda[\sigma_\mathrm{R}] = \frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}\int \mathrm{dLips}_\mathrm{LO}\Bigg[\left(\frac{3}{2} - \frac{m_t^2}{p_d p_t} - \frac{m_t^2}{p_t p_b}\right) - \frac{m_t^2}{p_d p_t} p_d^\mu \left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu}\right) - \frac{m_t^2}{p_t p_b} p_b^\mu \left(\frac{\partial}{\partial p_b^\mu} + \frac{\partial}{\partial p_t^\mu}\right)\Bigg] F_\mathrm{LO}. \quad (13)$$

$$\mathcal{T}_\lambda[\sigma_V] = -\frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}\int \mathrm{dLips}_\mathrm{LO}\Bigg[\mathrm{Tr}\left[\not{p}_t \mathrm{N} \not{p}_b \bar{\mathrm{N}}\right] + \left(\frac{2 p_t p_b - m_t^2}{p_t p_b} - \frac{m_t^2}{p_t p_b} p_b^\mu \left(\frac{\partial}{\partial p_b^\mu} + \frac{\partial}{\partial p_t^\mu}\right)\right) F_\mathrm{LO}\Bigg], \quad (14)$$

Renormalisation contribution

- we need to renormalise the heavy-quark wavefunction and mass in the on-shell scheme:

$$\begin{aligned} Z_m =& 1 + \frac{C_F g_s^2 m_t^{-2\epsilon}\Gamma(1+\epsilon)}{(4\pi)^{d/2}}\left[-\frac{3}{\epsilon} - 4 + \frac{2\pi\lambda}{m_t} + \mathcal{O}\left(\frac{\lambda^2}{m_t^2}\right)\right], \\ Z_2 =& 1 + \frac{C_F g_s^2 m_t^{-2\epsilon}\Gamma(1+\epsilon)}{(4\pi)^{d/2}}\left[-\frac{1}{\epsilon} - 4 + 4\ln\frac{m_t}{\lambda} + \frac{3\lambda\pi}{m_t} + \mathcal{O}\left(\frac{\lambda^2}{m_t^2}\right)\right]. \end{aligned} \tag{15}$$

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- the renormalisation contribution reads

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& + m_t \mathrm{Tr}\left[(\not{p}_t + m_t)\mathrm{N}\not{p}_b \frac{\partial \bar{\mathrm{N}}}{\partial m_t}\right]\Bigg].
\end{aligned}
\tag{16}
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Top quark mass scheme redefinition

- we now perform a mass scheme redefinition from pole-mass scheme to short-distance mass scheme

$$m_t = \tilde{m}_t \left(1 - \frac{C_F \alpha_s}{2\pi} \frac{\pi\lambda}{m_t}\right) . \qquad (17)$$

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$$\begin{aligned} \sigma_{\rm LO}(m_t) - \sigma_{\rm LO}(\tilde{m}_t) = \delta\sigma^{\rm expl}_{\rm mass} + \delta\sigma^{\rm impl}_{\rm mass} = \frac{C_F \alpha_s}{2\pi} \frac{\pi \lambda}{m_t} \int {\rm dLips}_{\rm LO} \times \\ \times \left[\frac{m_t^2}{p_d p_t} \left[1 + p_d^\mu \left(\frac{\partial}{\partial p_d^\mu} - \frac{\partial}{\partial p_t^\mu} \right)\right] F_{\rm LO} - m_t {\rm Tr}\left[1 \mathrm{N} \not{p}_b \bar{\mathrm{N}}\right] \right. \\ \left. - m_t {\rm Tr}\left[(\not{p}_t + m_t) \left(\frac{\partial \mathrm{N}}{\partial m_t} \not{p}_b \bar{\mathrm{N}} + \mathrm{N} \not{p}_b \frac{\partial \bar{\mathrm{N}}}{\partial m_t} \right)\right] \right]. \end{aligned} \tag{18}$$

Combined contributions

- combining all contributions, we obtain in the short-distance mass scheme:

$$\delta\sigma_{\mathrm{NLO}} = \sigma_R + \sigma_V + \sigma_{\mathrm{ren}} + \delta\sigma_{\mathrm{mass}}^{\mathrm{expl}} + \delta\sigma_{\mathrm{mass}}^{\mathrm{impl}} \tag{19}$$

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- what about observables, e.g. kinematic distributions, ... ?

Kinematic observables

- observable that depends on the momentum of the top quark

$$O_X = \int \mathrm{d}\sigma \, X(q_t). \tag{21}$$

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- can write in two pieces:

$$\begin{aligned} \mathcal{T}_\lambda[O_X^{(1)}] &= \mathcal{T}_\lambda \left[\int \mathrm{d}\sigma \; \left(X(p_t) + \frac{\partial X(p_t)}{\partial p_t^\mu} \frac{p_t k}{p_t p_d} p_d^\mu \right) \right], \\ \mathcal{T}_\lambda[O_X^{(2)}] &= -\mathcal{T}_\lambda \left[\int \mathrm{d}\sigma \; \frac{\partial X(p_t)}{\partial p_t^\mu} \; k^\mu \right]. \end{aligned} \tag{23}$$

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$$O_X = \int \mathrm{d}\sigma_{\mathrm{LO}}\left[X(p_t) + \frac{\alpha_s C_F}{2\pi}\frac{\pi\lambda}{m_t}\;\left(p_t^\mu - \frac{2m_t^2}{p_b p_t}p_b^\mu\right)\frac{\partial X(p_t)}{\partial p_t^\mu}\right]$$

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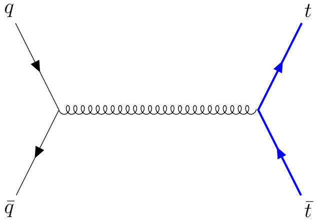
- for observables, linear power corrections **do not vanish** within the short-distance mass scheme

Top quark pair production at the Tevatron

- the dominant channel for $t\bar{t}$ production at the Tevatron is via $q\bar{q}$ fusion



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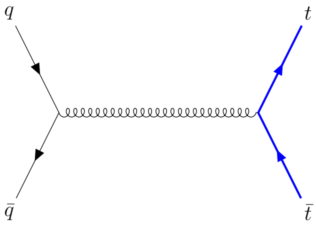

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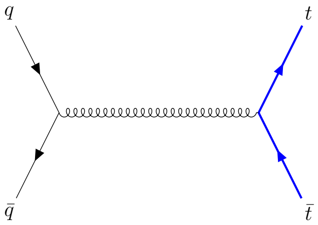

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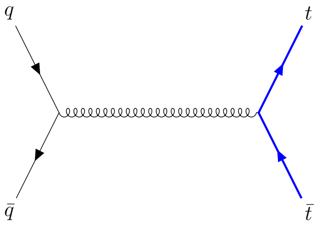

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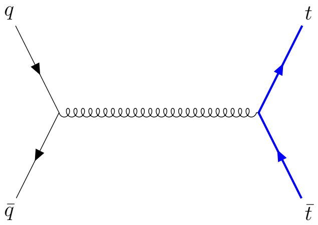

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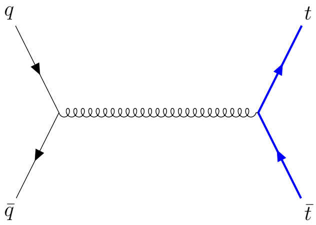

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Top quark pair production at the Tevatron: $p_{t\perp}$, y_t, $s_{t\bar{t}}$

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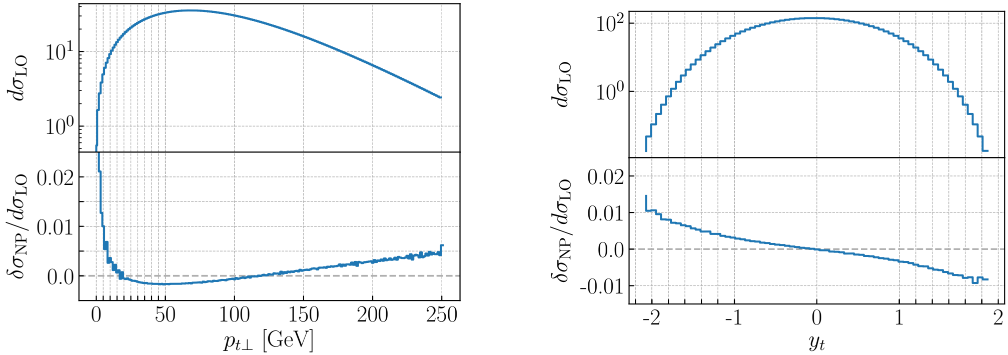

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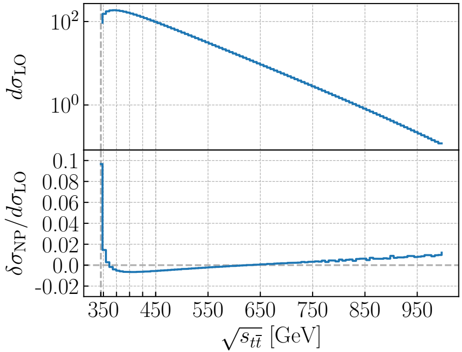

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Part III

Conclusions & Outlook

Summary

- improved understanding of analytic structure of linear power corrections
- computed linear power corrections for event-shape variables (C-parameter, thrust)
- for top quark production processes, **there are <u>no</u> linear power corrections** to total cross-sections within the short-distance mass scheme ($\overline{\text{MS}}$-scheme)
- however for observables ($p_{t\perp}$, y_t, $s_{t\bar{t}}$), **there <u>are</u> linear power corrections** within the short-distance mass scheme ($\overline{\text{MS}}$-scheme)
- computed expressions for observables, e.g. $p_{t\perp}$, y_t and $s_{t\bar{t}}$, ...
- similar results and conclusions for other (abelian) type processes ...

Thank you for attention!