NNLO QCD corrections to SIDIS: unpolarized & polarized

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Loops and Legs in Quantum Field Theory

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in collaboration with **S. Goyal, R. N. Lee, S. Moch, V. Pathak, V. Ravindran**

Electron Ion Collider

a machine to look inside the nucleus

EIC will take precision snapshots of the internal structures of the protons and neutrons, allowing us a better understanding of the strongest force in nature

A precise theoretical description of our current understanding (the Standard Model) is also necessary to find any agreement/disagreement with precise experimental data

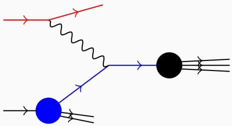
The inclusive/semi-inclusive deep inelastic scattering (DIS/SIDIS) plays a crucial role. Parton model connects the partonic cross-section to the hadronic one through PDFs & FFs. We compute the partonic cross-section using the framework of perturbative QCD order by order in α_s.

Higher order corrections are essential to

- achieve sufficiently/comparably precise theoretical estimates
- reduce the uncertainties arising from the factorization scales

Semi-Inclusive DIS

see the talk by J. Blümlein for DIS



SIDIS

$$l + H \to l + H' + X$$

Phase-space:
Final state hadron is tagged!
Extra constrain on the phase-space

$$dPS|_{\text{SIDIS}} = dPS|_{\text{DIS}} \times \delta\left(z' - \frac{p_a \cdot p_b}{p_a \cdot q}\right)$$

DIS

$$l + H \to l + X$$

Phase-space:
All final states are fully integrated!

The hadronic part is characterized by structure functions (F_i). The hadronic SFs are expressed with the convolution of partonic finite coefficient functions ($\mathcal{F}_{i,ab}$), PDFs and FFs.

Parton model & perturbative expansion

$$F_i = x^{i-1} \sum_{a,b} \int_x^1 \frac{dx_1}{x_1} f_a(x_1, \mu_F^2) \int_z^1 \frac{dz_1}{z_1} D_b(z_1, \mu_F^2) \times \mathcal{F}_{i,ab}\left(\frac{x}{x_1}, \frac{z}{z_1}, Q^2, \mu_F^2\right)$$

$\Downarrow$

the finite coefficient functions which can be computed perturbatively

In QCD, we have a series expansion of the partonic cross sections in strong coupling constant α_s:

$$\begin{aligned}
\mathcal{F}_{ab}(z) &= \mathcal{F}_{ab}^{(0)} \sum_{m=0}^{\infty} \alpha_s^m \, \mathcal{F}_{ab}^{(m)}(z) \\
&= \mathcal{F}_{ab}^{(0)} \left[1 + \alpha_s \mathcal{F}_{ab}^{(1)}(z) + \alpha_s^2 \mathcal{F}_{ab}^{(2)}(z) + \alpha_s^3 \mathcal{F}_{ab}^{(3)}(z) + \cdots\right]
\end{aligned}$$

We are interested in the second order correction

Parton model & perturbative expansion: polarized

We consider the asymmetry between the parallel and anti-parallel spin orientations of the colliding electron and hadron. $\Delta f_a = f_{a(\uparrow)/H(\uparrow)} - f_{a(\downarrow)/H(\uparrow)}$ are the spin-dependent PDFs.

$$g_1 = \sum_{a,b} \int_x^1 \frac{dx_1}{x_1} \Delta f_a(x_1, \mu_F^2) \int_z^1 \frac{dz_1}{z_1} D_b(z_1, \mu_F^2) \times \mathcal{G}_{1,ab}\left(\frac{x}{x_1}, \frac{z}{z_1}, Q^2, \mu_F^2\right)$$

$\Downarrow$

the finite coefficient functions which can be computed perturbatively

$$\mathcal{G}_{ab}(z) = \mathcal{G}_{ab}^{(0)} \sum_{m=0}^{\infty} \alpha_s^m \, \mathcal{G}_{ab}^{(m)}(z)$$
$$= \mathcal{G}_{ab}^{(0)} \left[1 + \alpha_s \mathcal{G}_{ab}^{(1)}(z) + \alpha_s^2 \mathcal{G}_{ab}^{(2)}(z) + \alpha_s^3 \mathcal{G}_{ab}^{(3)}(z) + \cdots\right]$$

$\nearrow$

We are interested in the second order correction

State-of-the-art

NLO and beyond

Altarelli, Ellis, Martinelli, Pi (1979);
Furmanski and Petronzio (1982);
de Florian, Stratmann and Vogelsang (1998); (polarized)
Daleo and Sassot (2003);

NNLO

Goyal, Lee, Moch, Pathak, NR, Ravindran (2023, 2024) ⇒ This talk
Bonino, Gehrmann, Löchner, Schönwald, Stagnitto (2024) ⇒ Next talk by Bonino

Resummation & Parton shower

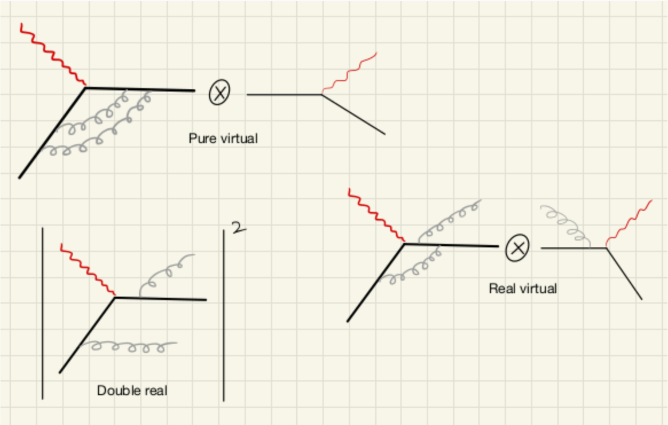
Cacciari and Catani (2001);
Anderle, Ringer and Vogelsang (2013);
Abele, de Florian, and Vogelsang (2021,2022);
Borsa, Jäger (2024); ⇒ Talk by Borsa

This talk is based on

S. Goyal, S. Moch, V. Pathak, NR, V. Ravindran
`NNLO QCD corrections to semi-inclusive DIS`,
(arXiv:2312.17711 [hep-ph]).

S. Goyal, R. Lee, S. Moch, V. Pathak, NR, V. Ravindran
`NNLO QCD corrections to polarized semi-inclusive DIS`,
(arXiv:2404.09959 [hep-ph]).

Schematic diagrams for NNLO contributions to SIDIS



Each individual contribution is divergent : $\frac{1}{\epsilon}$ in dimensional regularization

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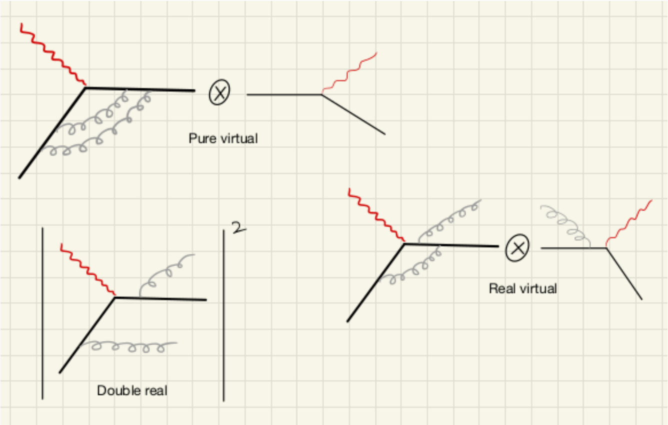

Sum of all degenerate processes: KLN theorem & mass factorization

Computational procedure

$d = 4 + \epsilon$

- Diagrammatic approach -> `QGRAF` to generate Feynman diagrams
- In-house `FORM` routines for algebraic manipulation : *Lorentz, Dirac and Color algebra*
- Projectors: different projectors for unpolarized and polarized cases
- Reverse unitarity : phase-space integrals to loop integrals

$$\delta(k^2 - m^2) \sim \frac{1}{2\pi i}\left(\frac{1}{k^2 - m^2 - i0} - \frac{1}{k^2 - m^2 + i0}\right)$$

- Decomposition of the dot products to obtain scalar integrals
- Identity relations among scalar integrals : IBPs (using LiteRed)
- Algebraic linear system of equations relating the integrals

$\Downarrow$

Master integrals (MIs)

- Computation of MIs : Parametric integration & Method of diff. eqn. (generic & canonical)
- UV renormalization and mass factorization
- Numerical evaluation using suitable PDFs and FFs

Solving master integrals

The main challenges are the MIs appearing in the double real emissions. We follow three different techniques to compute them.

Parametric integration

[Zijlstra, van Neerven]

Conventional method by choosing an appropriate frame of reference

In the end, we are left with the following type of angular integration

$$\int_0^\pi d\theta \int_0^\pi d\phi \frac{(\sin\theta)^{d-3}(\sin\phi)^{d-4}}{(a+b\cos\theta)(A+B\cos\theta+C\cos\phi\sin\theta)}$$

- For $a=b$ and $A^2=B^2+C^2$ (which is the case for the RR MIs in leading-color), this angular integration results into simple hypergeometric functions.
- For the rest of the MIs, either $a\neq b$ or $A^2\neq B^2+C^2$ which may result into functions of higher complexity. So, we use the method of DE to solve these MIs.
- However, we consider the threshold limit of these MIs and compute them using parametric integration which will be used as the boundary conditions for the DE.

The method of differential equations

A Feynman integral is a function of spacetime dimension d and kinematic invariants x, z.

$$J_i = \mathcal{N} \int \frac{d^d l_1}{(2\pi)^d} \, \frac{d^d l_2}{(2\pi)^d} \, \frac{1}{l_1^2 l_2^2 ((l_1 - l_2)^2 - m^2)(l_1 - q)^2 (l_2 - q)^2} \equiv f(d,\, x,\, z)$$

The idea is to obtain a differential eqn. for the integral $w.r.t.\ x,\ z$ and solve it.

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$$\frac{d}{dz} J_i = \text{some combinations of integrals}$$

$$\Downarrow \text{ IBP identities/reduction}$$

$$= \sum_{j=1}^{n} c_{ij} J_j$$

c_{ij}'s are rational function of $d,\ x$ and z.

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$$d_z \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ \vdots \\ J_n \end{pmatrix} = \begin{bmatrix} \bullet & \bullet & \bullet & \bullet & \cdots & \bullet \\ \bullet & \bullet & \bullet & \bullet & \cdots & \bullet \\ \bullet & \bullet & \bullet & \bullet & \cdots & \bullet \\ \bullet & \bullet & \bullet & \bullet & \cdots & \bullet \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \bullet & \bullet & \bullet & \bullet & \cdots & \bullet \end{bmatrix} \begin{pmatrix} J_1 \\ J_2 \\ J_3 \\ J_4 \\ \vdots \\ J_n \end{pmatrix}$$

$$d_z \mathbb{J} = \mathbb{A}(d,\, z)\mathbb{J}$$

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The bullets ($\bullet$) indicate a non-zero rational function of $d,\ x$ and z.

To solve such a system, it would be best to organize it in such a way that it diagonalizes, or at least it takes a block-triangular form. Then, it can be solved using bottom-up approach.

The results are obtained in terms of iterated integrals (HPLs/GPLs).

The method of differential equations: canonical form

[Henn (2013)]

We have a 21 × 21 system of DEs, on which we perform canonical transformation using `Libra`

$$
\begin{pmatrix}
\frac{ep}{(-1+xp)\,xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{ep}{(-1+xp)\,xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{ep}{(-1+xp)\,xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{ep}{(-1+xp)\,xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{2\,ep}{xp} & 0 & 0 & 0 & -\frac{ep}{2\,xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{16\,ep}{-1+xp} & \frac{ep\,(-1+zp)}{(-1+xp)\,(-xp+zp)} & 0 & 0 & -\frac{4\,ep}{xp-zp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{16\,ep}{xp-xp^2} & \frac{ep\,(-1+zp)}{(-1+xp)\,(xp-zp)} & 0 & 0 & \frac{4\,ep\,zp}{xp^2-xp\,zp} & 0 & -\frac{ep}{xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{8\,ep}{xp-xp^2} & \frac{ep\,(1+xp)}{4\,(-1+xp)\,xp} & 0 & 0 & 0 & 0 & 0 & -\frac{ep}{2\,xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{4\,ep}{(-1+xp)\,xp} & \frac{ep}{4-4\,xp} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{ep}{2\,xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{8\,ep\,(-1+xp)\,(-1+zp)\,\sqrt{zp}}{\sqrt{xp}\,(xp+zp+xp^2\,zp+xp\,zp^2)} & -\frac{ep\,\sqrt{zp}}{\sqrt{xp}\,(xp+zp)} & 0 & 0 & 0 & 0 & 0 & \frac{ep\,(-1+xp)\,(-1+zp)\,\sqrt{zp}}{\sqrt{xp}\,(xp+zp+xp^2\,zp+xp\,zp^2)} & 0 & -\frac{ep\,(xp+2\,zp+xp\,zp^2)}{2\,xp\,(xp+zp+xp^2\,zp+xp\,zp^2)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{16\,ep\,(xp+2\,zp+xp\,zp^2)}{xp\,(xp+zp+xp^2\,zp+xp\,zp^2)} & \frac{ep\,(-xp+zp)}{xp\,(xp+zp)} & 0 & 0 & 0 & 0 & 0 & \frac{2\,ep\,(-1+xp^2)\,zp}{xp\,(xp+zp+xp^2\,zp+xp\,zp^2)} & 0 & -\frac{ep\,(-1+xp)\,(-1+zp)\,\sqrt{zp}}{\sqrt{xp}\,(xp+zp+xp^2\,zp+xp\,zp^2)} & -\frac{ep}{xp+xp^2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{16\,ep\,(-1+zp)}{(-1+xp)\,(xp-zp)} & -\frac{ep\,(-2+xp+zp)}{(-1+xp)\,(xp-zp)} & 0 & 0 & 0 & 0 & 0 & \frac{2\,ep}{xp-zp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{4\,ep\,(-1+xp)\,\sqrt{-1-xp^2+xp\,(-2+4\,zp)}}{xp\,(1+xp^2+xp\,(2-4\,zp))} & -\frac{ep\,(1+3\,xp)\,\sqrt{-1-xp^2+xp\,(-2+4\,zp)}}{4\,xp\,(1+xp^2+xp\,(2-4\,zp))} & 0 & 0 & -\frac{ep\,(-1+xp)\,\sqrt{-1-xp^2+xp\,(-2+4\,zp)}}{xp\,(1+xp^2+xp\,(2-4\,zp))} & 0 & 0 & \frac{ep\,(-1+xp)\,\sqrt{-1-xp^2+xp\,(-2+4\,zp)}}{2\,xp\,(1+xp^2+xp\,(2-4\,zp))} & 0 & 0 & 0 & 0 & -\frac{ep\,(1+xp-2\,zp)}{1+xp^2+xp\,(2-4\,zp)} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{16\,ep}{xp} & \frac{ep}{xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{2\,ep\,(-1+xp)\,\sqrt{-1-xp^2+xp\,(-2+4\,zp)}}{xp\,(1+xp^2+xp\,(2-4\,zp))} & -\frac{ep}{xp+xp^2} & 0 & 0 & 0 & 0 & 0 & 0 \\
\frac{16\,ep\,(-1+zp)}{xp\,\sqrt{1+(-2+4\,xp)\,zp+zp^2}} & -\frac{ep\,(1+xp-zp+3\,xp\,zp)}{2\,(-1+xp)\,xp\,\sqrt{1+(-2+4\,xp)\,zp+zp^2}} & 0 & 0 & 0 & 0 & 0 & \frac{ep\,(-1+zp)}{xp\,\sqrt{1+(-2+4\,xp)\,zp+zp^2}} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{ep\,(-1+zp)^2}{xp\,(1+(-2+4\,xp)\,zp+zp^2)} & 0 & 0 & 0 & 0 & 0 \\
\frac{16\,ep}{xp-xp^2} & -\frac{ep}{2\,xp} & 0 & 0 & 0 & 0 & 0 & \frac{ep}{xp} & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{ep\,(-1+zp)}{xp\,\sqrt{1+(-2+4\,xp)\,zp+zp^2}} & 0 & 0 & 0 & 0 & 0 \\
-\frac{16\,ep\,(xp-2\,zp+xp\,zp)}{(-1+xp)\,xp\,(xp-zp)} & \frac{ep\,(-zp+xp\,(-1+2\,zp))}{(-1+xp)\,xp\,(xp-zp)} & 0 & 0 & 0 & 0 & 0 & -\frac{2\,ep\,zp}{xp^2-xp\,zp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{ep}{xp} & 0 & 0 & 0 \\
\frac{16\,ep\,(-1+zp^2)}{xp+zp+xp^2\,zp+xp\,zp^2} & \frac{ep\,(xp^2+zp-3\,xp\,(1+zp))}{2\,(-1+xp)\,xp\,(xp+zp)} & 0 & 0 & 0 & 0 & 0 & \frac{ep\,(zp+xp^2\,zp+xp\,(-1+3\,zp^2))}{xp\,(xp+zp+xp^2\,zp+xp\,zp^2)} & \frac{2\,ep}{xp} & \frac{ep\,(1+xp)\,\sqrt{zp}\,(1+zp)}{\sqrt{xp}\,(xp+zp+xp^2\,zp+xp\,zp^2)} & 0 & 0 & 0 & 0 & -\frac{ep\,(1+zp)}{(-1+xp)\,\sqrt{1+(-2+4\,xp)\,zp+zp^2}} & 0 & 0 & \frac{ep}{(-1+xp)\,xp} & 0 & 0 \\
\frac{16\,ep\,zp}{xp^2-xp\,zp} & -\frac{ep\,(xp+zp)}{xp\,(xp-zp)} & 0 & 0 & 0 & 0 & 0 & \frac{2\,ep\,zp}{xp^2-xp\,zp} & -\frac{2\,ep}{xp} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{ep}{(-1+xp)\,xp} & 0 \\
0 & \frac{ep}{xp-zp} & 0 & 0 & \frac{4\,ep}{xp-zp} & 0 & 0 & -\frac{2\,ep}{xp} & -\frac{2\,ep}{xp} & 0 & 0 & 0 & -\frac{2\,ep\,(1+xp)\,\sqrt{-1-xp^2+xp\,(-2+4\,zp)}}{xp\,(1+xp^2+xp\,(2-4\,zp))} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{ep}{(-1+xp)\,xp}
\end{pmatrix}
$$

After getting the ϵ-form, we performed path-ordered integrations. Note that we have 4 square-roots in addition to 10 kernels in x' and z'. As mentioned earlier, we obtained the boundaries at $x' \to 1$ and $z' \to 1$. We have computed all the MIs in this method.

The method of differential equations: generic

Considering the MIs as given by LiteRed, we find a few sub-systems which are coupled. And, as expected, the homogeneous solutions contain the square-roots. The goal is to avoid them.

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Iterated integrals are one-dimensional. They can be computed with great precision in a short amount of time. Besides, they have the following properties:

(a) Shuffle algebra : Allows to obtain a basis for a set of iterated integrals. Reduction to such a basis is extremely effective to reduce the computation time by few times.

(b) Scaling invariance : Allows to convert the limit of these integrals from kinematical variables (z) to constants (1). This makes the integration really precise.

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First few MIs were solved in terms of GPLs

$\Downarrow$

What happens when we have (multiple) square-roots?!

Rationalization

Rationalizable

- Find a suitable transformation

Let's consider $\sqrt{(1-z')^2 + 4x'z'}$. We can use the transformation $x' \to (1-u^2)/4/t/(1-t), z' \to 1 - 1/t$ for this.

Partly-rationalizable

- A single transformation can not rationalize all square-roots simultaneously.

$\Downarrow$

Square-roots will be present in the iterated integrals.

1) We can accept 'the fact' and evaluate them with appropriate analytic continuation.

or

2) Instead of using a single transformation rule to rationalize them, we write the system (each MI) as sum of functions of dependent variables and treat them separately. As a result, each sub-system has alphabet with 'good' letters with different argument.

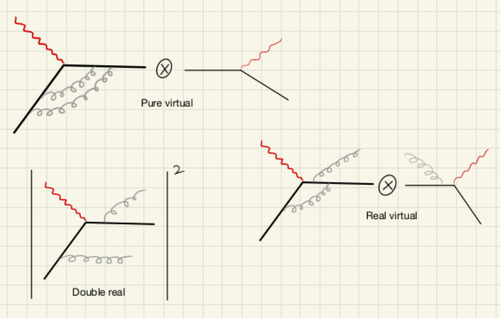
Solving master integrals: summary

- We followed three different techniques to compute the MIs for double real emissions.
- We have perfect agreement among the results from different methods.

- For real-virtual MIs also, we compute them using parametric method, where we use exact expressions for one-loop integrals followed by parametric phase-space integration. We also use method of DE and find perfect agreement.

To obtain the finite partonic cross-section

Combine everything, UV renormalization and mass factorization



- We compute relevant Feynman diagrams & corresponding Feynman integrals analytically.
- After summing all contributions for the Feynman diagrams, we can express the results in terms of MPLs with square-root arguments.
- The resultant diagrams contain two types of divergences: dimensional and kinematical

Combine everything, UV renormalization and mass factorization

- The contributions also have divergences for $x' \to 1$ and $z' \to 1$. We use

$$(1-x)^{-1+n\epsilon} = \frac{1}{n\epsilon}\delta(1-x) + \sum_{k=0}^{\infty} \frac{\epsilon^k}{k!} \left[\frac{\ln^k(1-x)}{(1-x)}\right]_+$$

  where

$$\int_0^1 dx f[x] g[x]_+ = \int_0^1 dx (f[x] - f[1]) g[x]$$

- Of course, for our case, we have double distributions.

- We also find functions that require special care for analytic continuation. We use partial fractioning and theta function to separate different sectors and implement correct analytic continuation. For example,

$$(z' - x')^{\epsilon} = |z' - x'|^{\epsilon} (\theta(z' - x') + (-1 + i\varepsilon)^{\epsilon} \theta(x' - z'))$$

- The divergences at $x' = z'$ or $x' + z' = 1$ are spurious and eventually they cancel.

Combine everything, UV renormalization and mass factorization

unpolarized

- The sources of dimensional divergences are of UV & IR type. UV renormalization takes care of the UV divergences. In case of unpolarized SIDIS, the UV renormalization is trivial.
- KLN theorem ensures the cancellation of all soft and some collinear divergences. The residual collinear divergences are absorbed by mass factorization, using appropriate splitting kernels.

$$\frac{\hat{\sigma}_{I,ab}(\epsilon)}{x'^{I-1}} = \Gamma_{c\leftarrow a}(\mu_F^2,\epsilon) \otimes \mathcal{F}_{I,cd}(\mu_F^2,\epsilon)\tilde{\otimes}\tilde{\Gamma}_{d\leftarrow b}(\mu_F^2,\epsilon)$$

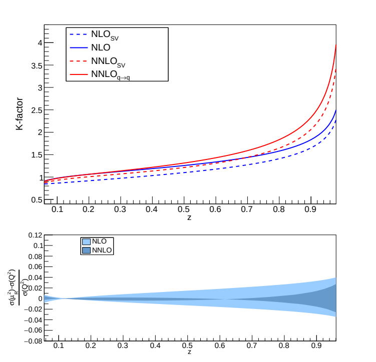
polarized

- The γ_5 in polarized SIDIS needs special care. We use Larin's prescription and we appropriately multiply the finite renormalization constants.
- For polarized case, we use the spin-dependent Altarelli-Parisi kernels for initial states. Since, the partonic cross-section is computed in Larin's scheme, the polarized space-like kernels also need to be considered in the same scheme. For final states, the spin-averaged time-like kernels are considered in the $\overline{\text{MS}}$ scheme.
- In the end, we transform our results obtained in Larin's scheme to $\overline{\text{MS}}$ scheme.

Results!

Results

- We have computed full NNLO corrections to unpolarized SIDIS (all processes) and the polarized SIDIS where the initial state is polarized.
- In both cases, we find good perturbative convergence of the NNLO corrections, and reduction of renormalization and factorization scale dependence.



The K-factors for corrections from different perturbative order for quark initiated process to a fragmenting quark. We have shown here only the non-singlet contributions for NNLO.

Detailed phenomenology is on the way!

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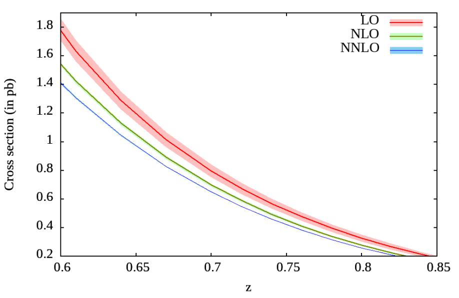

The contributions from different perturbative orders in polarized SIDIS with $\sqrt{s} = 140 GeV$. We have integrated over x between 0.40 to 0.62 and y between 0.30 to 0.50.

Detailed phenomenology is on the way!

Checks

- The master integrals were computed using different methods with perfect agreement!
- Mass factorization (universal) removes all remaining infrared singularities!
- Successful checks with available results in the threshold limit! SV and NSV results match with the literature for F_1, F_2 and g_1.
- The non-singlet part of g_1 is equal to F_3 which we have explicitly checked.
- The constraint (z') can be integrated in our analytic result. We found perfect agreement with the fully inclusive result.

- We also have perfect agreement for unpolarized SIDIS with the results by Bonino et al.

talk by Bonino

Concluding remarks!

- EIC will unravel the mysteries of strong force. Theoretical precision studies are extremely necessary to fully exploit the EIC data.
- Our current (well-tested) theoretical understanding (the SM) is constrained by its perturbative nature and hence, higher order perturbative corrections are necessary to achieve precise theoretical predictions.

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- In this talk, we have presented the first results on NNLO QCD corrections to unpolarized and polarized SIDIS.
- The NNLO predictions will be extremely important to extract PDFs and FFs very precisely.
- Precision predictions for polarized PDFs will be crucial to solve the proton spin puzzle.

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- Aside the phenomenological impact of the result, it also sets a milestone for the computational technique.

Thank you for your attention!