# Higgs Self-Coupling and Yukawa Corrections to Higgs Boson Pair Production

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Loops and Legs — Wittenberg, 15. April 2024

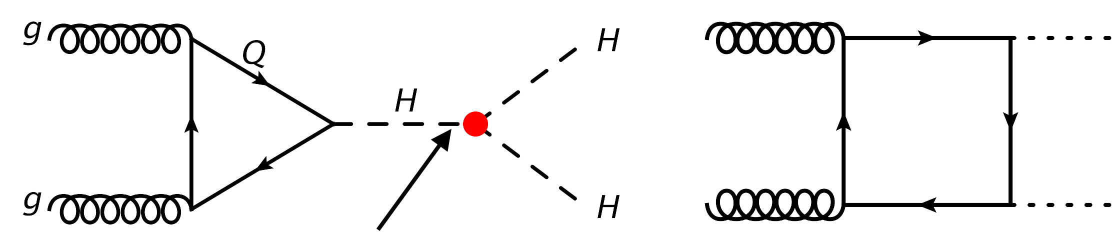
in Collaboration with Gudrun Heinrich, Stephen Jones, Thomas Stone, Augustin Vestner

# Motivation

Measurements of Higgs boson pair production is important

$\rightarrow$ direct relation to Higgs potential

$\rightarrow$ test mechanism of EW symmetry breaking



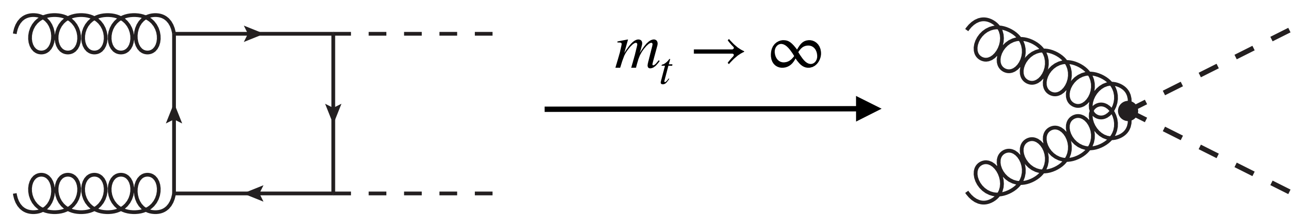

$$V(\Phi) = \frac{1}{2}\mu^2\Phi^2 + \frac{1}{4}\lambda\Phi^4 \xrightarrow[\text{breaking}]{\text{EW symmetry}} \frac{m_H^2}{2}H^2 + \frac{m_H^2}{2v}H^3 + \frac{m_H^2}{8v^2}H^4$$

## Electroweak corrections

- typically $\mathcal{O}(\text{few}\,\%)$, but can be larger on differential level
- challenging calculation, with many diagrams and masses

# NLO QCD corrections

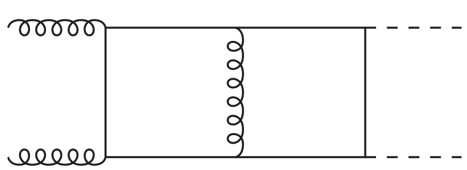
full SM




heavy-top limit

NLO QCD

[Borowka, Greiner, Heinrich, Jones, MK, Schlenk, Schubert, Zirke 16]
[Baglio, Campanario, Glaus, Mühlleitner, Ronca, Spira, Streicher 18]

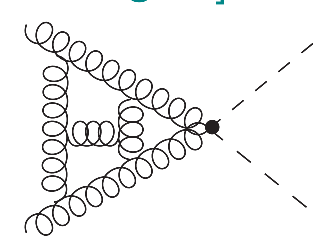


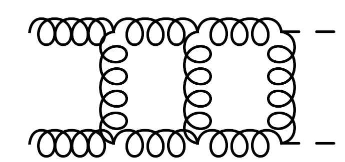
NNLO HTL

[De Florian, Mazzitelli 13]
[Grigo, Melnikov, Steinhauser 14]

N³LO HTL

[Chen, Li, Shao, Wang 19]





NNLO HTL ⊗ NLO QCD

[Grazzini, Heinrich, Jones, Kallweit, MK, Lindert, Mazzitelli 18]

N³LO HTL ⊗ NLO QCD

[Chen, Li, Shao, Wang 19]

→ NLO QCD corrections to $gg \to HH$ production known with high accuracy

# NLO EW progress

recently, huge progress towards NLO EW corrections to $gg \to HH$

- partial results:
  Effects of quartic Higgs coupling [Bizoń, Haisch, Rottoli 18,24]
  Higgs self-coupling corrections [Borowka, Duhr, Maltoni, Pagani, Shivaji, Zhao 19]
  Top-Yukawa corrections, using HTL for parts of the calculation [Mühlleitner, Schlenk, Spira 22]

- approximate results:
  Top-Yukawa corrections in the high-energy limit [Davies, Mishima, Schönwald, Steinhauser, Zhang, 22]
  EW corrections in large-$m_t$ limit [Davies, Schönwald, Steinhauser, Zhang, 23] → talk by Hantian Zhang

- full EW corrections [Bi, Huang, Huang, Ma, Yu 23]

this talk: EW corrections due to Higgs Self-Coupling and Yukawa corrections

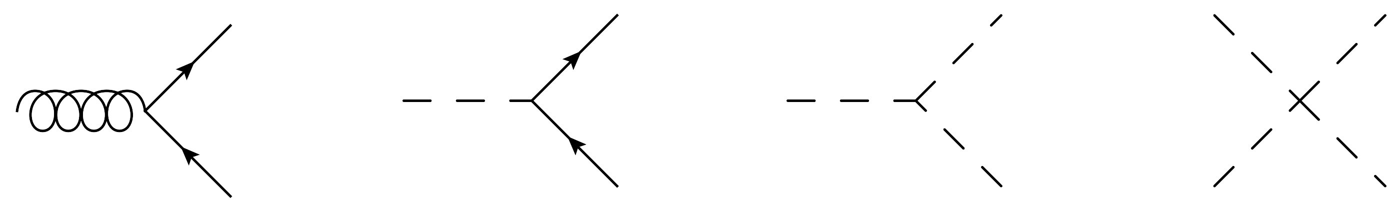
# Self-Coupling & Yukawa Corrections

We calculate Higgs Self-Coupling and Yukawa corrections to $gg \to HH$ production

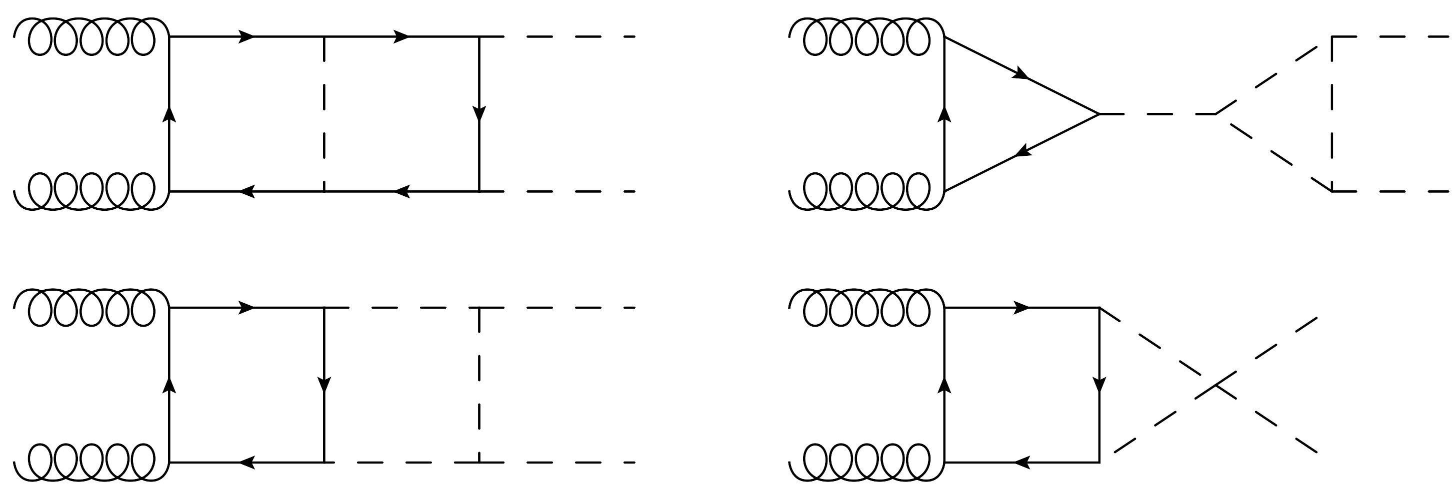
$\hat{=}$ gauge-less limit $\quad g_1, g_2 \to 0 \quad \Rightarrow \quad$ EW gauge bosons decouple

interactions

in unitary gauge



example diagrams



no real radiation: since $gg \to HHH$

- finite
- different experimental signature

# Amplitude Structure

Form Factor decomposition of $gg \to HH$ amplitude: [Glover, van der Bij `88]

$$\mathcal{M}_{ab} = \delta_{ab}\, \varepsilon_1^{\mu} \varepsilon_2^{\nu}\, \mathcal{M}_{\mu\nu}$$
$$\mathcal{M}^{\mu\nu} = F_1(s,t,m_h^2,m_t^2;d)\, T_1^{\mu\nu} + F_2(s,t,m_h^2,m_t^2;d)\, T_2^{\mu\nu}$$

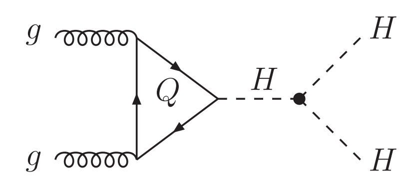
with

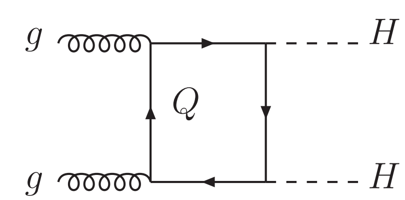
$$T_1^{\mu\nu} = g^{\mu\nu} - \frac{p_1^{\nu}\, p_2^{\mu}}{p_1 \cdot p_2}$$
$$T_2^{\mu\nu} = g^{\mu\nu} + \frac{1}{p_T^2\,(p_1 \cdot p_2)} \left\{ m_h^2\, p_1^{\nu}\, p_2^{\mu} - 2\,(p_1 \cdot p_3)\, p_3^{\nu}\, p_2^{\mu} - 2\,(p_2 \cdot p_3)\, p_3^{\mu}\, p_1^{\nu} + 2\,(p_1 \cdot p_2)\, p_3^{\nu}\, p_3^{\mu} \right\}$$

corresponding to helicity amplitudes

$$\mathcal{M}^{+-} = \mathcal{M}^{-+} = -\, F_1$$

+




$$\mathcal{M}^{++} = \mathcal{M}^{--} = -\, F_2$$

obtained using projectors $F_i = P_i^{\mu\nu} \mathcal{M}_{\mu\nu}$

checked by 2 independent calculations, using:
- alibrary [V. Magerya]
- Reduze 2 [v. Manteuffel, Studerus]

keep dependence on coupling constants $g_{Ht\bar{t}}\,, \quad g_{H^3}\,, \quad g_{H^4} \quad \to$ can be used for EFT studies

# Integral Reduction

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The Loop Integrals are reduced to Master Integrals using IBP relations [Tkachov 81; Chetyrkin 81]

$$\int \mathrm{d}^d p_i \frac{\partial}{\partial p_i^\mu} \left[ q^\mu \, \mathbf{I}'(p_1, \ldots, p_l; k_1, \ldots, k_m) \right] = 0$$

Using the programs
- Kira [Klappert, Lange, Maierhöfer, Usovitsch]
- FireFly [Klappert, Klein, Lange]
- Ratracer [V. Magerya]

} use finite-field methods to avoid large intermediate expressions [von Manteuffel, Schabinger 14]

↓ (Ratracer) record each arithmetic operation performed during Gaussian elimination, use this 'trace' to speed up black-box probes

We obtained the full symbolic reduction

- depending on 5 parameters: $s, t, m_h^2, m_t^2, d$
  currently using simplified version for evaluation of amplitude, with $\frac{m_h^2}{m_t^2} = \frac{12}{23}$ fixed $\rightarrow m_h = 125\,\mathrm{GeV}, \quad m_t = 173.1\,\mathrm{GeV}$
- 494 master integrals, up to 11 masters/sector
- size of reduced amplitude: 8.5 GB (with $m_h/m_t$ fixed)
  100 GB (full $m_h$, $m_t$ dependence)

# Improved Basis of Master Integrals

- Use a (quasi-)finite basis of master integrals [von Manteuffel, Panzer, Schabinger 14]
  - simplifies numerical evaluation of integrals
  - poles in ε only in coefficients
  - requires integrals in shifted dimensions [Bern, Dixon, Kosower 92; Tarasov 96; Lee 10]

- Further improvements of integral basis to achieve:
  (by trying different basis choices for each sector)
  - *d*-dependence factorizes from kinematic dependence in denominators of reduction coefficients $\frac{N(s,t,d)}{D_1(d)D_2(s,t)}$
    [Smirnov, Smirnov `20; Usovitsch `20]
  - simple denominator factors $D_1$, $D_2$
  - avoid poles in coefficients of integrals in top-level sectors as far as possible
    - $\rightarrow$ no poles in 7-propagator sectors
    - $\rightarrow$ no poles in non-planar sectors
  - avoid poles in DEQs

$\rightarrow$ huge impact on evaluation time

$$\mathcal{O}(100\,\mathrm{h}) \quad \rightarrow \quad \mathcal{O}(5\,\mathrm{min})$$

# pySecDec

[Heinrich, Jones, MK, Magerya, Olsson, Schlenk]

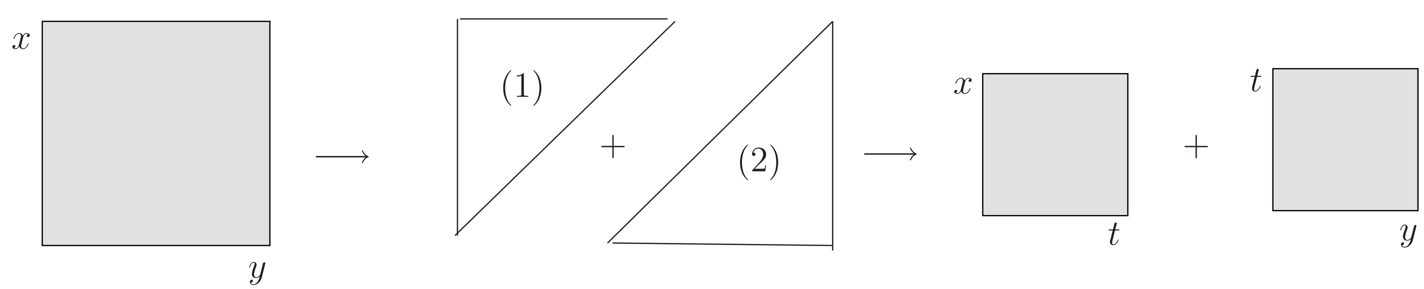
A toolbox for the calculation of dimensionally regulated parameter integrals

Available at
github.com/gudrunhe/secdec
Documentation:
secdec.readthedocs.io

Method:

- Sector decomposition [Binoth, Heinrich `00]

  factorizes overlapping singularities




$$\int_0^1 dx\,dy \frac{1}{(x+y)^{2+\varepsilon}}[\theta(x-y)+\Theta(y-x)] = \int_0^1 dx\,dt \frac{1}{x^{1+\varepsilon}(1+t)^{2+\varepsilon}} + \int_0^1 dy\,dt \frac{1}{y^{1+\varepsilon}(1+t)^{2+\varepsilon}}$$

- Subtraction of poles & expansion in $\varepsilon$

$$\int_0^1 dx\, x^{-1-\varepsilon} g(x) = -\frac{1}{\varepsilon} g(0) + \int_0^1 dx\, x^{-1-\varepsilon}\,[g(x)-g(0)]$$

- Contour deformation [Soper 00; Binoth et al. 05; Nagy, Soper 06, Anastasiou et al. 07; Borowka et al. 12]

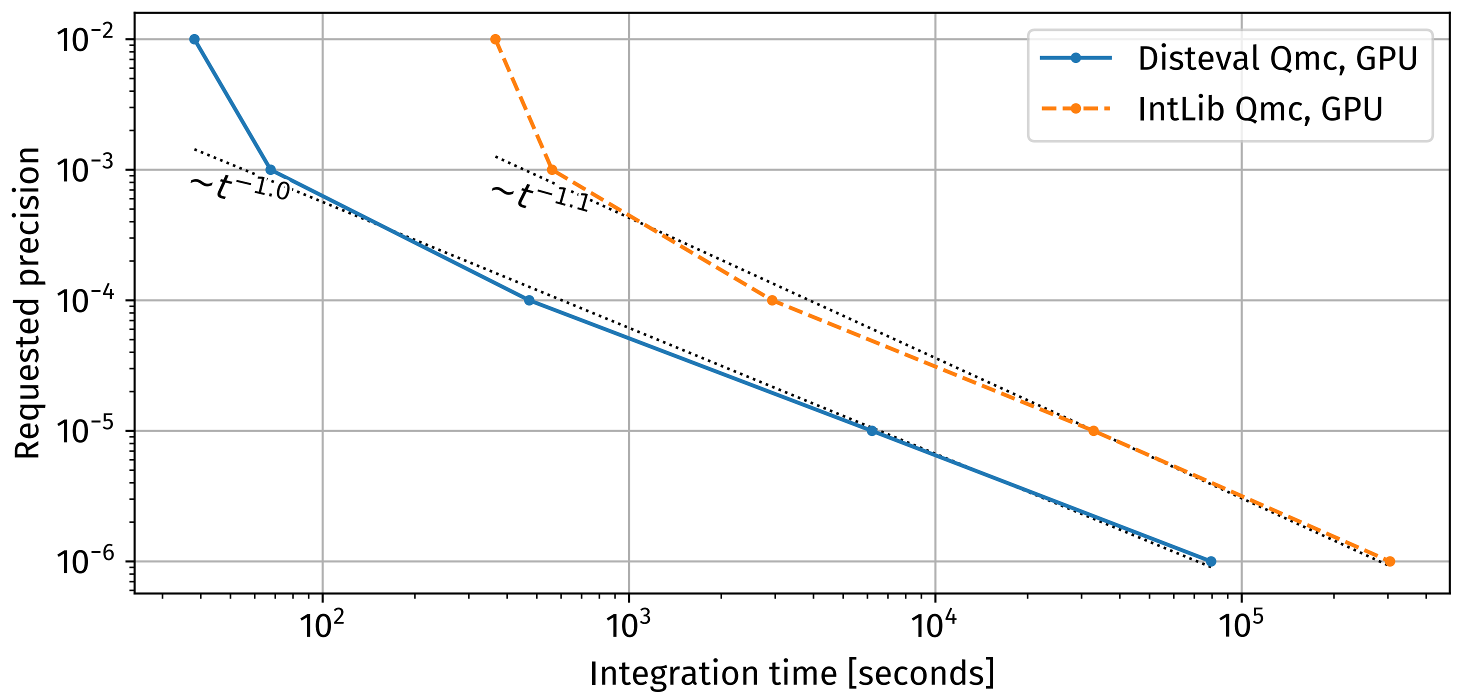
  analytic continuation from Euclidean to physical region

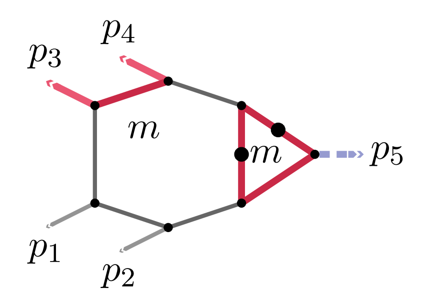
$\rightarrow$ finite integrals at each order in $\varepsilon$

$\rightarrow$ numerical integration possible

# pySecDec — New in version 1.6

- New Quasi-Monte Carlo evaluator 'Disteval'
  up to 10x faster than previous versions, due to
  - better utilization of GPU
  - SIMD instructions on CPU
  - various code improvements




- Improved handling of coefficients of master integrals
  - coefficients now parsed with GiNaC $\rightarrow$ more flexible, use rational numbers to avoid precision loss
  - sums of integrals can be named
- Auto-detect if extra regulators required for expansion-by-region
- Construction-free Median QMC lattices

# Quasi-Monte Carlo Integration

Review: Dick, Kuo, Sloan 13
First application to loop integrals:
Li, Wang, Yan, Zhao 15

Integration using rank-1 lattice rule

$$I[f] \approx I_k = \frac{1}{N} \cdot \sum_{i=1}^{N} f(\mathbf{x}_{i,k}), \quad \mathbf{x}_{i,k} = \left\{ \frac{i \cdot \mathbf{z}}{N} + \mathbf{\Delta}_k \right\}$$

$\{\ldots\} =$ fractional part ($\rightarrow x \in [0;1[$)

$\mathbf{\Delta}_k =$ randomized shifts

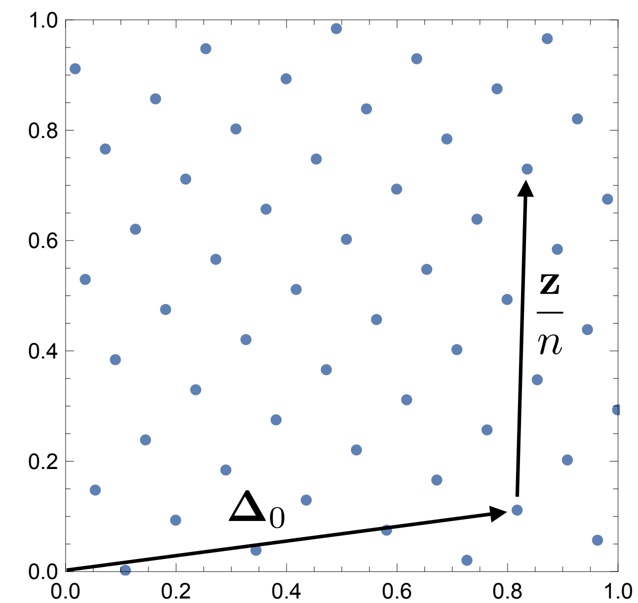
$\rightarrow$ $m$ different estimates of Integral: $I_1, \ldots, I_m$

$\rightarrow$ error estimate of result

$\mathbf{z} =$ generating vector

constructed component-by-component Nuyens `07

minimizing worst-case error $\epsilon_\gamma$

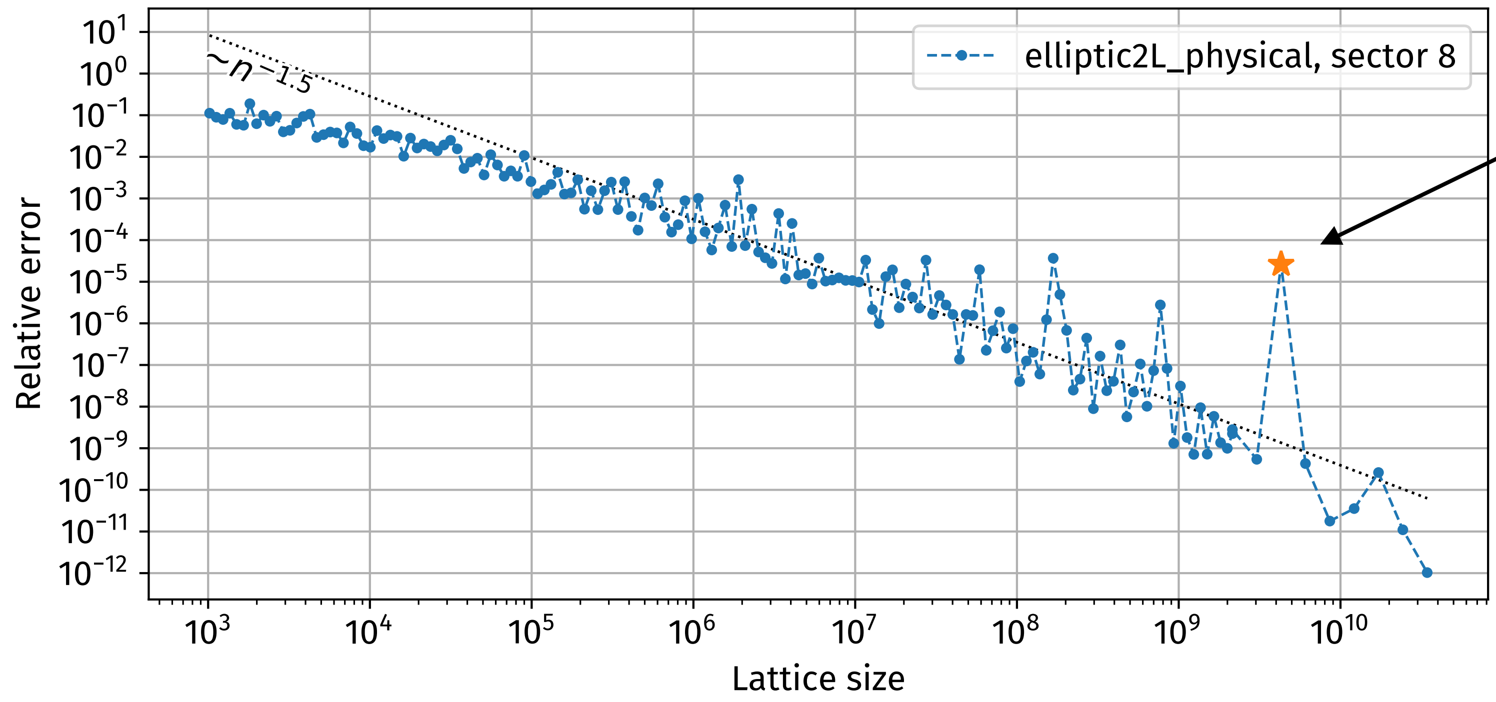



$\rightarrow$ integration error scales as $\mathcal{O}(n^{-1})$ or better

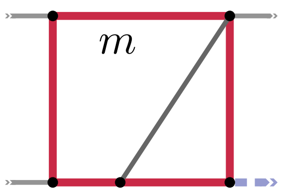
# Quasi-Monte Carlo Integration

pre-computed generating vectors $\mathbf{z}$ with component-by-component construction:

- lattice size limited by largest generating vector
- guaranteed $\mathcal{O}(n^{-1})$ scaling,
  but can encounter 'unlucky' combination of lattice & integrand




unlucky lattice
for this integrand



# Quasi-Monte Carlo Integration

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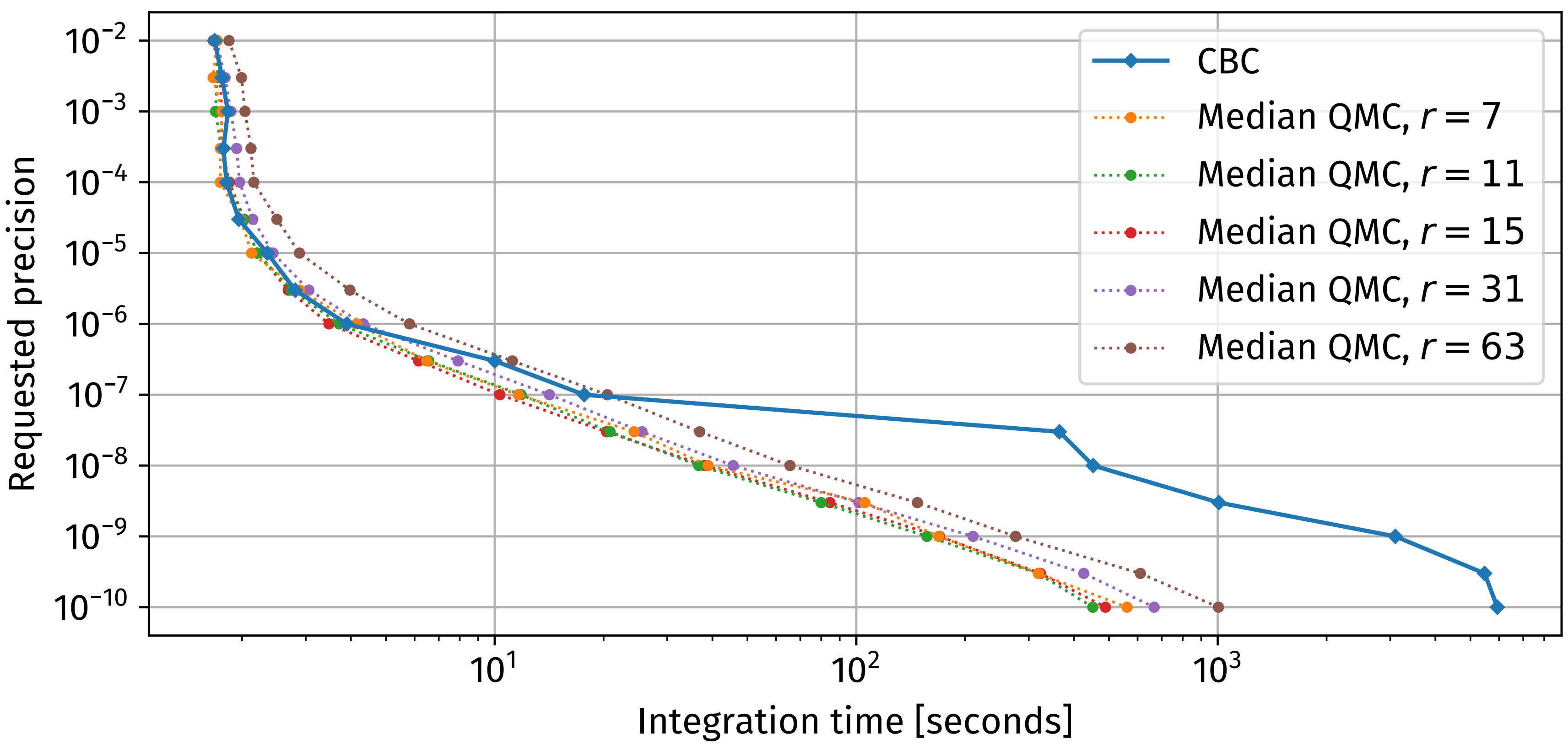
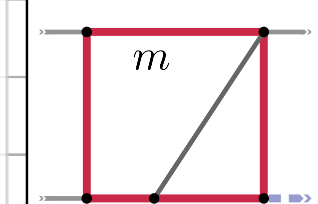
- lattice size limited by largest generating vector
- guaranteed $\mathcal{O}(n^{-1})$ scaling,
  but can encounter 'unlucky' combination of lattice & integrand

alternative method (since pySecDec v.1.6): Construction-free median QMC [Goda, L'Ecuyer 22]

- choose $r$ generating vectors $\mathbf{z}_i$
  with entries chosen randomly from $\{1 \leq z \leq n-1 \mid \gcd(z, n) = 1\}$
- calculate $I_i = \int f$ for each generating vector $\mathbf{z}_i$
- select $\mathbf{z}_i$ corresponding to $\text{median}(I_1, \ldots, I_r)$ for integration (with $m$ random shifts)

➡ same scaling as $\mathbf{z}$ from CBC construction (with high probability)



# Code Generation & Evaluation

Generation of integration library, e.g. for 1-loop amplitude:

```
import pySecDec as psd
if __name__ == '__main__':
    # int1 = box(p1,p2,p3)
    int1 = psd.LoopPackage(
        name = "box1",
        loop_integral = psd.loop_integral.LoopIntegralFromPropagators(
            propagators = ["k**2-mt2", "(k+p1)**2-mt2", "(k+p1+p2)**2-mt2", "(k+p1+p2+p3)**2-mt2"],
            loop_momenta = ["k"], external_momenta = ["p1", "p2", "p3"],
            replacement_rules = [('p1*p1', 0), ('p1*p2', "s/2"), ...]))
    coeff_F1_int1 = "4*mt2**2*(8*mt2-s-2*mh2)"
    coeff_F2_int1 = "..."

    # int2 = box(p2,p1,p3)
    int2 = psd.LoopPackage(
        name = "box2",
        loop_integral = psd.loop_integral.LoopIntegralFromPropagators(
            propagators = ["k**2-mt2", "(k+p2)**2-mt2", "(k+p1+p2)**2-mt2", "(k+p1+p2+p3)**2-mt2"],
            loop_momenta = ["k"], external_momenta = ["p1", "p2", "p3"],
            replacement_rules = [('p1*p1', 0), ('p1*p2', "s/2"), ... ]))
    coeff_F1_int2 = "..."
    coeff_F2_int2 = "..."

    # int 3, ...

    # define form factors as sums of (coeff * integral)
    psd.sum_package('HH1L',
        [int1, int2, ...],
        coefficients = {"F1": [coeff_F1_int1, coeff_F1_int2, ...],
                        "F2": [coeff_F2_int1, coeff_F2_int2, ...]},
        regulators = ["eps"],
        requested_orders = [0],
        real_parameters = ['s', 't', 'mh2', 'mt2']
    )
```

definition of 1st loop-integral

← coefficients, may also depend on $\varepsilon$

define form factors as weighed sum of integrals

$$F_i = \sum_j c_{ij} I_j$$

#sampling points per integral will automatically be optimized during evaluation

Compilation: `make -C HH1L disteval SECDEC_WITH_CUDA_FLAGS="-arch=sm_80"`

→ enable GPU support

Evaluation: `python3 -m pySecDec.disteval HH1L/disteval/HH1L.json --epsrel=0.001 s=... t=... ...`

# Renormalization & Input Parameters

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Renormalization:

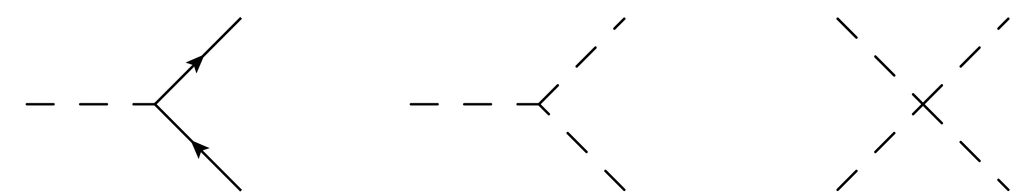
- on-shell renormalization for masses and fields

$$H_0 = Z_H^{1/2} H \qquad m_{H,0}^2 = Z_{m_H^2} m_H^2$$
$$t_0 = Z_t^{1/2} t \qquad m_{t,0} = Z_{m_t} m_t$$

- vev renormalization according to $G_\mu$ scheme

  $v \to v + \Delta v$ [see e.g. Biekötter, Pecjak, Scott, Smith 22]

  with $M_Z, M_W \to 0$, corresponding to gauge-less limit
- Fleischer-Jegerlehner tadpole prescription



$\sim \frac{m_t}{v}$ $\qquad \sim \frac{m_H^2}{v}$ $\qquad \sim \frac{m_H^2}{v^2}$

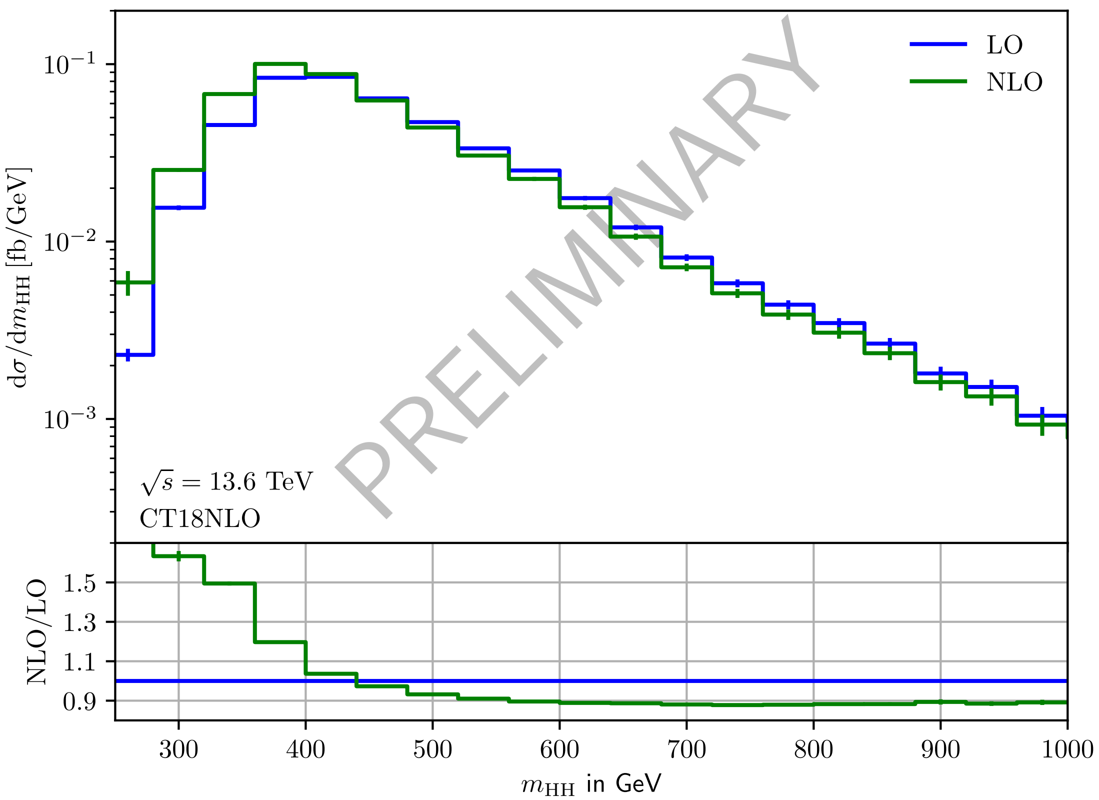
$\rightarrow$ corresponds to expansion in $\frac{1}{v}$

Input parameters:

- $\sqrt{s} = 13.6\,\mathrm{TeV}$
- $m_H = 125\,\mathrm{GeV}, \quad m_t = \sqrt{23/12}\, m_H \approx 173.1\,\mathrm{GeV}$
- CT18nlo pdf & $\alpha_s$, with $\mu_R = \mu_F = \frac{m_{HH}}{2}$
- $G_F = 1.166379 \cdot 10^{-5}\,\mathrm{GeV}^{-2}$

# Results

$m_{HH}$ distribution:




$\rightarrow$ $+50\,\%$ correction at $m_{HH} \approx 2\,m_t$

$-10\,\%$ correction at large $m_{HH}$

Technical Details:

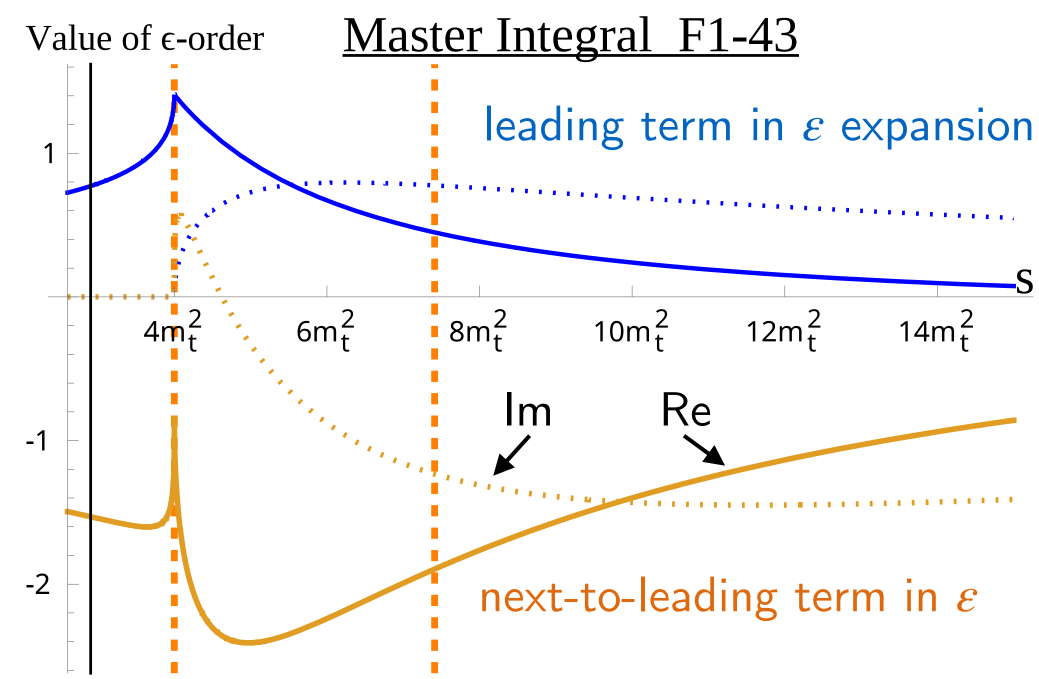
- $\sim$2000 sampling points
  distributed according to unweighted LO events
- median integration time: 5 min. for 0.1% precision
  using 1 Nvidia Tesla A100 GPU
  for some points close to $m_{HH} \approx 2\,m_t$ : 1d for $\mathcal{O}(1-10\,\%)$
- $\sim$5 min. to parse coefficients on 8 CPU cores
  - using rationalized kinematics $s, t$
    to avoid numerical instabilities
  - full $d$-dependence kept in coefficients
    $\rightarrow$ can be improved by expansion in $\varepsilon$

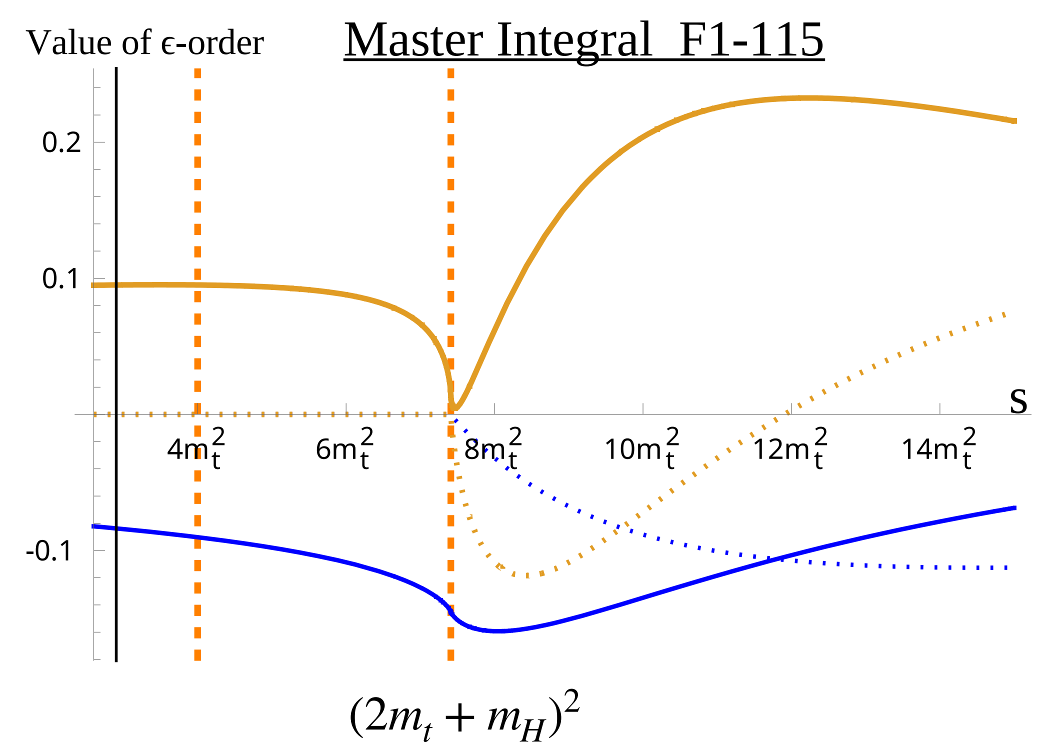
# Alternative Method

We also consider the solution of the DEQs [A. Kotikov 91]
as generalized series expansions [F. Moriello 19]
using the package DiffExp [M. Hidding 20]

- might be useful for phase-space regions where convergence with pySecDec slow
- can produce slices in phase space
- agreement of pySecDec and DiffExp
  verified for various integrals
- For some sectors, we need to set
  HomogeneousSolve → DontExpand
  IntegrationStrategy → VariationOfParameters

→ use original system $\partial_x \vec{g} = \mathbf{M}\vec{g}$ instead of $\partial_x^j \vec{g} = \mathbf{M}^{(j)}\vec{g}$
don't expand $\mathbf{M}$
→ works, but much slower,
further investigations needed (work in progress)




Thomas Stone, Higgs 2023

# Summary & Outlook

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## Higgs Self-Coupling and Yukawa Corrections to HH Production

- Calculation using various state-of-the-art tools:
  - full symbolic reduction using Kira, Firefly & Ratracer
    depending on 5 variables $s, t, m_H, m_t, d$
    494 master integrals
  - loop integrals evaluated numerically using pySecDec
    5 min. median integration time using 1 Nvidia Tesla A100 GPU
- Large positive correction at $m_{HH} \approx 2\,m_t$ ; $-10\,\%$ at large $m_{HH}$
- Future Plans:
  - comparison to EW corrections by other groups
  - full NLO EW corrections
  - include EFT operators
  - study b-mass effects

# Backup

# NLO QCD

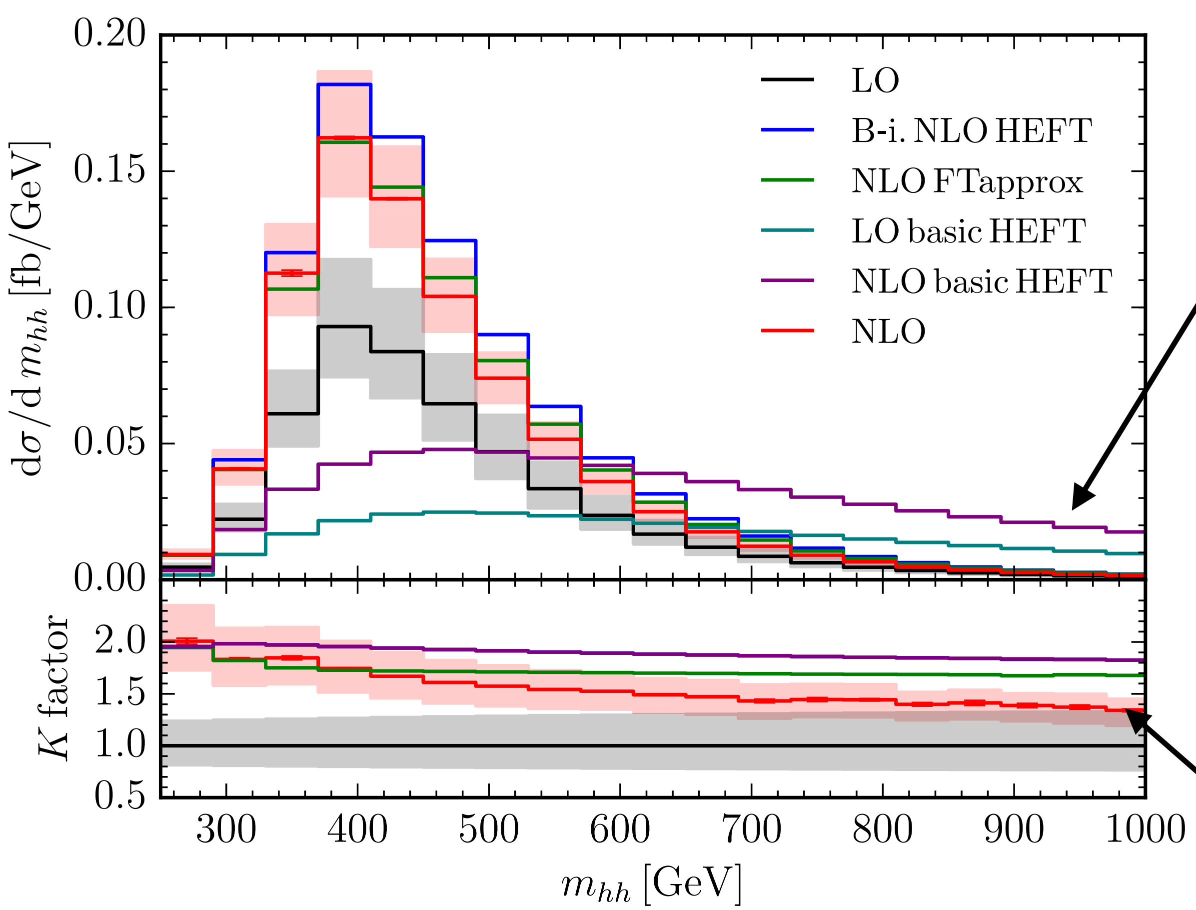
| $\sqrt{s}$ | LO | B-i. NLO HEFT | NLO FT$_{approx}$ | NLO |
|---|---|---|---|---|
| 14 TeV | $19.85^{+27.6\%}_{-20.5\%}$ | $38.32^{+18.1\%}_{-14.9\%}$ | $34.26^{+14.7\%}_{-13.2\%}$ | $32.91^{+13.6\%}_{-12.6\%}$ |
| 100 TeV | $731.3^{+20.9\%}_{-15.9\%}$ | $1511^{+16.0\%}_{-13.0\%}$ | $1220^{+11.9\%}_{-10.7\%}$ | $1149^{+10.8\%}_{-10.0\%}$ |

-14% wrt. NLO HEFT

-4% wrt. NLO FT$_{\text{approx}}$

## FTapprox

LO and real radiation with full $m_t$ dependence,
approximate virtual corrections via $$d\sigma_V^{\text{FT}_{\text{approx}}}(m_t) \approx \frac{d\sigma_B^{\text{SM}}(m_t)}{d\sigma_B^{\text{HTL}}(m_t \to \infty)} d\sigma_V^{\text{HTL}}(m_t \to \infty)$$




HTL predicts wrong shape of distribution

FT$_{\text{approx}}$ gives significant improvements compared to HTL

large top mass effects in virtual contributions in high $m_{hh}$ region

large dependence of K-factor on $m_{hh}$

# NNLO & $N^3LO$

Grazzini, Heinrich, Jones, Kallweit, MK, Lindert, Mazzitelli 18

combination with NNLO $(m_t \to \infty)$

$\to$ approx. $m_t$ dependence at NNLO

3 different methods:

1) $\text{NNLO}_{\text{NLO-i}}$

   rescale NLO by $K_{\text{NNLO}} = \text{NNLO}_{\text{HEFT}}/\text{NLO}_{\text{HEFT}}$

2) $\text{NNLO}_{\text{B-proj}}$

   project all real radiation contributions to Born configuration, rescale by $\text{LO}/\text{LO}_{\text{HEFT}}$

3) $\text{NNLO}_{\text{FTapprox}}$

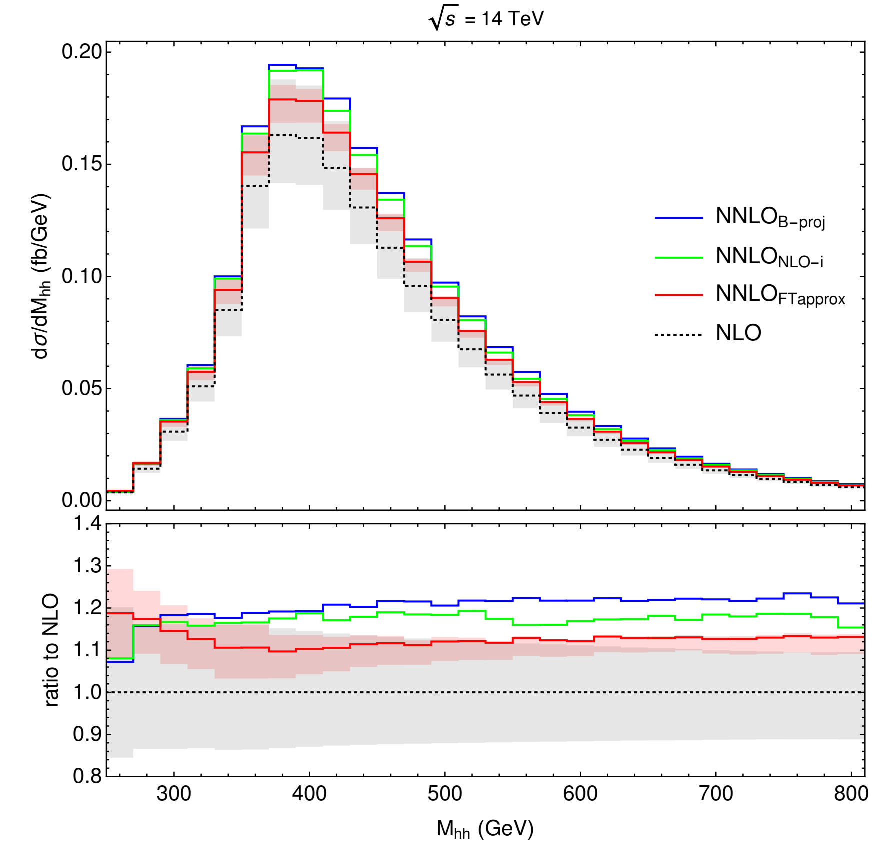
   calculate $\text{NNLO}_{\text{HEFT}}$ and for each multiplicity rescale by

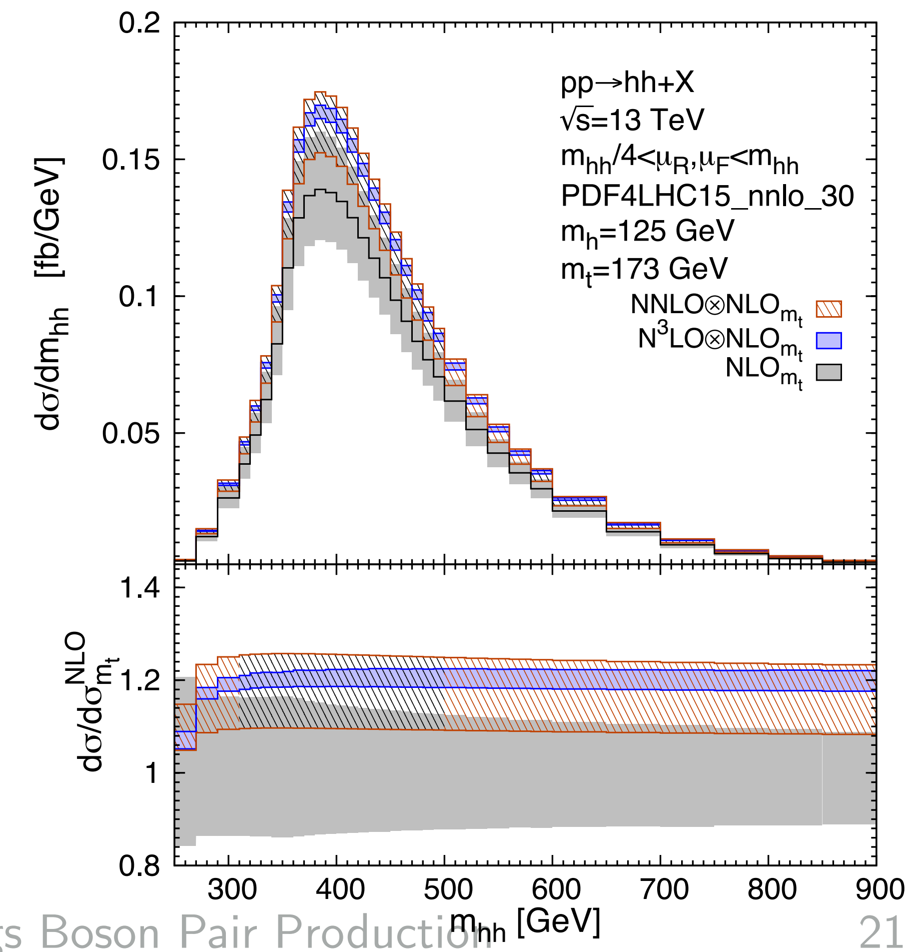
$$\mathcal{R}(ij \to HH+X) = \frac{\mathcal{A}^{\text{Born}}_{\text{Full}}(ij \to HH+X)}{\mathcal{A}^{(0)}_{\text{HEFT}}(ij \to HH+X)}$$

even $N^3LO_{\text{NLO-i}}$ is known

Chen, Li, Shao, Wang 19

| $\sqrt{s}$ | 13 TeV |
|---|---|
| $\text{NLO}_{m_t}$ | $27.56^{+14\%}_{-13\%}$ |
| $\text{NNLO}\oplus\text{NLO}_{m_t}$ | $32.16^{+5.9\%}_{-5.9\%}$ |
| $\text{NNLO}_{\text{B-i}}\oplus\text{NLO}_{m_t}$ | $33.08^{+5.0\%}_{-4.9\%}$ |
| $\text{NNLO}\otimes\text{NLO}_{m_t}$ | $32.47^{+5.3\%}_{-7.8\%}$ |
| $\text{N}^3\text{LO}\oplus\text{NLO}_{m_t}$ | $33.06^{+2.1\%}_{-2.9\%}$ |
| $\text{N}^3\text{LO}_{\text{B-i}}\oplus\text{NLO}_{m_t}$ | $34.17^{+1.9\%}_{-4.6\%}$ |
| $\text{N}^3\text{LO}\otimes\text{NLO}_{m_t}$ | $33.43^{+0.66\%}_{-2.8\%}$ |



# Mass Scheme Uncertainties

So far, all results used OS renormalization of $m_t$,

but also other schemes, e.g. $\overline{\text{MS}}$ valid $\rightarrow$ additional mass scheme uncertainty

NLO predictions in $\overline{\text{MS}}$ scheme

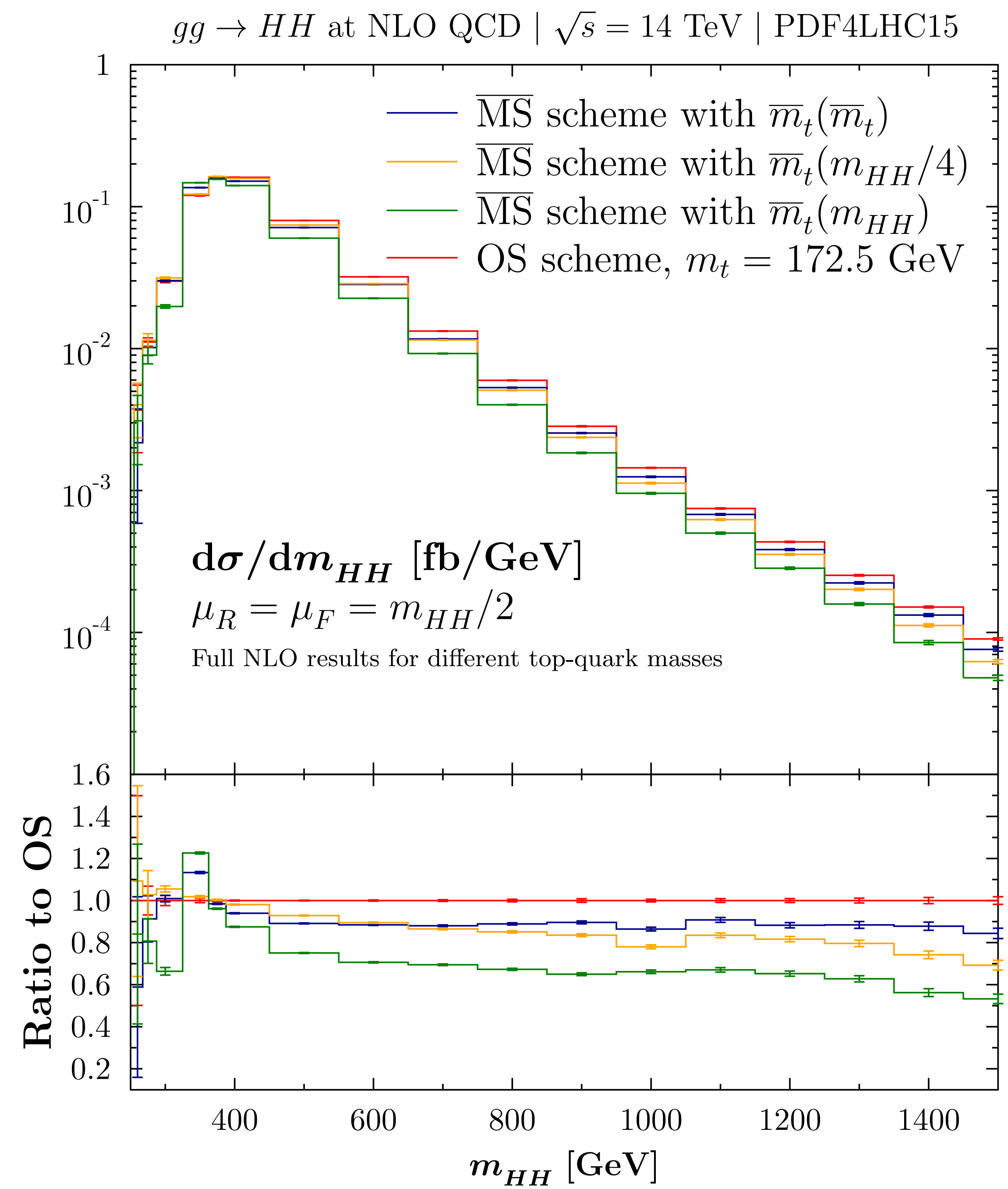
Baglio, Campanario, Glaus, Mühlleitner, Spira, Streicher 19,20

$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=300\text{ GeV}} = 0.0312(5)^{+9\%}_{-23\%}\,\text{fb/GeV}$$

$$\frac{d\sigma(gg \to HH)}{dQ}\Big|_{Q=1200\text{ GeV}} = 0.000435(4)^{+0\%}_{-30\%}\,\text{fb/GeV}$$

large scheme uncertainties at large $m_{HH}$

(larger than $\mu_R, \mu_F$ dependence)



# full HH EW

Bi, Huang, Huang, Ma, Yu `23