MAX-PLANCK-INSTITUT
FÜR PHYSIK

Top quark mass effect in Z boson pair production through gluon fusion

Based on *arXiv:2101.12095* and works in progress with Christian Brønnum-Hansen and Marius Wiesemann.

Chen-Yu Wang | 2024-04-16 | Loops and Legs 2024

Outline

1. Motivation

2. Two-loop $gg \to ZZ$ with full m_t dependence

- Numerical IBP reduction
- Master integral evaluations in the kinematics space

3. Conclusion & Outlook

Motivation

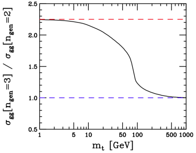
- Higher experimental precision at the LHC requires more accurate theoretical predictions.
  $\rightarrow$ **more loops**
- As $\sqrt{s}$ increases, massive particles in the loops become more important.
  $\rightarrow$ **more masses**
- Precision wish list *Amoroso et al. 2020*

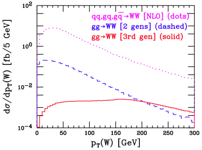
process	known	desired
⋮	⋮	⋮
$pp \to H + t\bar{t}$	$\text{NLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$	NNLO_{QCD}
⋮	⋮	⋮
$pp \to VV'$	$\text{NNLO}_{\text{QCD}} + \text{NNLO}_{\text{EW}}$ $+ \text{NLO}_{\text{QCD}}$ (gg channel)	NNLO_{QCD} (gg channel with massive loops)
⋮	⋮	⋮
$pp \to t\bar{t}$	$\text{NNLO}_{\text{QCD}} + \text{NLO}_{\text{EW}}$ NLO_{QCD} (with decays) NLO_{EW} (with decays)	NNLO_{QCD} (with decays) *Czakon, Mitov, et al. 2021*
⋮	⋮	⋮

Motivation

- background to $pp \to H^* \to WW/ZZ$, interference $\to$ Higgs width *Campbell et al. 2011a; Kauer and Passarino 2012; Caola and Melnikov 2013; Campbell et al. 2014; Azatov et al. 2015*
- anomalous gauge couplings

- gg channel: loop-induced: enters σ_{ppVV} at NNLO, enhanced by gluon flux & event selection *Binoth et al. 2006; Campbell et al. 2011b*
- massless NLO contribution: *Caola, Melnikov, Röntsch, et al. 2015, 2016*
  $\geq 50\%$ to σ_{ggZZ}, 50% to σ_{ggWW}
  $6-8\%$ to σ_{ppZZ}, 2% to σ_{ppWW}
- 3rd generation increases massless LO σ_{ggWW} by $10-13\%$ *Binoth et al. 2006; Campbell et al. 2011a*
- dominant contribution for high p_T *Campbell et al. 2011a*





Campbell et al. 2011a

$gg \to ZZ$: Progress

- LO: one-loop *Glover and Bij 1989; Kao and Dicus 1991; Duhrssen et al. 2005; Binoth et al. 2006*
- NLO real: one-loop *Agrawal and Shivaji 2012; Melia et al. 2012; Campanario et al. 2013*
- NLO virtual s-channel Higgs: $gg \to H^* \to VV$ *Spira et al. 1995; Harlander and Kant 2005; Binoth et al. 2006; Aglietti et al. 2007; Anastasiou et al. 2007*
- NLO virtual: two-loop massless: $gg \to VV$ *Caola, Henn, et al. 2015; Manteuffel and Tancredi 2015*
- NLO virtual: two-loop massive:
  - region expansions: $gg \to ZZ$ *Melnikov and Dowling 2015; Gröber et al. 2019; Davies, Mishima, Steinhauser, and Wellmann 2020*
  - full m_t dependence: $gg \to ZZ$ *Agarwal, Jones, and Manteuffel 2020; Brønnum-Hansen and Wang 2021*
- Phenomenology studies: *Caola, Melnikov, Röntsch, et al. 2015; Caola, Dowling, et al. 2016; Grazzini et al. 2016; Alioli, Ferrario Ravasio, et al. 2021; Buonocore et al. 2022; Agarwal, Jones, Kerner, et al. 2024.*

Analytic or alternative

Analytic

- deeper understanding (e.g. Parke-Taylor)
- fast, precise evaluation (e.g. Li_n, ${}_pF_q$, $K(\lambda)$, MPL, ...)
- $gg \to ZZ$ [Federico Coro], $Vgq\bar{q}$ [Cesare Carlo Mella]

Alternatives

- region expansions: $gg \to ZZ$ Melnikov and Dowling 2015; Davies, Mishima, Steinhauser, and Wellmann 2020, $gg \to ZH$ Davies, Mishima, and Steinhauser 2020, $gg \to HH$ Davies, Herren, et al. 2022; Davies, Schönwald, et al. 2023 [Hantian Zhang]
- sector decomposition: $gg \to HH$ Borowka, Greiner, et al. 2016 [Matthias Kerner], $gg \to ZZ$ Agarwal, Jones, and Manteuffel 2020, $gg \to ZH$ Chen, Heinrich, et al. 2020
- differential equations: $gg \to t\bar{t}$ Chen, Czakon, et al. 2018, $gg \to H$ Czakon and Niggetiedt 2020; Niggetiedt and Usovitsch 2024 [Marco Niggetiedt], $gg \to HH$ Davies, Herren, et al. 2022 [Joshua Davies]
- dispersion relation: $e^+e^- \to ZH$ Song and Freitas 2021; Freitas and Song 2023 [Ayres Freitas]
- combined: $gg \to HH$ Davies, Heinrich, et al. 2019, $gg \to ZH$ Chen, Davies, et al. 2022

Analytic or numeric?

Analytic

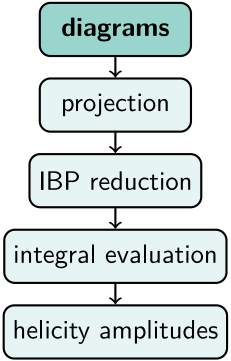
- deeper structure (e.g. Parke-Taylor)
- wider applications (e.g. changing parameters)

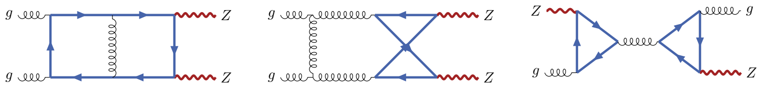
Alternatives

$$\left.\begin{array}{l}\text{fast numerical method that produces \textbf{arbitrary precision} result}\\ \text{good \textbf{interpolation} algorithm / importance sampling}\end{array}\right\} \Rightarrow \begin{array}{l}\text{fast, precise evaluation}\\ \text{in the parameter space}\end{array}$$

- Numeric IBP reduction: set masses and kinematic variables to rational numbers
- Numeric DE evaluation: simple boundary condition + arbitrary precision *Lee et al. 2018; Liu, Ma, and Wang 2018; Abreu et al. 2020; Hidding 2020; Moriello 2020*

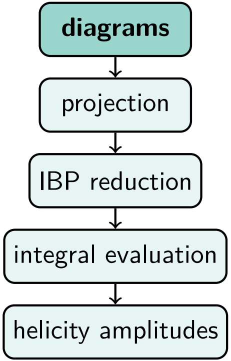
Diagrams

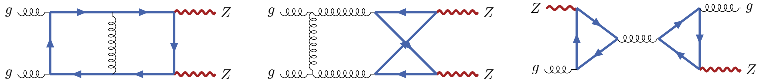




- Diagrams generated in `QGRAF` *Nogueira 1993* and processed in `FORM` *Vermaseren 2000; Kuipers et al. 2015; Ruijl et al. 2017*
- Decomposition in colour factors: $A = \delta^{AB}\left[C_A A^{[C_A]} + C_F A^{[C_F]} + A^{[\triangle^2]}\right]$
- γ_5 scheme:
  - $\triangle^2$ diagrams: Larin scheme *Larin and Vermaseren 1991; Moch et al. 2015.*
  - two-loop ZZ: naive scheme, thanks to Furry's theorem.
- IR structure: $A^{(2)}(\epsilon,\mu) = I^{(1)}(\epsilon,\mu)A^{(1)}(\epsilon,\mu) + F^{(2)}(\epsilon,\mu)$. *Catani 1998*

Diagrams





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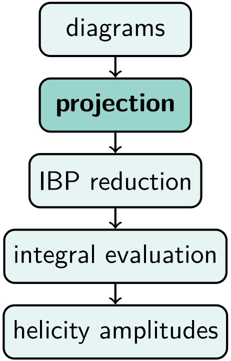
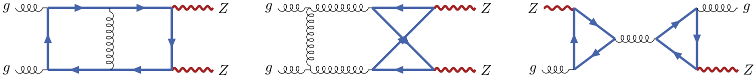
Projection



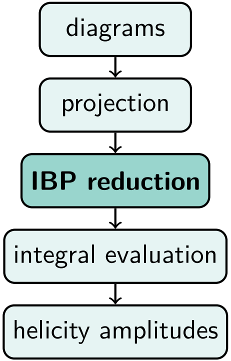


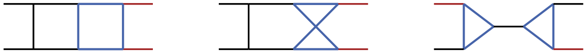
- Projection onto 18 tensor structures *Binoth et al. 2006*

$$A = \sum_{l=1}^{18} A_l \, T_l$$

- Find symmetry relations with `REDUZE 2` *Manteuffel and Studerus 2012.*

IBP reduction: KIRA & FireFly



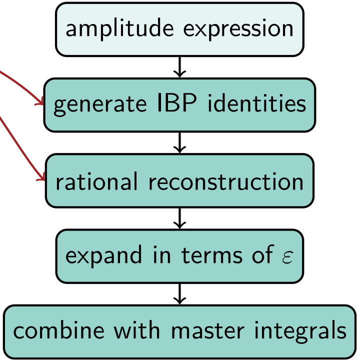


Reduction performed **at each phase space point** with KIRA *Klappert, Lange, et al. 2020; Lange et al. 2021* & FireFly *Klappert and Lange 2020; Klappert, Klein, et al. 2021:*

- 21 families with highest rank 4 $\rightarrow$ 205 MIs + crossings.
- Parametric only in d, numeric (s, t, m_t, m_Z):
  - $m_t = 173$ GeV, $m_Z = 91.1876$ GeV.
  - Rational values for s, t.
- Avoid non-factorisable denominators *Smirnov and Smirnov 2020; Usovitsch 2020.*
- Comparing with (semi-)analytical reduction:
  - Lower the memory/storage consumption significantly.
  - Straightforward parallelisation.

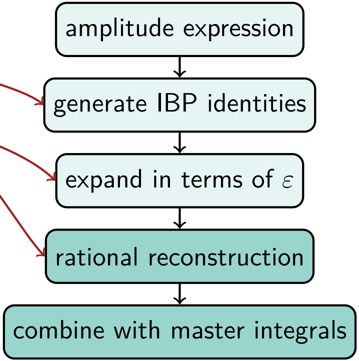
IBP reduction: KIRA & FireFly

- Numerical reduction with KIRA & FireFly.
- For each phase space point: ~ 3 CPU hours, ~ 2 GB memory.
- Rooms for improvement:
  - IBP system only needs to be generate once $\Rightarrow$
    - Generate analytic IBP system with KIRA.
  - IBP system contains redundant relations $\Rightarrow$
    - Simplify the system with RATRACER. [Vitaly Magerya]
  - We don't need the full rational expression in terms of d $\Rightarrow$
    - Expand in terms of ε before reconstruction with RATRACER.
    - Truncate irrelevant coefficients according to MI evaluations.

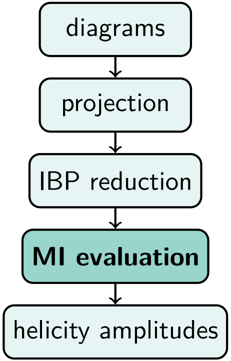


IBP reduction: `KIRA` & `RATRACER` & `FireFly`

- Generate the IBP system analytically with `KIRA`.
- Simplify, expand and numerically reconstruct the system with `RATRACER` & `FireFly`.
- For each phase space point: ~ 3 CPU minutes, < 1 GB memory.
- Benefits:
  - No more time wasted on IBP system generation.
  - Slimer IBP system to reconstruct.
  - Purely numerical reconstruction.
  - Only relevant coefficients are reconstructed.



Master integral evaluation



- Based on the **auxiliary mass flow method** *Liu, Ma, and Wang 2018; Liu, Ma, Tao, et al. 2020; Liu and Ma 2022*

$$I \propto \lim_{\eta \to 0^+} \int \prod_{i=1}^{2} \mathrm{d}^d k_i \prod_{a=1}^{9} \frac{1}{[q_a^2 - (m_a^2 - i\eta)]^{\nu_a}}$$

- Add an imaginary part to the **top quark mass**

$$m_t^2 \to m_t^2 - i\eta.$$

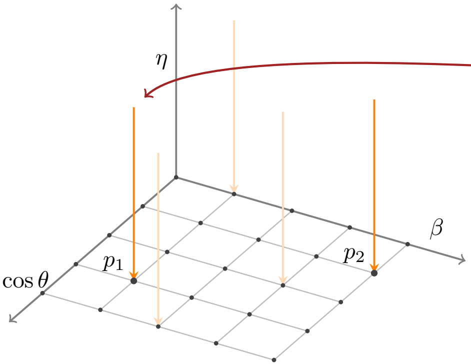
- Solve differential equations w.r.t the mass

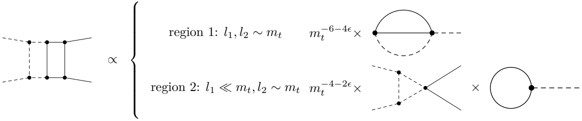
$$\partial_x \boldsymbol{I} = \boldsymbol{M}\boldsymbol{I}, \quad x \propto -i\eta$$

with boundary condition at $x \to -i\infty$. Physical mass at $x \to 0$.

Master integral evaluation

$$s = \frac{4m_Z^2}{1-\beta^2}, \quad t = m_Z^2 - \frac{s}{2}\left(1 - \beta\cos\theta\right).$$

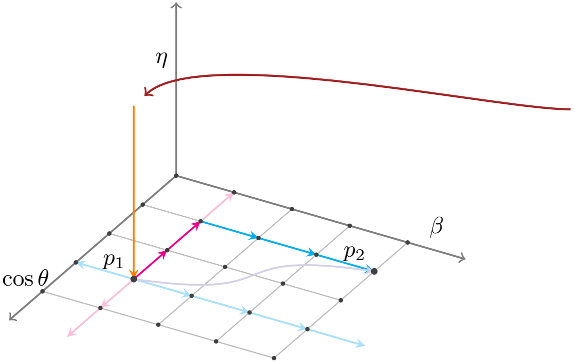


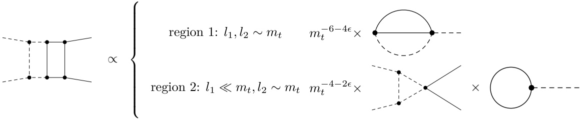


- For each phase space point: 20 digits $\sim$ 1 CPU hour.
- Arbitrary precision ensures stable evaluation in the singular region.
- Straightforward parallelisation.
- Solving differential equation in the kinematic space.

Faster master integral evaluation

$$s = \frac{4m_Z^2}{1-\beta^2}, \quad t = m_Z^2 - \frac{s}{2}(1-\beta\cos\theta)\,.$$





- Solve for a few reference points from the boundary conditions, then transport in the parametric space.
- For rectangular grid, series expansion along one dimension enables faster evaluation.

Faster master integral evaluation

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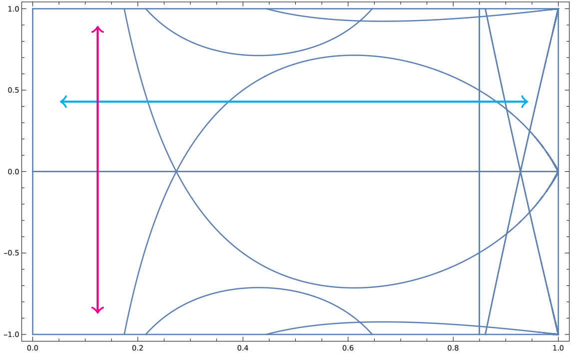


Singularities in the parametric space $(\beta, \cos\theta)$.

- Solve for a few reference points from the boundary conditions, then transport in the parametric space.
- For rectangular grid, series expansion along one dimension enables faster evaluation.
- For each phase space point: 20 digits $\sim$ 1 CPU minute on average.

Cross-checking

- $\sqrt{s} \approx 210$ GeV
- $\theta \approx 114°$
- $\mu = m_Z$

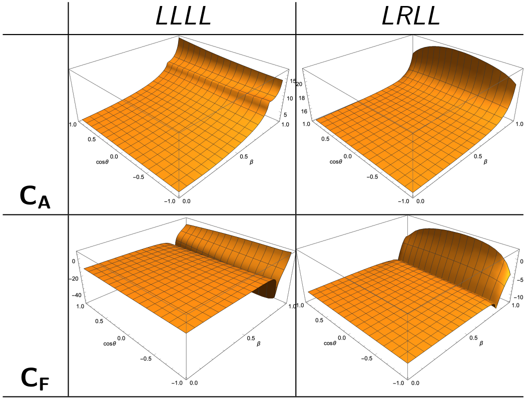
$\mathbf{C_A}$		ϵ^{-2}	ϵ^{-1}
LLLL	$A^{(2)}/A^{(1)}$	$1.0000000000008 - 7.6 \cdot 10^{-13} i$	$0.8304916142577 + 3.229874368770 i$
	IR pole	1.0000000000000	$0.8304916142539 + 3.229874368771 i$
LRLL	$A^{(2)}/A^{(1)}$	$1.0000000000009 - 1.4 \cdot 10^{-12} i$	$0.2359507533599 + 2.885154863850 i$
	IR pole	1.0000000000000	$0.2359507533772 + 2.885154863852 i$

- IR divergence matches Catani's IR operator *Catani 1998.*

$$I^{(1)}(\epsilon,\mu)A^{(1)}(\epsilon,\mu) = \frac{e^{\epsilon\gamma_E}}{\Gamma(1-\epsilon)}\left(\frac{C_A}{\epsilon^2} + \frac{\frac{11}{6}C_A}{\epsilon}\right)\left(\frac{\mu^2 e^{i\pi}}{s}\right)^{\epsilon} A^{(1)}(\epsilon,\mu)$$

- Cross-checking:
  - Master intergrals against `pySecDec` *Borowka, Heinrich, Jahn, Jones, Kerner, Schlenk, and Zirke 2018; Borowka, Heinrich, Jahn, Jones, Kerner, and Schlenk 2019*
  - Finite terms against *Agarwal, Jones, and Manteuffel 2020* analytic IBP reduction & sector decomposition.
  - Finite terms against *Davies, Mishima, Steinhauser, and Wellmann 2020* low- and high-energy expansions.
  - Form factors against `ggvvamp` *Manteuffel and Tancredi 2015* in the massless m_t limit.

Helicity amplitudes



- $\frac{2\,\mathrm{Re}\left[F^{(2)}A^{(1)\star}\right]}{|A^{(1)}|^2}$ as a function of energy β and scattering angle $\cos\theta$.
- Massive boson polarisation vectors written in terms of decay currents

$$\epsilon^{*\mu}_{3,L} = \langle 5|\gamma^\mu|6], \qquad \epsilon^{*\mu}_{4,L} = \langle 7|\gamma^\mu|8].$$

Conclusion & Outlook

- We improved our calculation of two-loop helicity amplitudes for $gg \to ZZ$ with full m_t dependence.
- **Numeric IBP reduction** is efficient in practice for complicated multi-scale processes.
- **Auxiliary mass flow** method and **solving differential equation in the kinematic space** provide an efficient and precise way to evaluate multi-loop integrals.
- We are implementing the top quark mass corrections in the `POWHEG-BOX` **framework** *Nason 2004; Frixione et al. 2007; Alioli, Nason, et al. 2010; Ježo and Nason 2015*

Thank you for your attention!