Progress towards full Next-to-Leading Logarithmic Accuracy to Scattering at High Energy

including relevance of NLL for describing e.g. $d\sigma/dp_\perp$ and related observables

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with E. Byrne, A. Maier, J. Smillie and the rest of HEJ,
G. Falcioni, M. Rosca recently joined

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Introduction

Outline of talk:

1. Amplitudes in the High Energy Limit
2. From amplitudes to cross sections
3. Progress towards full next-to-leading logarithmic accuracy

High Energy Jets:

- **Factorisation of matrix elements** using **currents** retains analytic properties such as **crossing symmetries**
- systematic **power expansion of QCD amplitudes** for real emissions
- all-order **leading and sub-leading logarithmic corrections** Focus of this talk
- **matching**, **results**. . .

Regge theory

Regge theory describes scattering from a **central potential** in terms of the projections on Legendre polynomial and states of **definite orbital angular momentum** (partial wave analysis)

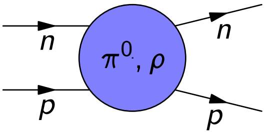
The analysis of **analytic scattering amplitudes** in terms of Regge Theory: Regge (1959)

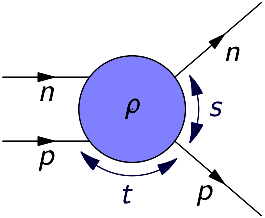
$$\mathcal{M} = \sum_i \Gamma_i(t)\ (s)^{j_i}$$

At **large energies** s, the contribution from particle of **highest spin j dominates**

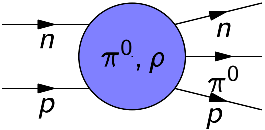
$$\mathcal{M} \to \Gamma(t)\ (s)^j$$

Regge limit: $s \gg -t$ or $s \gg p_t^2$





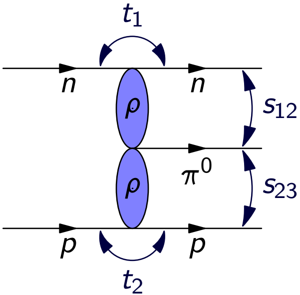
Large s of course leads to the possibility of **multi-particle production**



Multi-Regge limit: Brower, DeTAR, Weis (1974)

$s_{12}, s_{23} \gg p_{t_i}^2, |t_i|, \quad |t_i| \sim |t_j|, \quad |p_{t_i}| \sim |p_{t_j}|$

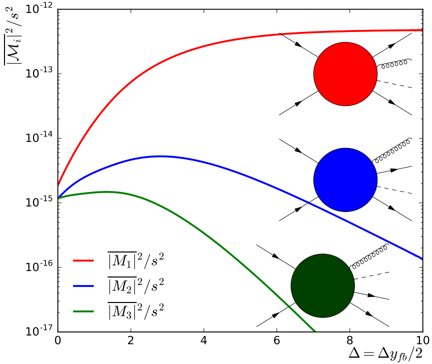
$$\mathcal{M} = s_{12}^j s_{23}^j \, \Gamma(t_1, t_2, s/(s_{12}s_{23}))$$

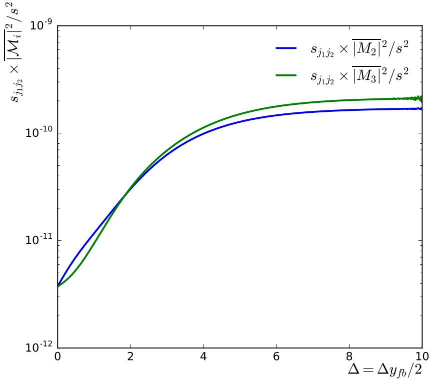


No underlying theory for strong interactions; derives constraints on the high energy behaviour based on the constraints from an **analytic scattering amplitude**.

Scaling of QCD Amplitudes

The scaling extends to QCD amplitudes involving also Higgs bosons, W, Z and photon production.

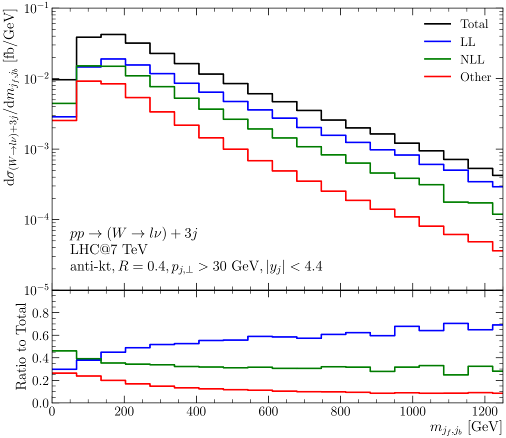


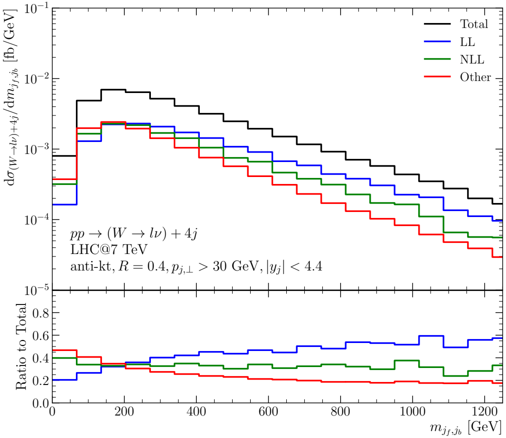


The **scaling** for different kinematic evaluations of the same amplitude is exactly as predicted by Regge theory applied to the **planar graph** connecting the rapidity-ordered configuration.

M. Heil, A. Maier, J.M. Smillie, JRA, arXiv:1706.01002

Cross sections vs logarithmic ordering

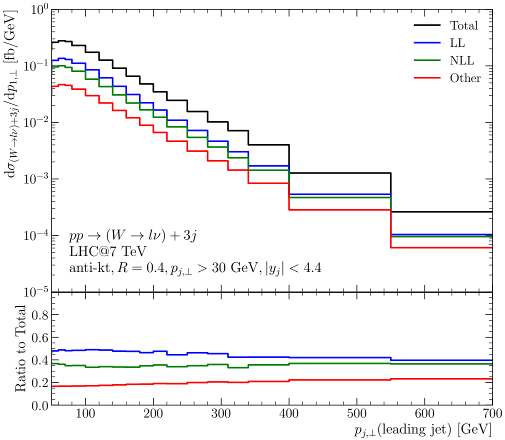


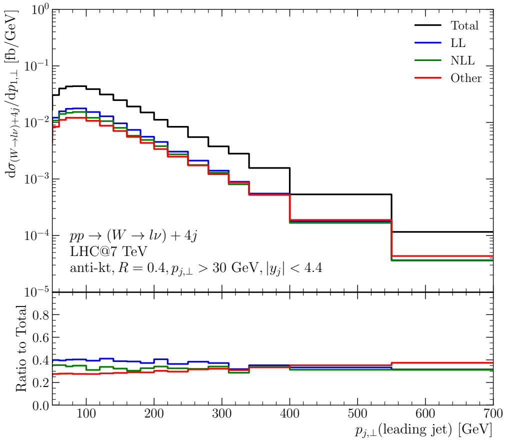


$pp \to W3j$ J. Black, H. Brooks, A. Maier, J.M. Smillie, JRA, arXiv:2012.10310 $pp \to W4j$

The cross sections really do follow the logarithmic ordering.

Logartihmic split for other observables





pp $\to$ *W3j* J. Black, H. Brooks, A. Maier, J.M. Smillie, JRA, arXiv:2012.10310 *pp* $\to$ *W4j*

The logarithmic ordering is less good for p_t-based observables – expect NLL to be as important as LL, and therefore these corrections are necessary.

Perurbative Corrections in the High Energy Limit

Building approximations used for all-order evaluations. Standard approach:

$qQ \to qQ : \overline{|\mathcal{M}|}^2 \propto \frac{s^2 + u^2}{t^2}.$

In the high energy limit $s \sim u \gg t \to -k_\perp^2$ so $\hat{\sigma} \propto \frac{1}{k_{1,\perp}^2 k_{2,\perp}^2}$.

In the limit where all $s_{ij} \gg p_t^2$ $2 \to n$ (in LL configurations) factorise: $\hat{\sigma} \propto \prod_i^n \frac{1}{k_{i,\perp}^2}$.

Since the $2 \to 3$ **real emision** perturbative corrections have $|M|^2/s^2 \to \mathrm{constant}$ for large $\Delta y_{fb} \sim \log(s/p_t^2)$, integration over the rapidity of the middle parton will contribute a correction $\alpha_s \Delta y_{fb} \sim \alpha_s \log(s/p_t^2)$.

The other orderings of momenta (and other processes) contribute sub-leading corrections which can be included at next-to-leading order.

Perurbative Corrections in the High Energy Limit

The virtual corrections also exhibit universal logarithmic terms in the **colour octet** channel

$$\mathcal{A}_4^{1-loop}(\bar{q},\bar{Q};Q,q) = g^4\left[\left(\delta_{i_1 i_3}\delta_{i_2 i_4} - \frac{1}{N_c}\delta_{i_1 i_4}\delta_{i_2 i_3}\right) a_{4;1}(1,2;3,4)\right] + \delta_{i_1 i_3}\delta_{i_2 i_4} a_{4;2}(1,2;3,4)$$

$$a_{4;1}(-,-;+,+) = c_\Gamma\ a_{4;0}(-,-;+,+)\ F_{a,1}^{--}(\varepsilon, s_{!2}, s_{13}, s_{14})$$

$$F_{a;1}^{--}(\varepsilon, s_{12}, s_{13}, s_{14}) = \left(-\frac{\mu^2}{s_{14}}\right)^{\varepsilon}\left\{N_c\left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} + \frac{11}{3} - \frac{2}{\varepsilon}\log\frac{s_{14}}{s_{12}} + \frac{13}{9} + \pi^2\right] + \cdots\right\} - \frac{1}{\varepsilon}\beta_0$$

Logarithmic structure predicted to all orders (BFKL, Regge, VDD,. . .)**.**

Control perturbative corrections of $\alpha_s^n \log^n(s/p_t^2)$ (leading logarithm) and $\alpha_s^{n+1}\log^n(s/p_t^2)$ (NLL)

Regge Theory and QCD

QCD allows for the calculation of the scattering amplitudes. The amplitudes are still **analytic**, and a Regge analysis can be applied. The amplitude can be reconstructed (to ensure logarithmic accuracy of the cross section) by effective vertices. These **building blocks** can be **calculated in QCD**.

These are the impact factors and kernels in the BFKL language. So what is different with High Energy Jets?

Some of the problems of standard analysis: Simplest example includes just one gluon exchange $qQ \to qQ : \overline{|\mathcal{M}|}^2 \propto \frac{s^2+u^2}{t^2} \to \frac{2s^2}{t^2}, \hat{\sigma} \propto \prod_i^n \frac{1}{k_{i,\perp}^2}$. HE limit does not describe well even this simplest process within the phase space relevant for LHC phenomenology.

Analytic Constraints

How did the high energy limit get it so wrong for such a simple process?
Need a method to "analytically reconstruct" the amplitude from the understanding of the high energy behaviour.

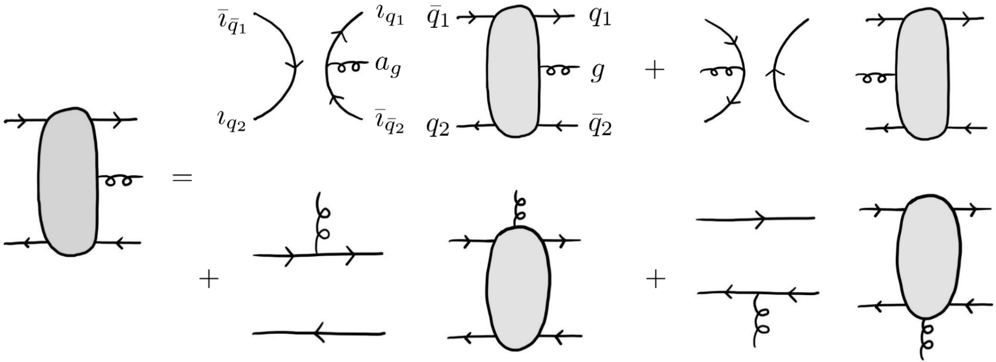
$$\mathcal{M} \propto j_\mu(p_a, p_1)\, j^\mu(p_b, p_2)/t$$

- Simple description; each current depends on momenta of relevant quarks only.
- Same helicity: contributes s, opposite contributes u.
- **Lorentz invariance**
- Ensures **crossing symmetry**

Will require these constraints on the "analytic reconstruction" by insisting on contractions of currents. Higher logarithmic corrections will require two-particle production currents...

Calculation of two-particle current

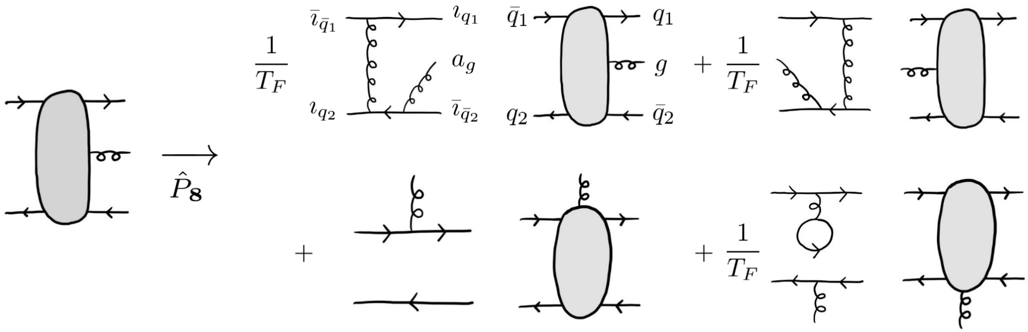
Extract two gluon current from 2 → 3 amplitude (see thesis of E. Byrne):



$$\hat{P}_{\mathbf{8}\,\imath_1\bar{\imath}_1\imath_2\bar{\imath}_2} = \frac{1}{T_F} T^{a_t}_{\imath_1\bar{\imath}_1} T^{a_t}_{\imath_2\bar{\imath}_2}$$

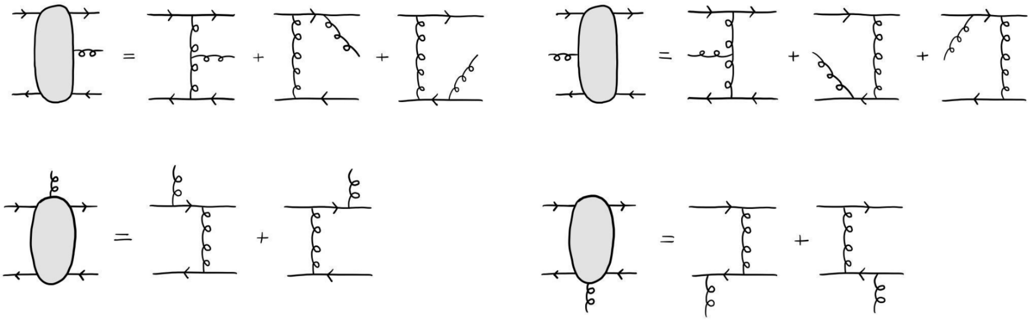
Use colour basis which allows projection of the colour octet

Calculation of two-particle current, II



Last term has colour contribution 0.

Calculation of two-particle current, III



Most contributions already kinematically factorised. For some helicity configurations the factorisation is exact.

Calculation of two-particle current, IV

$$M(\bar{q}_1^{\ominus}, q_1^{\oplus}, \bar{q}_2^{\ominus}, q_2^{\oplus}, g^{\lambda_g}) = J_{\bar{q}qg^*\,\mu_{t_1}}(p_{\bar{q}1}^{\ominus}, p_{q1}^{\oplus}, p_{t_1}) \left(\frac{-i}{t_1}\right) J^{\mu_{t_1}}_{\bar{q}qgg^*}(p_{\bar{q}2}^{\ominus}, p_{q2}^{\oplus}, p_g^{\lambda_g}, -p_{t_1}),$$

$$J^{\mu_{t_1}}_{\bar{q}qgg^*}(p_{\bar{q}2}^{\ominus}, p_{q2}^{\oplus}, p_g^{\lambda_g}, -p_{t_1}) = -ig_s^2 \epsilon_{\mu_g}(p_g^{\lambda_g}, p_r)\Bigg\{ \frac{1}{s_{q_2 g}} [q_2|\mu_g(q_2+g)\mu_{t_1}|\bar{q}_2\rangle$$

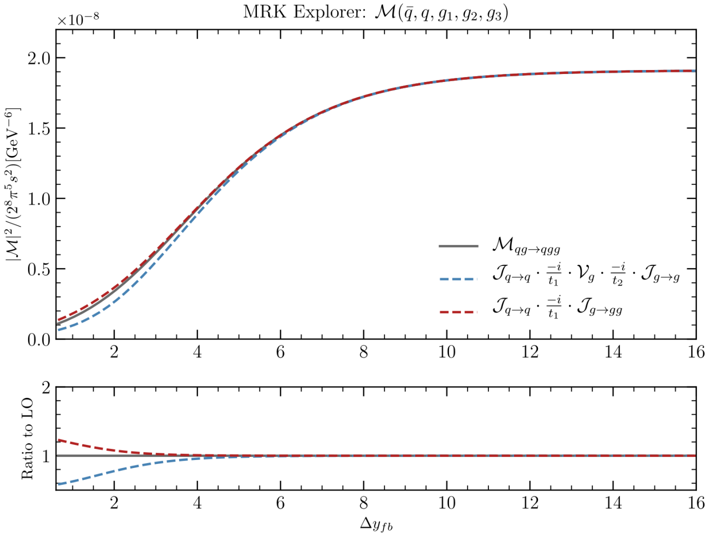
$$+ \frac{1}{t_2}\left(\eta^{\mu_{t_1}\mu_{t_2}}(p_{t_1}+p_{t_2})^{\mu_g} - 2p_g^{\mu_{t_2}}\eta^{\mu_{t_1}\mu_g} + 2p_g^{\mu_{t_1}}\eta^{\mu_{t_2}\mu_g} - t_1\frac{2p_{\bar{q}1}^{\mu_g}}{s_{\bar{q}_1 g}}\right)[q_2|\mu_{t_2}|\bar{q}_2\rangle\Bigg\}.$$

Gauge invariance: J evaluated with $\epsilon_{\mu_g} \to p_g^{\mu}$ is 0.

Final state particles distributed at rapidities $-\Delta/2, 0, \Delta/2$

Asymptotic limit fails to describe region relevant for LHC experiment.

But in the Multi-Regge-Kinematic limit (large invariant mass between all particles) just leading log accuracy (supplemented by analyticity) receives good description for all Δy



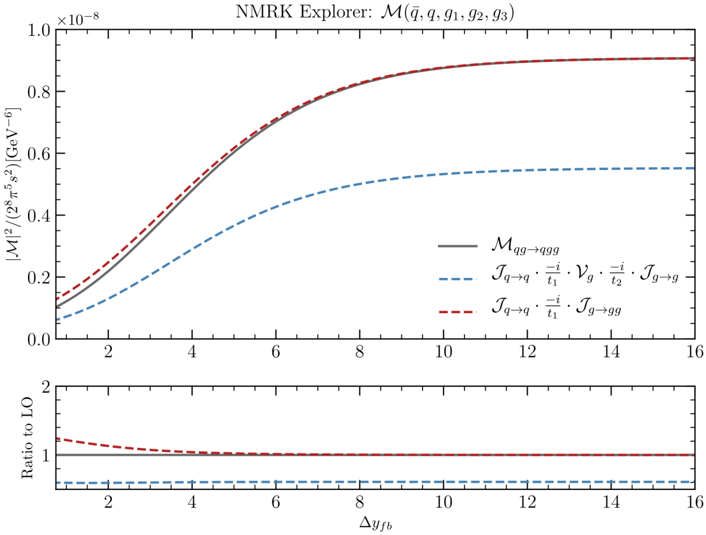
E. Byrne

Results for two-particle current

Final state particles distributed at rapidities $-\Delta/2-0.25, -\Delta/2+0.25, \Delta/2$

Asymptotic limit fails to describe region relevant for LHC experiment.

But in the Quasi-Multi-Regge-Kinematic limit (large invariant mass requirement dropped for exactly one pair) next-to-leading log accuracy (supplemented by analyticity) achieves good description for all Δy



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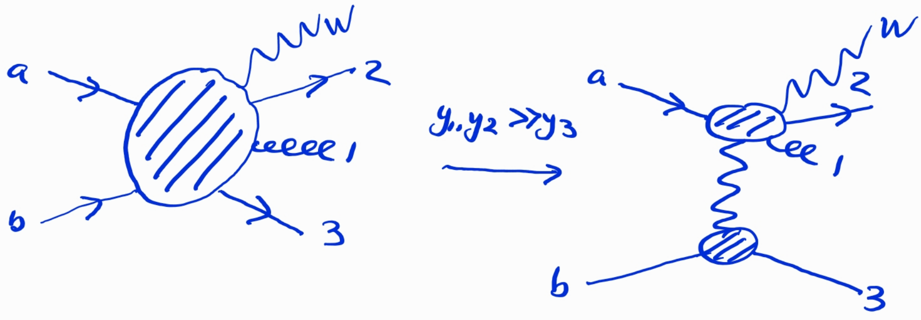
NLL components for Reggeisation

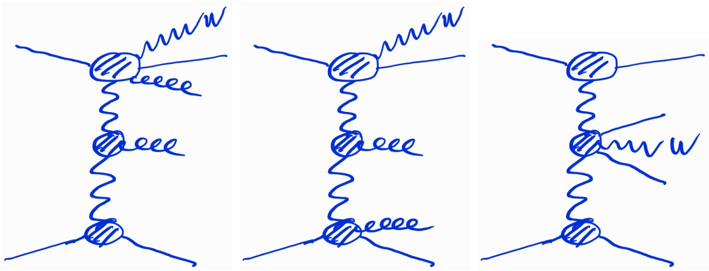
Consider again $pp \to W3j$. Next-to-leading logarithmic corrections arise e.g. from the amplitudes in the quasi-multi-Regge-kinematic limit, where the invariant mass between one pair of partons is not large.



Amplitude expressed as $\mathcal{M} = I^\mu(a, w, 1, 2)\ J_\mu(b, 3)$. Full crossing symmetry, Lorentz and gauge invariance in each component. $I^\mu(a, w, 1, 2)$ calculated by projection onto colour octet exchange in the t-channel.

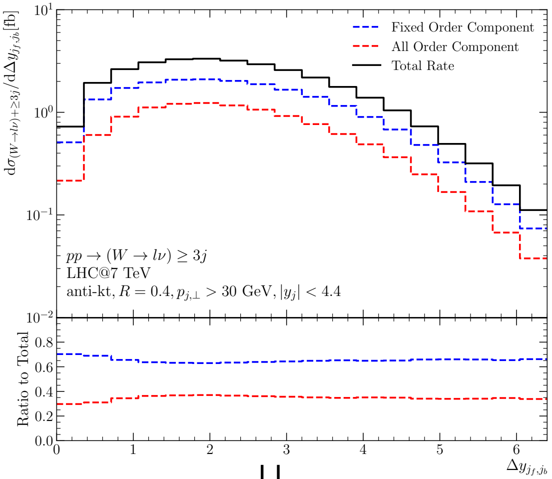
Higher Order Corrections

Can calculate higher order corrections with NLL components by explicit MC integration over the regulated amplitudes, represented by a Reggeised graph

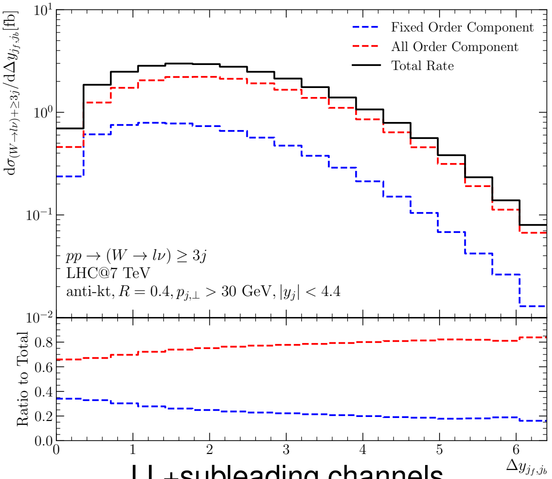


Virtual corrections encoded in the t-channel propagators.
Sub-processes and phase space points not reached with LL or NLL Reggeised description are treated at fixed order.
Investigate impact on p_t-distributions (traditionally hard to describe correctly in high energy framework)

Impact of NLL corrections for W3J



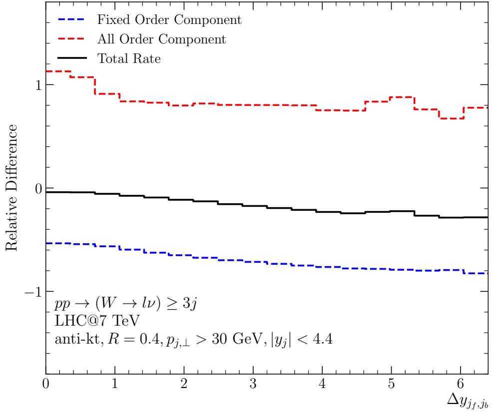
LL



LL+subleading channels

J. Black, H. Brooks, A. Maier, J.M. Smillie, JRA, arXiv:2012.10310

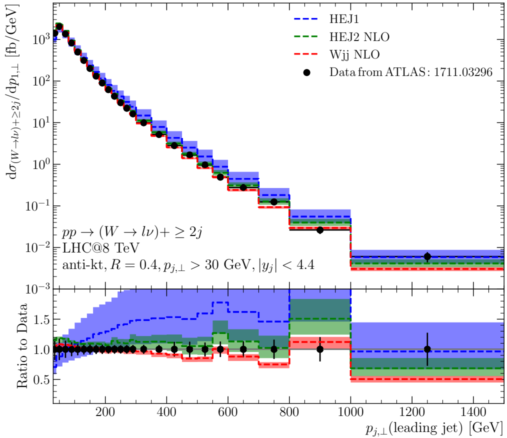
Impact of NLL corrections for W3J



Much less fixed order matching, much bigger resummation component. Final result of the inclusive distribution changes by up to 25%.

J. Black, H. Brooks, A. Maier, J.M. Smillie, JRA, arXiv:2012.10310

Comparison to Data (WJJ)



The NLL terms included and improvement in matching are sufficient to ensure the predictions agree well with data even in the most difficult regions of phase space.

arXiv:2012.10310

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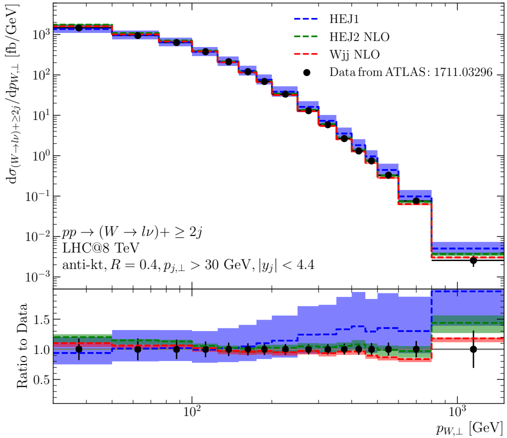

The NLL terms included and improvement in matching are sufficient to ensure the predictions agree well with data even in the most difficult regions of phase space. arXiv:2012.10310

Conclusions

Unsurprisingly, the inclusion of sub-leading logarithms leads to

- small changes in the leading regions of phase space
- a better description in sub-leading regions of phase space

Hall-marks of a well-behaved perturbative expansion.
Further improvements ongoing, calculation of full NLL currents with both virtual and real corrections using FKS regularisation.