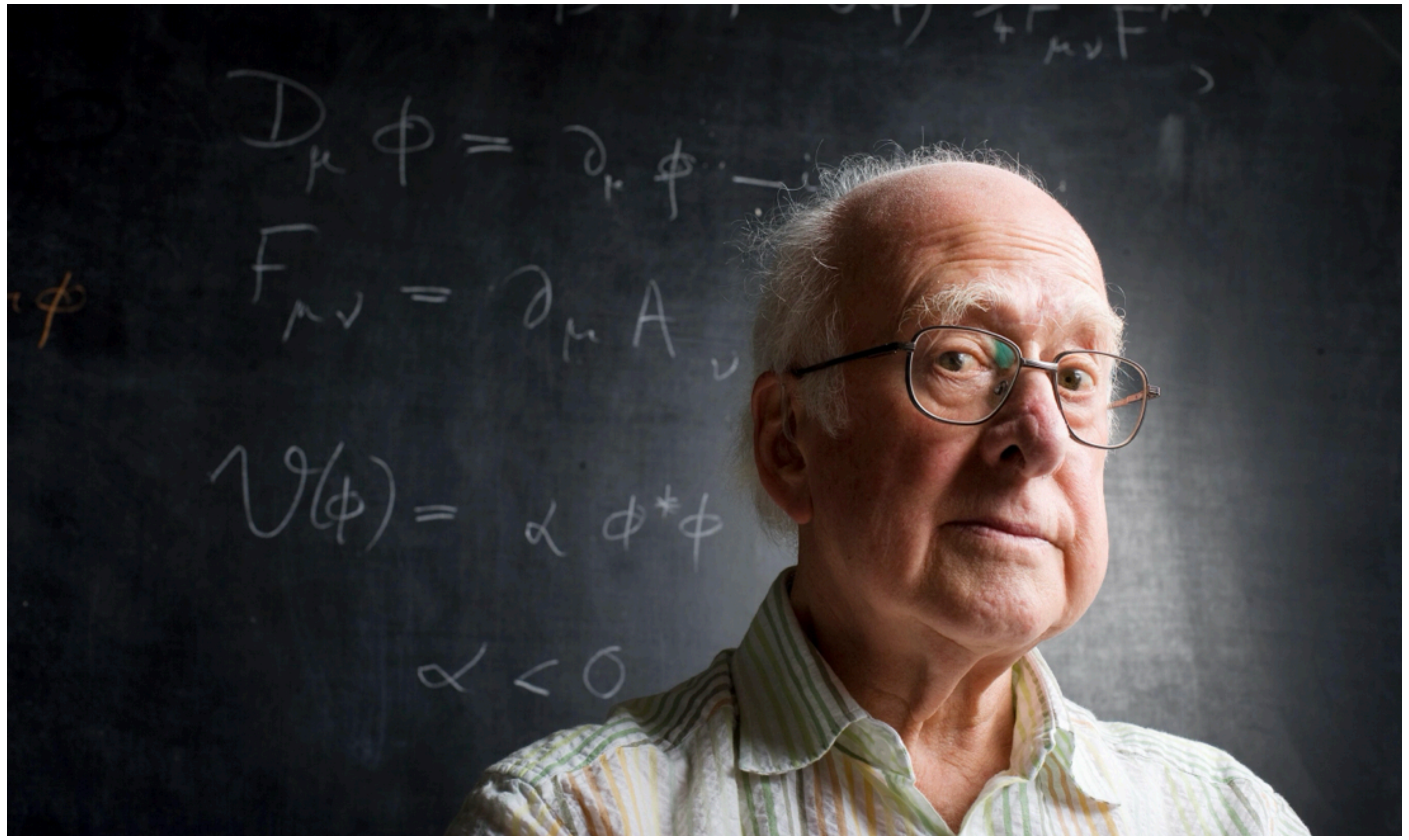
Loops and Legs, Wittenberg, April 2024

Peter Higgs



Memory Wall: https://www.online-tribute.com/PeterHiggs

Articles in the Press: https://higgs.ph.ed.ac.uk/peter-higgs/

Elusive: How Peter Higgs Solved the Mystery of Mass - Frank Close

Loops and Legs, Wittenberg, April 2024

Regge poles & cuts and the Lipatov vertex

Einan Gardi
Higgs Centre for Theoretical Physics, University of Edinburgh

New results on 2 to 3 scattering with Giulio Falcioni, Calum Milloy, Leonardo Vernazza and Samuel Abreu

The high-energy limit of 2 → 2 gauge-theory amplitudes

- **Simplification at leading power in t/s:** helicity is conserved; t-channel exchange is dominant

Reggeization:
(Regge-pole)

$$\frac{s}{t} \longrightarrow \frac{s}{t}\left(\frac{s}{-t}\right)^{\alpha_g(t)}$$

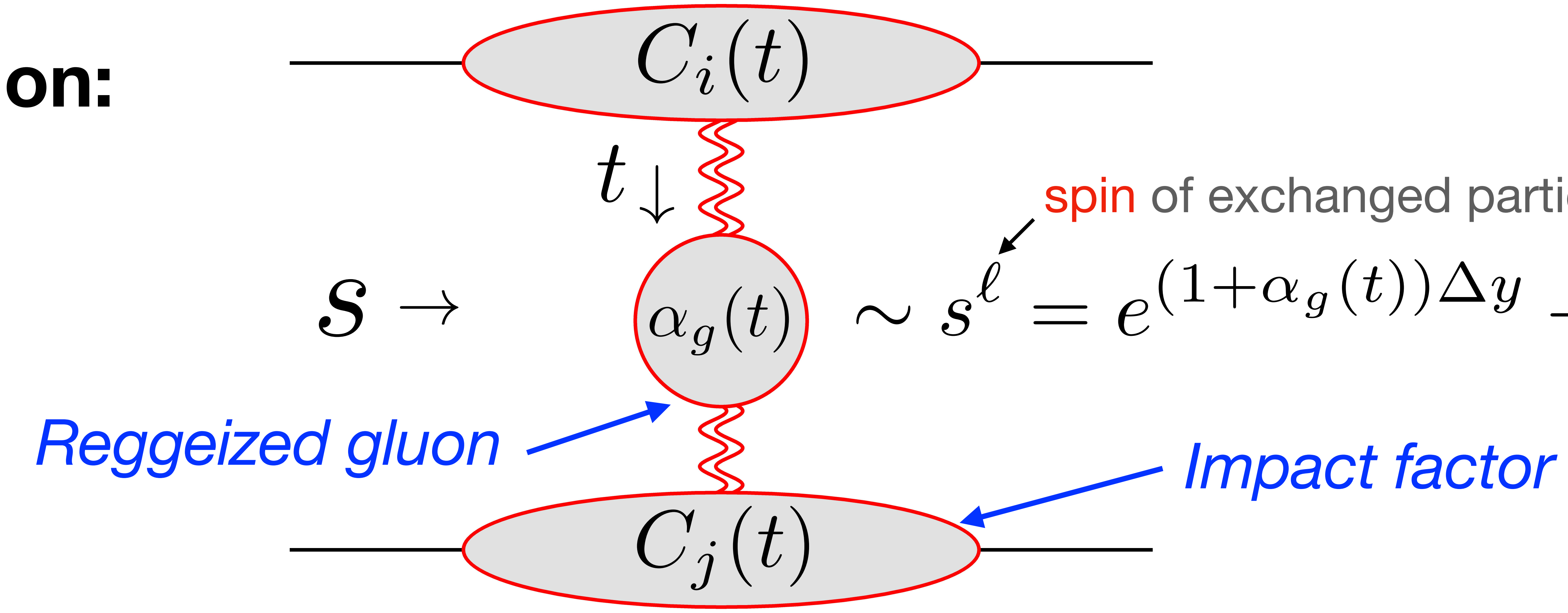
gluon Regge trajectory:

$$\alpha_g(t) = -\alpha_s \mathbf{T}_t^2 (\mu^2)^\epsilon \int \frac{d^{2-2\epsilon}k_\perp}{(2\pi)^{2-2\epsilon}} \frac{q_\perp^2}{k_\perp^2 (q_\perp - k_\perp)^2} + \mathcal{O}(\alpha_s^2)$$

$$= \frac{\alpha_s}{\pi}\mathbf{T}_t^2 \left(\frac{-t}{\mu^2}\right)^{-\epsilon} \frac{B_0(\epsilon)}{2\epsilon} + \mathcal{O}(\alpha_s^2)$$

$$B_0(\epsilon) = e^{\epsilon\gamma_E}\frac{\Gamma^2(1-\epsilon)\Gamma(1+\epsilon)}{\Gamma(1-2\epsilon)} = 1 - \frac{\zeta_2}{2}\epsilon^2 - \frac{7\zeta_3}{3}\epsilon^3 + \ldots$$

Factorization:



spin of exchanged particle

$$\sim s^{\ell} = e^{(1+\alpha_g(t))\Delta y} + \ldots$$

- Regge-pole factorization amounts to a **relation** between gg→gg, qg→qg, qq→qq
- This holds for the **real part** of the amplitude through NLL.
  Beyond that it is violated by **non-planar** corrections associated with **multi-Reggeon** exchange forming **Regge cuts.** These effects are now better understood.

2 → 2 amplitudes: signature and reality properties

- Defining **signature** *even* and *odd* amplitudes under $s \leftrightarrow u$

$$\mathcal{M}^{(\pm)}(s,t) = \tfrac{1}{2}\Big(\mathcal{M}(s,t) \pm \mathcal{M}(-s-t,t)\Big)$$

- The spectral representation of the amplitude implies:

$$\mathcal{M}^{(+)}(s,t) = i\int_{\gamma-i\infty}^{\gamma+i\infty} \frac{dj}{2\sin(\pi j/2)}\, a_j^{(+)}(t)\, e^{jL}\,,$$

$$\mathcal{M}^{(-)}(s,t) = \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{dj}{2\cos(\pi j/2)}\, a_j^{(-)}(t)\, e^{jL}\,,$$

with $\left(a_{j^*}^{\pm}(t)\right)^* = a_j^{\pm}(t)$

$$L \equiv \log\left|\frac{s}{t}\right| - i\frac{\pi}{2} = \frac{1}{2}\left(\log\frac{-s-i0}{-t} + \log\frac{-u-i0}{-t}\right)$$

- Expanding the amplitude in the signature-symmetric log, L, the coefficients in $\mathcal{M}^{(+)}$ are imaginary, while in $\mathcal{M}^{(-)}$ real.

[See 1701.05241 Caron-Huot, EG, Vernazza]

The singularity structure of 2 → 2 amplitudes in the complex angular momentum plane: pole vs. cut

- The signature-odd amplitude admits

$$\mathcal{M}^{(-)}(s,t) = \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{dj}{2\cos(\pi j/2)}\, a_j^{(-)}(t)\, e^{jL}\,,$$

	singularity	amplitude asymptotics
pole	$a_j^{(-)}(t) \simeq \dfrac{1}{j-1-\alpha(t)}$	$\mathcal{M}^{(-)}(s,t)\vert_{\text{Regge pole}} \simeq \dfrac{\pi}{\sin\frac{\pi\,\alpha(t)}{2}}\dfrac{s}{t}\, e^{L\,\alpha(t)} + \ldots,$
cut	$a_j^{(-)}(t) \simeq \dfrac{1}{(j-1-\alpha(t))^{1+\beta(t)}}$	$\mathcal{M}^{(-)}(s,t)\vert_{\text{Regge cut}} \simeq \dfrac{\pi}{\sin\frac{\pi\,\alpha(t)}{2}}\dfrac{s}{t}\dfrac{1}{\Gamma\left(1+\beta(t)\right)} L^{\beta(t)}\, e^{L\,\alpha(t)} + \text{subleading logs}$

- Reggeization of the **signature-odd** amplitude (NLL): a manifestation of a pure **Regge pole**.

Signature, number of Reggeons and t-channel colour flow

- The signature odd and even sectors decouple

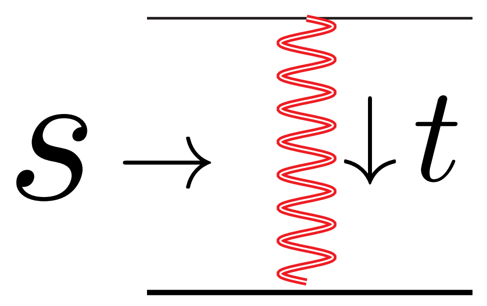
$$\mathcal{M}_{ij\to ij} \xrightarrow{\text{Regge}} \mathcal{M}^{(-)}_{ij\to ij} + \mathcal{M}^{(+)}_{ij\to ij}$$

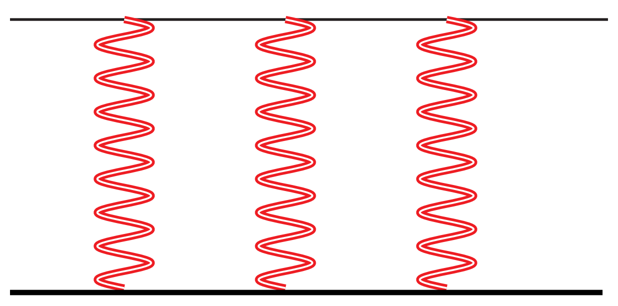
- odd/even signature amplitude is governed by the exchange of an odd/even number of Reggeons.

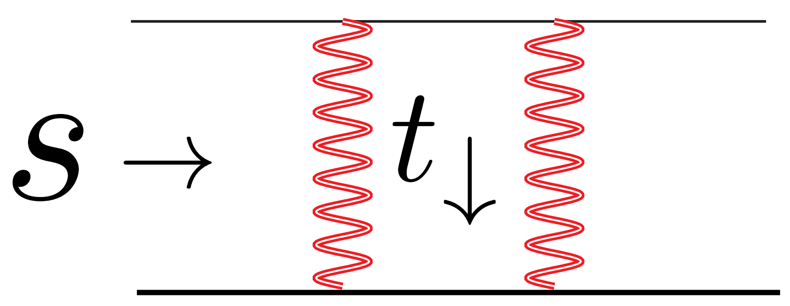
- Bose symmetry in *gg* → *gg* correlates odd/even signature with odd/even colour representations in the *t* channel.

$(\mathbf{8}\otimes\mathbf{8})_{gg}$

odd	$8_a \oplus (10 \oplus \overline{10})$	even	$0 \oplus 1 \oplus 8_s \oplus 27$







More generally we use channel colour operators: $\mathbf{T}_t^2$ is even, $\mathbf{T}^2_{s-u} \equiv \dfrac{\mathbf{T}_s^2 - \mathbf{T}_u^2}{2}$ is odd

Signature-odd amplitudes: Regge-pole factorisation and its breaking

Regge factorization and violation:

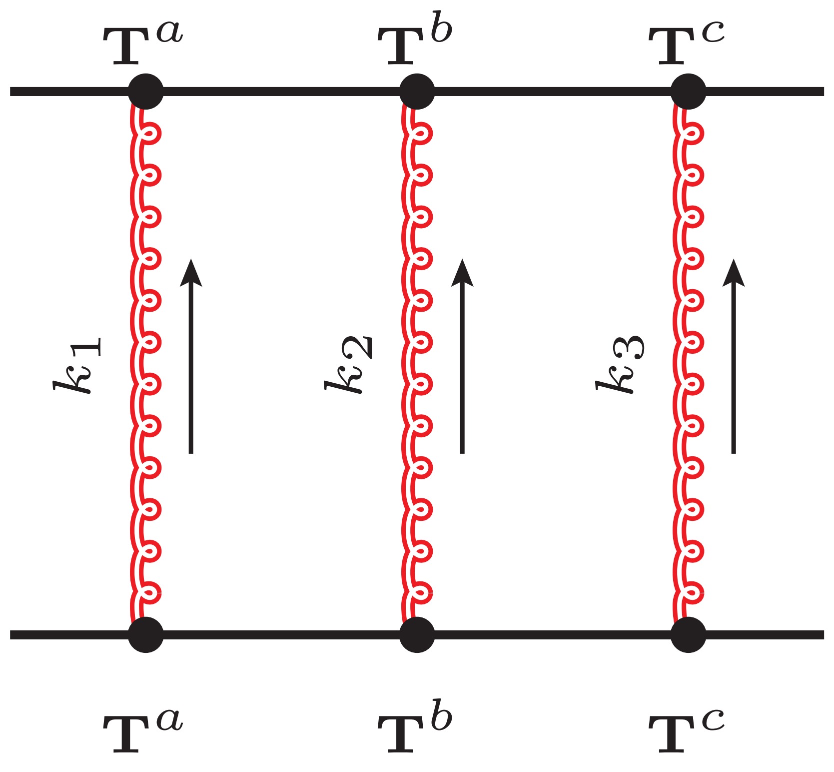
$$\mathcal{M}^{(-)}_{ij\to ij} = C_i(t)\, e^{\alpha_g(t)\, C_A\, L}\, C_j(t)\mathcal{M}^{\rm tree}_{ij\to ij} + \mathrm{MR}$$

Regge factorisation breaking (starting at 2 loops) can be inferred from comparing qq, qg, gg amplitudes

[Del Duca, Glover ’01]

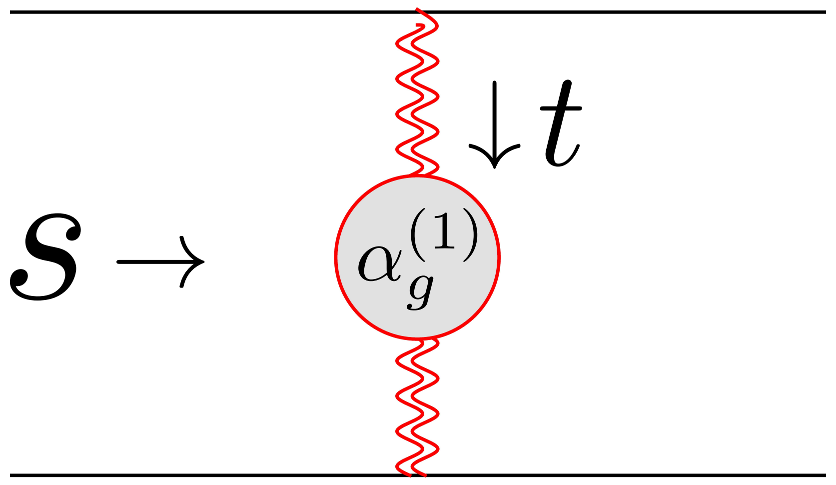
[Del Duca, Falcioni, Magnea, Vernazza ’14]

But until recently unknown how to account for it



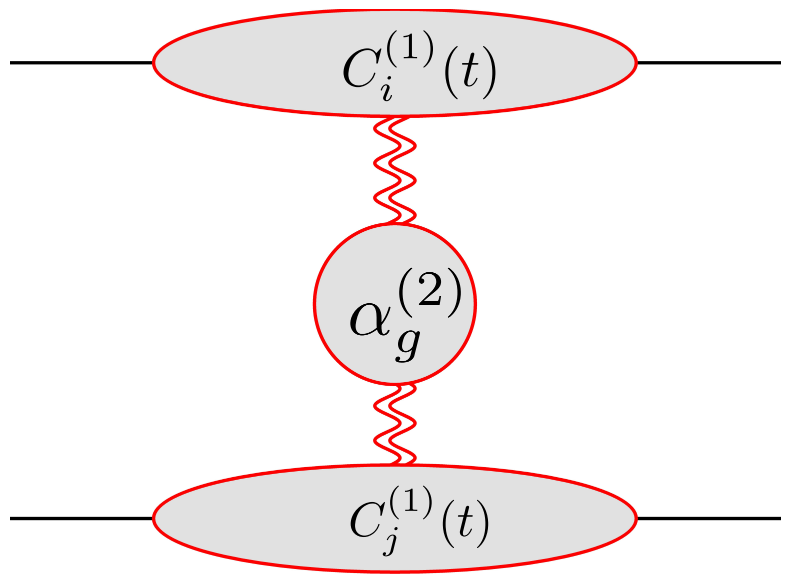
Colour **octet** exchange in the t channel: single Reggeon

LL $\alpha_s^n \log^n\left(\frac{s}{-t}\right)$



one-loop Regge trajectory

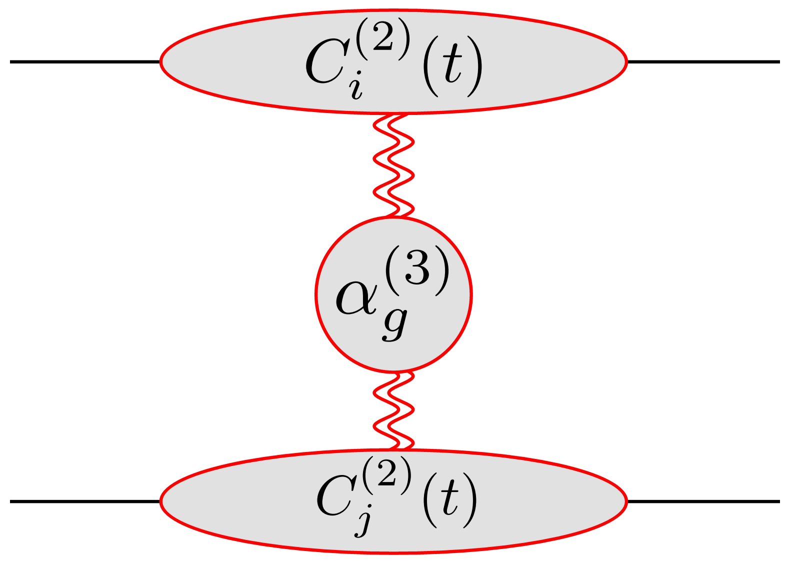
NLL $\alpha_s^n \log^{n-1}\left(\frac{s}{-t}\right)$



two-loop Regge trajectory

one-loop impact factors

NNLL $\alpha_s^n \log^{n-2}\left(\frac{s}{-t}\right)$



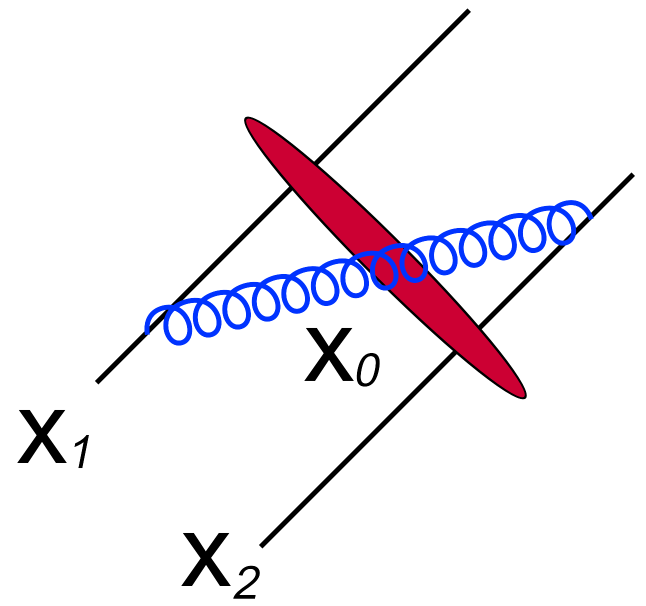
three-loop Regge trajectory

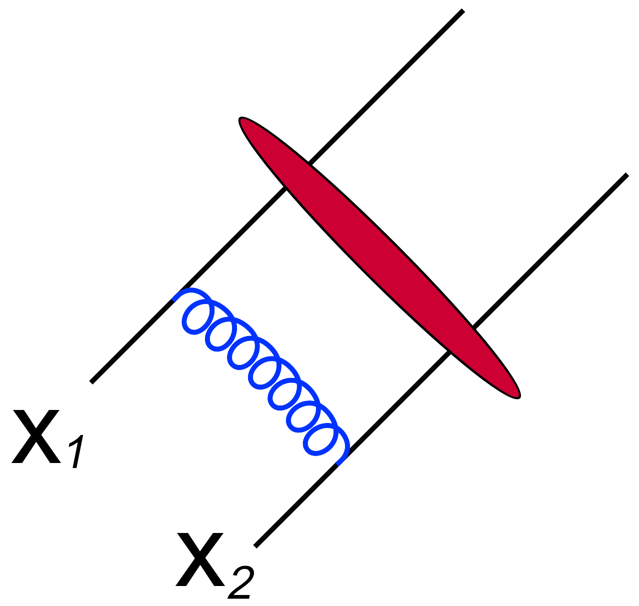
two-loop impact factors

Non-linear rapidity evolution equations

- The colliding particles are replaced by (sets of) infinite lightlike Wilson lines

$$U(\mathbf{x}) = \mathcal{P}\exp\left\{ig_s\int_{-\infty}^{\infty} dx^+ A_+^a(x^+, x^- = 0; \mathbf{x})T^a\right\}$$





- Rapidity evolution equation [Balitsky-JIMWLK]

$$-\frac{d}{d\eta}\left[U(\mathbf{x}_1)\ldots U(\mathbf{x}_n)\right] = H\left[U(\mathbf{x}_1)\ldots U(\mathbf{x}_n)\right]$$

$$H = \frac{\alpha_s}{2\pi^2}\int d\mathbf{x}_i d\mathbf{x}_j d\mathbf{x}_0 \frac{\mathbf{x}_{0i}\cdot\mathbf{x}_{0j}}{\mathbf{x}_{0i}^2\mathbf{x}_{0j}^2}\left(T_{i,L}^a T_{j,L}^a + T_{i,R}^a T_{j,R}^a - U_{\mathrm{adj}}^{ab}(\mathbf{x}_0)(T_{i,L}^a T_{j,R}^b + T_{j,L}^a T_{i,R}^b)\right)$$

$$T_{i,L}^a \equiv T^a U(\mathbf{x}_i)\frac{\delta}{\delta U(\mathbf{x}_i)}, \qquad T_{i,R}^a \equiv U(\mathbf{x}_i)T^a\frac{\delta}{\delta U(\mathbf{x}_i)}$$

Provides complete separation between the light-cone directions and the transverse plane: **2-dimensional dynamics**

Towards an effective theory: Defining the Reggeon

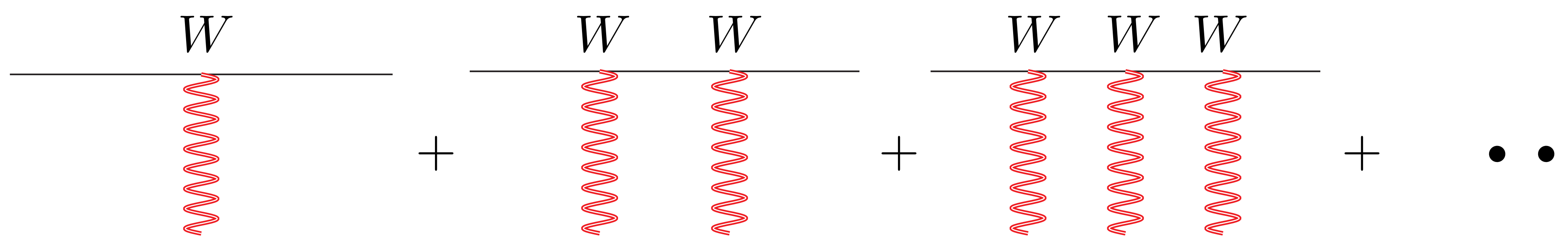
- In the perturbative regime $U(\mathbf{x}) \simeq 1$ it is natural to expand in terms of W Simon Caron-Huot (2013)

$$U(\mathbf{x}) = \mathcal{P} \exp\left\{ i g_s \int_{-\infty}^{\infty} dx^+ A_+^a(x^+, x^- = 0; \mathbf{x}) T^a \right\} = e^{i g_s T^a W^a(\mathbf{x})}.$$

W sources a Reggeon

- Scattered particles are expanded in states of a definite number of Reggeons

$$|\psi_i\rangle \equiv \frac{Z_i^{-1}}{2p_1^+} a_i(p_4) a_i^\dagger(p_1)|0\rangle \sim g_s\,|W\rangle + g_s^2\,|WW\rangle + g_s^3\,|WWW\rangle + \dots =$$



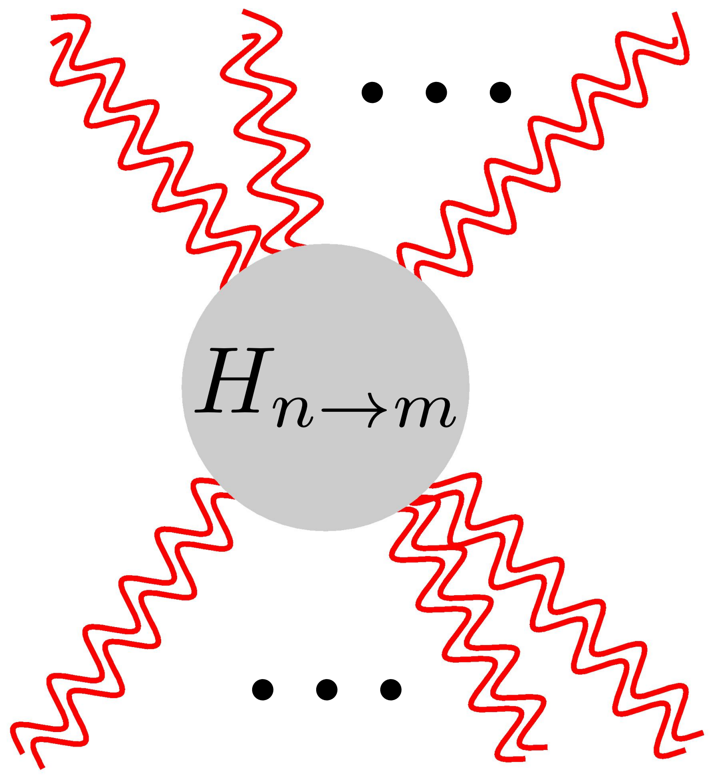
- Amplitudes are governed by rapidity evolution between the target and projectile:

$$\frac{i(Z_i Z_j)^{-1}}{2s} \mathcal{M}_{ij \to ij} = \langle \psi_j | e^{-HL} | \psi_i \rangle$$

$$-\frac{d}{d\eta}\,|\psi_i\rangle = H\,|\psi_i\rangle$$

$$H \begin{pmatrix} W \\ WW \\ WWW \\ \dots \end{pmatrix} \equiv \begin{pmatrix} H_{1\to1} & 0 & H_{3\to1} & \dots \\ 0 & H_{2\to2} & 0 & \dots \\ H_{1\to3} & 0 & H_{3\to3} & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix} \begin{pmatrix} W \\ WW \\ WWW \\ \dots \end{pmatrix}$$

n Reggeons



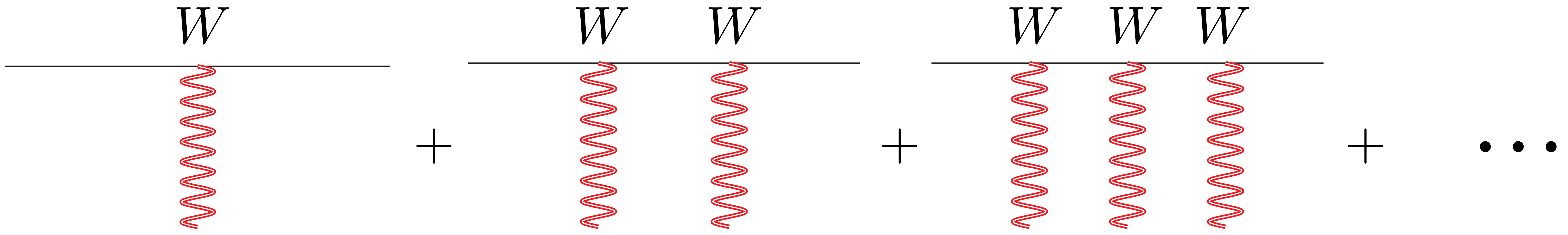
m Reggeons

- Each action of the Hamiltonian generates an extra power of the high-energy log L

Computing multi-Regge exchanges using non-linear rapidity evolution

1701.05241 Caron-Huot, EG, Vernazza

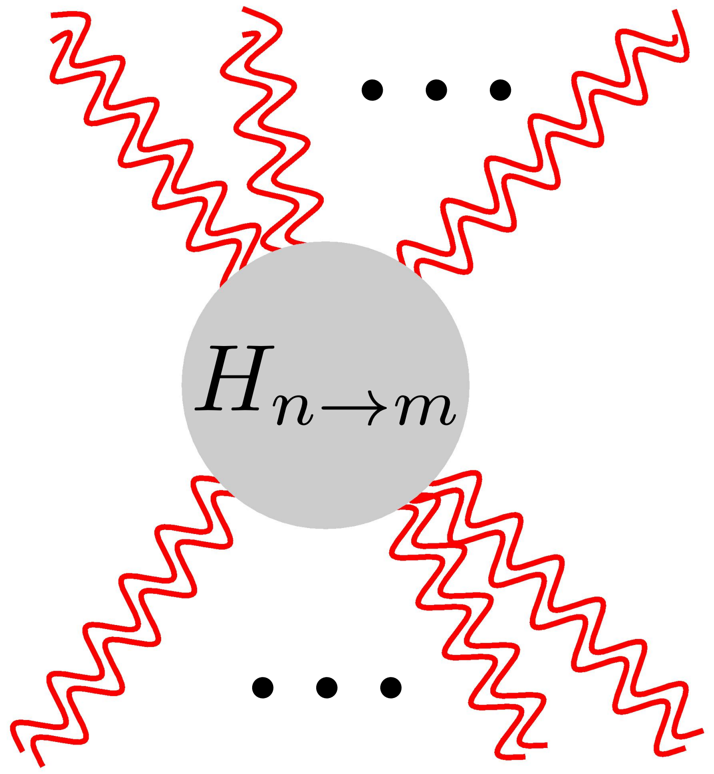
Projectile $|\psi_i\rangle =$



n Reggeons

n Reggeons to *m* Reggeons transition Hamiltonian [1701.05241]

$$\sum_{n,m}$$

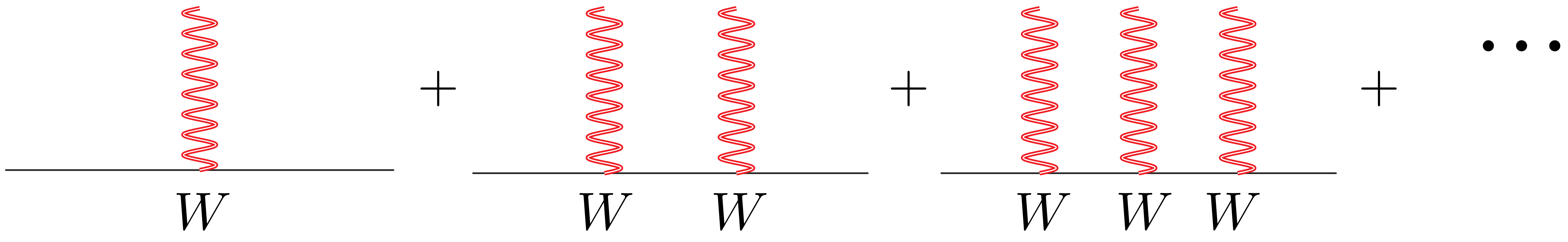


m Reggeons

$$H\begin{pmatrix} W \\ WW \\ WWW \\ \dots \end{pmatrix} \equiv \begin{pmatrix} H_{1\to1} & 0 & H_{3\to1} & \dots \\ 0 & H_{2\to2} & 0 & \dots \\ H_{1\to3} & 0 & H_{3\to3} & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}\begin{pmatrix} W \\ WW \\ WWW \\ \dots \end{pmatrix}$$

$$\sim \begin{pmatrix} g_s^2 & 0 & g_s^4 & \dots \\ 0 & g_s^2 & 0 & \dots \\ g_s^4 & 0 & g_s^2 & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}\begin{pmatrix} W \\ WW \\ WWW \\ \dots \end{pmatrix}$$

Target $\langle\psi_j| =$

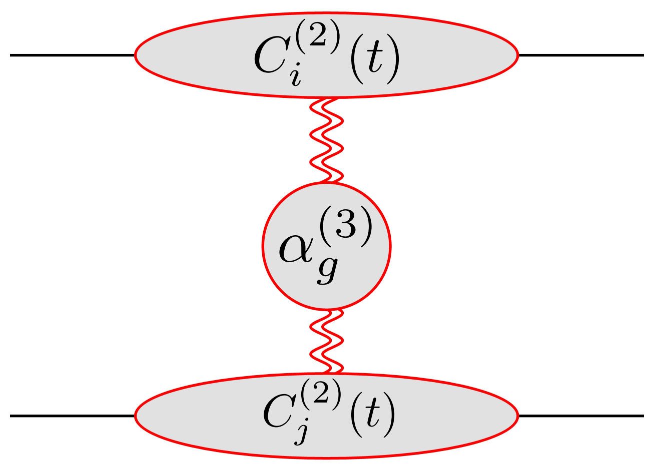


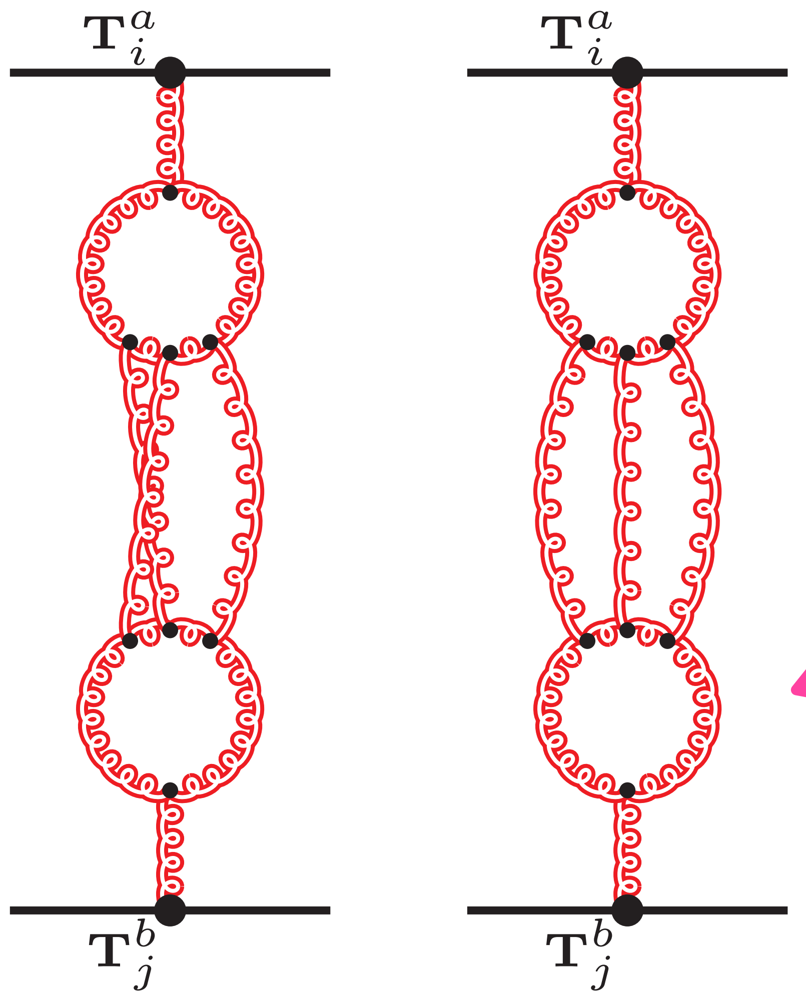
Signature-odd 2 → 2 amplitudes: understanding the NNLL tower

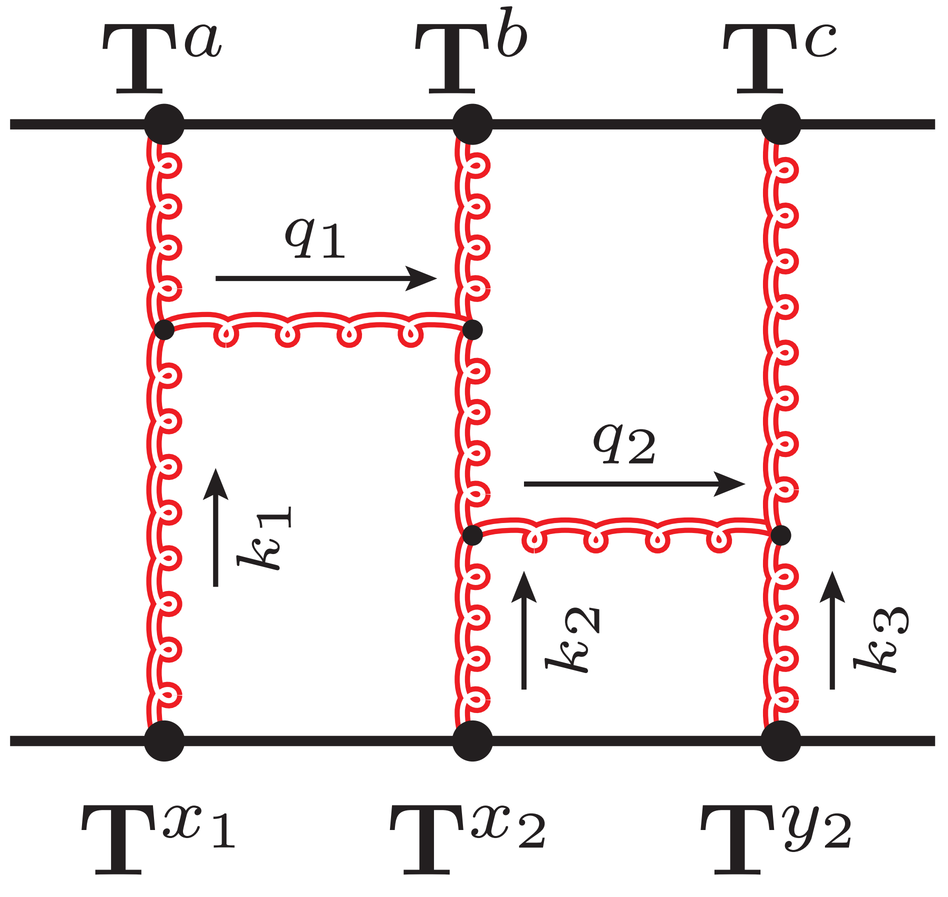
- Using **non-linear** rapidity evolution, the NNLL tower is determined to all orders in terms of **one** and **three** Reggeon exchanges

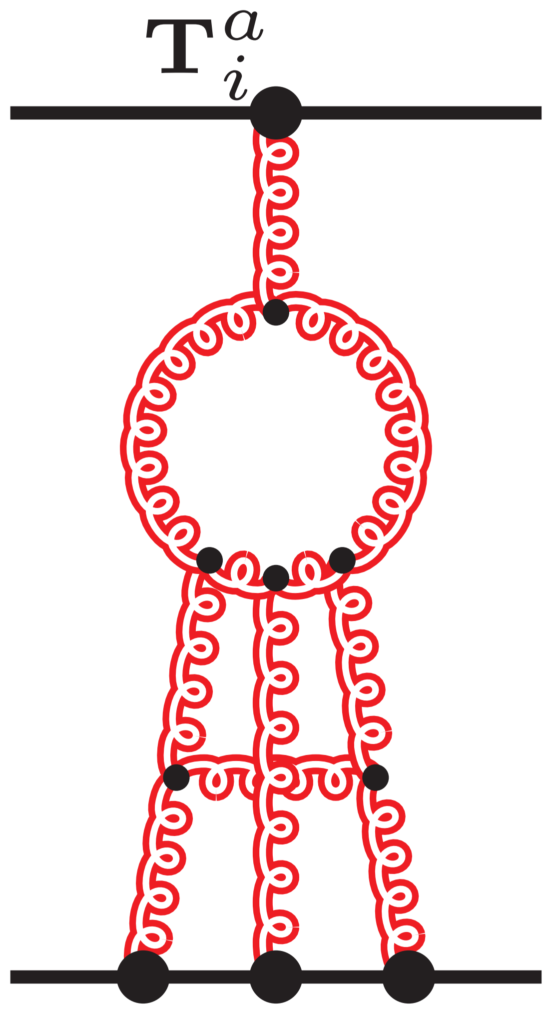
- Expanding in $X \equiv \frac{\alpha_s}{\pi} r_\Gamma L$

$$\frac{i}{2s}\hat{\mathcal{M}}^{(-),\text{NNLL}}_{ij\to ij} = \left(\frac{\alpha_s}{\pi}\right)^2 \Big\{ \langle j_1|i_1\rangle^{\text{NNLO}} + r_\Gamma^2\pi^2 \sum_{k=0}^{\infty} \frac{(-X)^k}{k!} \Big[\langle j_3|\hat{H}^k_{3\to3}|i_3\rangle + \Theta(k\geq 1)\left[\langle j_1|\hat{H}_{3\to1}\hat{H}^{k-1}_{3\to3}|i_3\rangle + \langle j_3|\hat{H}^{k-1}_{3\to3}\hat{H}_{1\to3}|i_1\rangle\right] + \Theta(k\geq 2)\langle j_1|\hat{H}_{3\to1}\hat{H}^{k-2}_{3\to3}\hat{H}_{1\to3}|i_1\rangle \Big]^{\text{LO}} \Big\}$$









Caron-Huot, EG, Vernazza
JHEP 06 (2017) 016
Falcioni, EG, Milloy, Vernazza
Phys. Rev. D 103 (2021) L111501

- All diagrams computed to four loops

Signature odd 2 → 2 amplitude at NNLL: Regge pole and cut

Requiring that the Regge cut is strictly non-planar fixes the separation between Regge pole vs. Regge cut

$$\mathcal{M}^{(-)}_{ij\to ij} = \underbrace{\mathcal{M}^{(-)\,\mathrm{SR}}_{ij\to ij} + \mathcal{M}^{(-)\,\mathrm{MR}}_{ij\to ij}\Big|_{\mathrm{planar}}} + \mathcal{M}^{(-)\,\mathrm{MR}}_{ij\to ij}\Big|_{\mathrm{nonplanar}}$$

$$= \mathcal{M}^{(-)\,\mathrm{pole}}_{ij\to ij} + \mathcal{M}^{(-)\,\mathrm{cut}}_{ij\to ij}$$

$$\mathcal{M}^{(-)\,\mathrm{pole}}_{ij\to ij} = C_i(t)\, e^{\alpha_g(t)\, C_A\, L}\, C_j(t) \mathcal{M}^{\mathrm{tree}}_{ij\to ij}$$

Falcioni, EG, Maher, Milloy, Vernazza
Phys.Rev.Lett. 128 (2022) 13, 13;
JHEP 03 (2022) 053

$\mathcal{M}^{(-)\,\mathrm{MR}}_{ij\to ij}\Big|_{\mathrm{planar}}$ must be **universal** (gg, gq, qq) to be absorbed in the factorizing pole term.

$\mathcal{M}^{(-)\,\mathrm{MR}}_{ij\to ij}\Big|_{\mathrm{planar}}$ ***cannot*** contribute beyond 3 loops: the NNLL Regge pole term has **no** free parameters!

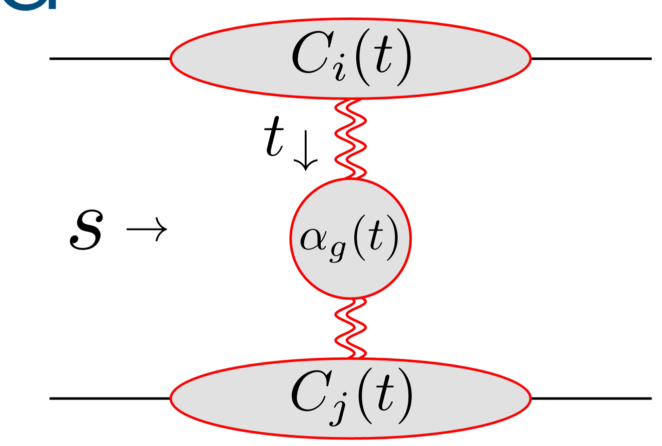
Indeed, at 4 loops **planar Multi Regge contributions to conspire to cancel!**

Signature odd amplitude at NNLL: properties of Regge pole and cut

All-order structure through NNLL for any gauge theory, any representation:

$$\mathcal{M}^{(-)}_{ij\to ij} = Z_i(t)\,\bar{D}_i(t)\,Z_j(t)\,\bar{D}_j(t)\left[\left(\frac{-s}{-t}\right)^{C_A\alpha_g(t)} + \left(\frac{-u}{-t}\right)^{C_A\alpha_g(t)}\right]\mathcal{M}^{\text{tree}}_{ij\to ij} + \sum_{n=2}^{\infty} a^n L^{n-2}\mathcal{M}^{(\pm,n,n-2)\,\text{cut}}$$

Regge pole-factorized



- single Reggeon; colour octet
- dominant in planar limit
- parameters at NNLL are fully fixed by matching to qq scattering amplitudes [Caola, Chakraborty, Gambuti, von Manteuffel, Tancredi. JHEP 10 (2021) 206]
  Consistent with other channels (gg, qg)
  Falcioni et al. Phys.Rev.Lett. 128 (2022) 13, 13
  Caola et al. Phys.Rev.Lett. 128 (2022) 21, 21

Regge cut: breaks factorization

- multiple Reggeons; various colour reps.
- suppressed in planar limit
- proportional to π^2
- no dependence on the matter content: the same for any gauge theory!
- Sensitive to soft singularities beyond the dipole formula.

Regge-pole factorisation for multi-leg amplitudes in MRK

Multi-Regge Kinematics (MRK)

4-momentum $p = (p^+, p^-; \mathbf{p})$

target $p_1 = (0, p_1^-; \mathbf{0})$

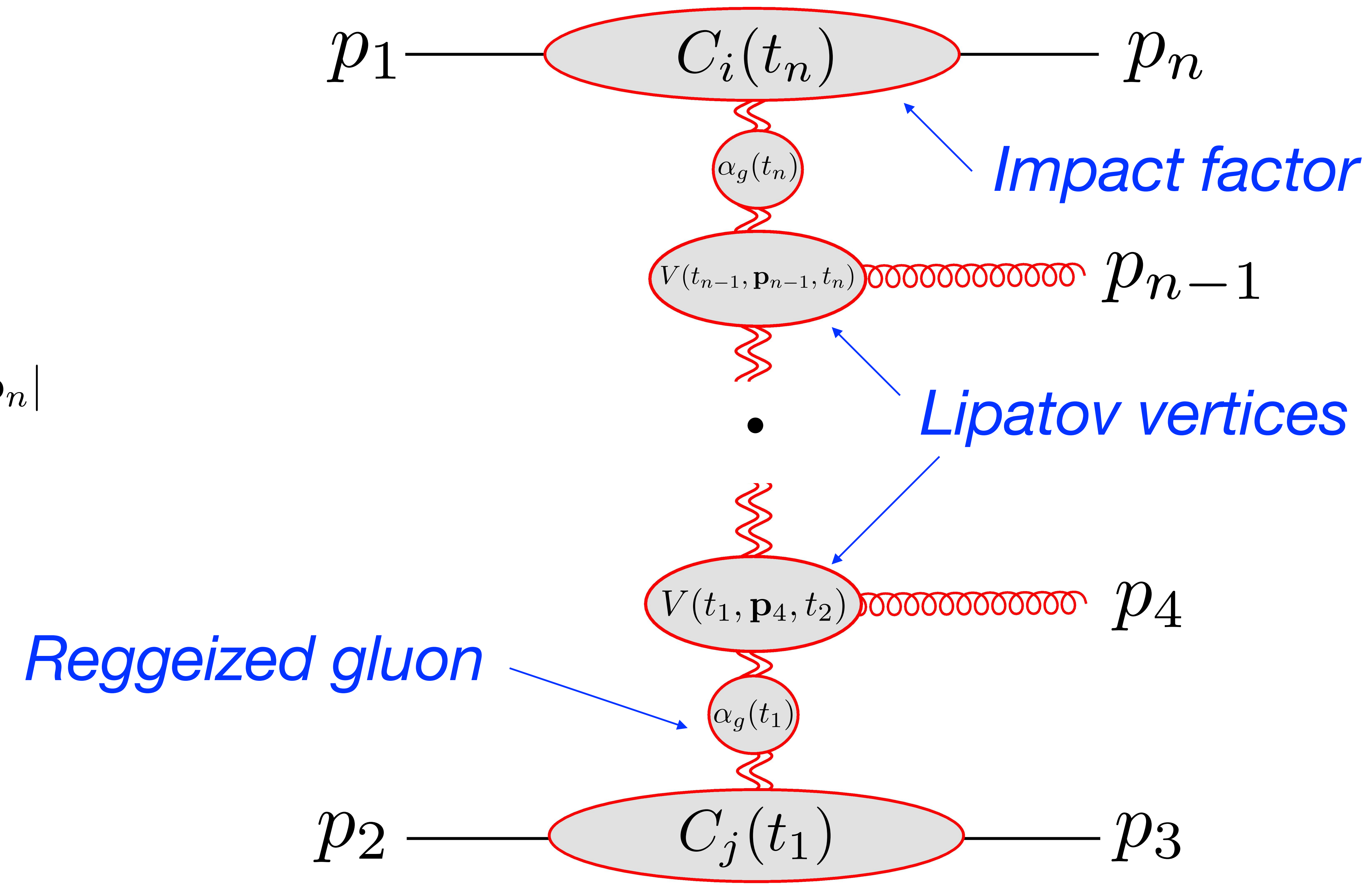
projectile $p_2 = (p_2^+, 0; \mathbf{0})$

strong hierarchy of light-cone components $p_3^+ \gg p_4^+ \gg ... \gg p_n^+$

$p_3^- \ll p_4^- \ll ... \ll p_n^-$

no ordering of transverse components $|\mathbf{p}_3| \sim |\mathbf{p}_4| \sim ... \sim |\mathbf{p}_n|$

Regge (pole) factorization in MRK



Regge (pole) factorization holds in MRK for the dispersive (real part) of the amplitudes through NLL; established using unitarity [Fadin et al. 2006]

Planar limit:

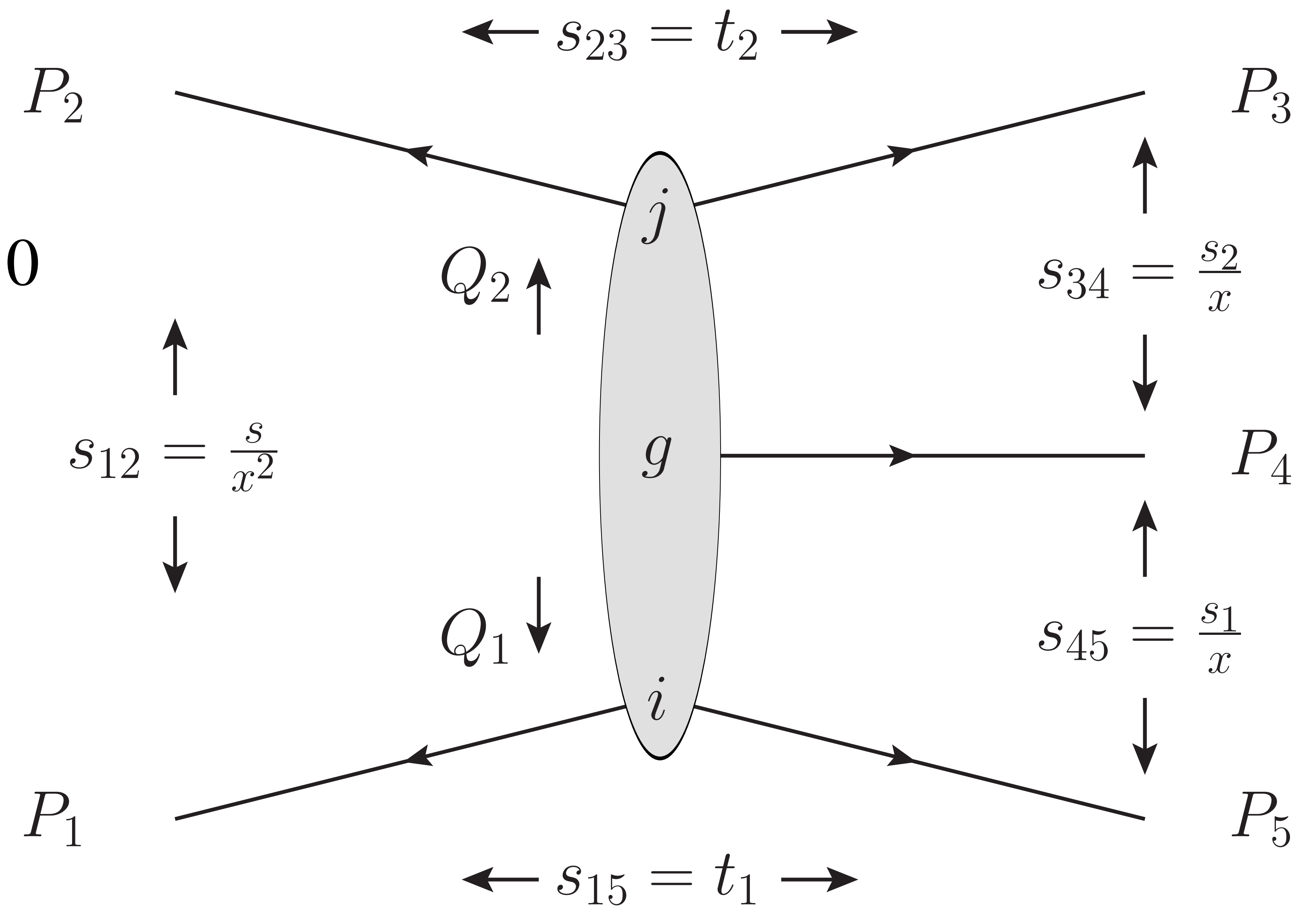
- Four- and five-point planar amplitudes have only Regge poles. Essential for the BDS ansatz in SYM.
- Six and higher point planar amplitudes have also Regge cuts in some special kinematic regions [Bartels, Lipatov, Sabio Vera (2008)]. All multiplicity planar results available [Del Duca *et al.* (2019)]

$2 \to 3$ amplitudes in multi-Regge kinematics

Del Duca, Duhr, Glover (2009); Caron-Huot, Chicherin, Henn, Zhang, Zoia, JHEP 10 (2020) 188;
Fadin, Fucilla, Papa (2023); Abreu, EG, Falcioni, Milloy and Vernazza — to appear

- Multi-Regge kinematics:

$$s_{12} \to \frac{s_{12}}{x^2} \qquad s_{45} \to \frac{s_1}{x} \qquad s_{34} \to \frac{s_2}{x} \qquad s_{15} \to t_1 \qquad s_{23} \to t_2 \qquad \text{for} \quad x \to 0$$



- Signature symmetry operations:

$$(1 \leftrightarrow 5) \quad \to \quad \{s \to -s, \quad s_{45} \to -s_{45}\},$$
$$(2 \leftrightarrow 3) \quad \to \quad \{s \to -s, \quad s_{34} \to -s_{34}\}.$$

- t-channel colour basis - diagonal operators:

$$\mathbf{T}^2_{t_1} \equiv (\mathbf{T}_1 + \mathbf{T}_5)^2$$
$$\mathbf{T}^2_{t_2} \equiv (\mathbf{T}_2 + \mathbf{T}_3)^2$$

Signature-preserving operator on line i, j: $\quad \mathbf{T}_{(++)} = (\mathbf{T}^a_1 + \mathbf{T}^a_5) \cdot (\mathbf{T}^a_2 + \mathbf{T}^a_3),$

Signature-preserving on line i, inverting on j: $\quad \mathbf{T}_{(+-)} = (\mathbf{T}^a_1 + \mathbf{T}^a_5) \cdot (\mathbf{T}^a_2 - \mathbf{T}^a_3),$

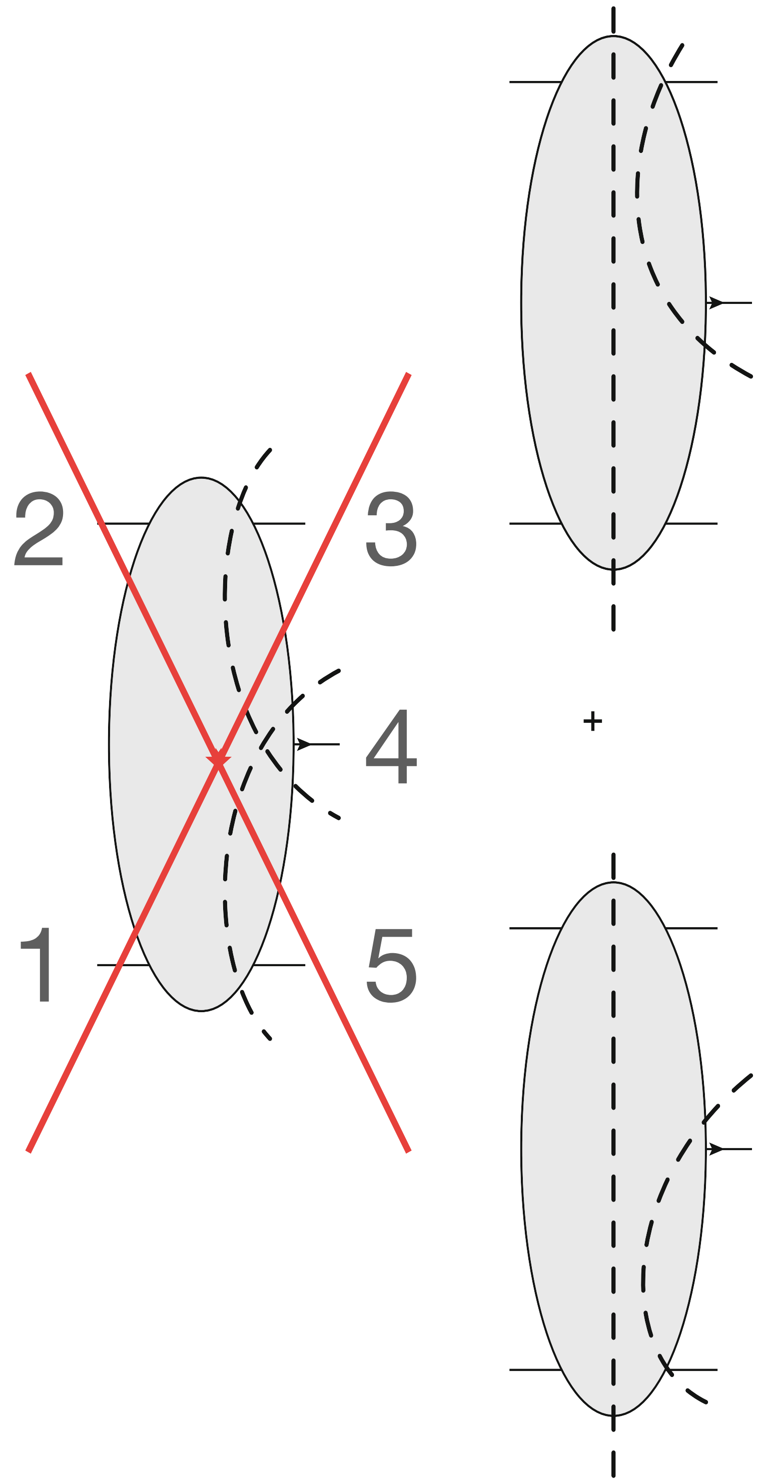
Signature-preserving on line j, inverting on i: $\quad \mathbf{T}_{(-+)} = (\mathbf{T}^a_1 - \mathbf{T}^a_5) \cdot (\mathbf{T}^a_2 + \mathbf{T}^a_3),$

Signature-inverting operator on lines i, j: $\quad \mathbf{T}_{(--)} = (\mathbf{T}^a_1 - \mathbf{T}^a_5) \cdot (\mathbf{T}^a_2 - \mathbf{T}^a_3).$

Odd-Odd 2 → 3 amplitude: discontinuity structure

- Steinmann relations forbid unitarity cuts in partially overlapping channels.

- Allowed iterated discontinuities: compatible with the signature $\quad s_{12}$ and s_{45} or s_{12} and s_{34}



- All-order factorization formula for 2 → 3 amplitudes in Multi-Regge kinematics in terms of two real-valued vertex functions

$$\frac{\mathcal{M}^{(-,-)}_{ij\to i'gj'}\Big|^{\text{1-Reggeon}}}{\mathcal{M}^{\text{tree}}_{ij\to i'gj'}} = c_i(t_1,\tau)\frac{1}{4}\Bigg\{\left[\left(\frac{s_{34}}{\tau}\right)^{\omega_2-\omega_1}+\left(\frac{-s_{34}}{\tau}\right)^{\omega_2-\omega_1}\right]\left[\left(\frac{s}{\tau}\right)^{\omega_1}+\left(\frac{-s}{\tau}\right)^{\omega_1}\right]v_R(t_1,t_2,|\mathbf{p}_4|^2,\tau)+ \left[\left(\frac{s_{45}}{\tau}\right)^{\omega_1-\omega_2}+\left(\frac{-s_{45}}{\tau}\right)^{\omega_1-\omega_2}\right]\left[\left(\frac{s}{\tau}\right)^{\omega_2}+\left(\frac{-s}{\tau}\right)^{\omega_2}\right]v_L(t_1,t_2,|\mathbf{p}_4|^2,\tau)\Bigg\}c_j(t_2,\tau)$$

[Bartels (1980), Fadin and Lipatov (1993),…, Fadin, Fucilla, Papa (2023)]

Odd-Odd 2 → 3 amplitude

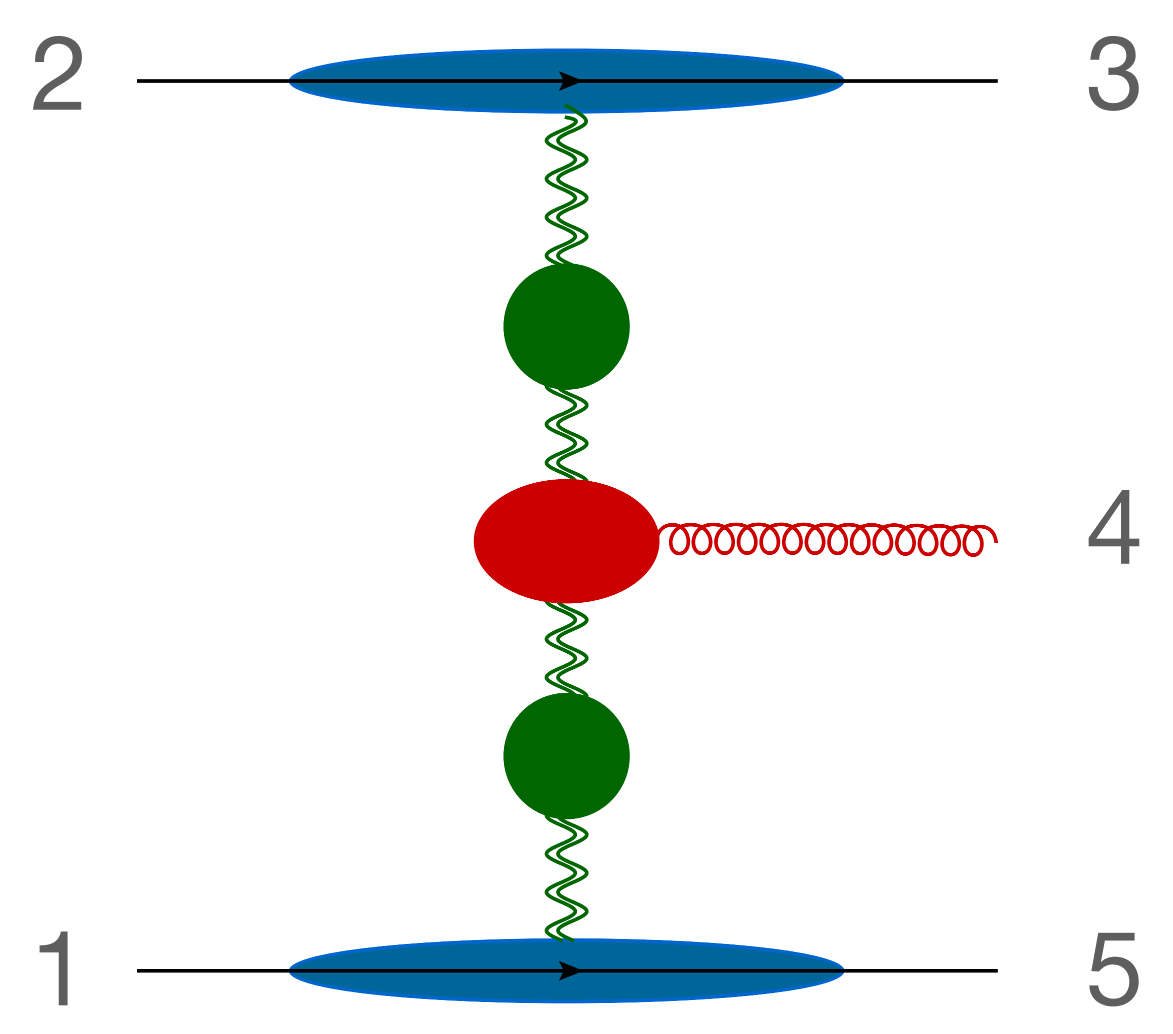
- All-order factorization formula for 2 → 3 amplitudes in Multi-Regge kinematics in terms of two real-valued vertex functions v_R, v_L

$$\omega_1 = C_A \alpha_g(t_1), \qquad \omega_2 = C_A \alpha_g(t_2)$$

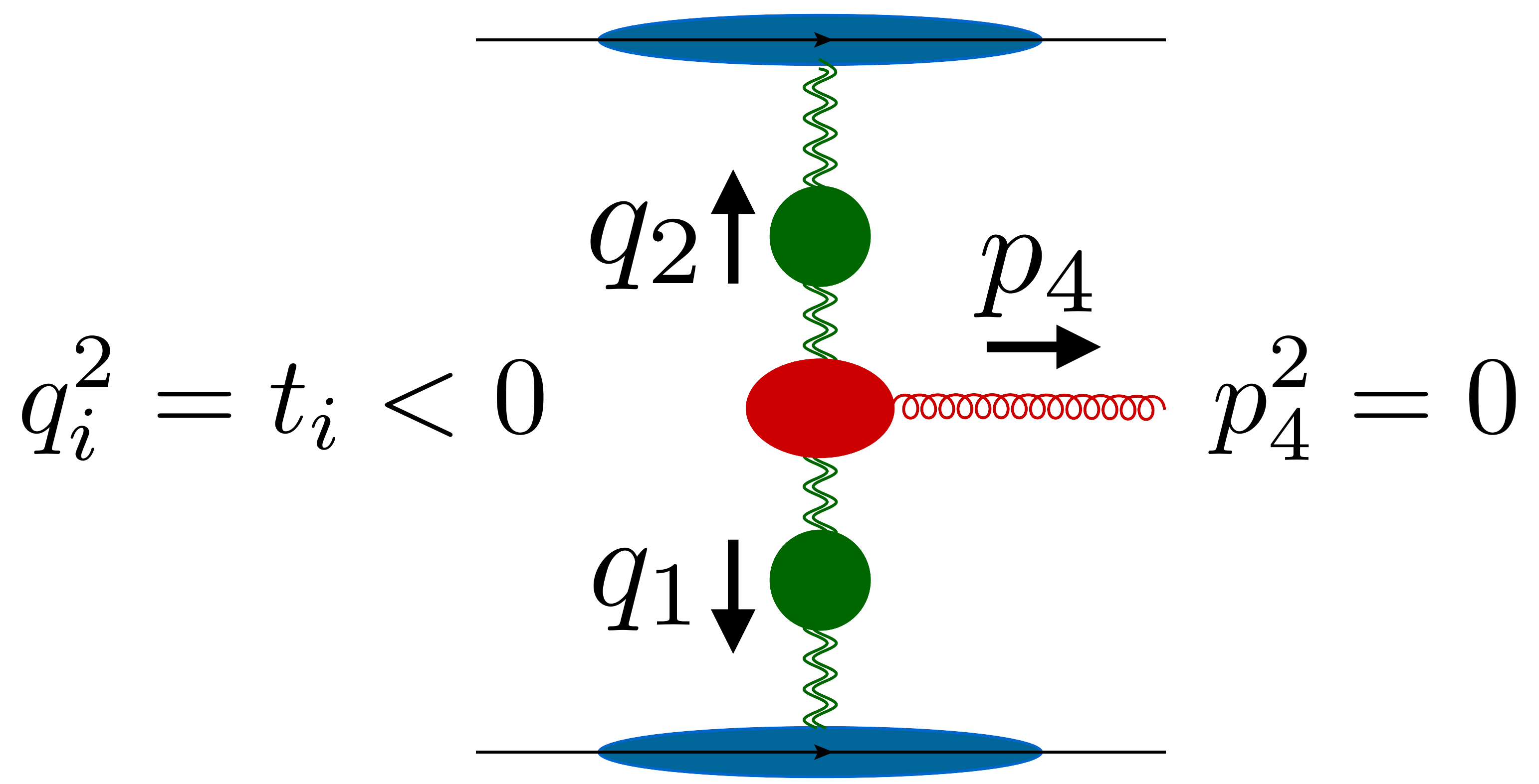
$$\frac{\mathcal{M}^{(-,-)}_{ij\to i'gj'}\Big|^{\text{1-Reggeon}}}{\mathcal{M}^{\text{tree}}_{ij\to i'gj'}} = c_i(t_1,\tau)\frac{1}{4}\Bigg\{\left[\left(\frac{s_{34}}{\tau}\right)^{\omega_2-\omega_1} + \left(\frac{-s_{34}}{\tau}\right)^{\omega_2-\omega_1}\right]\left[\left(\frac{s}{\tau}\right)^{\omega_1} + \left(\frac{-s}{\tau}\right)^{\omega_1}\right] v_R(t_1,t_2,|\mathbf{p}_4|^2,\tau) +$$
$$\left[\left(\frac{s_{45}}{\tau}\right)^{\omega_1-\omega_2} + \left(\frac{-s_{45}}{\tau}\right)^{\omega_1-\omega_2}\right]\left[\left(\frac{s}{\tau}\right)^{\omega_2} + \left(\frac{-s}{\tau}\right)^{\omega_2}\right] v_L(t_1,t_2,|\mathbf{p}_4|^2,\tau)\Bigg\} c_j(t_2,\tau)$$

- **Equivalently**: a single complex-valued vertex rapidity variables absorb a phase: $\eta_1 = \log\frac{s_{45}}{\tau} - \frac{i\pi}{4}, \qquad \eta_2 = \log\frac{s_{34}}{\tau} - \frac{i\pi}{4}$

$$\frac{\mathcal{M}^{(-,-)}_{ij\to i'gj'}\Big|^{\text{1-Reggeon}}}{\mathcal{M}^{\text{tree}}_{ij\to i'gj'}} = c_i(t_1,\tau)\, e^{\omega_1\,\eta_1}\, v(t_1,t_2,\mathbf{p}_4^2,\tau)\, e^{\omega_2\,\eta_2}\, c_j(t_2,\tau)$$



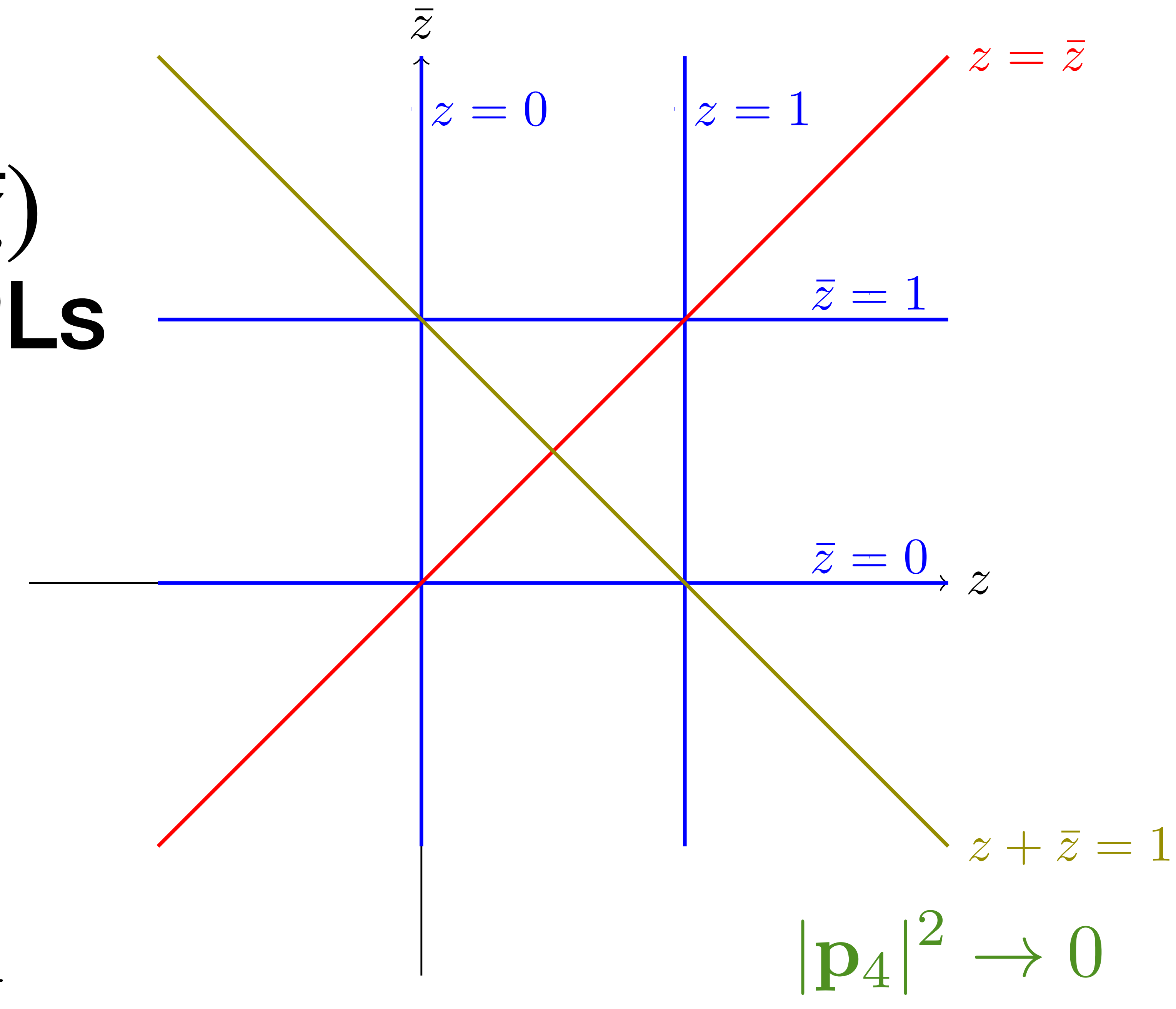
Complex-valued vertex: properties



$$v(t_1, t_2, |\mathbf{p}_4|^2, \tau) = \frac{\mathcal{M}^{(-,-)}_{ij\to i'gj'}\Big|^{\text{1-Reggeon}}}{c_i(t_1,\tau)\, e^{\omega_1 \eta_1}\, e^{\omega_2 \eta_2}\, c_j(t_2,\tau) \mathcal{M}^{\text{tree}}_{ij\to i'gj'}}$$

- Euclidean 2-dim momenta: $\frac{-t_1}{|\mathbf{p}_4|^2} = (1-z)(1-\bar{z}), \qquad \frac{-t_2}{|\mathbf{p}_4|^2} = z\bar{z}$

$$\frac{(\mathbf{q}_1\cdot\mathbf{q}_2)^2 - |\mathbf{q}_1|^2|\mathbf{q}_2|^2}{|\mathbf{p}_4|^2/4} = (z-\bar{z})^2 \to 0$$

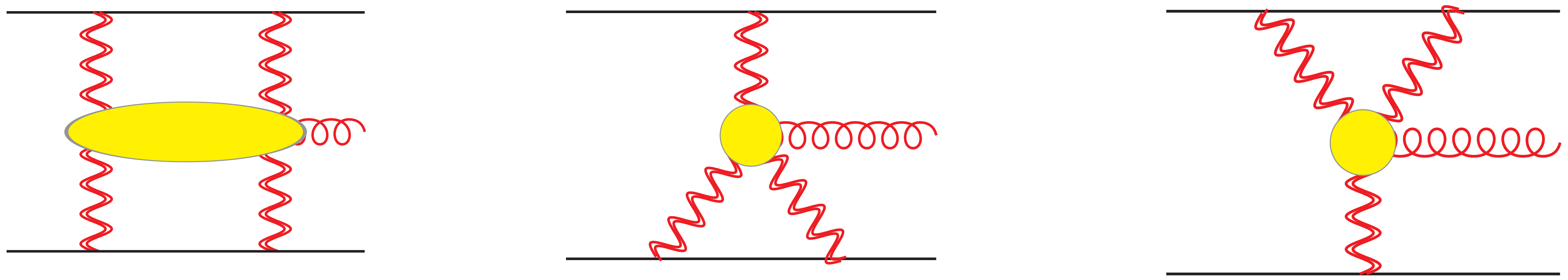


- Absence of discontinuities in physical kinematics $z = \bar{z}^*$ (Euclidean 2-dim) implies that the transcendental functions $f(z,\bar{z})$ in the complex vertex $v(t_1, t_2, |\mathbf{p}_4|^2)$ should be **Single-Valued GPLs**
- The **reality** condition of v_R, v_L implies that these functions obey: $f(z,\bar{z}) = f^*(\bar{z}, z)$
- **Target-Projectile symmetry** implies $f(z,\bar{z}) = f(1-\bar{z}, 1-z)$
- Symbol alphabet: $\{z,\ \bar{z},\ 1-z,\ 1-\bar{z},\ z-\bar{z},\ 1-z-\bar{z}\}$
- Rational factors have <u>spurious</u> singularities on the lines $z=\bar{z},\ z+\bar{z}=1$

2 → 3 amplitudes at one loop: multi-Reggeon contributions

- A new feature compared to 2 → 2 scattering: even and odd signature mix

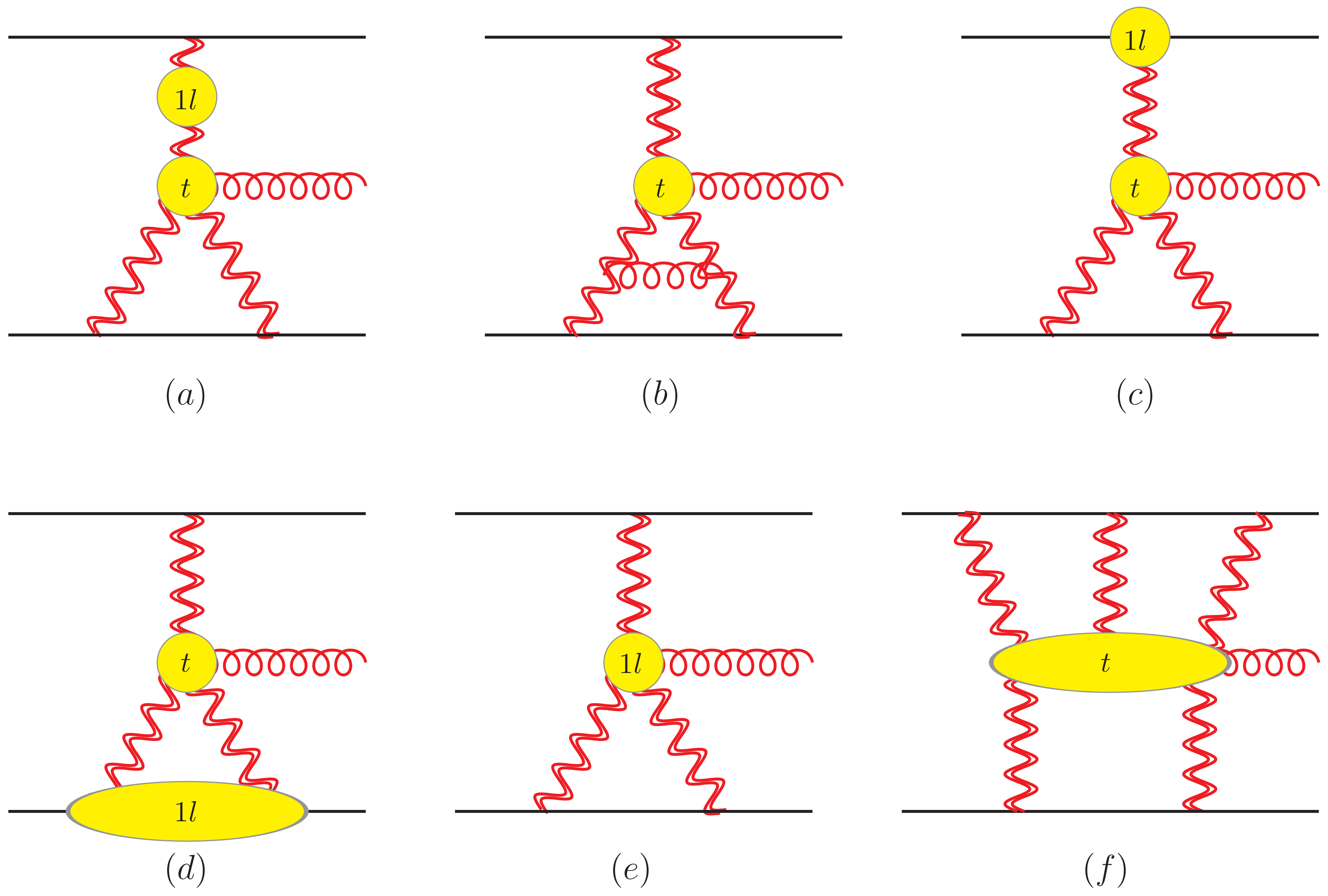
$$\begin{aligned}\mathcal{M}^{\mathrm{MR}\,(1)}_{ij\to i'gj'} &= \mathcal{M}^{(1)}_{\mathcal{R}^2 g\mathcal{R}^2} + \mathcal{M}^{(1)}_{\mathcal{R}g\mathcal{R}^2} + \mathcal{M}^{(1)}_{\mathcal{R}^2 g\mathcal{R}} \\ &= \frac{i\pi}{4}\Bigg\{\frac{1}{\epsilon}\left(\mathbf{T}_{(--)} + \mathbf{T}_{(+-)} + \mathbf{T}_{(-+)}\right) \\ &\qquad + \log\frac{p_4^2}{p_3^2 p_5^2}\mathbf{T}_{(--)} + \log\frac{p_3^2}{p_4^2 p_5^2}\mathbf{T}_{(-+)} + \log\frac{p_5^2}{p_3^2 p_4^2}\mathbf{T}_{(+-)} + \mathcal{O}(\epsilon)\Bigg\}\mathcal{M}^{\mathrm{tree}}_{ij\to i'gj'}\end{aligned}$$



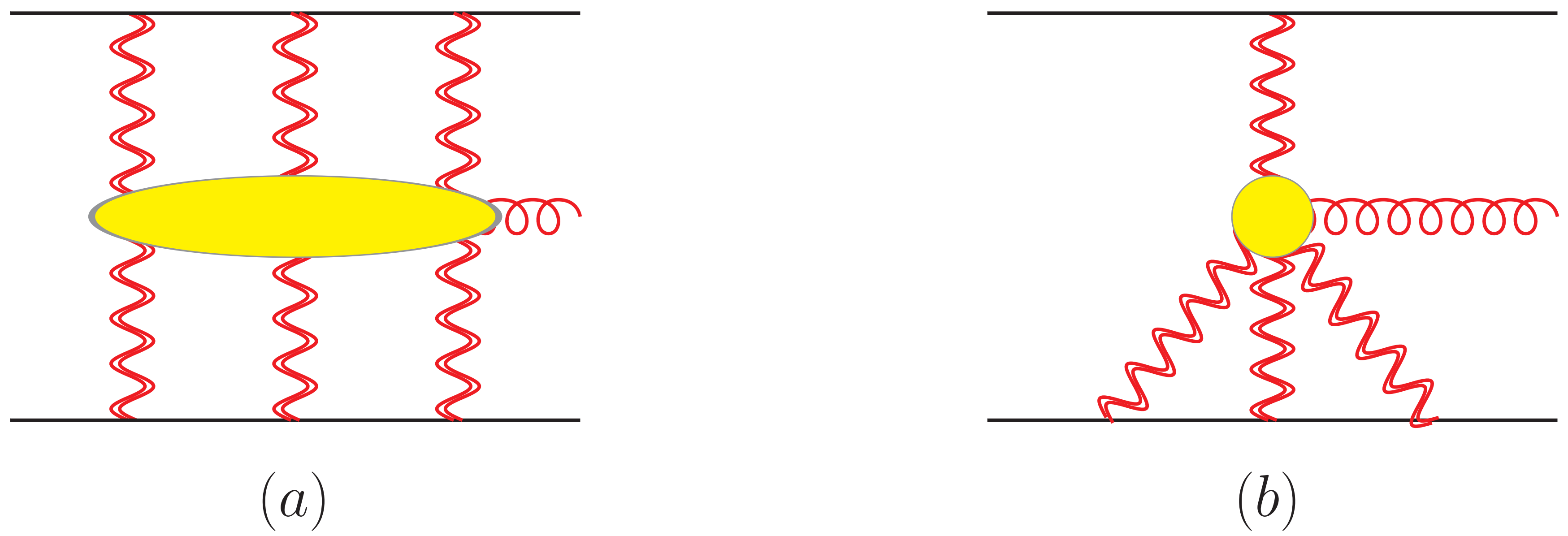
- But as in 2 → 2 scattering, at one loop multi-Reggeon exchanges do not affect the dispersive (odd-odd signature) part of the amplitude.

Multiple-Reggeon effect in 2 → 3 scattering

- At two loops there are many contributions of mixed odd-even signature



- But importantly, **there are odd-odd contributions from multi-Reggeon exchange**

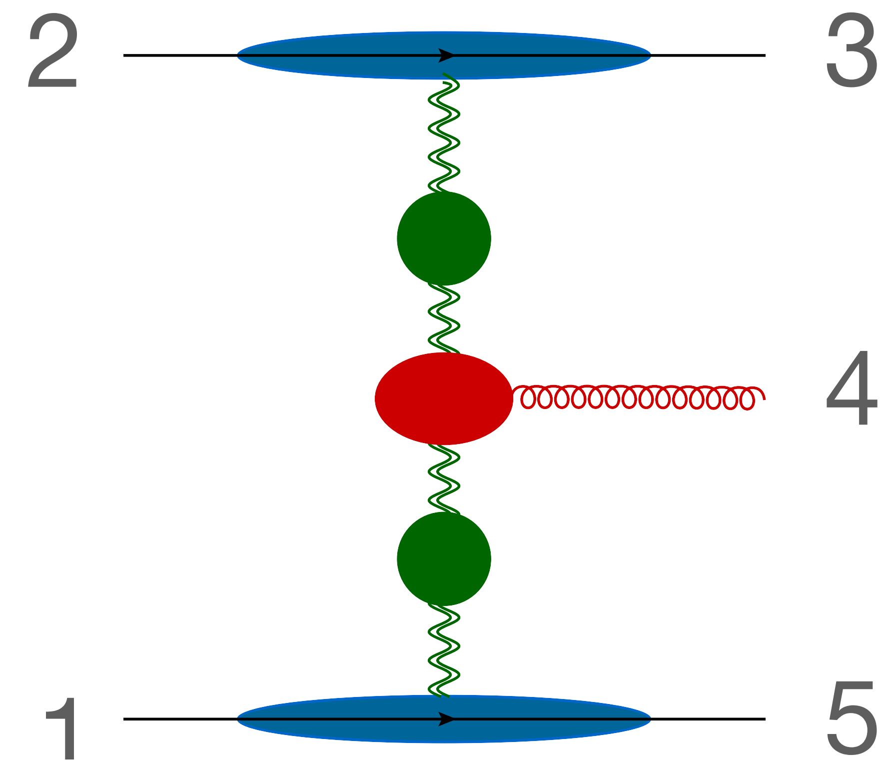


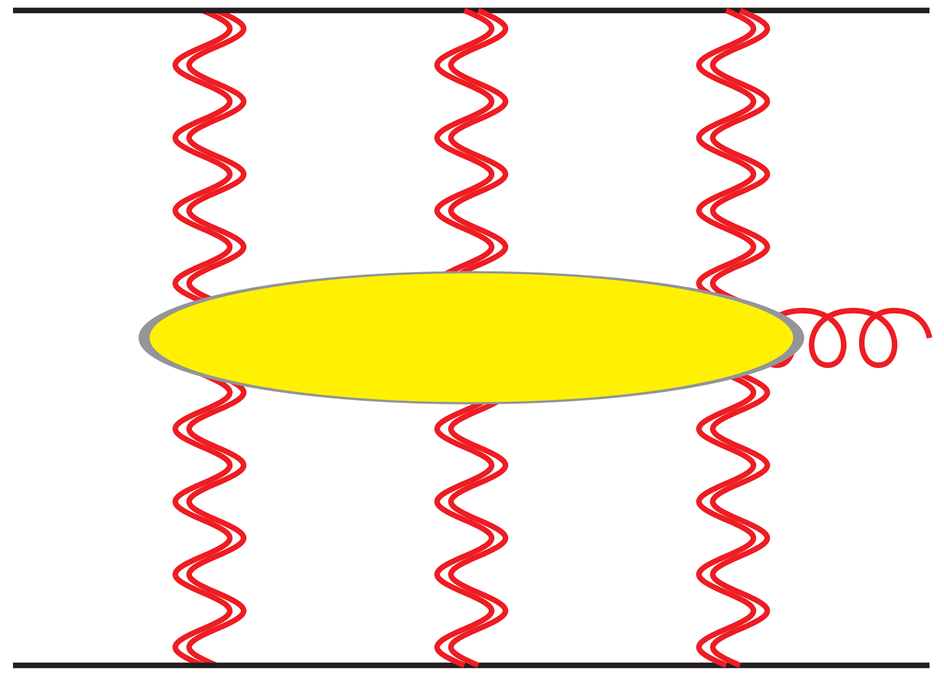
These break factorization!

Signature odd-odd 2 → 3 amplitude

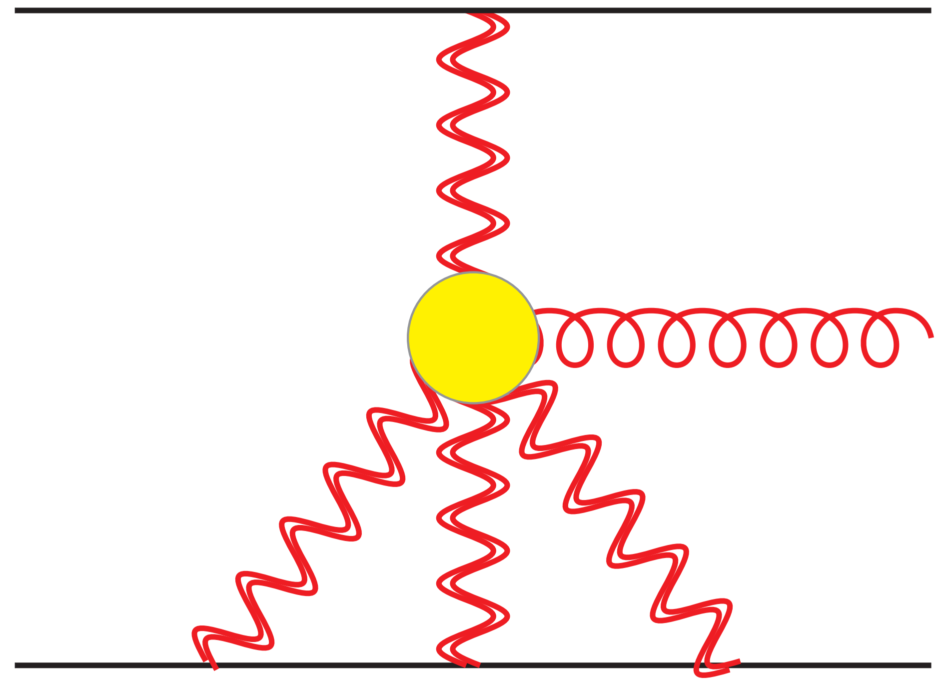
- Two-loop contributions of odd-odd signature

$$\mathcal{M}^{(-,-)\,(2)}_{ij\to i'gj'} = \mathcal{M}^{(2)}_{\mathcal{R}g\mathcal{R}} + \mathcal{M}^{(2)}_{\mathcal{R}^3 g\mathcal{R}^3} + \mathcal{M}^{(2)}_{\mathcal{R}g\mathcal{R}^3} + \mathcal{M}^{(2)}_{\mathcal{R}^3 g\mathcal{R}}$$





(a)



(b)

$$\begin{aligned} C_{\mathcal{R}^3 g\mathcal{R}^3} &= \mathbf{T}_i^{\{a,b,c\}}\, i f^{ca_4 d}\, \mathbf{T}_j^{\{a,b,d\}} \\ &= \frac{1}{144}\left\{9\mathbf{T}^2_{(--)} + \mathbf{T}^2_{(++)} + 4N_c\mathbf{T}_{(++)} + 3\left(\mathbf{T}^2_{(-+)} + \mathbf{T}^2_{(+-)}\right)\right\}\mathcal{C}^{(0)}_{ij} \\ &= \begin{cases} \frac{1}{72}\left(N_c^2 - 6 + \frac{18}{N_c^2}\right) c^{[8,8]_a} & \text{for } qq \\ \frac{1}{72}\left(N_c^2 + 6\right) c^{[8,8_a]_a} & \text{for } qg \\ \frac{1}{72}\left(N_c^2 + 36\right) c^{[8_a,8_a]} - \frac{1}{4}\sqrt{N_c^2 - 4}\, c^{[10,\overline{10}]_1} & \text{for } gg \end{cases} \end{aligned}$$

$$\begin{aligned} C_{\mathcal{R}g\mathcal{R}^3} &= \mathbf{T}_i^b f^{bck} f^{kge} f^{eda_4}\mathbf{T}_j^{\{c,d,g\}} \\ &= \frac{1}{24}\left(2N_c\,\mathbf{T}_{(++)} + 2(\mathbf{T}_{(++)})^2 + 6(\mathbf{T}_{(-+)})^2\right)\mathcal{C}^{(0)}_{ij} \\ &= \begin{cases} \left(\frac{N_c^2}{24} + \frac{3}{2}\right) c^{[8_a,8_a]} - \frac{3\sqrt{N_c^2-4}}{4\sqrt{2}} c^{[10+\overline{10},8_a]} & \text{for } gg \\ \left(\frac{N_c^2}{24} + \frac{1}{4}\right) c^{[8,8]_a} & \text{for } qq \\ \left(\frac{N_c^2}{24} + \frac{1}{4}\right) c^{[8,8_a]_a} & \text{for } qg \\ \left(\frac{N_c^2}{24} + \frac{3}{2}\right) c^{[8,8_a]_a} - \frac{3\sqrt{N_c^2-4}}{4\sqrt{2}} c^{[8,10+\overline{10}]} & \text{for } gq \end{cases} \end{aligned}$$

Factorizable and non-factorizable contributions in 2 → 3 amplitudes

- The [8,8] component of the Multi-Reggeon (MR) amplitude, split into Regge-factorizable (planar) terms and non-factorizable terms

$$\mathcal{M}^{(2),\,[8,8]}_{\rm MR} = \frac{(i\pi)^2}{72}\left(\frac{\mu^2}{|\mathbf{p}_4|^2}\right)^{2\epsilon}\mathcal{M}^{(0),\,[8,8]} \times \begin{cases} (N_c^2+36)F_{\rm fact}(z,\bar z) & \text{for } gg \\ N_c^2 F_{\rm fact}(z,\bar z) \,+\, F^{qq}_{\text{non-fact}}(z,\bar z) & \text{for } qq \\ N_c^2 F_{\rm fact}(z,\bar z) \,+\, F^{qg}_{\text{non-fact}}(z,\bar z) & \text{for } qg \end{cases}$$

$$F_{\rm fact}(z,\bar z) = \frac{1}{\epsilon^2} - \frac{1}{2\epsilon}\log|z|^2|1-z|^2 + 3\,D_2(z,\bar z) - \zeta_2 + \frac{5}{4}\log^2|z|^2 + \frac{5}{4}\log^2|1-z|^2 - \frac{1}{2}\log|z|^2\log|1-z|^2$$

$D_2(z,\bar z)$ is the Block-Wigner Dilogarithm

$$F^{qq}_{\text{non-fact}} = \frac{9}{\epsilon}\log|z|^2|1-z|^2 + \frac{9}{2}\Big(12D_2(z,\bar z) - \log^2|z|^2 - 2\log|z|^2\log|1-z|^2 - \log^2|1-z|^2\Big)$$
$$+\frac{3}{N_c^2}\left(\frac{3}{\epsilon^2} - \frac{6}{\epsilon}\log|z|^2|1-z|^2 - 18D_2(z,\bar z) + 6\log^2|z|^2 + 3\log|z|^2\log|1-z|^2 + 6\log^2|1-z|^2 - \frac{\pi^2}{2}\right)$$

$$F^{qg}_{\text{non-fact}} = \frac{27}{2\epsilon^2} - \frac{9}{\epsilon}\Big(2\log|z|^2 - 3\log|1-z|^2\Big) + \frac{9}{4}\Big(48D_2(z,\bar z) + 10\log^2|z|^2 - 8\log|z|^2\log|1-z|^2 - \pi^2\Big)$$

Abreu, EG, Falcioni, Milloy and Vernazza — to appear

Regge poles & cuts and the Lipatov vertex

(1) Rapidity evolution equations (2 dim!) facilitate efficient computation in the (multi) Regge limit

NLL for signature even 2 → 2 amplitudes (all orders)

NNLL for signature odd 2 → 2 amplitudes (so far to four loops)

NNLL for signature odd-odd 2 → 3 amplitudes (so far to two loops)

(2) **Regge-pole factorization violations in 2 → 2 and 2 → 3 amplitudes** - Regge cut contributions - **are non-planar**

(3) Based on (1), (2) and recent 3-loop 4-point calculations **we now know all Regge-pole parameters to 3 loops.**

(4) Based on (1), (2) and (3) and recent 2-loop 5-point calculations* **we can determine the 2-loop Lipatov vertex in QCD.**

*G. De Laurentis, H. Ita , M. Klinkert, V. Sotnikov 2311.10086, 2311.18752,

and B. Agarwal, F. Buccioni, F. Devoto, G. Gambuti, A. von Manteuffel, L. Tancredi, 2311.09870.

(5) The Lipatov Vertex is one of the building blocks of **NNLO BFKL Kernel**. Most others will be available soon.

Great prospects to further exploiting the interplay the Regge limit, fixed-order computations and the study of IR singularities