Antenna Subtraction for Final-State Radiation @ N^3LO

Petr Jakubčík

with X. Chen, M. Marcoli and G. Stagnitto

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Loops & Legs, 18/04/2024

Motivation

high-lumi run → (sub) 1% level precision for

① precision observables → α_s, PDF fitting

② prominent QCD background → new physics searches

for these, N^3LO is the new NNLO!

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N^3LO phenomenology: so far crossings of colour singlet production

H (ggF, bbF, VBF)	Anastasiou, Duhr, Dulat, Mistlberger, Pelloni; Dreyer, Karlberg, Hirschi; Chen, Gehrmann...	DY (Z/γ^*, W)	Duhr, Dulat, Mistlberger; Camarda, Cieri, Ferrera; Chen, Gehrmann; Neumann, Campbell
HH	Dreyer, Karlberg; Chen, Li, Shao, Wang		
VH	Baglio, Duhr, Mistlberger, Szafron		
$e^+e^- \to t\bar{t}$	Chen, Guan, He, Liu, Ma		

... and many others in the last decade!

✓ $\mathcal{O}(1\%)$ level ✓ significant improvement

but jets are the only QCD object available in high-E experiments!

$\Rightarrow$ formulation of subtraction scheme

MEs @ fixed pert. order $\rightarrow$ real — implicitly singular

MEs @ fixed pert. order $\rightarrow$ virtual — manifestly singular

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factorisation $\rightarrow$ process dependent $\times$ universal functions

describe & integrate over extra radiation

$\rightarrow$ expose poles $\xrightarrow[\text{cancel}]{}$ finite PS integrands

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MEs @ fixed pert. order → real — implicitly singular

→ virtual — manifestly singular

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antenna [Gehrmann, G.-de Ridder, Glover ...]

ColorfulNNLO [Somogyi, Trocsanyi, Del Duca ...]

sector improved residue [Czakon, Heymann, Mitov]

nested soft-collinear [Caola, Röntsch, Melnikov ...]

local analytic sector [Magnea, Maina, Pelliccioli, Signorile-S., Torrielli, Uccirati]

geometric IR [Herzog]

... and others @ NNLO

no general scheme formulated & applied @ N^3LO yet

antenna subtraction

good candidate for extension:

- analytic pole cancellation
- hadrons initial & jets final state
- efficient implementation in NNLOJET

paradigm:

"antenna function"

$\boxed{\text{SIMPLE MATRIX ELEMENTS}} \times \boxed{\text{REDUCED MULTIPLICITY MATRIX ELEMENTS}} = \boxed{\text{SUBTRACTION TERMS FOR REAL(-VIRT) RADIATION}}$

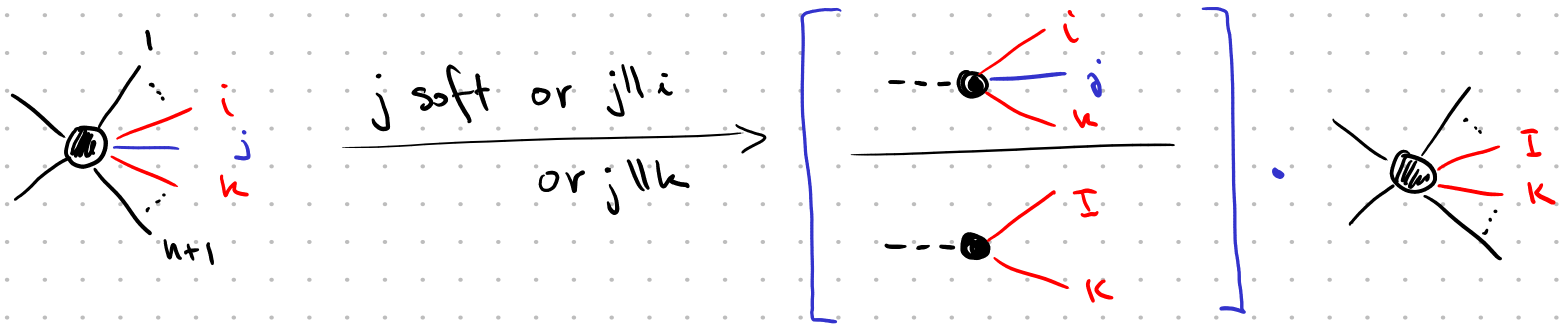
✓ have correct limits
(antennas smoothly interpolate between many limits)

✓ can be integrated analytically over PS

NLO

REAL

$X_3^0 \tilde{M}_n^0$



three parton tree level antenna $X_3^0(i,j,k)$

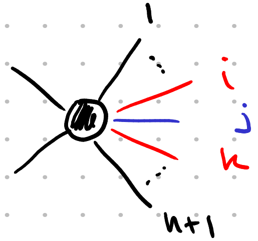
VIRTUAL

reduced matrix elem. $\tilde{M}^{\text{loops}}_{\text{hard partons}}$

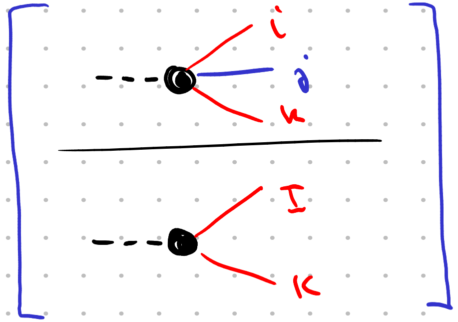
unintegrated antena $X^{\text{loops}}_{\text{emissions}+2}$

integrated antenna $\mathcal{X}$

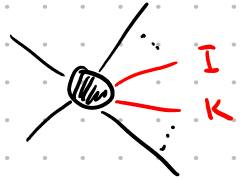
	NLO	
REAL	$X_3^0 \tilde{M}_n^0$	see diagrams below
VIRTUAL	$X_3^0 \tilde{M}_n^0$	see diagrams below



$\xrightarrow[\text{or } j \parallel k]{j \text{ soft or } j \parallel i}$

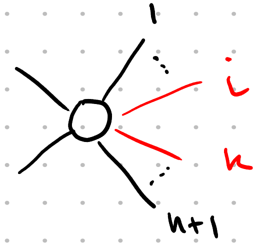


$\cdot$



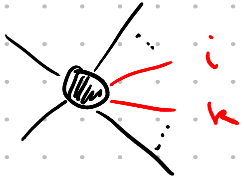
three parton tree level antenna $X_3^0(i,j,k)$

$\downarrow$ analytic PS integration $\int d\phi_3$

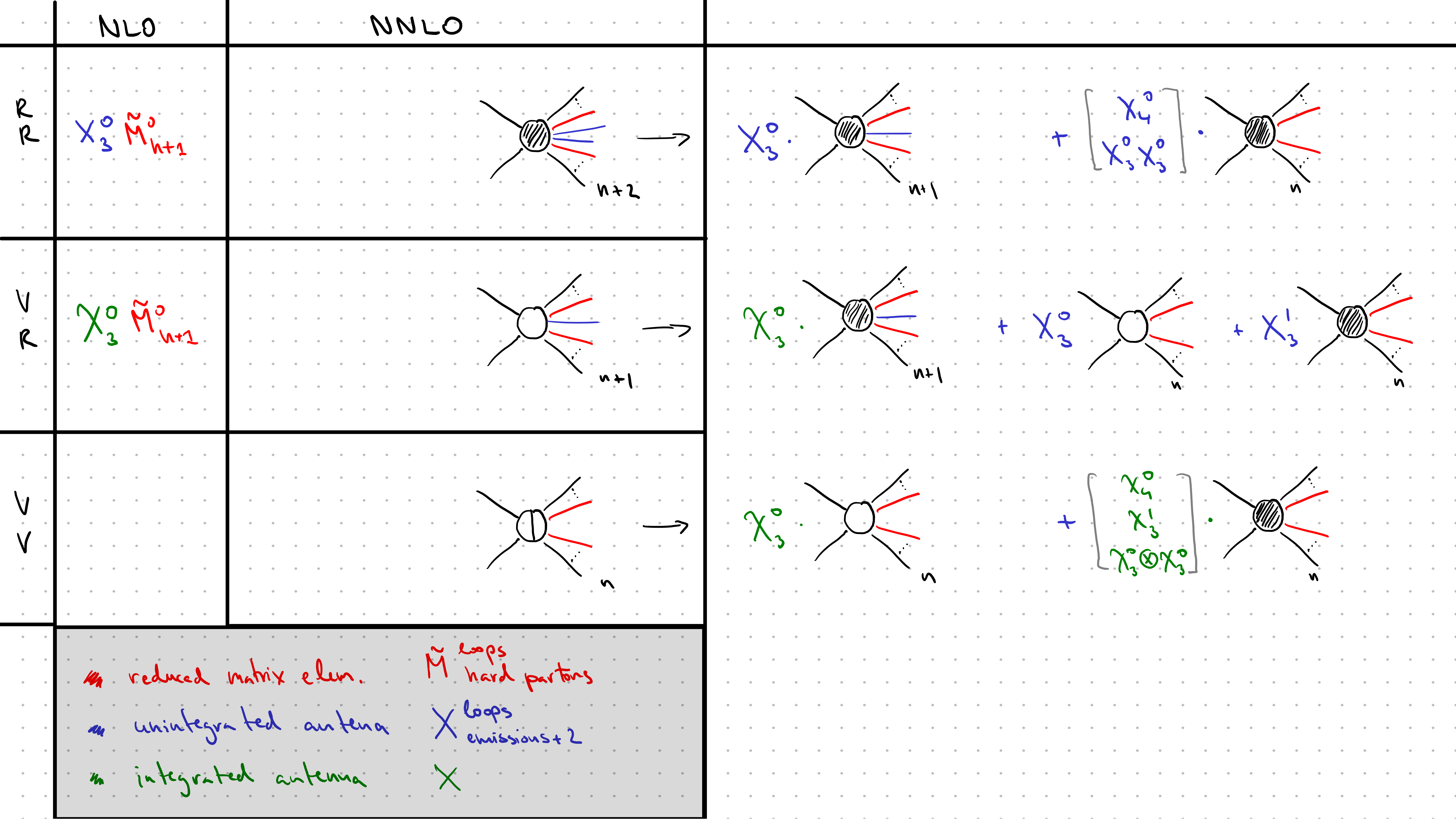


poles agree

$X_3^0(s_{ik}) \cdot$



- reduced matrix elem. — $\tilde{M}$ loops, hard partons
- unintegrated antena — X loops, emissions + 2
- integrated antenna — X



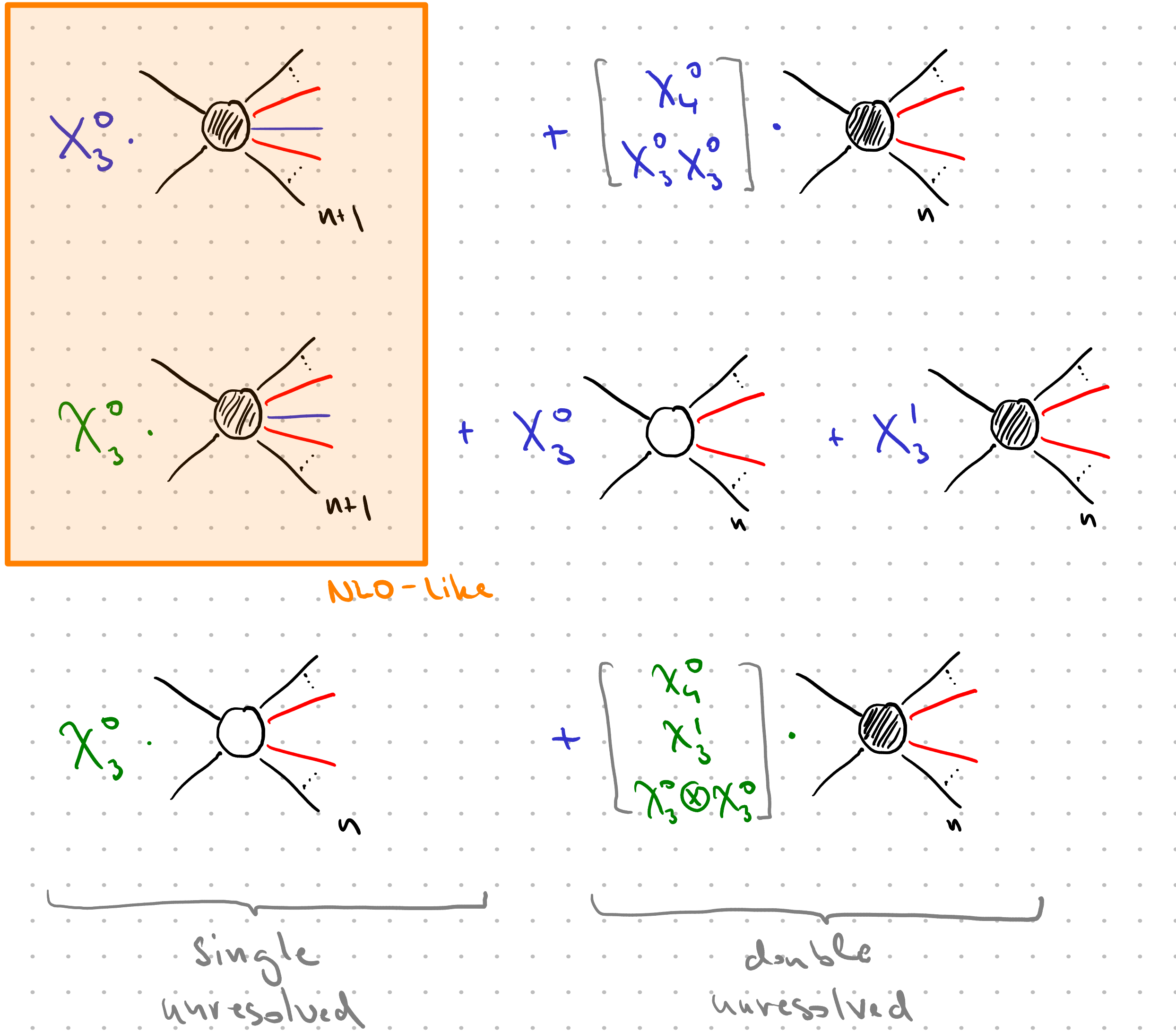
NLO
NNLO
R R
$X_3^0 \tilde{M}^0_{n+1}$
n+2
$X_3^0 \cdot$
n+1
$+ \begin{bmatrix} X_4^0 \\ X_3^0 X_3^0 \end{bmatrix} \cdot$
n
V R
$X_3^0 \tilde{M}^0_{n+1}$
n+1
$X_3^0 \cdot$
n+1
$+ X_3^0$
n
$+ X_3^1$
n
V V
n
$X_3^0 \cdot$
n
$+ \begin{bmatrix} X_4^0 \\ X_3^1 \\ X_3^0 \otimes X_3^0 \end{bmatrix} \cdot$
n
reduced matrix elem.
$\tilde{M}$ loops hard partons
unintegrated antena
X loops emissions+2
integrated antenna
X

	NLO	NNLO
RR	$X_3^0 \tilde{M}^0_{n+1}$	$(X_4^0 - X_3^0 X_3^0)\,\tilde{M}^0_n$
VR	$\mathcal{X}_3^0 \tilde{M}^0_{n+1}$	$(X_3^1 + \mathcal{X}_3^0 X_3^0)\,\tilde{M}^0_n + X_3^0 \tilde{M}^1_n$
VV		$(\mathcal{X}_4^0 + \mathcal{X}_3^1 + \mathcal{X}_3^0 \mathcal{X}_3^0)\,\tilde{M}^0_n + \mathcal{X}_3^0 \tilde{M}^1_n$

reduced matrix elem. $\tilde{M}^{\text{loops}}_{\text{hard partons}}$

unintegrated antenna $X^{\text{loops}}_{\text{emissions}+2}$

integrated antenna $\mathcal{X}$



	NLO	NNLO	N^3LO (a sketch)
RRR	$X_3^0 \tilde{M}^0_{n+2}$	$(X_4^0 - X_3^0 X_3^0)\, \tilde{M}^0_{n+1}$	$(X_5^0 - X_4^0 X_3^0 - X_3^0 X_4^0 + X_3^0 X_3^0 X_3^0)\, \tilde{M}^0_n$
VRR	$X_3^0 \tilde{M}^0_{n+2}$	$(X_3^1 + X_3^0 X_3^0)\, \tilde{M}^0_{n+1}$ $+ X_3^0 \tilde{M}^1_{n+1}$	$(X_4^1 - X_3^1 X_3^0 + X_3^0 X_3^0 X_3^0 + X_3^0 X_4^0)\, \tilde{M}^0_n$ $+ (X_4^0 + X_3^0 X_3^0)\, \tilde{M}^1_n$
VVR		$(X_4^0 + X_3^1 + X_3^0 X_3^0)\, \tilde{M}^0_{n+1}$ $+ X_3^0 \tilde{M}^1_{n+1}$	$(X_3^2 + X_3^0 X_3^1 + X_3^1 X_3^0 + X_4^0 X_3^0 + X_3^0 X_3^0 X_3^0)\, \tilde{M}^0_n$ $+ (-X_3^1 + X_3^0 X_3^0)\, \tilde{M}^1_n$ $+ X_3^0 \tilde{M}^2_n$
VVV	reduced matrix elem. $\tilde{M}^{\text{loops}}_{\text{hard partons}}$ unintegrated antena $X^{\text{loops}}_{\text{emissions}+2}$ integrated antenna X		$(X_5^0 + X_4^1 + X_3^2 + X_4^0 X_3^0 + X_3^1 X_3^0 + X_3^0 X_3^0 X_3^0)\, \tilde{M}^0_n$ $+ (X_4^0 + X_3^1 + X_3^0 X_3^0)\, \tilde{M}^1_n$ $+ X_3^0 \tilde{M}^2_n$

How to obtain the X_i^j and X_i^j?

- capture real & virtual QCD radiation from any pair of hard partons

→ decays of colour singlets with no QCD vertex @ Born

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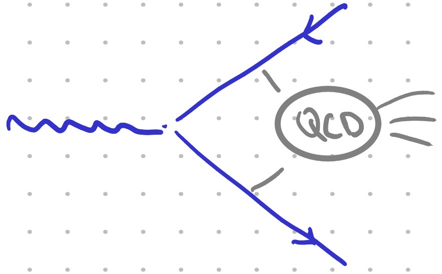
$\rightarrow$ decays of colour singlets with no QCD vertex @ Born:

quark-antiquark: $\gamma^* \to q\bar{q}$



$\gamma^* \otimes \text{QCD}$

(or $H \overset{\text{Yukawa}}{\underset{m_b=0}{\otimes}} \text{QCD}$)

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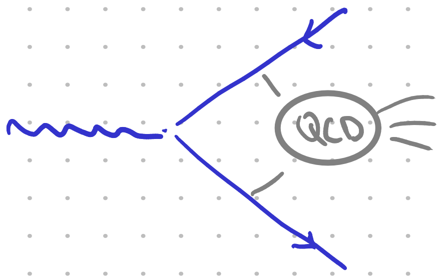
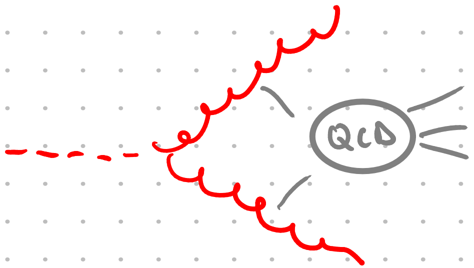
gluon-gluon: $H \rightarrow gg$





$\gamma^* \otimes \mathrm{QCD}$

$\left(\text{or } H \underset{m_b=0}{\overset{\text{Yukawa}}{\otimes}} \mathrm{QCD}\right)$

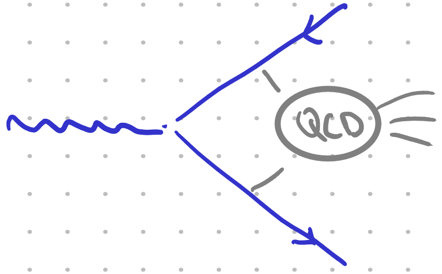
$H \overset{\text{HEFT}}{\otimes} \mathrm{QCD}$

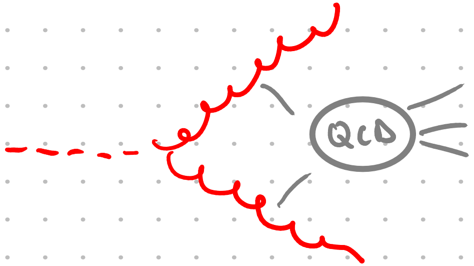
How to obtain the X_i^j and X_i^j?

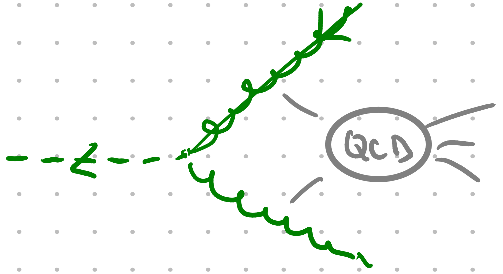
- capture real & virtual QCD radiation from any pair of hard partons

$\rightarrow$ decays of colour singlets with no QCD vertex @ Born:

quark-antiquark: $\gamma^* \rightarrow q\bar{q}$ gluon-gluon: $H \rightarrow gg$ quark-gluon: $\chi \rightarrow \tilde{g}g$



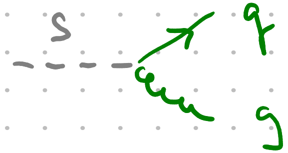




$\gamma^* \otimes \text{QCD}$ $H \overset{\text{HEFT}}{\otimes} \text{QCD}$? ? ?

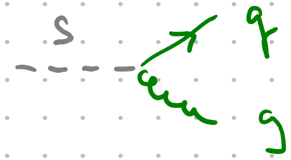
$\left(\text{or } H \underset{m_b=0}{\overset{\text{Yukawa}}{\otimes}} \text{QCD}\right)$

quark-gluon antenna

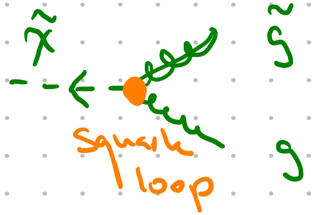
wish: 

+ QCD

quark-gluon antenna

wish: 

+ QCD

allowed: 

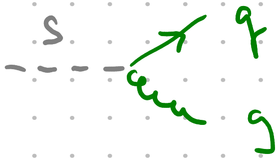
+ (N=1 SYM)

quark-gluon antenna

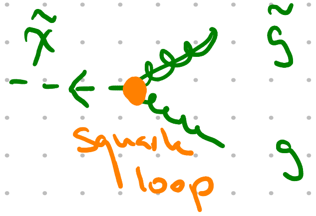
compromise: $\boxed{\underset{\text{EFT}}{\tilde{\chi} \otimes} \ \text{YM with } \begin{array}{l} N_F \text{ fundamental fermions} \\ N_A \text{ adjoint fermions} \end{array}}$

set $N_A = 0$

set $N_F = 0$, $N_A = 1$

wish: 

$+$ QCD

allowed: 

$+$ ($\mathcal{N}=1$ SYM)

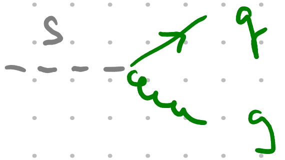
quark-gluon antenna

compromise: $\boxed{\tilde{\chi} \underset{\text{EFT}}{\otimes} \text{YM with } \begin{matrix} N_F \text{ fundamental fermions} \\ N_A \text{ adjoint fermions} \end{matrix}}$

set $N_A = 0$

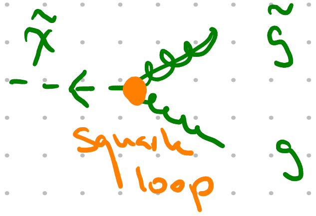
set $N_F = 0$, $N_A = 1$

wish:



+ QCD

allowed:



+ ($N=1$ SYM)

initial-final antennas:

- unintegrated related by kin. crossing
- integration non-trivial [Fontana, Gehrmann, Schönwald]

talk!

$\Rightarrow$ Task:

(generally known but recomputed) compute matrix elements for decay of $\gamma^* / H / \tilde{X} \rightarrow$ 2, 3, 4, 5 partons at 3, 2, 1, 0 loops } unintegrated antennas

$\Rightarrow$ Task:

(generally known but recomputed)

compute matrix elements for
decay of $\gamma^* / H / \tilde{X} \to$ 2, 3, 4, 5 partons
at 3, 2, 1, 0 loops

} unintegrated antennas

(done)

integrate each over respective phase spac

} integrated antennas

$\Rightarrow$ Task:

generally known but recomputed

compute matrix elements for

decay of $\gamma^* / H / \tilde{\chi} \rightarrow$ 2, 3, 4, 5 partons

at 3, 2, 1, 0 loops

unintegrated antennas

cumbersome

(done)

integrate each over respective phase spac

integrated antennas

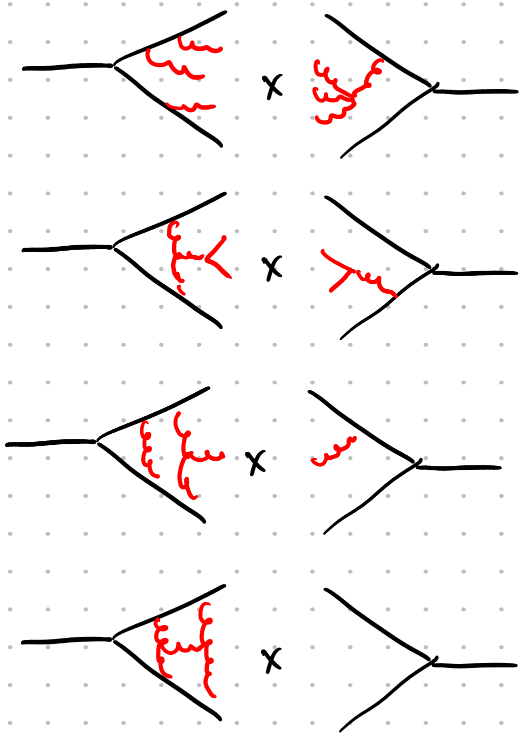
Organization of the calculation

RRR M_5^0

VRR M_4^1

VVR M_3^2

VVV M_2^3



matrix elements for "unintegrated" antenna functions

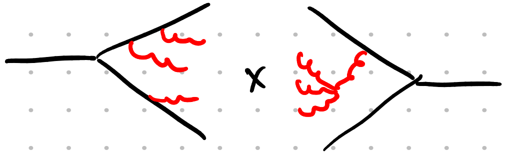
break down by:

- every partonic final state
- loop configurations (1×1, 2×0)

Organization of the calculation

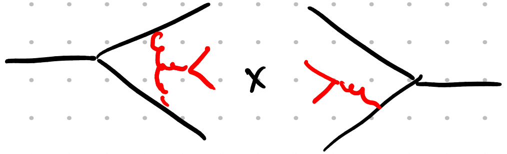
"integrated" antenna functions

RRR $\int d\phi_5 \, M_5^0$

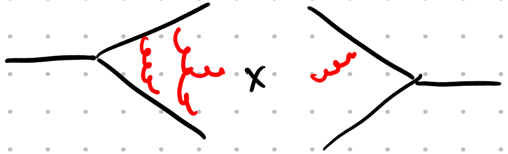


Singularities due to soft/collinear radiation

VRR $\int d\phi_4 \, M_4^1$

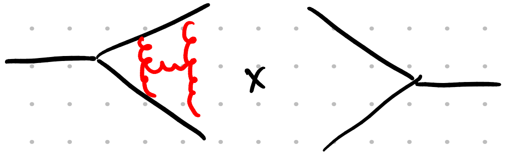


VVR $\int d\phi_3 \, M_3^2$



ε poles

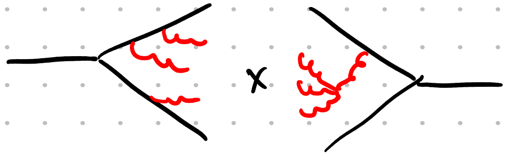
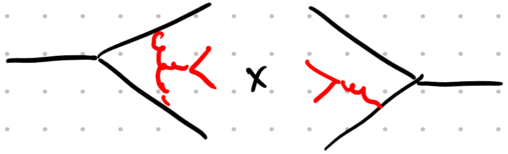
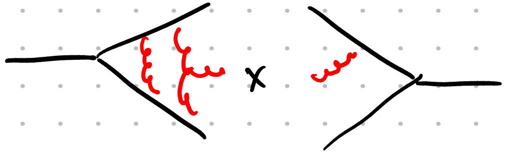
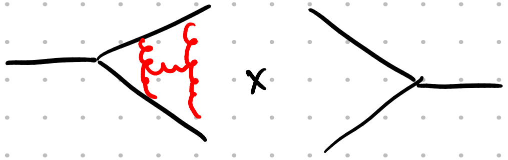
VVV $\int d\phi_2 \, M_2^3$



divergent loop integration

Organization of the calculation

"integrated" antenna functions

RRR	$\int d\phi_5\, M_5^0$		Singularities due to soft / collinear radiation
VRR	$\int d\phi_4\, M_4^1$		
VVR	$\int d\phi_3\, M_3^2$		ε poles
VVV	$\int d\phi_2\, M_2^3$		divergent loop integration

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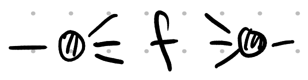
"R-ratio" $= \sigma^{(3)}$

total cross-section
for production of hadrons

optical theorem

$\sum_f d\Pi_f$ ()

$=$ Im ()

finite by KLN theorem

⇒ consider QCD corrections to singlet self energies

1. UV poles

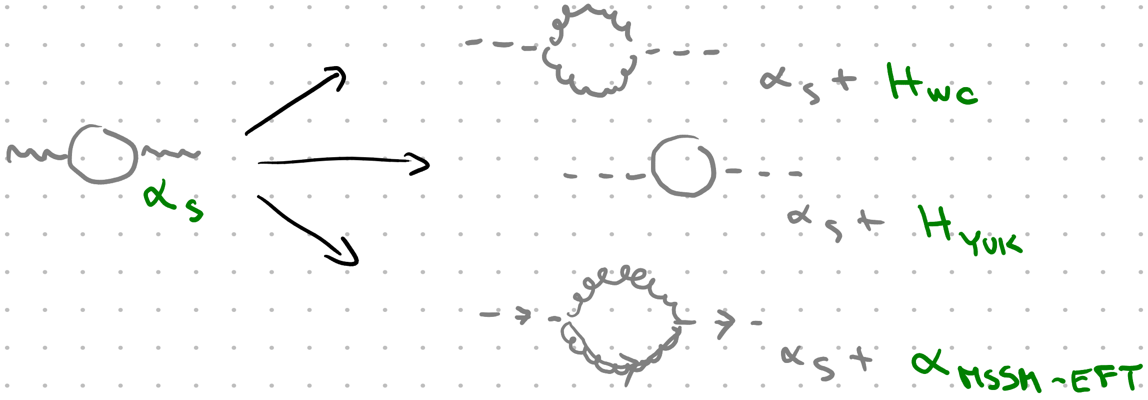
→ renormalization constants

2. finite parts

→ inclusive xsection (check)

3. all physical cuts

→ integrated antenna functions



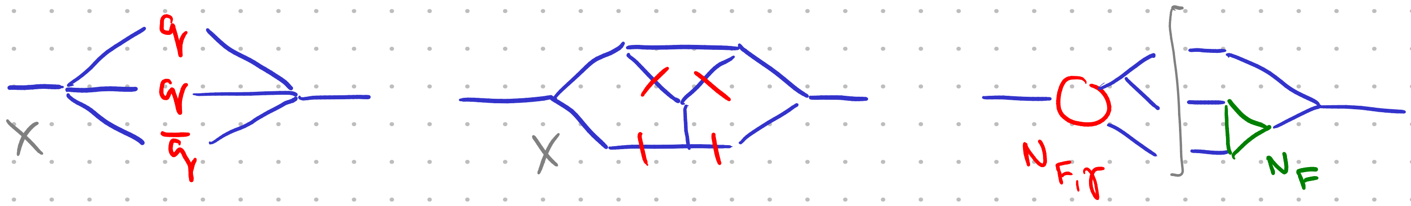
loop & PS integration at once!

[Anastasiou, Melnikov '02] reverse unitarity: $2\pi i\,\delta^+(p^2) \longrightarrow \frac{1}{p^2 - i0} - \frac{1}{p^2 + i0}$

⊕ leverage multiloop methods (IBP reduction)

MIs for cuts of 4 loop SEs known [Gituliar, Magerya, Pikelner, 2019] book-keeping

1 [select physical configurations & tag & assign cut integral families



reduction

2 [Reduze [Manteuffel, Studerus '12] for IBP reduction of cut topologies

FORCER [Ruijl, Ueda, Vermaseren '19] for reduction of four-loop self-energies

tensor rank ≤ 4 (γ) or 5 (H, X) $\rightarrow$ a month on 40 cores

integration

3 [~ 30 master integrals per layer

+ some NNLO integrals extended to weight 6 with AMFLOW [Liu, Ma, '22]

$A_4^1(1_q, 2_g, 3_g, 4_g, 5_{\bar{q}})$

catalogue

$\longleftrightarrow$

$\mathcal{A}_4^1$

$$\frac{1}{\epsilon^2}\left(\frac{4s_{12}+2s_{13}+3s_{14}}{s_{24}s_{34}} - \frac{2}{s_{13}} + \frac{6s_{12}+3s_{14}+3s_{23}}{s_{13}s_{24}} - \frac{2}{s_{34}} + \dots \right.$$
$$\left. + \frac{s_{12}s_{34}+s_{13}s_{34}+s_{14}s_{34}}{s_{24}\left(s_{23}+s_{24}+s_{34}\right)^2} + \frac{2s_{12}^3-4s_{34}s_{12}^2+3s_{34}^2s_{12}-s_{34}^3}{s_{13}s_{24}\left(s_{13}+s_{14}+s_{34}\right)\left(s_{23}+s_{24}+s_{34}\right)}\right)$$
$$+\frac{1}{\epsilon}\sum_k r_k(s_{ij})\left\{1, \log(s_{13}), \log(s_{24}), \log(s_{34})\right\}$$
$$+\sum_k r_k(s_{ij})\left\{1, \log, \log^2, \mathrm{Li}_2\right\} + \mathcal{O}(\epsilon)$$

$$+\frac{1}{\epsilon^6}\left(-\frac{41}{36}\right) + \frac{1}{\epsilon^5}\left(-\frac{311}{36}\right) + \frac{1}{\epsilon^4}\left(-\frac{54325}{1296} + \frac{1151}{432}\pi^2\right)$$
$$+\frac{1}{\epsilon^3}\left(-\frac{1590017}{7776} + \frac{23519}{1296}\pi^2 + \frac{380}{9}\zeta_3\right)$$
$$+\frac{1}{\epsilon^2}\left(-\frac{7646353}{7776} + \frac{1461895}{15552}\pi^2 + \frac{66593}{216}\zeta_3 - \frac{16537}{10368}\pi^4\right)$$
$$+\frac{1}{\epsilon}\left(-\frac{84015367}{17496} + \frac{44125165}{93312}\pi^2 + \frac{2192809}{1296}\zeta_3\right.$$
$$\left. - \frac{7973}{960}\pi^4 - \frac{7435}{72}\pi^2\zeta_3 + \frac{40319}{90}\zeta_5\right)$$
$$-\frac{20052623335}{839808} + \frac{163823405}{69984}\pi^2 + \frac{22820177}{2592}\zeta_3 - \frac{4992721}{124416}\pi^4$$
$$-\frac{22493}{32}\pi^2\zeta_3 + \frac{1335263}{360}\zeta_5 + \frac{4433837}{13063680}\pi^6 - \frac{64345}{72}\zeta_3^2 + \mathcal{O}(\epsilon)$$

build (part of) RRV subtraction term $\longleftrightarrow$ cancel (same) poles at virt. level

$A_4^1(1_q, 2_g, 3_g, 4_g, 5_{\bar{q}})$ catalogue $\longleftrightarrow$ $\mathcal{A}_4^1$

$$\frac{1}{\epsilon^2}\left(\frac{4s_{12}+2s_{13}+3s_{14}}{s_{24}s_{34}} - \frac{2}{s_{13}} + \frac{6s_{12}+3s_{14}+3s_{23}}{s_{13}s_{24}} - \frac{2}{s_{34}} + \dots \right.$$
$$\left. + \frac{s_{12}s_{34}+s_{13}s_{34}+s_{14}s_{34}}{s_{24}\left(s_{23}+s_{24}+s_{34}\right)^2} + \frac{2s_{12}^3 - 4s_{34}s_{12}^2 + 3s_{34}^2 s_{12} - s_{34}^3}{s_{13}s_{24}\left(s_{13}+s_{14}+s_{34}\right)\left(s_{23}+s_{24}+s_{34}\right)}\right)$$
$$+\frac{1}{\epsilon}\sum_k r_k(s_{ij})\,\{1, \log(s_{13}), \log(s_{24}), \log(s_{34})\}$$
$$+\sum_k r_k(s_{ij})\left\{1, \log, \log^2, \mathrm{Li}_2\right\} + \mathcal{O}(\epsilon)$$

$$+\frac{1}{\epsilon^6}\left(-\frac{41}{36}\right) + \frac{1}{\epsilon^5}\left(-\frac{311}{36}\right) + \frac{1}{\epsilon^4}\left(-\frac{54325}{1296} + \frac{1151}{432}\pi^2\right)$$
$$+\frac{1}{\epsilon^3}\left(-\frac{1590017}{7776} + \frac{23519}{1296}\pi^2 + \frac{380}{9}\zeta_3\right)$$
$$+\frac{1}{\epsilon^2}\left(-\frac{7646353}{7776} + \frac{1461895}{15552}\pi^2 + \frac{66593}{216}\zeta_3 - \frac{16537}{10368}\pi^4\right)$$
$$+\frac{1}{\epsilon}\left(-\frac{84015367}{17496} + \frac{44125165}{93312}\pi^2 + \frac{2192809}{1296}\zeta_3 \right.$$
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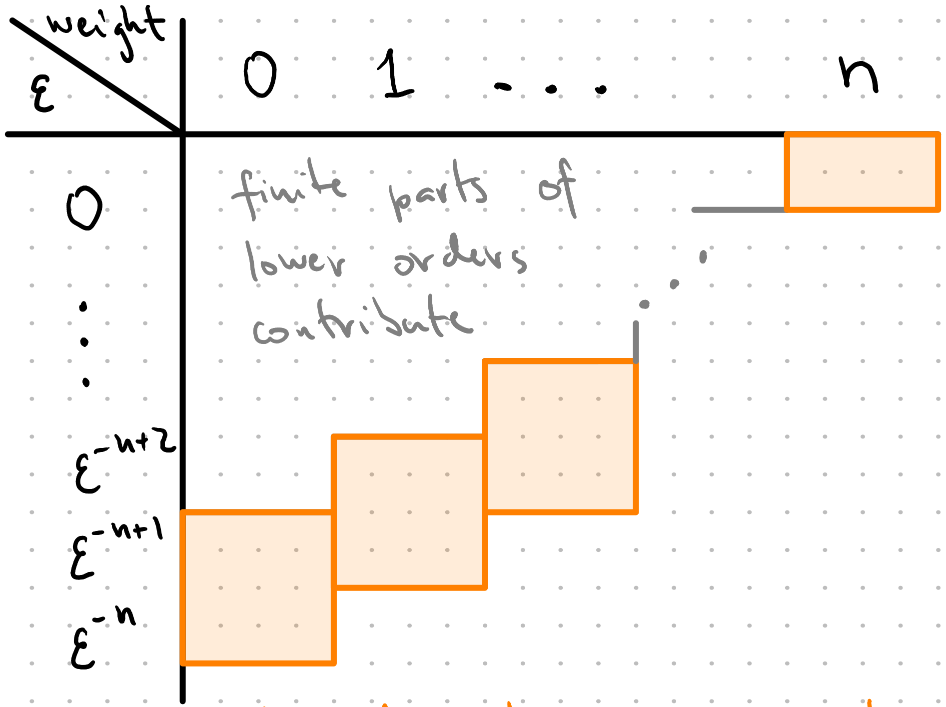
build (part of) RRV subtraction term $\longleftrightarrow$ cancel (some) poles at virt. level

- ✓ cancellation of IR poles
- ✓ recovery of R-ratios for $\gamma^* \to q\bar{q}$, $H \to gg$, $H \to b\bar{b}$
- ✓ match the corresponding form factors
- ✓ (re)derive renormalization of couplings
- ✓ new soft/collinear anomalous dimensions for $\chi \to \tilde{g}g$

$\gamma^* \to q\bar{q}$ and $H \to b\bar{b}$

for every colour layer:



two deepest appearing poles agree

@ NNLO: interpretation in terms of integration of double unresolved limits

[Braun-White, Glover, Fox, '21-'23]

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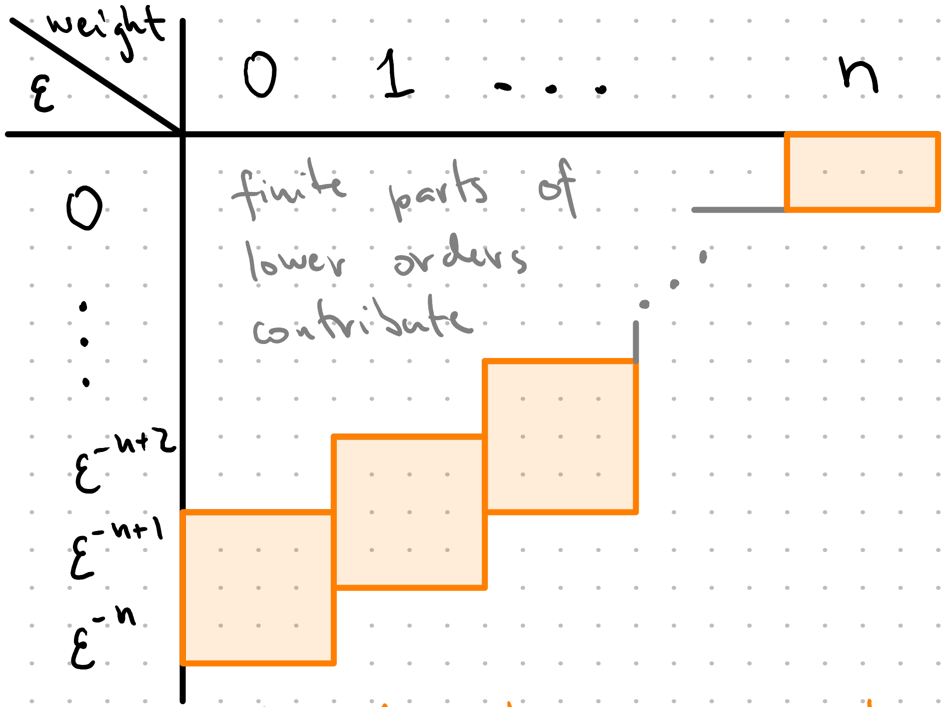


two deepest appearing poles agree

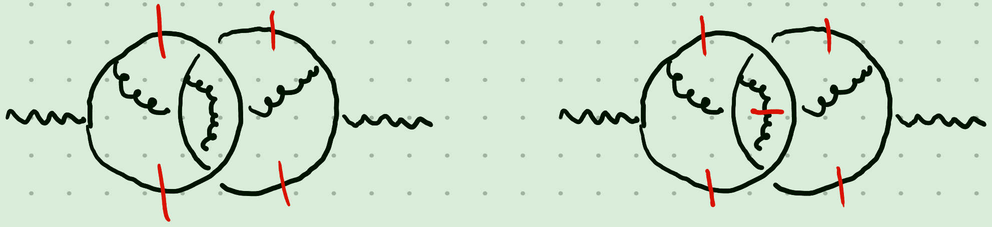
@ NNLO: interpretation in terms of integration of double unresolved limits

[Braun-White, Glover, Fox, '21-'23]

unique to $\gamma^* \to q\bar{q}$ @ N^3LO: singlet

$$\propto \left(\sum_q e_q\right)^2 d^{abc} d^{abc} = \left(\sum_q e_q\right)^2 \frac{N-4}{N}$$

at $\mathcal{O}(\varepsilon^{-1})$ in X_4^1 and X_5^0



IR cancellation patterns

! understanding of cancellation of poles between real/virtual radiation in terms of integration of individual unresolved limits -> simpler subtraction

IR cancellation patterns

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	Final-state $\mathcal{I}$	N^2	N^0	N^{-2}	$N_F N$	$N_F N^{-1}$
VVV	$q\bar{q}$	$-\frac{1}{6}\frac{1}{\epsilon^6} - \frac{17}{8}\frac{1}{\epsilon^5}$	$+\frac{1}{3}\frac{1}{\epsilon^6} + \frac{23}{8}\frac{1}{\epsilon^5}$	$-\frac{1}{6}\frac{1}{\epsilon^6} - \frac{3}{4}\frac{1}{\epsilon^5}$	$+\frac{1}{4}\frac{1}{\epsilon^5}$	$-\frac{1}{4}\frac{1}{\epsilon^5}$
VVR	$q\bar{q}g$	$+\frac{29}{36}\frac{1}{\epsilon^6} + \frac{1663}{216}\frac{1}{\epsilon^5}$	$-\frac{5}{4}\frac{1}{\epsilon^6} - \frac{53}{6}\frac{1}{\epsilon^5}$	$+\frac{1}{2}\frac{1}{\epsilon^6} + \frac{9}{4}\frac{1}{\epsilon^5}$	$-\frac{20}{27}\frac{1}{\epsilon^5}$	$+\frac{7}{12}\frac{1}{\epsilon^5}$
VRR	$q\bar{q}gg$	$-\frac{41}{36}\frac{1}{\epsilon^6} - \frac{311}{36}\frac{1}{\epsilon^5}$	$+\frac{3}{2}\frac{1}{\epsilon^6} + \frac{217}{24}\frac{1}{\epsilon^5}$	$-\frac{1}{2}\frac{1}{\epsilon^6} - \frac{9}{4}\frac{1}{\epsilon^5}$	$+\frac{1}{2}\frac{1}{\epsilon^5}$	$-\frac{1}{3}\frac{1}{\epsilon^5}$
	$q\bar{q}q'\bar{q}' + q\bar{q}q\bar{q}$				$+\frac{13}{108}\frac{1}{\epsilon^5}$	$-\frac{11}{108}\frac{1}{\epsilon^5}$
RRR	$q\bar{q}ggg$	$+\frac{1}{2}\frac{1}{\epsilon^6} + \frac{311}{108}\frac{1}{\epsilon^5}$	$-\frac{7}{12}\frac{1}{\epsilon^6} - \frac{37}{12}\frac{1}{\epsilon^5}$	$+\frac{1}{6}\frac{1}{\epsilon^6} + \frac{3}{4}\frac{1}{\epsilon^5}$		
	$q\bar{q}q'\bar{q}'g + q\bar{q}q\bar{q}g$				$-\frac{7}{54}\frac{1}{\epsilon^5}$	$+\frac{11}{108}\frac{1}{\epsilon^5}$

- Abelian structures in subleading

$$\underbrace{\left(I^{(1)}_{q\bar{q}} - I^{(1)}_{q\bar{q}}\right)^k}_{\text{exponentiation of emissions}} \overset{!}{=} 0$$

- (+ − + −) pattern everywhere
- large numerical cancellations

etc...

IR cancellation patterns

- understanding of cancellation of poles between real/virtual radiation in terms of integration of individual unresolved limits -> simpler subtraction

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VVR	$q\bar{q}g$	$+\frac{29}{36}\frac{1}{\epsilon^6} + \frac{1663}{216}\frac{1}{\epsilon^5}$	$-\frac{5}{4}\frac{1}{\epsilon^6} - \frac{53}{6}\frac{1}{\epsilon^5}$	$+\frac{1}{2}\frac{1}{\epsilon^6} + \frac{9}{4}\frac{1}{\epsilon^5}$	$-\frac{20}{27}\frac{1}{\epsilon^5}$	$+\frac{7}{12}\frac{1}{\epsilon^5}$
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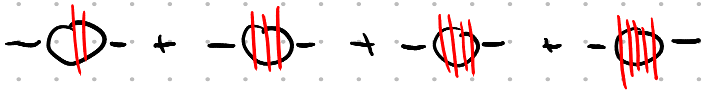
Stronger statement (Cutkosky):

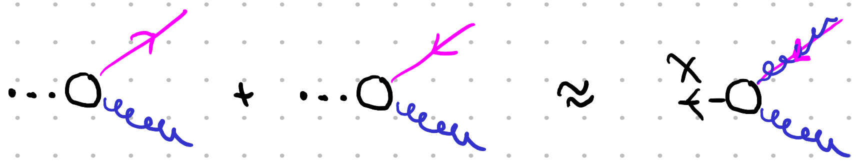
$$\mathrm{Disc}\left(\underbrace{-\!\!(4L)\!\!-}_{\text{1 diagram}}\right) = \sum$$



cancellation occurs individually over 100s of subsets of cut diagrams

Quark-Gluon antenna functions

with colour ordering :



adjoint fermion = "gluino"

successful at NLO and NNLO (after removing spurious limits such as $q \| \bar{q} \| q$)

Quark-Gluon antenna functions

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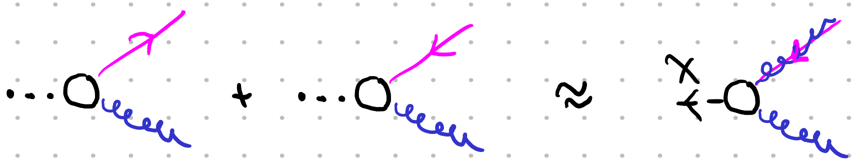

adjoint fermion = "gluino"

successful at NLO and NNLO (after removing spurious limits such as $g \parallel \bar{q} \parallel q$)

observed in IR structure @ N^3LO:

final states with adjoint and fundamental fermions differ only by

$$\boxed{\begin{aligned} T_F N_F &\to T_F N_F + C_A \frac{N_{\tilde{g}}}{2} \\ C_F N_{\tilde{g}} &\to C_A N_{\tilde{g}} \end{aligned}}$$

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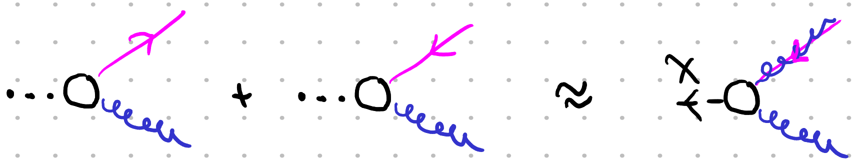

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BONUS 3 loop form factor gives soft/collinear and cusp anomalous dimensions

special case : result for N=1 SYM

$$\gamma_2^K = \left(\frac{11\zeta_3}{3} + \frac{11\pi^4}{90} - \frac{67\pi^2}{27} + \frac{245}{12}\right) C_A^2 + \left(-\frac{14\zeta_3}{3} + \frac{10\pi^2}{27} - \frac{209}{54}\right) C_A N_F$$
$$+ \left(4\zeta_3 - \frac{55}{12}\right) C_F N_F - \frac{2}{27} N_F^2 + \left(-\frac{2\zeta_3}{3} + \frac{10\pi^2}{27} - \frac{913}{108}\right) C_A^2 N_{\tilde{g}}$$
$$- \frac{2}{27} C_A^2 N_{\tilde{g}}^2 - \frac{4}{27} C_A N_F N_{\tilde{g}} .$$

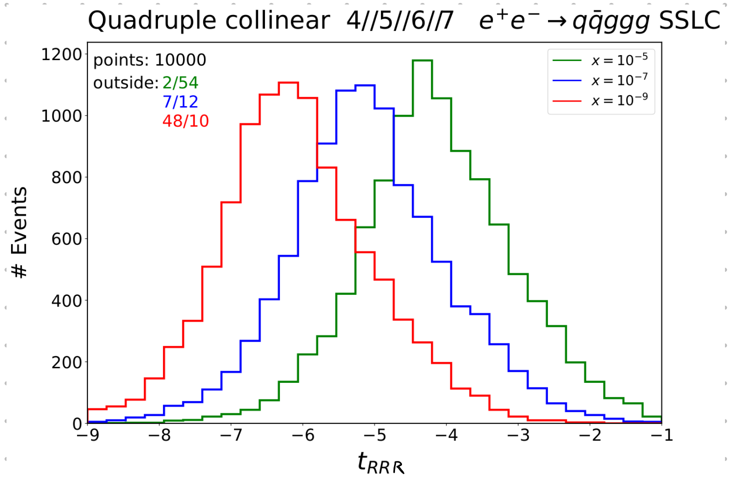
Applications

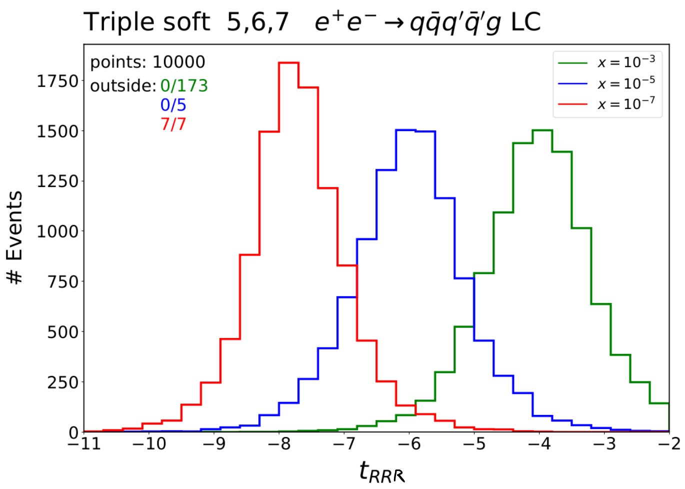
First feasible process : $e^+e^- \to 2$ jets

✓ amplitudes

✓ antennas

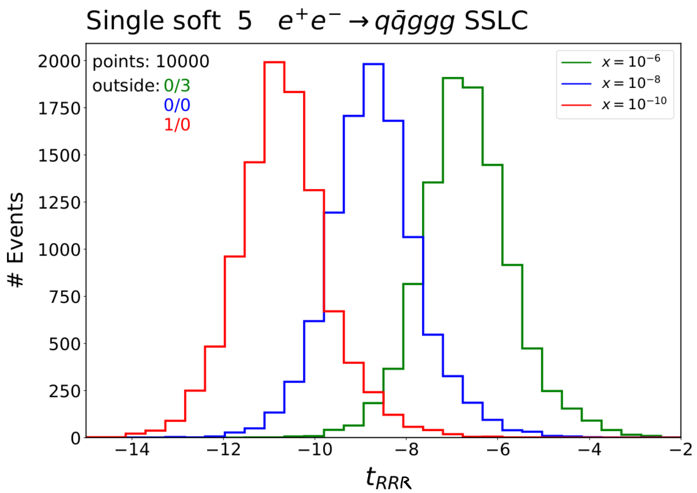
✓ RRR subtraction term





$\uparrow$

$\log_{10}\left(\left|1 - \frac{\mathrm{ME}}{\mathrm{SUB}}\right|\right)$



$e^+e^- \to 2$ jets

forward - backward asymmetry : perturbative corrections small

→ potential precision observable @ lin. collider

($\sin^2\vartheta_w$ sensitivity)

@ NNLO :

approx. in VV	not IR safe	
		[Altarelli, Lampe '93]
		[Ravindran, van Neerven '98]
		[Catani, Seymour '98]
✓	✓	[Weinzierl '06]

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\sigma_F = \int_0^1 d\cos\vartheta \, \frac{d\sigma}{d\cos\vartheta} \qquad \sigma_B = \int_{-1}^0 d\cos\vartheta \, \frac{d\sigma}{d\cos\vartheta}$$

between beam & flavoured jet axis

Future : higher multiplicity & hadronic initial states