Swiss National Science Foundation

Universität Zürich[UZH]

NNLO QCD predictions for **heavy-quark pair** production in association with **colourless particles**

Chiara Savoini

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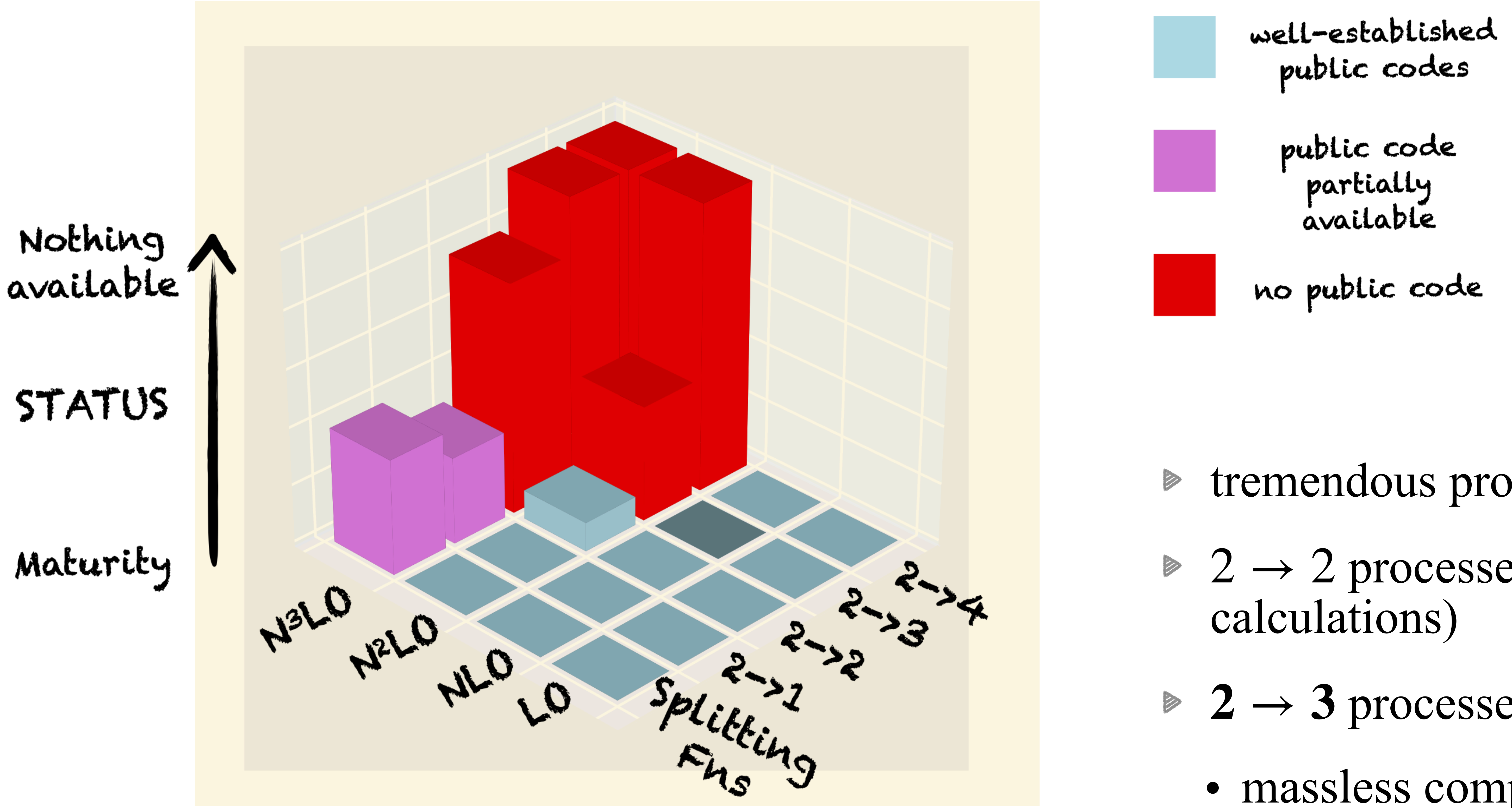
based on Phys.Rev.Lett. 130 (2023), Phys.Rev.D 107 (2023) and Phys.Rev.Lett. 131 (2023)
+ work in progress

Introduction



Luca Buonocore ©

- tremendous progress in the past ∼10 years!
- 2 → 2 processes at NNLO are under control (independent calculations)
- **2 → 3** processes at NNLO represent the **current frontier**
  - massless computations (up to one massive leg) basically done!

$pp \to \gamma\gamma\gamma$ [Chawdhry et al. (2019)] [Kallweit et al. (2020)]

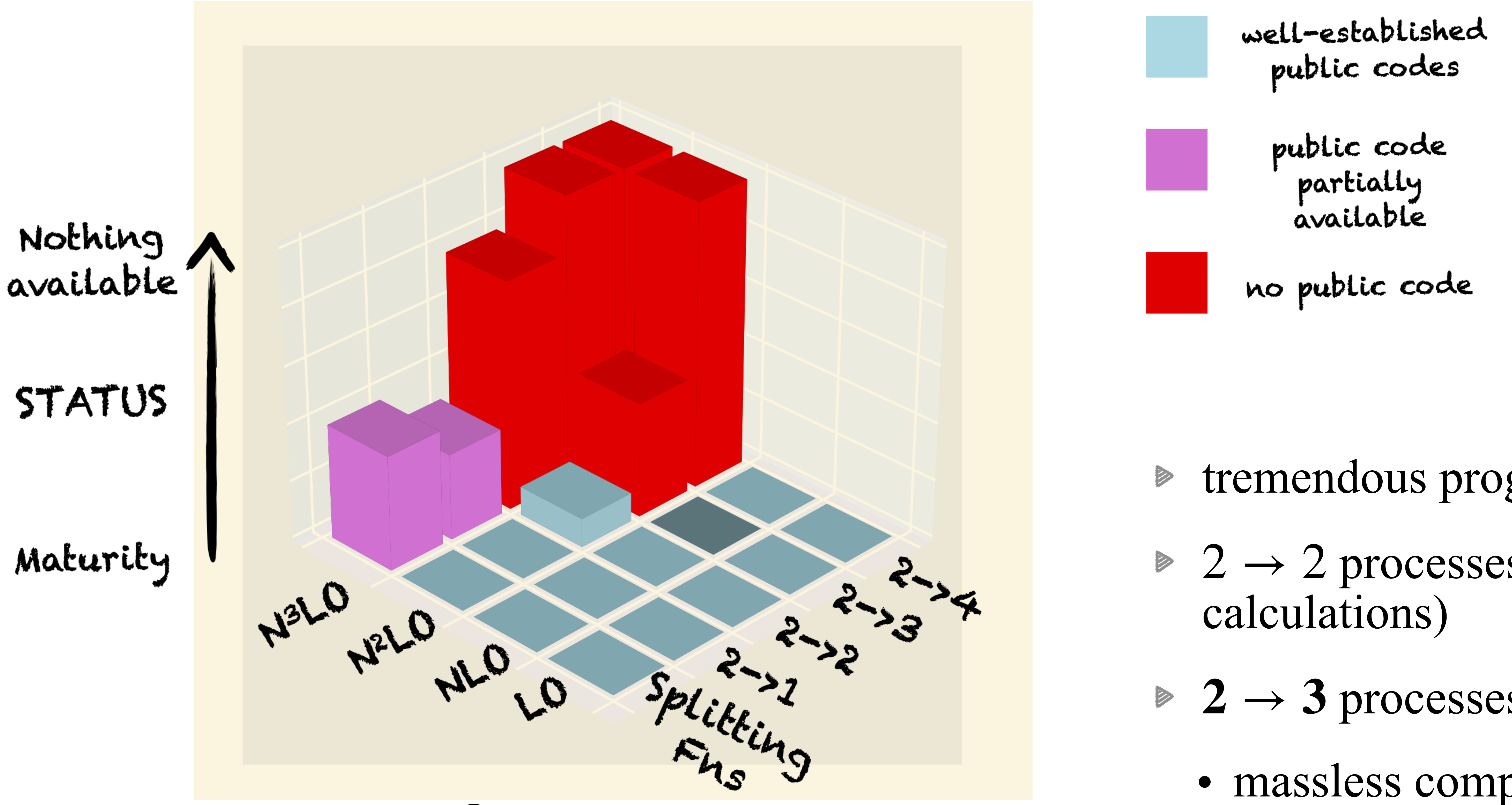
$pp \to \gamma\gamma j$ [Chawdhry et al. (2021)]

$pp \to jjj$ [Czakon et al. (2021)]

$pp \to Wb\bar{b}$ (5FS) [Hartanto et al. (2022)]

$pp \to \gamma jj$ [Badger et al. (2023)]

Introduction



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  - in this talk we will focus on $\mathbf{2 \to 3}$ processes with (more than one) **external massive legs**

$Wb\bar{b}$ $t\bar{t}H$ $t\bar{t}W$ $b\bar{b}4l$

very first results

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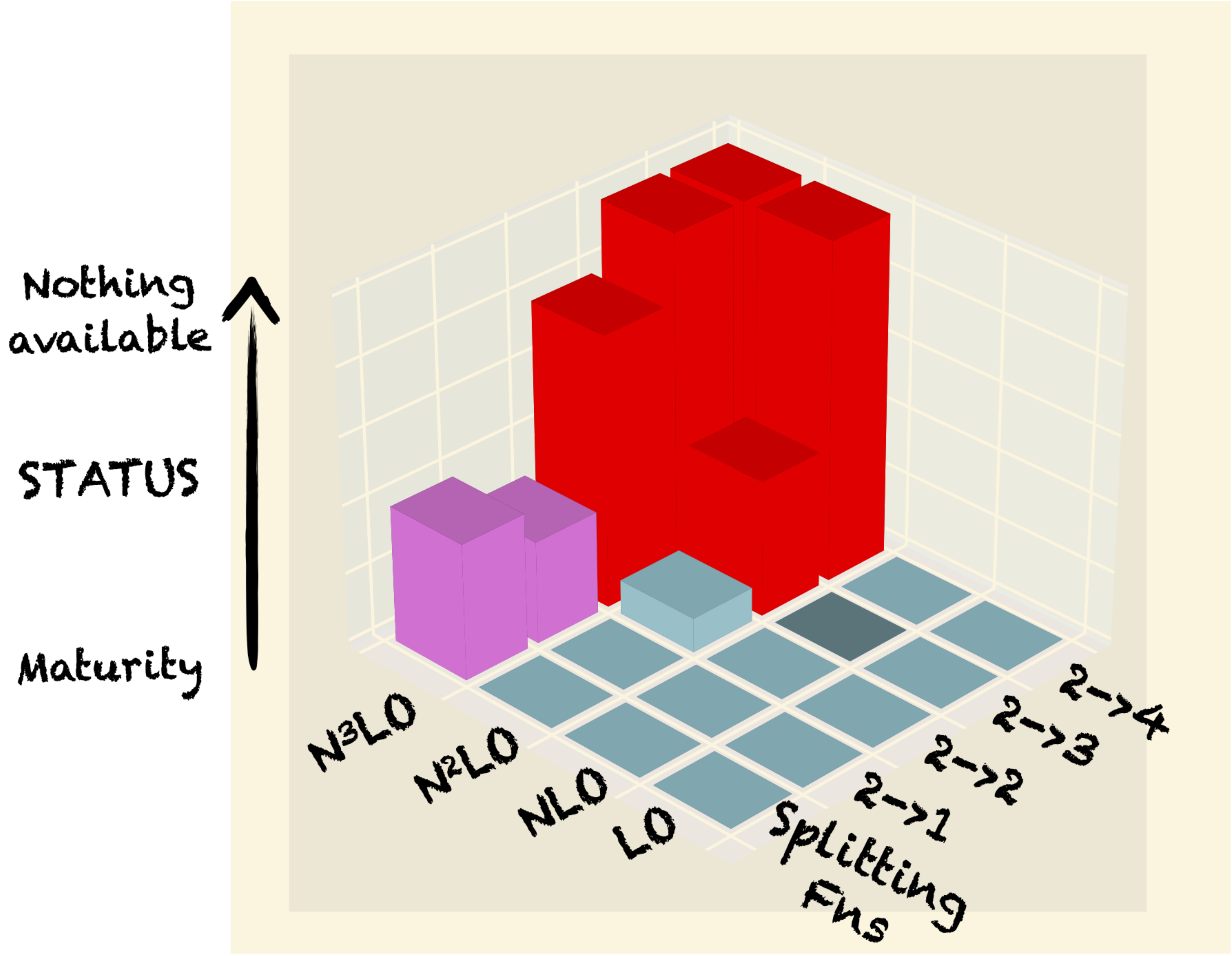

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well-established public codes

public code partially available

no public code

to complete an NNLO computation: crucial to construct an NNLO subtraction/slicing scheme and have the two-loop virtual amplitudes

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very first results

The framework: q_T-subtraction

[Catani, Grazzini (2007)]

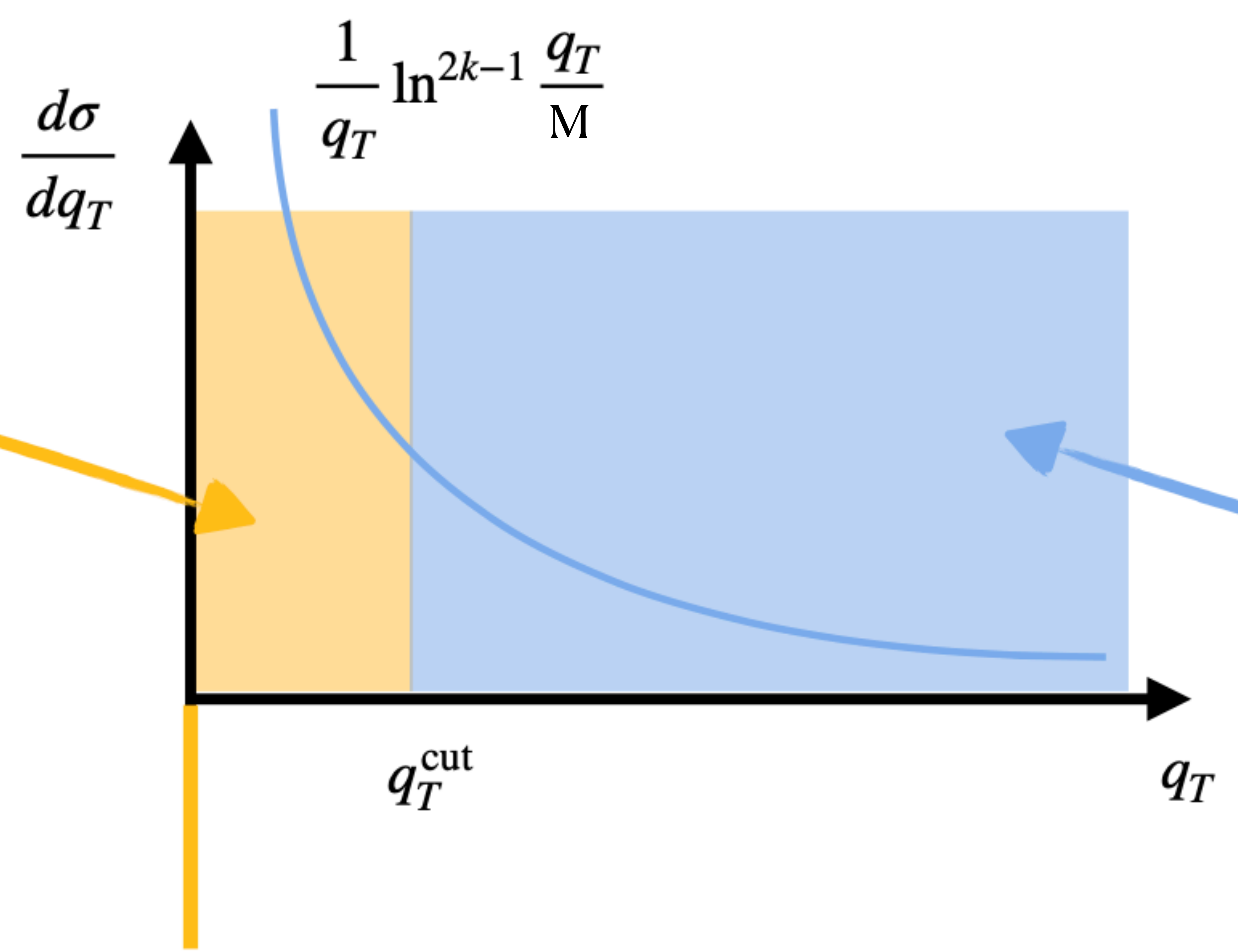
- cross section for the production of a triggered final state at N^{k}LO (in our case the triggered final state is $Q\bar{Q}F$)

crucial to keep the mass of the heavy quark m_Q

all emissions are unresolved

we can exploit the **QCD factorisation** of the matrix elements in the singular soft and/or collinear limits

ingredients from q_T - resummation



1 emission is always **resolved**

the complexity of the calculation is reduced by 1 order

logarithmic IR sensitivity to the cut

q_T is the transverse momentum of the $Q\bar{Q}F$ system

$$d\sigma_{N^kLO} = \mathscr{H}_{N^kLO} \otimes d\sigma_{LO} + [d\sigma^R_{N^{k-1}LO} - d\sigma^{CT}_{N^kLO}]_{q_T > q_T^{\rm cut}} + \mathcal{O}((q_T^{\rm cut})^p)$$

see Flavio's talk

missing power corrections

The framework: q_T -subtraction [Catani, Grazzini (2007)]

- master formula at NNLO

$$d\sigma_{NNLO} = \mathscr{H}_{NNLO} \otimes d\sigma_{LO} + [d\sigma^{R}_{NLO} - d\sigma^{CT}_{NNLO}]_{q_T > q_T^{\text{cut}}} + \mathcal{O}((q_T^{\text{cut}})^p)$$

☑ the required matrix elements can be computed with **automated tools** like OpenLoops2

[Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, Zoller (2019)]

☑ the remaining NLO-type singularities can be removed by applying a **local subtraction** method

[Catani, Seymour (1998)] [Catani, Dittmaier, Seymour, Trocsanyi (2002)]

☑ automatised implementation in the MATRIX framework, which relies on the efficient multi-channel Monte Carlo integrator MUNICH [Grazzini, Kallweit, Wiesemann (2017)]

The framework: q_T-subtraction

[Catani, Grazzini (2007)]

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$$d\sigma_{NNLO} = \mathscr{H}_{NNLO} \otimes d\sigma_{LO} + [d\sigma^{R}_{NLO} - d\sigma^{CT}_{NNLO}]_{q_T > q_T^{\text{cut}}} + \mathcal{O}((q_T^{\text{cut}})^p)$$

☑ non trivial ingredient: **two-loop soft function** for heavy-quark production

[Catani, Devoto, Grazzini, Mazzitelli (2023)]
[Devoto, Mazzitelli (in preparation)]

☑ all ingredients are known except for the **two-loop virtual amplitudes** contributing to the the hard-collinear coefficient

$$\mathscr{H}_{NNLO} = H^{(2)}\delta(1-z_1)\delta(1-z_2) + \delta\mathscr{H}^{(2)}(z_1, z_2)$$

where

$$H^{(2)} = \left.\frac{2\Re(\mathscr{M}^{(2)}_{fin}(\mu_{IR}, \mu_R)\mathscr{M}^{(0)*})}{|\mathscr{M}^{(0)}|^2}\right|_{\mu_R=\mu_{IR}=Q}$$

main bottleneck:
$2 \to 3$ two-loop amplitudes with internal and external massive legs are currently out of reach!

see Matteo's and Ben's talks

Q is the invariant mass of the $Q\bar{Q}F$ system

The framework: q_T-subtraction

[Catani, Grazzini (2007)]

- master formula at NNLO

$$d\sigma_{NNLO} = \mathscr{H}_{NNLO} \otimes d\sigma_{LO} + [d\sigma^{R}_{NLO} - d\sigma^{CT}_{NNLO}]_{q_T > q_T^{\mathrm{cut}}} + \mathcal{O}((q_T^{\mathrm{cut}})^p)$$

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Q is the invariant mass of the $Q\bar{Q}F$ system

idea: exploit the factorisation of the amplitude, in certain kinematical regimes, into a calculable factor and a simpler (available) amplitude

Massification

↓

$Wb\bar{b}$

[Buonocore, Devoto, Kallweit, Mazzitelli, Rottoli, **CS** (2022)]

Why is $Wb\bar{b}$ an interesting process?

- irreducible background to $WH(H \to b\bar{b})$, single top $\bar{b}t(t \to Wb)$ production, BSM searches
- test of perturbative QCD: 4FS vs 5FS, **modelling of flavoured jets**
- large NLO QCD corrections
- mandatory to include **NNLO QCD** corrections!

State of the art

- ☑ **NLO QCD** corrections in $p\bar{p}$ collisions (*massless bottom quarks*) [Ellis, Veseli (1999)]
- ☑ **NLO QCD** corrections in $p\bar{p}$ collisions and at the LHC (*massive bottom quarks*) [Febres Cordero, Reina, Wackeroth (2006) & (2009)]
- ☑ **NLO QCD** corrections (*4FS* + *5FS*) [Campbell, Ellis, Febres Cordero, Maltoni, Reina, Wackeroth, Willenbrock (2009)] [Campbell, Caola, Febres Cordero, Reina, Wackeroth (2011)]
- ☑ **NLO+PS** [Oleari, Reina (2011)] [Frederix, Frixione, Hirshi, Maltoni, Pittau, Torrielli (2011)]
- ☑ $Wb\bar{b}j$ at NLO with **POWHEG+MiNLO** [Luisoni, Oleari, Tramontano (2015)]
- ☑ NLO QCD corrections to $Wb\bar{b}$ + (up to 3 light jets) in 4FS [Anger, Febres Cordero, Ita, Sotnikov (2018)]
- ☑ Analytical **two-loop** W+4-parton **amplitudes** in LCA [Badger, Hartanto, Zoia (2021)] [Abreu, Febres Cordero, Ita, Klinkert, Page, Sotnikov (2021)]
- ☑ **NNLO QCD** corrections (*massless bottom quarks*) [Hartanto, Poncelet, Popescu, Zoia (2022)]

first NNLO calculation!

$Wb\bar{b}$ @NNLO: 4FS vs 5FS

If the bottom quarks are massless:

- care must be taken to ensure IR safety: the (usual) experimental definition of a flavoured jet is both soft and collinear unsafe
- **flavour-sensitive** jet **algorithms** must be employed [Banfi, Salam, Zanderighi (2006)] [Czakon, Mitov, Poncelet (2022)] [Caletti et al. (2022)] [Gauld, Huss, Stagnitto (2022)] [Caola et al. (2023)]

If the bottom quarks are treated as massive:

- the **mass** acts as the **physical IR regulator**: physical suppression in the double-soft and collinear limits
- any **standard flavour-blind** jet clustering **algorithm** can be used (in particular anti-k_T)
- direct comparison with experimental data is possible (unfolding corrections are limited to non-perturbative modelling and hadronisation)

CAVEATS:

- left over **logarithmic IR sensitivity** to the heavy-quark mass (at each perturbative order)
- calculations with massive quarks are challenging!

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two-loop 5-point amplitudes with more than one external massive leg have not yet been computed

but m_b is the smallest scale in the game, so we want to exploit this hierarchy!

Massification in a nutshell

[Moch, Mitov (2007)]

- massification relies on the **factorisation** properties of massless QCD amplitudes into a product of functions that organise the contributions of momentum regions relevant to the ϵ poles in the scattering amplitude [Sterman, Tejeda-Yeomans (2003)]

$$|\mathcal{M}_p\rangle = \mathcal{J}_0^{[p]}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \varepsilon\right) \mathcal{S}_0^{[p]}\left(\{k_i\}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \varepsilon\right) |\mathcal{H}_p\rangle$$

JET function: collinear contributions

SOFT function: coherent soft radiation

HARD function: short-distance dynamics

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- when the mass m is introduced, some of the **collinear singularities** are **screened** (*quasi-collinear* singularities)
- in the limit $m \ll Q$, the massive amplitude "shares" essential properties with the corresponding massless amplitude

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$$\mathcal{M}^{[\rm p],(m)}\left(\{k_i\}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \varepsilon\right) = \prod_{i \in \{\text{all legs}\}} \left(Z_{[i]}^{(m|0)}\left(\frac{m^2}{\mu^2}, \alpha_s(\mu^2), \varepsilon\right)\right)^{\frac{1}{2}} \times \mathcal{M}^{[\rm p],(m=0)}\left(\{k_i\}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \varepsilon\right)$$

up to $\mathcal{O}(m/Q)$

universal, perturbatively computable, ratio of massive and massless form factors

$$Z_{[i]}^{(m|0)}\left(\frac{m^2}{\mu^2}, \alpha_s, \varepsilon\right) = \mathcal{F}^{[i]}\left(\frac{Q^2}{\mu^2}, \frac{m^2}{\mu^2}, \alpha_s, \varepsilon\right)\left(\mathcal{F}^{[i]}\left(\frac{Q^2}{\mu^2}, 0, \alpha_s, \varepsilon\right)\right)^{-1}$$

see Vasily's talk: analogous procedure used in $Zb\bar{b}$ (4FS)

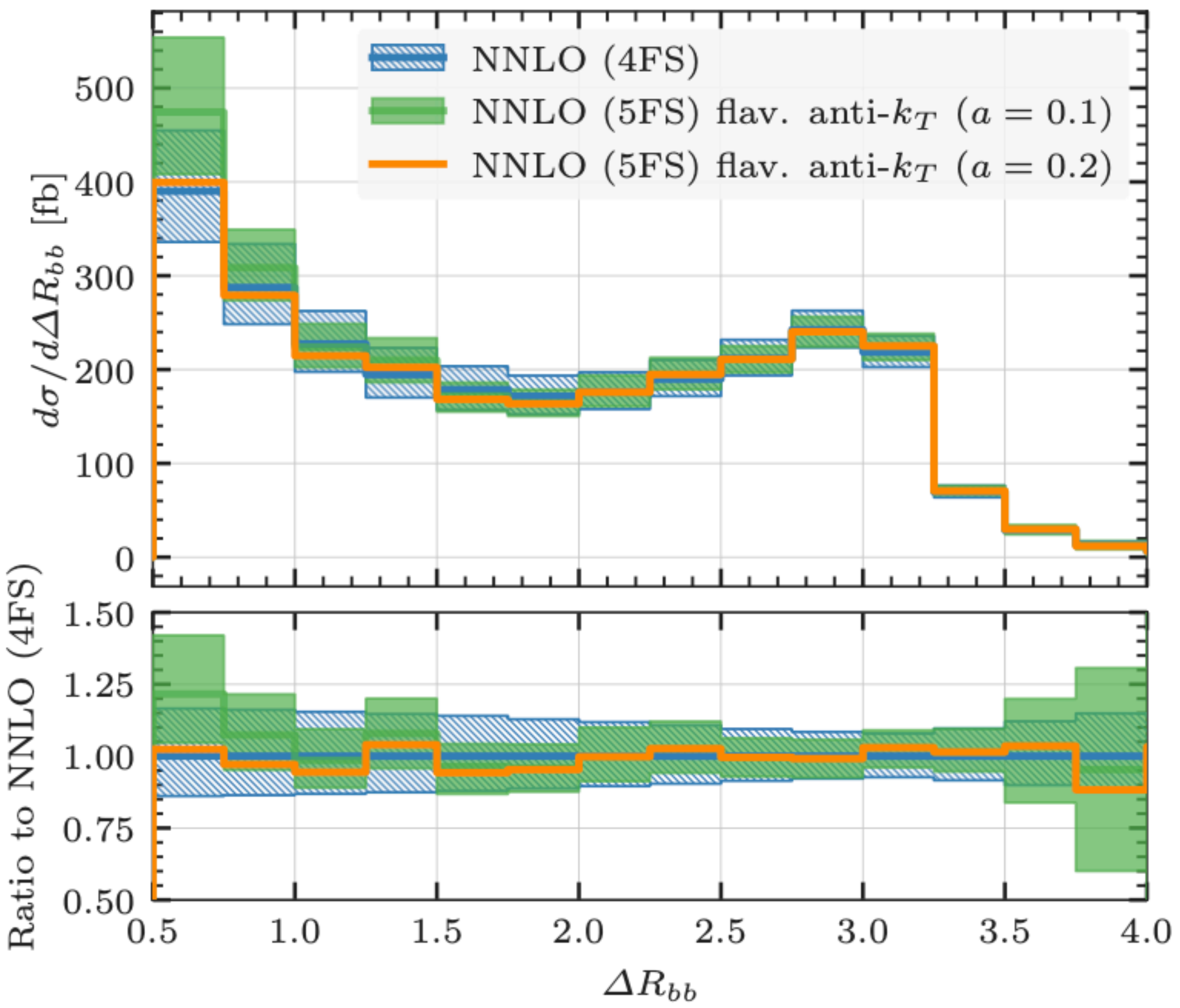
Results: comparison with 5FS

[CMS: arXiv 1608.07561]

setup: NNLO NNPDF31 4F, $\sqrt{s} = 8\,TeV$, $\mu_R = \mu_F = E_T(l\nu) + p_T(b_1) + p_T(b_2)$

$p_{T,l} > 30\,GeV \;\; |\eta_l| < 2.1$, $p_{T,j} > 25\,GeV \;\; |\eta_j| < 2.4$

$n_b = 2$ with $p_{T,b} > 25\,GeV \;\; |\eta_b| < 2.4$ (standard anti-k_T with $R = 0.5$)



order	σ^{4FS} [fb]	$\sigma^{5FS}_{a=0.05}$ [fb]	$\sigma^{5FS}_{a=0.1}$ [fb]	$\sigma^{5FS}_{a=0.2}$ [fb]
LO	$210.42(2)^{+21.4\%}_{-16.2\%}$	$262.52(10)^{+21.4\%}_{-16.1\%}$	$262.47(10)^{+21.4\%}_{-16.1\%}$	$261.71(10)^{+21.4\%}_{-16.1\%}$
NLO	$468.01(5)^{+17.8\%}_{-13.8\%}$	$500.9(8)^{+16.1\%}_{-12.8\%}$	$497.8(8)^{+16.0\%}_{-12.7\%}$	$486.3(8)^{+15.5\%}_{-12.5\%}$
NNLO	$652.8(1.6)^{+12.8\%}_{-11.0\%}$	$690(7)^{+10.9\%}_{-9.7\%}$	$677(7)^{+10.4\%}_{-9.4\%}$	$647(7)^{+9.5\%}_{-9.4\%}$

- comparison against the 5F massless computation [Poncelet et al. (2022)]
  - general **agreement within scale uncertainties** (with the massive calculation systematically lower)
  - good agreement for the largest value $a = 0.2$
  - the uncertainties due to variation of $m_b \in [4.2, 4.92]\,GeV$ are at **2%** level (smaller than the ones due to the variation of a, ~**7%**)

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[CMS: arXiv 1608.07561]

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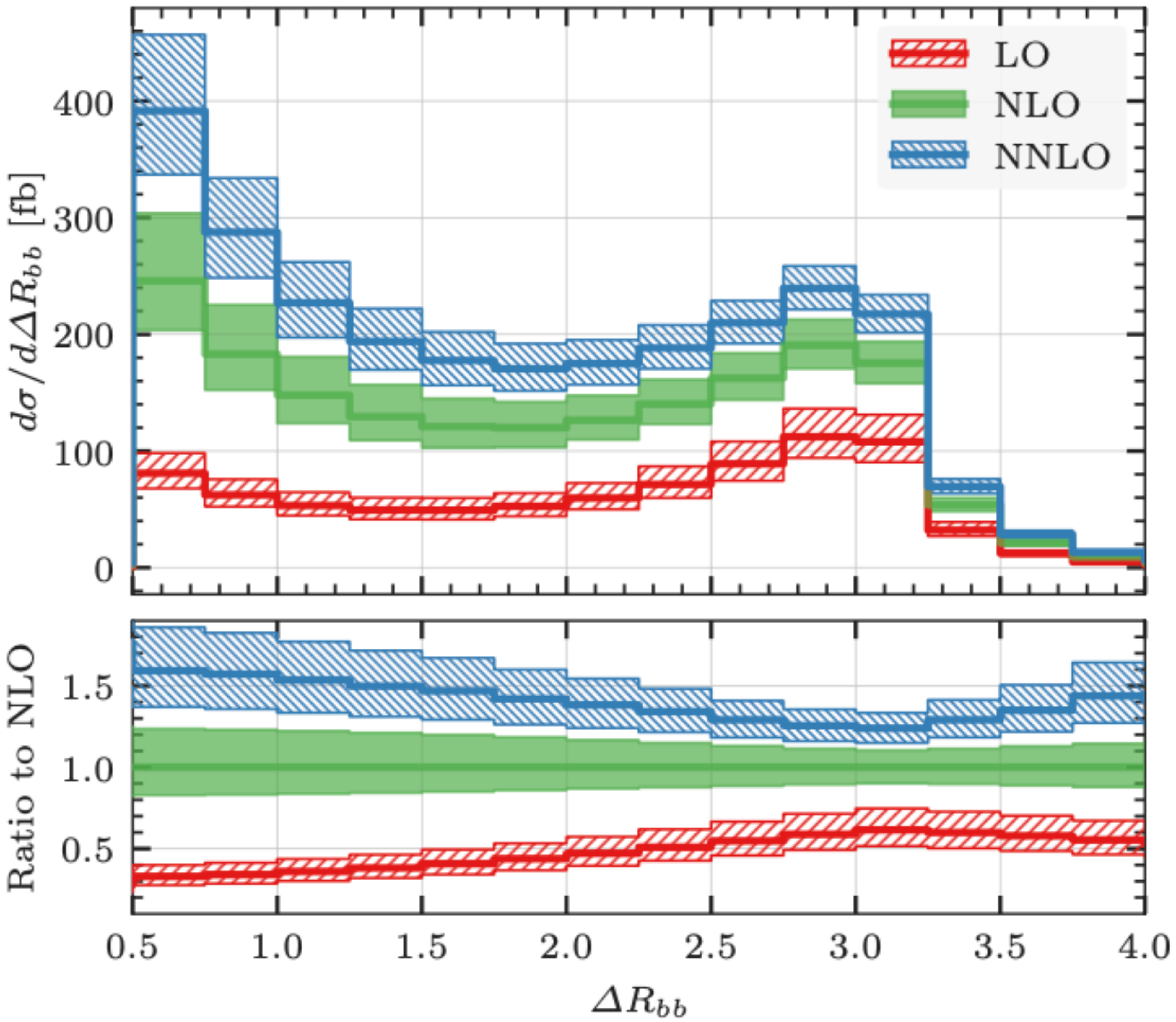

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- large positive NNLO corrections: **+40%**
- still **large** (but reduced) perturbative **uncertainties**
- other theoretical uncertainties are subdominant:
  - impact of the massification estimated at NLO $|1 - \Delta\sigma^{\mathrm{approx}}_{\mathrm{NLO}}/\Delta\sigma^{\mathrm{exact}}_{\mathrm{NLO}}| \approx 3\,\%$
  - the genuine part of the two-loop amplitudes in LCA amounts to $2\,\%$ of the total NNLO cross section

Soft approximation

$t\bar{t}H$

[Catani, Devoto, Grazzini, Kallweit, Mazzitelli, **CS** (2022)]

Why is $t\bar{t}H$ an interesting process?

- direct probe of the **top Yukawa coupling**
- the current **experimental accuracy** is $\mathcal{O}(20\%)$ but it is expected to go down to $\mathcal{O}(2\%)$ at the end of HL-LHC [CERN Yellow Report (2019)]
- the extraction of the $t\bar{t}H$ signal is limited by the theoretical uncertainties in the modelling of the backgrounds, mainly $t\bar{t}b\bar{b}$ and $t\bar{t}W + jets$
- current **theoretical predictions**: $\mathcal{O}(10\%)$ [LHC cross section WG (2016)]
- mandatory to include **NNLO QCD** corrections!

State of the art

- ☑ **NLO QCD** corrections (*on-shell top quarks*) [Beenakker, Dittmaier, Krämer, Plumper, Spira, Zerwas (2001,2003) [Reina, Dawson, Wackeroth, Jackson, Orr (2001,2003)]
- ☑ **NLO EW** corrections (*on-shell top quarks*) [Frixione, Hirschi, Pagani, Shao, Zaro (2015)]
- ☑ **NLO QCD** corrections (*leptonically decaying top quarks*) [Denner, Feger (2015)] [Stremmer, Malgorzata (2022)]
- ☑ **NLO QCD + EW** corrections (*off-shell top quarks*) [Denner, Lang, Pellen, Uccirati (2017)]
- ☑ current predictions based on: **NLO QCD + EW** corrections (*on-shell top quarks*), including **NNLL** soft-gluon resummation [Broggio et al.] [Kulesza et al.]
- ☑ **NNLO QCD** contributions for the **off-diagonal** partonic channels [Catani, Fabre, Grazzini, Kallweit (2021)]
- ☑ **complete NNLO QCD** predictions with approximated two-loop amplitudes [Catani, Devoto, Grazzini, Kallweit, Mazzitelli, **CS** (2022)]

first NNLO calculation!

use a SOFT-BOSON APPROXIMATION to estimate the order of magnitude of the double-virtual contribution

State of the art

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1. Two-loop amplitudes for ttH production: the quark-initiated Nf-part
Bakul Agarwal, Gudrun Heinrich, Stephen P. Jones, Matthias Kerner, Sven Yannick Klein, Jannis Lang, Vitaly Magerya, Anton Olsson

see Vitaly's talk

2. Two-loop QCD amplitudes for $t\bar{t}H$ production from boosted limit
Guoxing Wang, Tianya Xia, Li Lin Yang, Xiaoping Ye

6. One loop QCD corrections to $gg \to t\bar{t}H$ at $\mathcal{O}(\epsilon^2)$
Federico Buccioni, Philipp Alexander Kreer, Xiao Liu, Lorenzo Tancredi

HOT TOPIC !!

see Ben's talk

5. Two-Loop Master Integrals for Leading-Color $pp \to t\bar{t}H$ Amplitudes with a Light-Quark Loop
F. Febres Cordero, G. Figueiredo, M. Kraus, B. Page, L. Reina

Soft approximation in a nutshell

- **master formula** (at leading power) in the soft Higgs limit ($k \to 0$, $m_H \ll m_t$)

$$\lim_{k\to 0} \mathcal{M}_{t\bar{t}H}(\{p_i\}, k) = F(\alpha_s(\mu_R); m_t/\mu_R)\, J^{(0)}(k) \mathcal{M}_{t\bar{t}}(\{p_i\})$$

[Bärnreuther, Czakon, Fiedler (2013)]

soft limit of the scalar form factor for the heavy quark [Bernreuther et al. (2005)] [Blümlein et al. (2017)]

$$F(\alpha_s(\mu_R); m_t/\mu_R) = 1 + \frac{\alpha_s(\mu_R)}{2\pi}(-3C_F) + \left(\frac{\alpha_s(\mu_R)}{2\pi}\right)^2 \left(\frac{33}{4}C_F^2 - \frac{185}{12}C_F C_A + \frac{13}{6}C_F(n_L+1) - 6C_F\beta_0 \ln\frac{\mu_R^2}{m_t^2}\right) + \mathcal{O}(\alpha_s^3)$$

up to two-loop order

we assume that all heavy quarks involved in the process have the same mass

$$J^{(0)}(k) = \frac{m}{v}\sum_j \frac{m}{p_j \cdot k}$$

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why soft Higgs approximation

a careful assessment of the quality of the approximation is required

Results: systematic uncertainties

setup: NNLO NNPDF31, $m_H = 125GeV$, $m_t = 173.3GeV$, $\mu_R = \mu_F = (2m_t + m_H)/2$

	$\sqrt{s} = 13$ TeV		$\sqrt{s} = 100$ TeV	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
σ_{LO}	261.58	129.47	23055	2323.7
$\Delta\sigma_{\mathrm{NLO,H}}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\mathrm{NLO,H}}\vert_{\mathrm{soft}}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\mathrm{NNLO,H}}\vert_{\mathrm{soft}}$	−2.980(3)	2.622(0)	−239.4(4)	65.45(1)

- at NLO, difference of **5%** (**30%**) in $q\bar{q}$ (gg) channel
- at NNLO, the hard-virtual contribution is about **1%** of the LO cross section in gg and **2-3%** in $q\bar{q}$ small!
- our prescription to provide a conservative uncertainty is:
  - ☑ apply the approximation at a **different subtraction scale** (vary μ_{IR} by a factor 2 around Q); add the two-loop shift based on the exact tree-level and one-loop $t\bar{t}H$ amplitudes
  - ☑ take into account the NLO discrepancy and multiply it by a **tolerance factor 3**
  - ☑ combine **linearly** the gg and $q\bar{q}$ channels

Results: systematic uncertainties

setup: NNLO NNPDF31, $m_H = 125 GeV$, $m_t = 173.3 GeV$, $\mu_R = \mu_F = (2m_t + m_H)/2$

	$\sqrt{s} = 13$ TeV		$\sqrt{s} = 100$ TeV	
σ [fb]	gg	$q\bar{q}$	gg	$q\bar{q}$
$\sigma_{\rm LO}$	261.58	129.47	23055	2323.7
$\Delta\sigma_{\rm NLO,H}$	88.62	7.826	8205	217.0
$\Delta\sigma_{\rm NLO,H}\vert_{\rm soft}$	61.98	7.413	5612	206.0
$\Delta\sigma_{\rm NNLO,H}\vert_{\rm soft}$	−2.980(3)	2.622(0)	−239.4(4)	65.45(1)

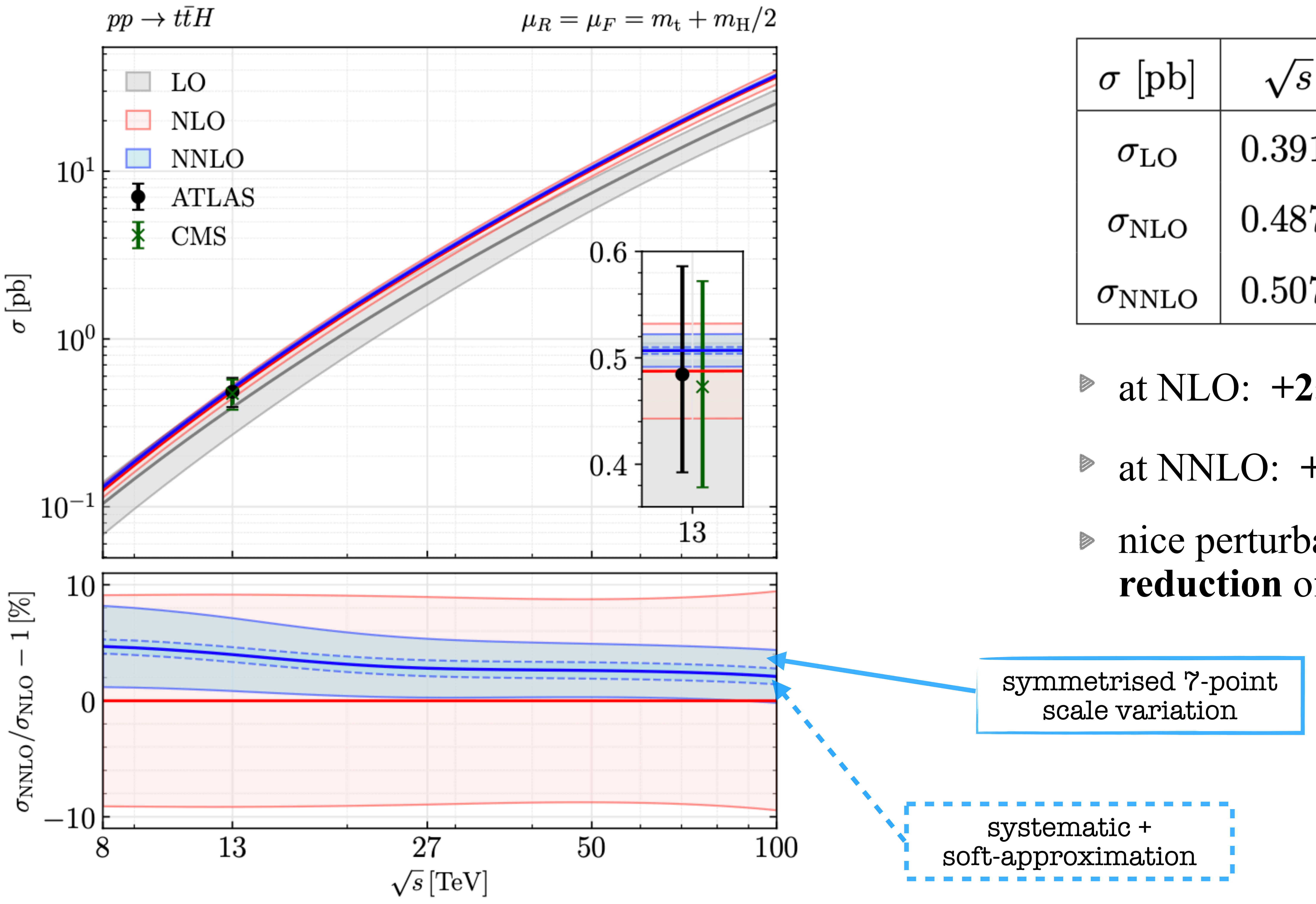
FINAL UNCERTAINTY:

$\pm 0.6\,\%$ on σ_{NNLO} , $\pm 15\,\%$ on $\Delta\sigma_{NNLO}$

- at NLO, difference of **5%** (**30%**) in $q\bar{q}$ (gg) channel
- at NNLO, the hard-virtual contribution is about **1%** of the LO cross section in gg and **2-3%** in $q\bar{q}$ small!
- our prescription to provide a conservative uncertainty is:
  - ☑ apply the approximation at a **different subtraction scale** (vary μ_{IR} by a factor 2 around Q); add the two-loop shift based on the exact tree-level and one-loop $t\bar{t}H$ amplitudes
  - ☑ take into account the NLO discrepancy and multiply it by a **tolerance factor 3**
  - ☑ combine **linearly** the gg and $q\bar{q}$ channels

Results: total XS

setup: NNLO NNPDF31, $m_H = 125 GeV$, $m_t = 173.3 GeV$, $\mu_R = \mu_F = (2m_t + m_H)/2$



σ [pb]	$\sqrt{s} = 13$ TeV	$\sqrt{s} = 100$ TeV
σ_{LO}	$0.3910^{+31.3\%}_{-22.2\%}$	$25.38^{+21.1\%}_{-16.0\%}$
σ_{NLO}	$0.4875^{+5.6\%}_{-9.1\%}$	$36.43^{+9.4\%}_{-8.7\%}$
σ_{NNLO}	$0.5070\,(31)^{+0.9\%}_{-3.0\%}$	$37.20(25)^{+0.1\%}_{-2.2\%}$

- at NLO: **+25 (+44)%** at $\sqrt{s} = 13\,(100)\,TeV$
- at NNLO: **+4 (+2)%** at $\sqrt{s} = 13\,(100)\,TeV$
- nice perturbative convergence with **significant reduction** of the theory uncertainties $\mathcal{O}(3\%)$

[ATLAS, Nature 607, 52 (2022)]
[CMS, Nature 607, 60 (2022)]

Soft approximation & massification

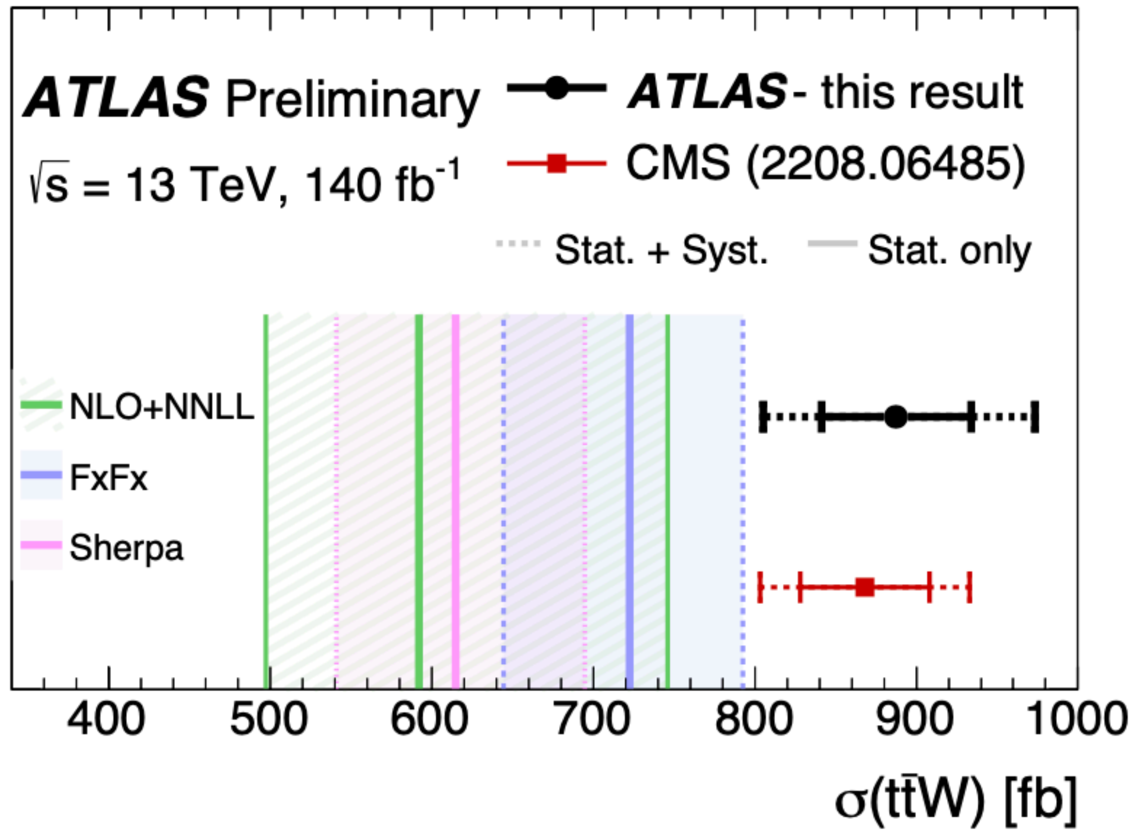
$t\bar{t}W$

[Buonocore, Devoto, Grazzini, Kallweit, Mazzitelli, Rottoli, **CS** (2023)]

Why is $t\bar{t}W$ an interesting process?

- relevant background for SM processes ($t\bar{t}H$, $t\bar{t}t\bar{t}$)
- multi-lepton signature relevant for BSM sources
- “**special**”: large NLO QCD and EW corrections
- well known **tension** between theory and experiments (excess at **1-2σ level**)
- current **NLO QCD + EW** predictions, supplemented with **multi-jet merging** are affected by relatively large uncertainties
- mandatory to include **NNLO QCD** corrections!

[CMS: arXiv 2208.06485]
[ATLAS-CONF-2023-019]



State of the art

- ☑ **NLO QCD** corrections (*on-shell top quarks*) [Badger, Campbell, Ellis (2010-2012)]
- ☑ **NLO QCD + EW** corrections (*on-shell top quarks and W*) [Frixione, Hirschi, Pagani, Shao, Zaro (2015)]
- ☑ inclusion of **soft gluon resummation at NNLL** [Broggio et al. (2016)] [Kulesza et al. (2019)]
- ☑ **NLO QCD** corrections (*full off-shell process, three charged lepton signature*) [Bevilacqua et al. (2020)] [Denner, Pellicciolі (2020)]
- ☑ combined **NLO QCD + EW** corrections (*full off-shell process, three charged lepton signature)* [Denner, Pellicciolі (2021)]
- ☑ current experimental measurements are compared with **NLO QCD + EW** (*on-shell)* predictions supplemented with **multi-jet merging** [Frederix, Tsinikos (2021)]
- ☑ **complete NNLO QCD + NLO EW** (*on-shell)* with approximated two-loop amplitudes

first NNLO calculation!

use both MASSIFICATION & SOFT-BOSON APPROXIMATION

Soft approximation & massification

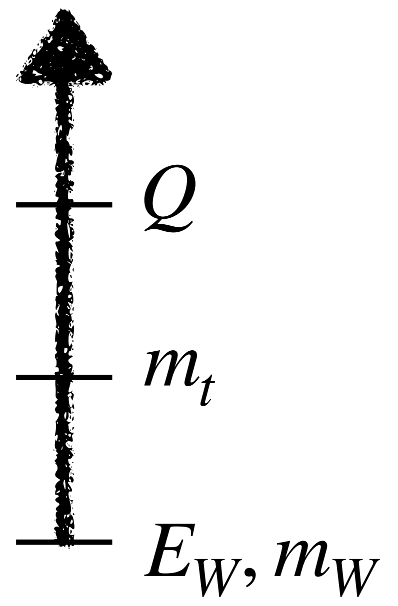
- good starting point: two rather different and complementary approximations of the exact two-loop virtual amplitudes
- **soft approximation**:
  - it works nicely in the case of $t\bar{t}H$, mainly due to the smallness of the approximated $H^{(2)}$ contribution [Catani et al. (2022)]
  - formally it is valid in the limit $E_W \to 0,\ m_W \ll m_t$ (which is not the case for a physical W boson …)

$$\mathcal{M}_{Wt\bar{t}}(\{p_i\}, p_W; \mu, \epsilon) \sim \frac{g_W}{\sqrt{2}} \left(\frac{p_2 \cdot \epsilon^*(p_W)}{p_2 \cdot p_W} - \frac{p_1 \cdot \epsilon^*(p_W)}{p_1 \cdot p_W} \right) \mathcal{M}^{L}_{t\bar{t}}(\{p_i\}; \mu, \epsilon) + \mathcal{O}(m_W/m_t, E_W/Q_{t\bar{t}})$$

[Bärnreuther, Czakon, Fiedler (2013)]
[Chen, Czakon, Poncelet (2017)]

Eikonal factor

reduced **polarised** $t\bar{t}$ amplitude



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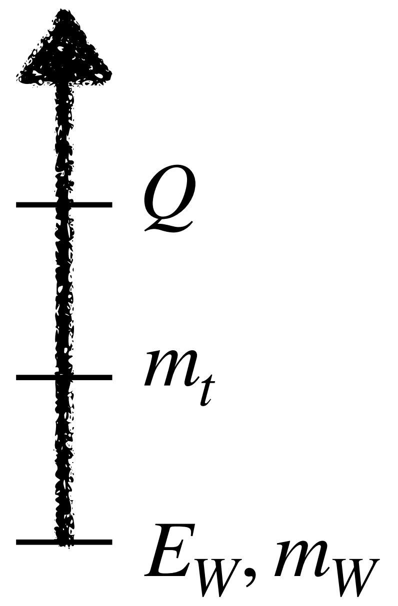
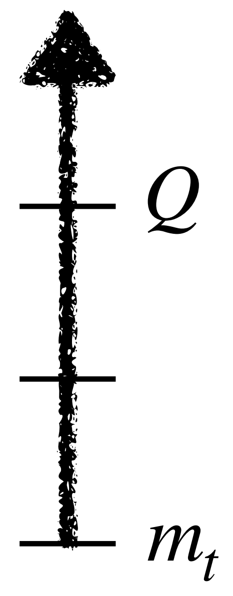


- **massification**: [Moch, Mitov (2007)]
  - it is fully justified in the case of $Wb\bar{b}$, due to the smallness of the bottom mass [Buonocore et al. (2022)]
  - formally it is valid in the limit $m_t \ll Q_{Wt\bar{t}}$ (which is not the case …)

$$\mathcal{M}_{Wt\bar{t}}(\{p_i\}, p_W; \mu, \epsilon) \sim Z^{(m_t|0)}_{[q]}(\alpha_s(\mu), m_t/\mu, \epsilon)\, \mathcal{M}^{(m_t=0)}_{Wt\bar{t}}(\{p_i\}, p_W; \mu, \epsilon) + \mathcal{O}(m_t^2/Q^2_{Wt\bar{t}})$$

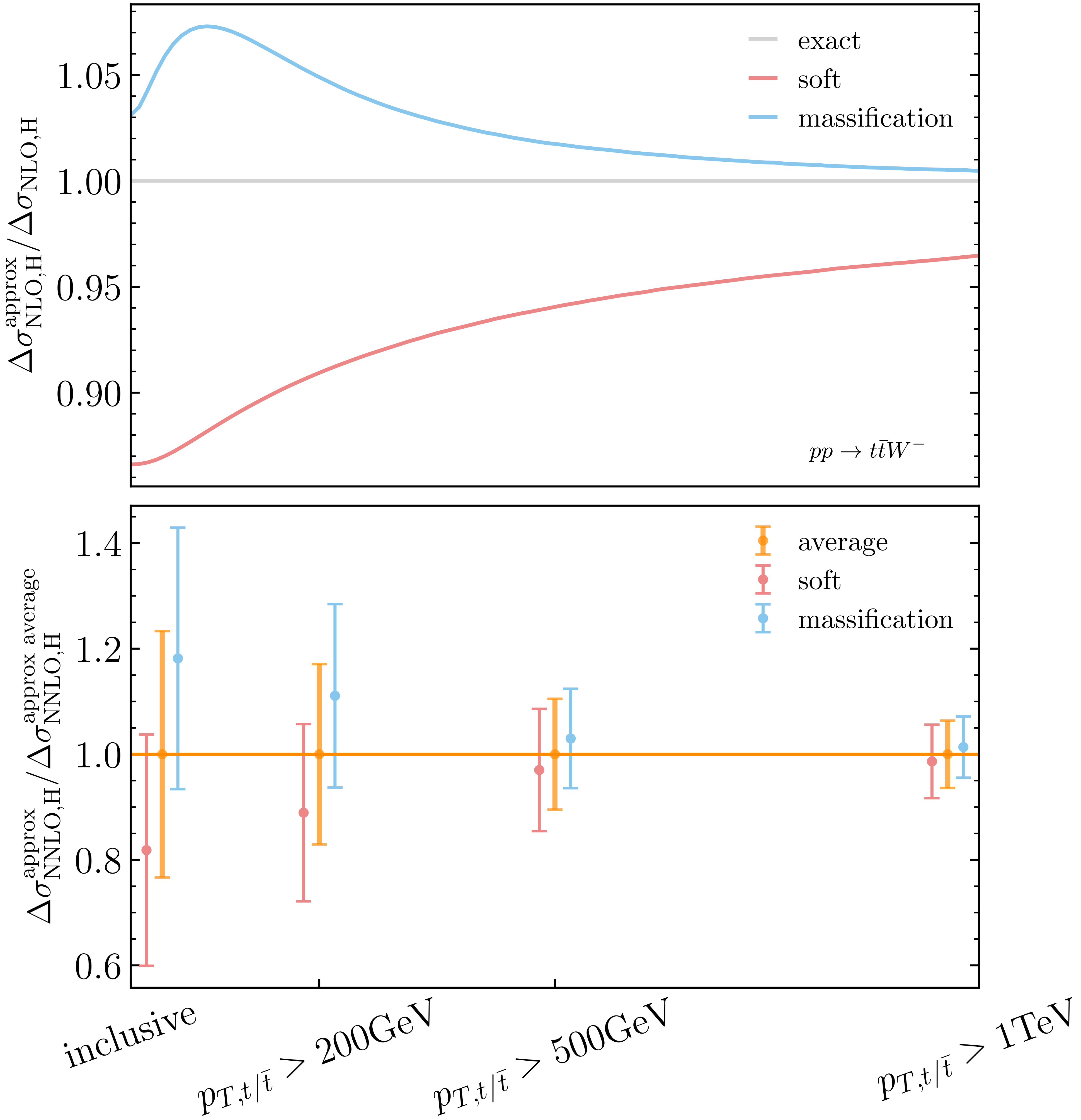
[Abreu at al. (2021)]
[Badger at al. (2021)]



DISCLAIMER:
none of the two approximations is (a priori) reasonable for the bulk of the events

Results: “best” prediction

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385 GeV$, $m_t = 173.2 GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$



- at NLO both approaches show a **remarkable good agreement** with the exact virtual coefficient (discrepancy within 15%)

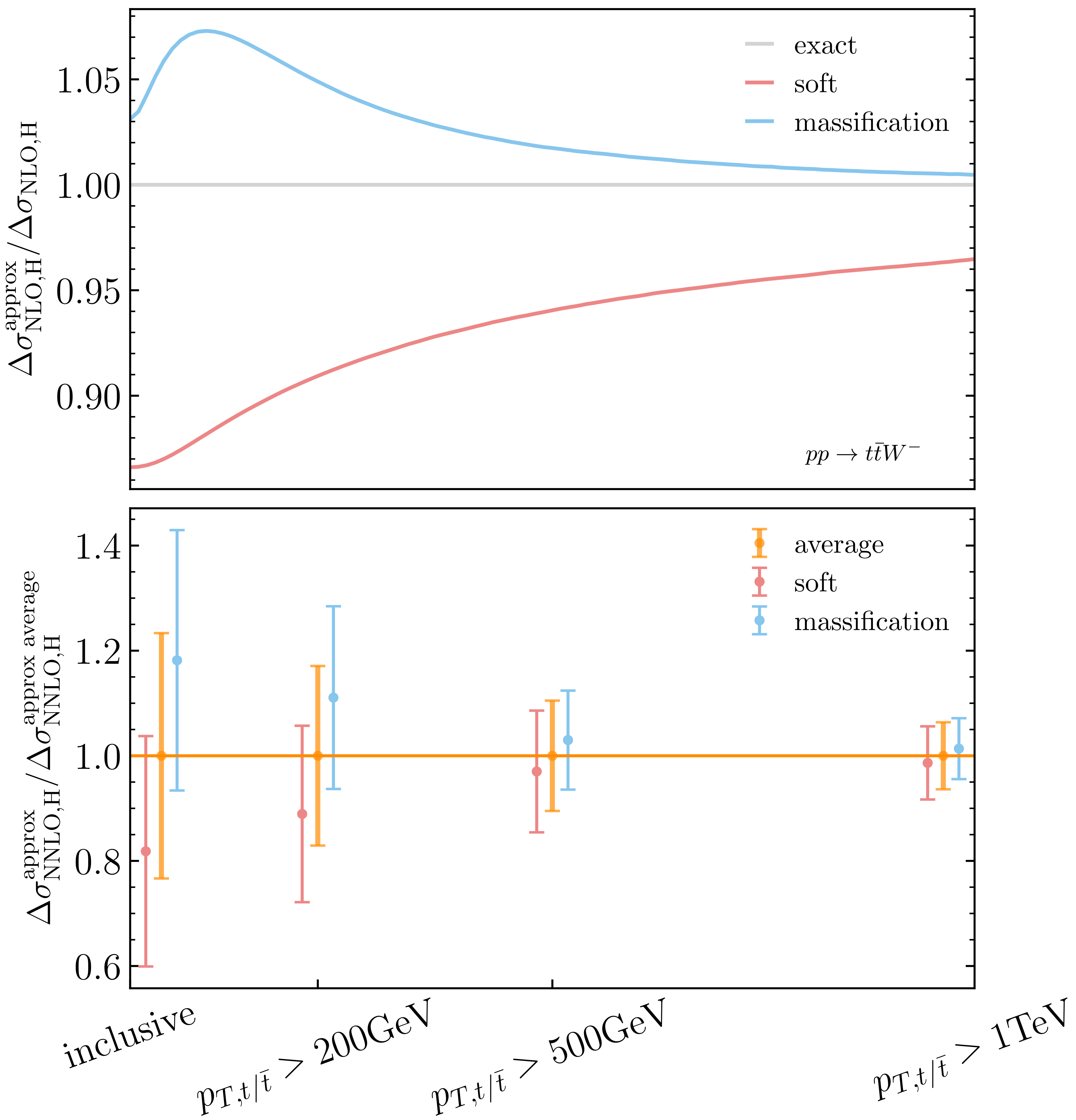
  agreement improved by the LO reweighting!

- at NNLO we define **our best prediction** as the **average** of the two approximated results
- the **conservative systematic uncertainty** on the approximated two-loop contribution is defined by linearly combining the uncertainties on the two approximations

  the uncertainty on each approximation is computed as the maximum between the NLO discrepancy and effects due to μ_{IR} scale variation

Results: "best" prediction

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385 GeV$, $m_t = 173.2 GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$



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- at NNLO we define **our best prediction** as the **average** of the two approximated results
- the **conservative systematic uncertainty** on the approximated two-loop contribution is defined by linearly combining the uncertainties on the two approximations
- the two-loop contribution turns out to be **6-7%** of the NNLO cross section

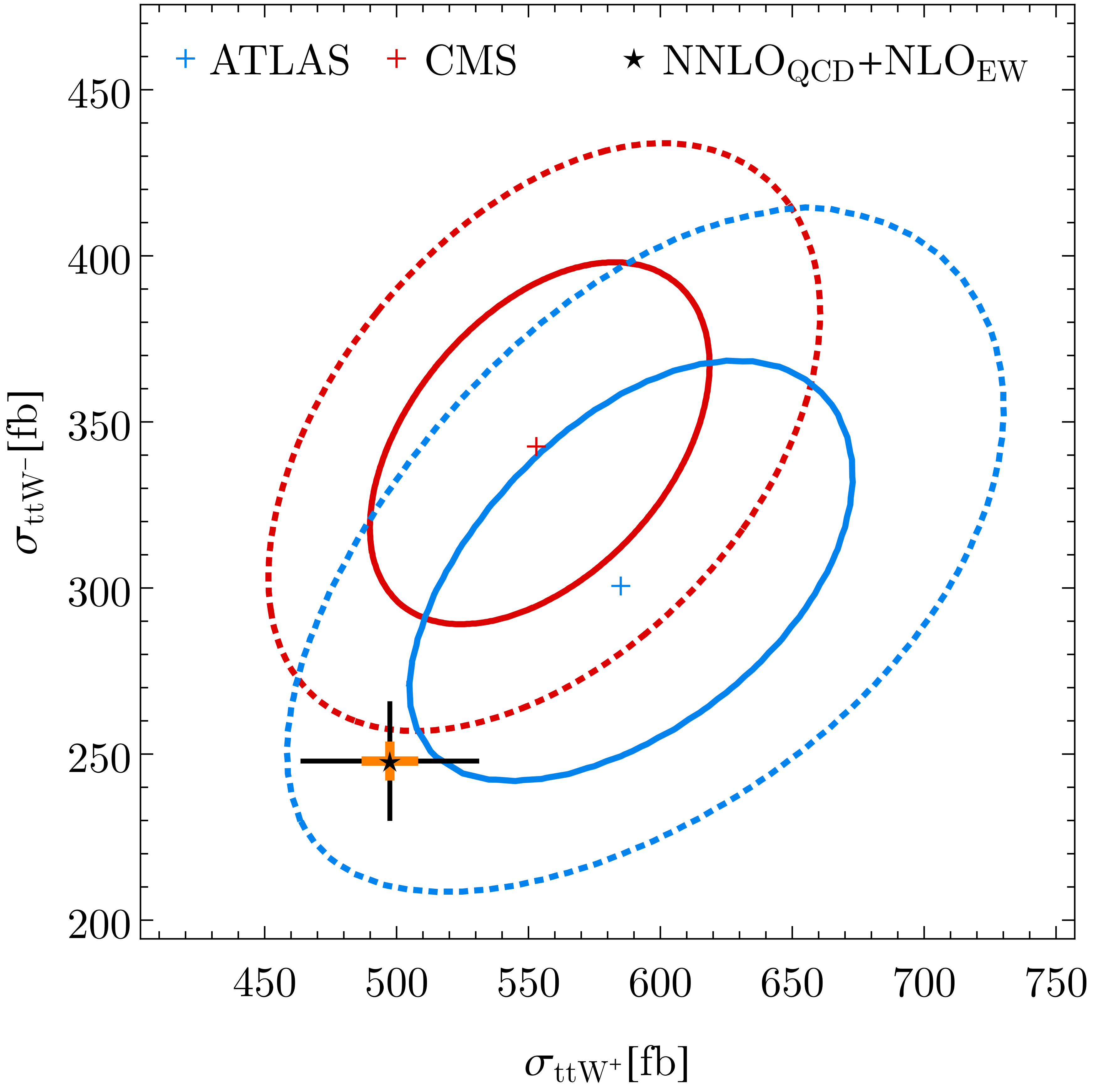
relatively sizeable!

FINAL UNCERTAINTY:

$\pm 1.8\,\%$ on σ_{NNLO}, $\mathcal{O}(25\%)$ on $\Delta\sigma_{NNLO,H}$

Results: comparison with data

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385 GeV$, $m_t = 173.2 GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$



	$\sigma_{t\bar{t}W^+}$ [fb]	$\sigma_{t\bar{t}W^-}$ [fb]	$\sigma_{t\bar{t}W}$ [fb]	$\sigma_{t\bar{t}W^+}/\sigma_{t\bar{t}W^-}$
LO$_{QCD}$	$283.4^{+25.3\%}_{-18.8\%}$	$136.8^{+25.2\%}_{-18.8\%}$	$420.2^{+25.3\%}_{-18.8\%}$	$2.071^{+3.2\%}_{-3.2\%}$
NLO$_{QCD}$	$416.9^{+12.5\%}_{-11.4\%}$	$205.1^{+13.2\%}_{-11.7\%}$	$622.0^{+12.7\%}_{-11.5\%}$	$2.033^{+3.0\%}_{-3.4\%}$
NNLO$_{QCD}$	$475.2^{+4.8\%}_{-6.4\%} \pm 1.9\%$	$235.5^{+5.1\%}_{-6.6\%} \pm 1.9\%$	$710.7^{+4.9\%}_{-6.5\%} \pm 1.9\%$	$2.018^{+1.6\%}_{-1.2\%}$
NNLO$_{QCD}$+NLO$_{EW}$	$497.5^{+6.6\%}_{-6.6\%} \pm 1.8\%$	$247.9^{+7.0\%}_{-7.0\%} \pm 1.8\%$	$745.3^{+6.7\%}_{-6.7\%} \pm 1.8\%$	$2.007^{+2.1\%}_{-2.1\%}$
ATLAS [11]	$585^{+6.0\%+8.0\%}_{-5.8\%-7.5\%}$	$301^{+9.3\%+11.6\%}_{-9.0\%-10.3\%}$	$890^{+5.6\%+7.9\%}_{-5.6\%-7.9\%}$	$1.95^{+10.8\%+8.2\%}_{-9.2\%\ -6.7\%}$
CMS [10]	$553^{+5.4\%+5.4\%}_{-5.4\%-5.4\%}$	$343^{+7.6\%+7.3\%}_{-7.6\%-7.3\%}$	$868^{+4.6\%+5.9\%}_{-4.6\%-5.9\%}$	$1.61^{+9.3\%+4.3\%}_{-9.3\%-3.1\%}$

- NNLO corrections lead to moderately higher rates (**+15%**)
- comparison against ATLAS and CMS data:
  - the **agreement stays** at the 1σ and 2σ level respectively
  - our result is **compatible** with FxFx: $\sigma^{\text{FxFx}}_{t\bar{t}W} = 722.4^{+9.7\%}_{-10.8\%}$ fb
  - reduction of the perturbative scale uncertainties

[ATLAS-CONF-2023-019]
[CMS: arXiv 2208.06485]

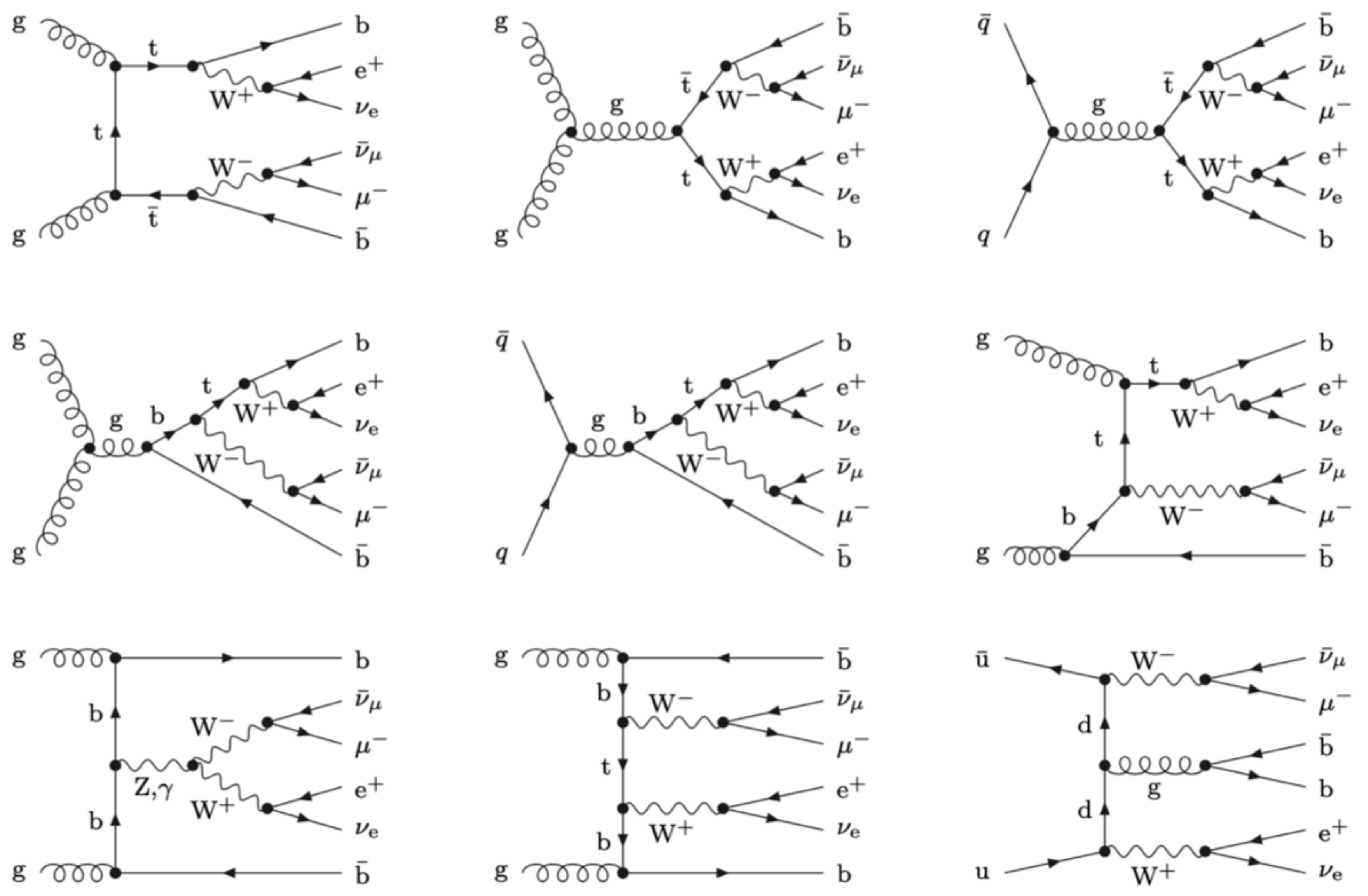
Beyond $2 \to 3$: off-shell $t\bar{t}$ production

NEW

- within the q_T -subtraction framework we can (in principle) deal with any process involving the production of a heavy-quark pair in association with a colourless system
- **ambitious goal**: provide NNLO QCD predictions for off-shell $t\bar{t}$ production, in the fully-leptonic decay $pp \to b\bar{b}e^+\nu_e\mu^-\bar{\nu}_\mu$ (4FS)
- this process represents a cornerstone of the physics programme at LHC
- NNLO QCD predictions in NWA are available (5FS) [Czakon et al. (2020)]
- off-shell contributions as well as single-top and non-resonant topologies could play a relevant role in the measurement of exclusive observables and in the extraction of m_t

can we deal with higher-multiplicity processes?

same QCD structure as $Q\bar{Q}V$ processes, with a much more involved phase-space ($2 \to 6$ at Born)



[Buonocore, Devoto, Grazzini, Kallweit, Lindert, Mazzitelli, **CS** (work in progress)]

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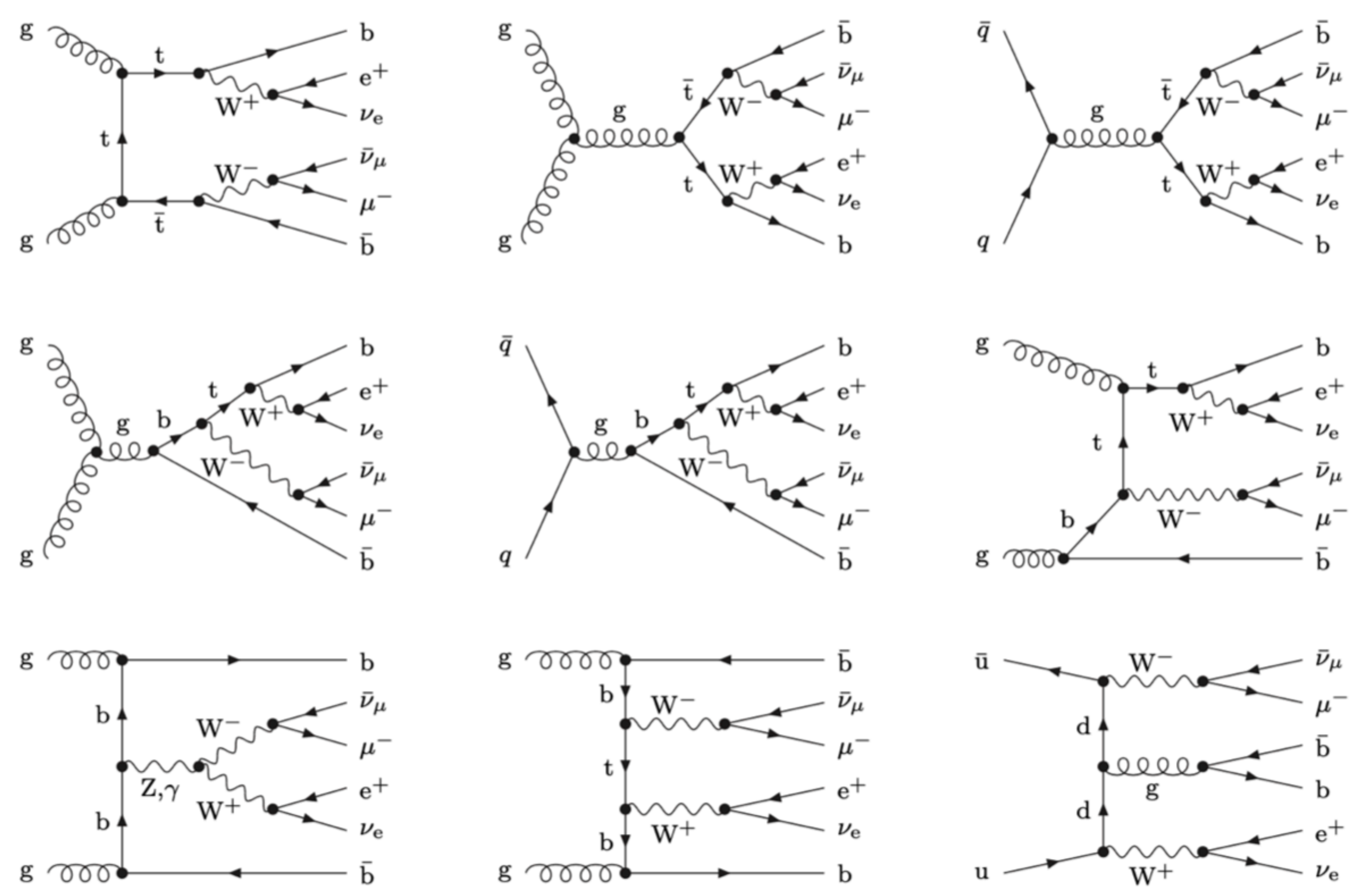


main bottleneck: two-loop QCD amplitudes for $2 \to 4$ with internal and external masses

idea: apply the double-pole approximation (DPA), only at the level of the virtual contribution

first approach to 6-point massless Feynman integrals in [Henn at al. (2024)]

[Buonocore, Devoto, Grazzini, Kallweit, Lindert, Mazzitelli, **CS** (work in progress)]

Beyond $2 \to 3$: off-shell $t\bar{t}$ production

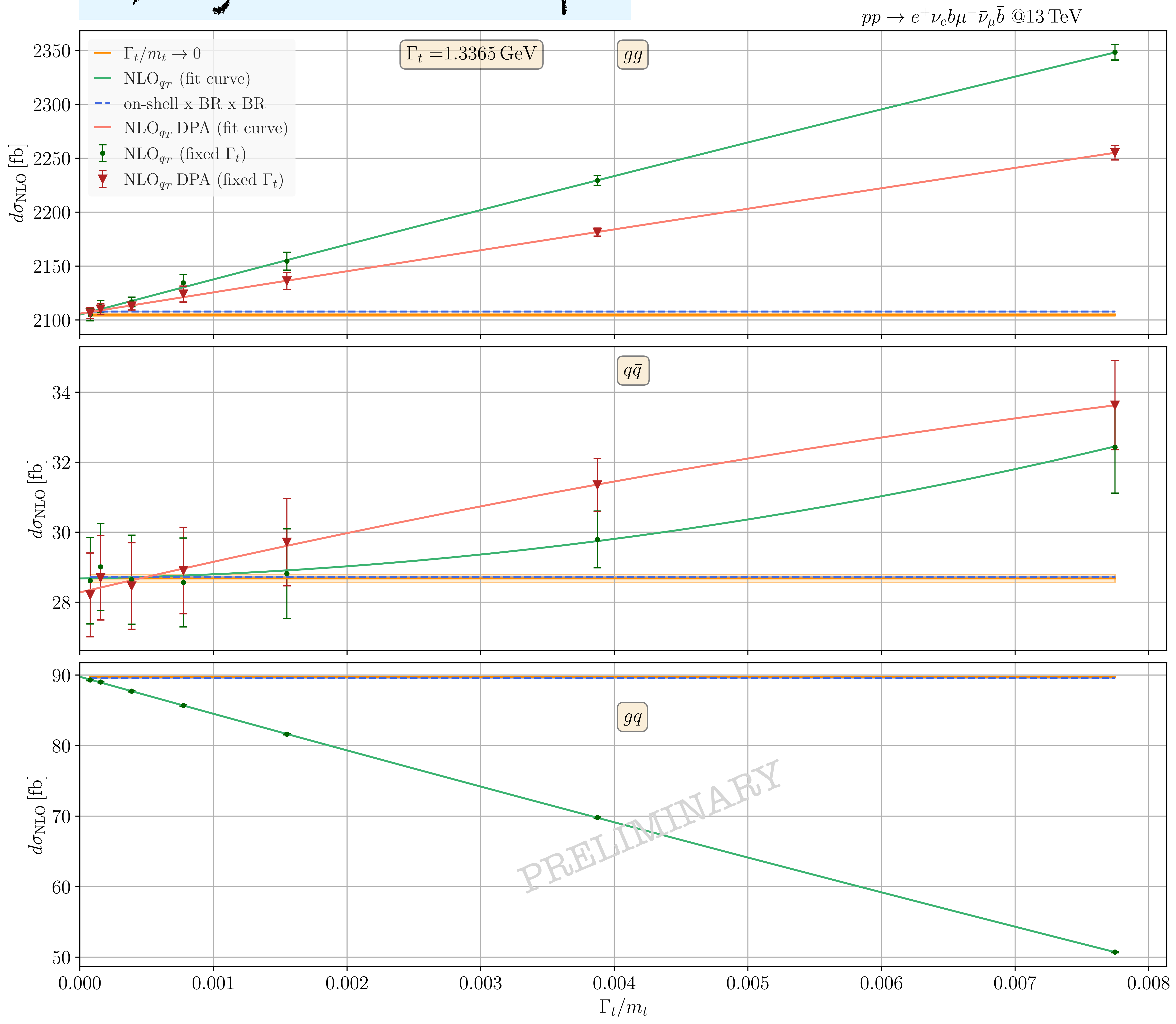
@NLO

- all ingredients are available
- **non-trivial proof** of:
  - the validity of our subtraction method
  - the numerical stability of MATRIX framework
- **full validation** of our results against the on-shell $t\bar{t}$ cross section, in the limit $\Gamma_t \to 0$
- results in DPA are in **very good agreement** with the exact ones ($\mathcal{O}(4\%)$ difference on $d\sigma_{NLO}$ for physical Γ_t)

both factorisable and non-factorisable corrections included!

- ready to produce differential distributions (also in the case of fiducial setups)

fully inclusive setup



[Buonocore, Devoto, Grazzini, Kallweit, Lindert, Mazzitelli, **CS** (work in progress)]

Beyond $2 \to 3$: off-shell $t\bar{t}$ production

@NNLO

▶ **off-diagonal** channels:

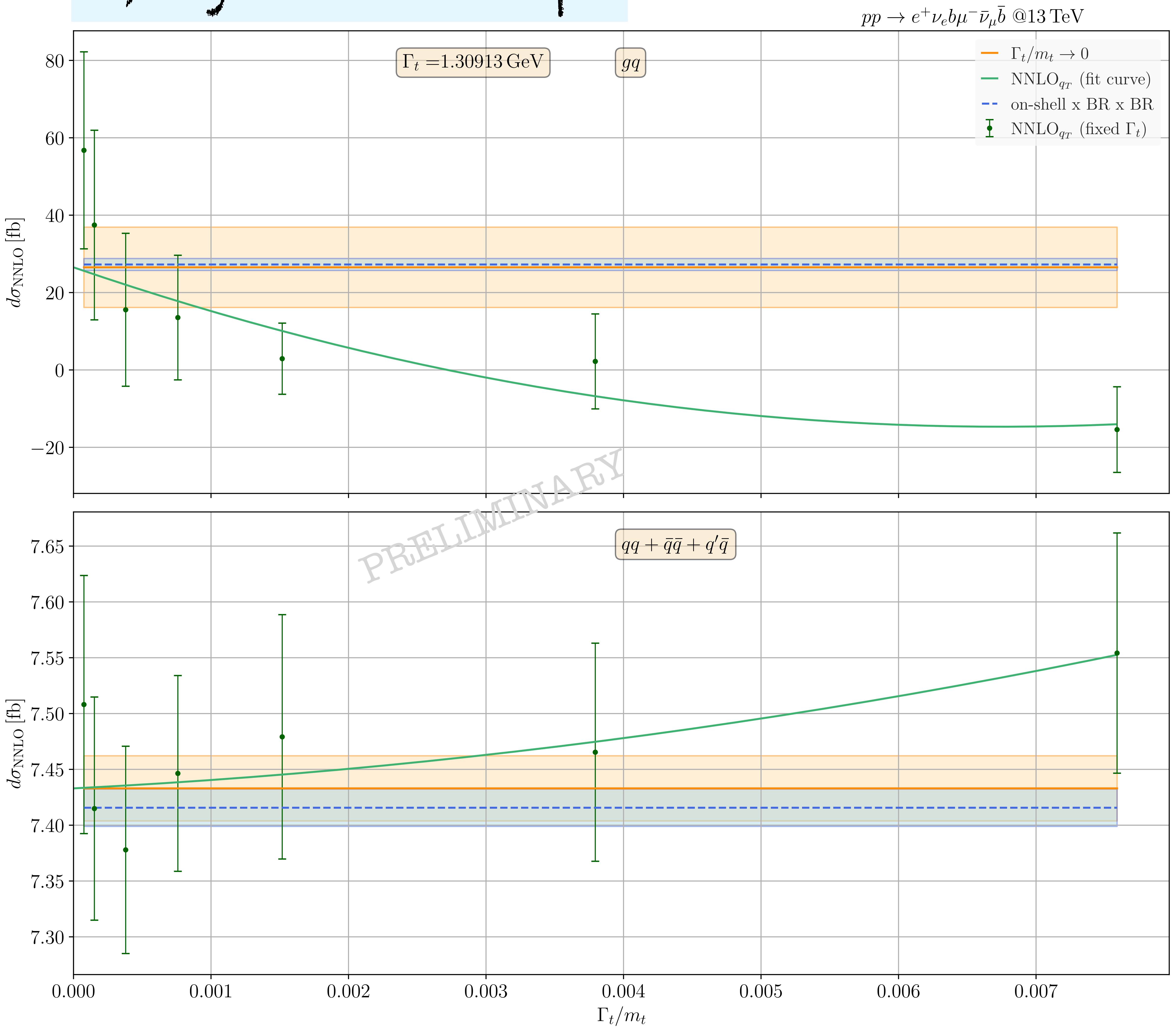
- all ingredients are available
- full validation — solved numerical issues related to the integration of the double-real contribution over a 8-particle phase space!
- non-trivial double extrapolation ($q_T^{\mathrm{cut}} \to 0$ and $\Gamma_t \to 0$)

▶ **diagonal** channels:

- cancellation of $\log(q_T^{\mathrm{cut}})$
- missing two-loop amplitudes — non-factorisable corrections not available
- not ready to present results…

slow but constant progress

fully inclusive setup



[Buonocore, Devoto, Grazzini, Kallweit, Lindert, Mazzitelli, **CS** (work in progress)]

Summary

- the expected precision of LHC data requires **NNLO QCD predictions** (crucial for precise phenomenology)
- the **current frontier** is represented by $2 \to 3$ processes with several massive external legs
- thanks to the progress in the q_T **-subtraction scheme** and QCD **5-point scattering amplitudes**,

  we were able to complete the first NNLO QCD calculation for $Wb\bar{b}$ (in 4FS), $t\bar{t}H$ and $t\bar{t}W$
- the approximation of the missing two-loop amplitudes is essentially based on two **factorisation approaches**:

  SOFT-BOSON APPROXIMATION **MASSIFICATION**
- we have achieved a **good control** of the systematic uncertainties and a **reduction** of the perturbative uncertainties
- we produced **phenomenological** results for the LHC:
  - $Wb\bar{b}$: large NNLO corrections ($+40\,\%$), more direct comparison with data (fewer ambiguities related to flavour tagging)
  - $t\bar{t}H$: moderate NNLO corrections ($+4\,\%$), small quantitative impact of the genuine double-virtual contribution
  - $t\bar{t}W$: the inclusion of NNLO QCD + NLO EW corrections cannot "solve" the tension with the data ($\sim 1\sigma$ - 2σ)
- beyond $2 \to 3$ processes: off-shell $t\bar{t}$ production
  - full validation at NLO and for the off-diagonal channels at NNLO

Outlook

- $t\bar{t}H$:
  - include **NLO EW** corrections as well as effects from soft-gluon resummation (matching between fixed-order and threshold resummation in progress within HWG)
  - explore **other approximations** (e.g. massification) for the double-virtual contribution
  - provide reliable predictions for **differential distributions** from low to high energies

- $Wb\bar{b}$:
  - possible matching with parton showers in order to reach **NNLO+PS** accuracy
  - combination of the 4FS and 5FS results à la FONLL (?)

- $b\bar{b}4l$:
  - complete the construction of DPA at NNLO for the **diagonal channels**

THANK YOU!! stay tuned!!

BACKUP SLIDES

IR safety and flavour tagging

Jet clustering algorithms consist in a sequence of **two-to-one recombination** steps. They are completely defined once the binary distance d_{ij} and the beam distance d_{iB} are given. For the family of k_T algorithms:

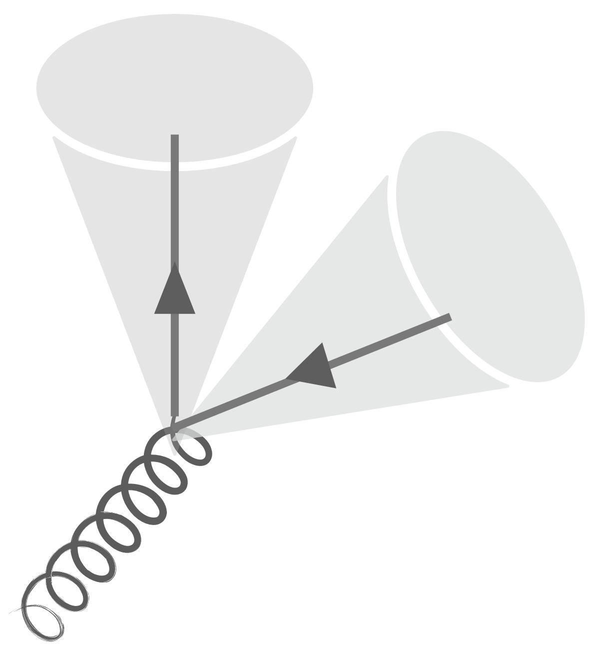
$$d_{ij} = \min(k_{T,i}^{2\alpha}, k_{T,j}^{2\alpha})\, R_{ij}^2 \qquad \text{with} \qquad R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2$$

$$d_{iB} = k_{T,i}^{2\alpha}$$

A crucial requirement (for the parton-level calculations) is **infrared (IR) safety**.

For **observables sensitive to the flavour assignment,** IR safety can be an issue:

- an obvious approach to defining the jet flavour would be to start from an existing jet algorithm and then define the net flavour content of each jet as the total number of quarks minus anti-quarks per each flavour
- this procedure leads to problems starting from relative order α_s^2. The problematic configurations are those in which a soft gluon splits into a widely separated $q\bar{q}$ pair.



Flavour aware jet algorithm: flavour k_T

[Banfi, Salam, Zanderighi (2006)]

first solution: modify the k_T-distance by taking into account that the matrix element is not divergent in the soft q limit

Remark: a **distance measure** should satisfy two main characteristics:

1. two particles should be considered close ($d_{ij} \to 0$) when there is a corresponding divergence in the matrix element
2. the measure should not introduce "spurious" extra closeness for a variation of the momenta that does not lead to any extra divergence in the matrix element

$$d_{ij}^{(F)} = (\Delta\eta_{ij}^2 + \Delta\phi_{ij}^2) \times \begin{cases} \max(k_{ti}^2, k_{tj}^2)\,, & \text{softer of } i,j \text{ is flavoured,} \\ \min(k_{ti}^2, k_{tj}^2)\,, & \text{softer of } i,j \text{ is flavourless,} \end{cases}$$

also the **beam distance** can be problematic, so it is modified as well:

$$d_{iB}^{(F)} = \begin{cases} \max(k_{ti}^2, k_{tB}^2)\,, & i \text{ is flavoured,} \\ \min(k_{ti}^2, k_{tB}^2)\,, & i \text{ is flavourless.} \end{cases}$$

with $k_{tB}(\eta) = \sum_i k_{ti}\left(\Theta(\eta_i - \eta) + \Theta(\eta - \eta_i)e^{\eta_i - \eta}\right)$ and $k_{t\bar{B}}(\eta) = \sum_i k_{ti}\left(\Theta(\eta - \eta_i) + \Theta(\eta_i - \eta)e^{\eta - \eta_i}\right)$

this algorithm prevents the unwanted soft-hard recombination if the softer pseudo-jet is flavoured while it still leads to soft-soft recombination

Flavour aware jet algorithm: flavour anti-k_T

[Czakon, Mitov, Poncelet (2022)]

second solution: modify the anti k_T-distance (with a *damping function*) without touching the particle-beam distance

Remark: in the wide-angle double-soft limit ($E_i, E_j \to 0$) with i and j of opposite-sign flavour :

1. $d_{ij} \to 0$ for arbitrary R_{ij}
2. $d_{ij} \to 0$ faster than the distance of either i or j to the remaining pseudo-jets

$$d_{ij}^{(F)} \equiv d_{ij} \times \begin{cases} \mathcal{S}_{ij}\,, & \text{if both } i \text{ and } j \text{ have non-zero flavour of opposite sign,} \\ 1\,, & \text{otherwise.} \end{cases}$$

with $\mathcal{S}_{ij} \equiv 1 - \theta\left(1-\kappa_{ij}\right)\cos\left(\frac{\pi}{2}\kappa_{ij}\right)$ with $\kappa_{ij} \equiv \frac{1}{a}\frac{k_{T,i}^2 + k_{T,j}^2}{2k_{T,\max}^2}$

$\mathcal{S}_{ij} \sim E^4 \;\; (E \to 0)$ where E is the energy of the harder quark in the soft wide-angle pair

technical parameter that controls the damping. In the limit $a \to 0$, standard anti-k_T is recovered

no need to modify the **beam distance**

this algorithm is not IR-safe (from the point of view of flavour tagging) beyond NNLO

Flavour aware jet algorithm: new ideas

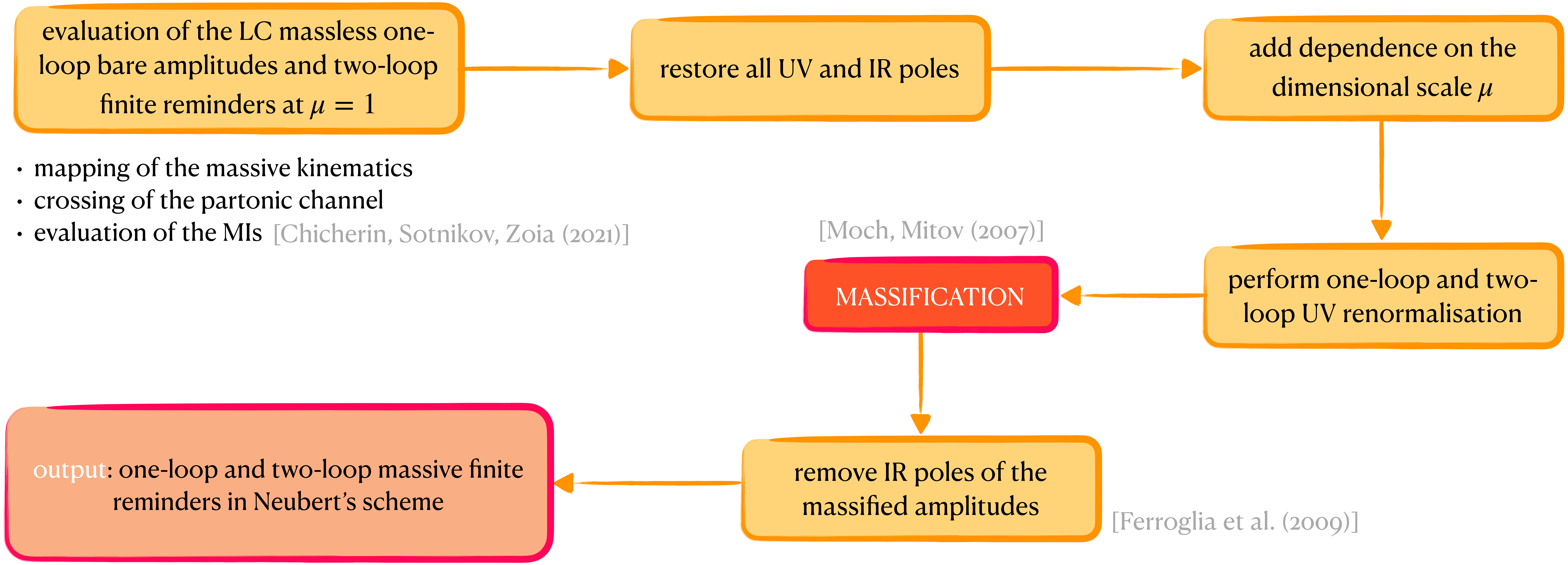
new recent solutions:

- **flavour dressing** [Gauld, Huss, Stagnitto (2022)]
  - the inputs to the flavour-dressing algorithm are a set of *flavour-agnostic jets* and a set of flavoured particles (*clusters*)
  - necessary to specify an *association criterion* and an *accumulation* one
  - IR safety guaranteed at any perturbative order
- **fragmentation approach** [Caletti et al. (2022)]
  - the idea is to use a jet clustering algorithm based on *Winner-Take-All* (WTA) recombination scheme
  - and define the flavour of a jet as the net flavour along the WTA axis (soft-safe but collinear-unsafe)
  - collinear divergences are reabsorbed into a perturbative WTA *fragmentation function*
- **interleaved flavour neutralisation** [Caola et al. (2023)]
  - similar to “flavour dressing”, the exact anti-k_T kinematics is preserved
  - but flavour information is integrated at each step of the clustering procedure (using IFN)
  - IR safety guaranteed at any perturbative order and useful to study jet substructures

WQQAmp: a massive C++ implementation

- **idea:** exploit the recently computed leading-colour **massless two-loop 5-point amplitudes** for W+4 partons [Abreu at al. (2022)]

WORKFLOW in a nutshell



evaluation time of $\mathcal{O}(4s)$ per phase space point

VALIDATION and CHECKS:

- stability of the two-loop massless amplitudes
- one-loop amplitudes in LCA tested against MCFM
- cancellation of the massified poles in LCA

Wbb: boosted setup

to take into account the multi-scale nature of the problem

[ATLAS: arXiv 2007.02873]

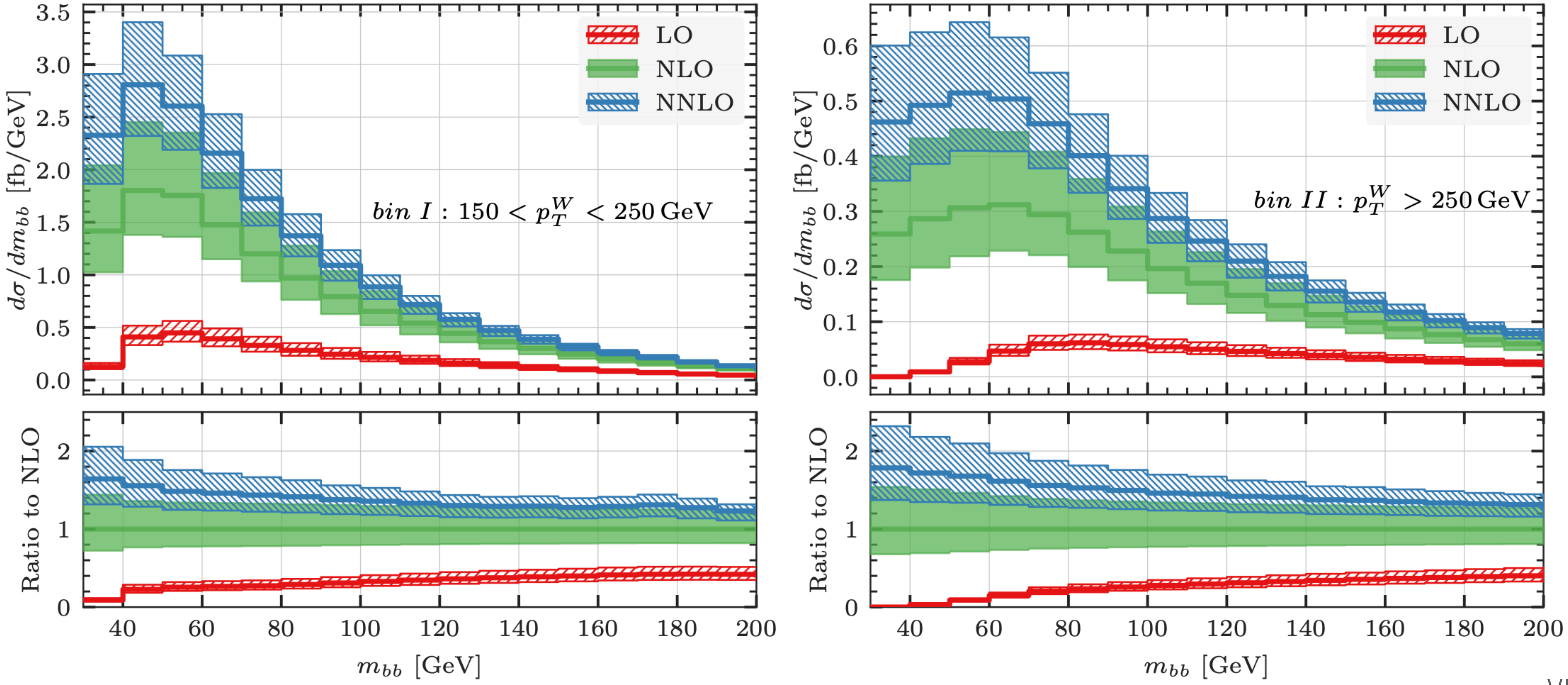
$p_{T,W} > 150\,GeV$

setup: NNLO NNPDF31 4F , $\sqrt{s} = 13.6\,TeV$, $\mu_R = \mu_F = H_T \cdot m_{bb}$

$p_{T,l} > 30\,GeV$ $|\eta_l| < 2.5$, $p_{T,j} > 20\,GeV$ $|\eta_j| < 2.5$ or $p_{T,j} > 30\,GeV$ $2.5 < |\eta_j| < 4.5$

$n_b = 2$ with $p_{T,b_1} > 45\,GeV$ $0.5 < \Delta R_{bb} < 2$ (standard anti-k_T with $R = 0.4$)

- **similar pattern** of NNLO corrections for both considered $p_{T,W}$ bins
- NNLO corrections are **not uniform** all over the m_{bb} spectrum (larger for smaller m_{bb} values)
- unreliable LO scale uncertainties
- **reduction** of the perturbative uncertainties at NNLO and **partial overlap** with the NNLO bands
- broader peak in *bin II*



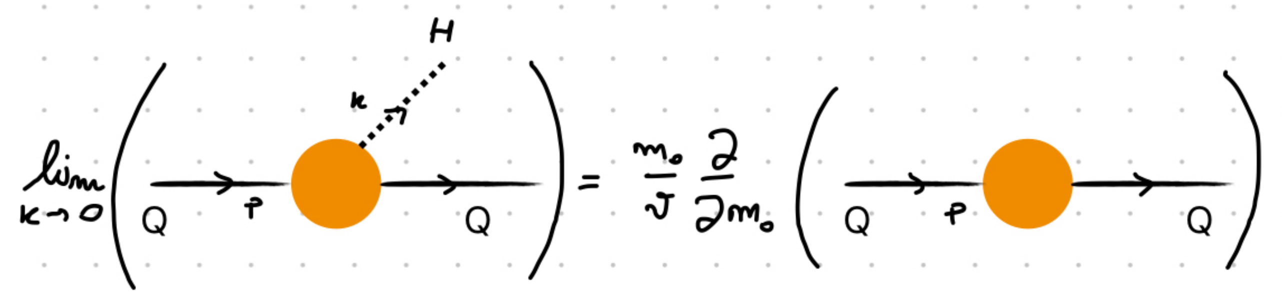
Soft Higgs approximation: more details

[Shifman, Vainshtein, Voloshin, Zakharov (1979)]

- the effective coupling can also be derived by exploiting Higgs **low-energy theorems** (LETs)

[Kniehl, Spira (1995)]

$$\lim_{k\to 0} \mathcal{M}^{bare}_{Q\to QH}(p,k) = \frac{1}{v}\frac{\partial}{\partial \log m_0} \mathcal{M}^{bare}_{Q\to Q}(p)\Big|_{p^2=m^2}$$



heavy-quark self-energy

In the soft limit, the Higgs boson is not a dynamical d.o.f.

Its effect is to shift the mass of the heavy quark:

$$m_0 \to m_0\left(1+\frac{H}{v}\right)$$

$$\mathcal{M}^{bare}_{Q\to Q}(p) = \bar{Q}_0\left\{m_0[-1+\Sigma_S(p)] + \not{p}\,\Sigma_V(p)\right\} Q_0$$

[Broadhurst, Grafe, Gray, Schilcher (1990)]
[Broadhurst, Gray, Schilcher (1991)]

$$\Sigma_S(p) = -\sum_{n=1}^{+\infty}\left[\frac{g_0^2}{(4\pi)^{D/2}(p^2)^{\epsilon}}\right]^n \left(A_n(m_0^2/p^2) - B_n(m_0^2/p^2)\right)$$

$$\Sigma_V(p) = -\sum_{n=1}^{+\infty}\left[\frac{g_0^2}{(4\pi)^{D/2}(p^2)^{\epsilon}}\right]^n B_n(m_0^2/p^2)$$

- renormalisation of the quark mass and wave function $m_0\bar{Q}_0 Q_0 = m\bar{Q}QZ_mZ_2$
- $\overline{MS}$ renormalisation of the strong coupling + decoupling of the heavy quark

[Chetyrkin, Kniehl, Steinhauser (1997)]

ttH: subtraction scale variation

- in order to test our prescription, we **vary the subtraction scale** μ at which we apply the soft factorisation formula
- the **renormalisation scale** μ_R is kept **fixed** at $Q_{t\bar{t}H}$ in the $t\bar{t}H$ amplitudes and at $Q_{t\bar{t}}$ in the $t\bar{t}$ ones
- the running terms are added exactly

gg channel @13TeV

approximation		$\sigma^{\text{VT only H1}}_{\text{NLO QCD}}$ [fb]		
		$\mu = Q/2$	$\mu = Q$	$\mu = 2Q$
exact		123.12 ± 0.04	88.61 ± 0.02	4.568 ± 0.013
		$\mu = Q_{proj}/2$	$\mu = Q_{proj}$	$\mu = 2Q_{proj}$
	$Q_{t\bar{t}}$	100.73 ± 0.03	61.98 ± 0.02	-26.308 ± 0.015
		$\mu = Q_{proj}/2 + (Q/2 \to Q)$	$\mu = Q_{proj} + (Q \to Q)$	$\mu = 2Q_{proj} + (2Q \to Q)$
	$Q_{t\bar{t}}$	66.24 ± 0.04	61.98 ± 0.02	57.76 ± 0.03

approximation		$\sigma^{\text{VT2 only H2 M2M0}}_{\text{NNLO QCD}}$ [fb]		
		$\mu = Q_{proj}/2$	$\mu = Q_{proj}$	$\mu = 2Q_{proj}$
	$Q_{t\bar{t}}$	13.114 ± 0.007	-2.977 ± 0.002	-29.03 ± 0.02
		$\mu = Q_{proj}/2 + (Q/2 \to Q)$	$\mu = Q_{proj} + (Q \to Q)$	$\mu = 2Q_{proj} + (2Q \to Q)$
	$Q_{t\bar{t}}$	1.882 ± 0.005	-2.977 ± 0.002	-3.715 ± 0.005
$\mathbf{F_2(Q)}$		$\mu = Q_{proj}/2 + (Q/2 \to Q)$	$\mu = Q_{proj} + (Q \to Q)$	$\mu = 2Q_{proj} + (2Q \to Q)$
	$Q_{t\bar{t}}$	0.378 ± 0.005	-4.487 ± 0.003	-5.222 ± 0.005

$gg : ^{+164\%}_{-25\%}$ at 13TeV (similar pattern $^{+142\%}_{-20\%}$ at 100TeV)

$Q = Q_{t\bar{t}H}$ $\qquad$ $Q_{proj} = Q_{t\bar{t}}$

$$\left(\frac{\alpha_s(\mu_R)}{\pi}\right)^n \left(H^{(n)}|_{\mu=\mu_R=Q}\right)|\mathcal{M}^{(0)}|^2 \to \left(\frac{\alpha_s(\mu_R)}{\pi}\right)^n \left(H^{(n)}_{soft}|_{\mu=\xi Q_{proj};\mu_R=Q_{proj}} + (\mu : \xi Q \to Q)\right)|\mathcal{M}^{(0)}|^2$$

where $n = 1{,}2$ and $\xi = \left\{\frac{1}{2}, 1, 2\right\}$

exact running terms

ttH: subtraction scale variation

- in order to test our prescription, we **vary the subtraction scale** μ at which we apply the soft factorisation formula
- the **renormalisation scale** μ_R is kept **fixed** at $Q_{t\bar{t}H}$ in the $t\bar{t}H$ amplitudes and at $Q_{t\bar{t}}$ in the $t\bar{t}$ ones
- the running terms are added exactly

$q\bar{q}$: $^{+4\%}_{-0\%}$ at 13TeV (similar pattern $^{+3\%}_{-0\%}$ at 100TeV)

$q\bar{q}$ channel @13TeV

approximation	$\sigma^{\text{VT only H1}}_{\text{NLO QCD}}$ [fb]			
		$\mu = Q/2$	$\mu = Q$	$\mu = 2Q$
exact		18.048 ± 0.006	7.825 ± 0.005	-13.32 ± 0.01
		$\mu = Q_{proj}/2$	$\mu = Q_{proj}$	$\mu = 2Q_{proj}$
	$Q_{t\bar{t}}$	18.380 ± 0.006	7.413 ± 0.005	-14.47 ± 0.01
		$\mu = Q_{proj}/2 + (Q/2 \to Q)$	$\mu = Q_{proj} + (Q \to Q)$	$\mu = 2Q_{proj} + (2Q \to Q)$
	$Q_{t\bar{t}}$	8.156 ± 0.007	7.413 ± 0.005	6.671 ± 0.008

approximation	$\sigma^{\text{VT2 only H2 M2M0}}_{\text{NNLO QCD}}$ [fb]			
		$\mu = Q_{proj}/2$	$\mu = Q_{proj}$	$\mu = 2Q_{proj}$
	$Q_{t\bar{t}}$	2.7703 ± 0.0014	2.607 ± 0.001	4.193 ± 0.002
		$\mu = Q_{proj}/2 + (Q/2 \to Q)$	$\mu = Q_{proj} + (Q \to Q)$	$\mu = 2Q_{proj} + (2Q \to Q)$
	$Q_{t\bar{t}}$	2.6956 ± 0.0014	2.607 ± 0.001	2.7099 ± 0.0015
$\mathbf{F_2(Q)}$		$\mu = Q_{proj}/2 + (Q/2 \to Q)$	$\mu = Q_{proj} + (Q \to Q)$	$\mu = 2Q_{proj} + (2Q \to Q)$
	$Q_{t\bar{t}}$	1.8432 ± 0.0008	1.7550 ± 0.0007	1.8565 ± 0.0006

$Q = Q_{t\bar{t}H}$ $\quad$ $Q_{proj} = Q_{t\bar{t}}$

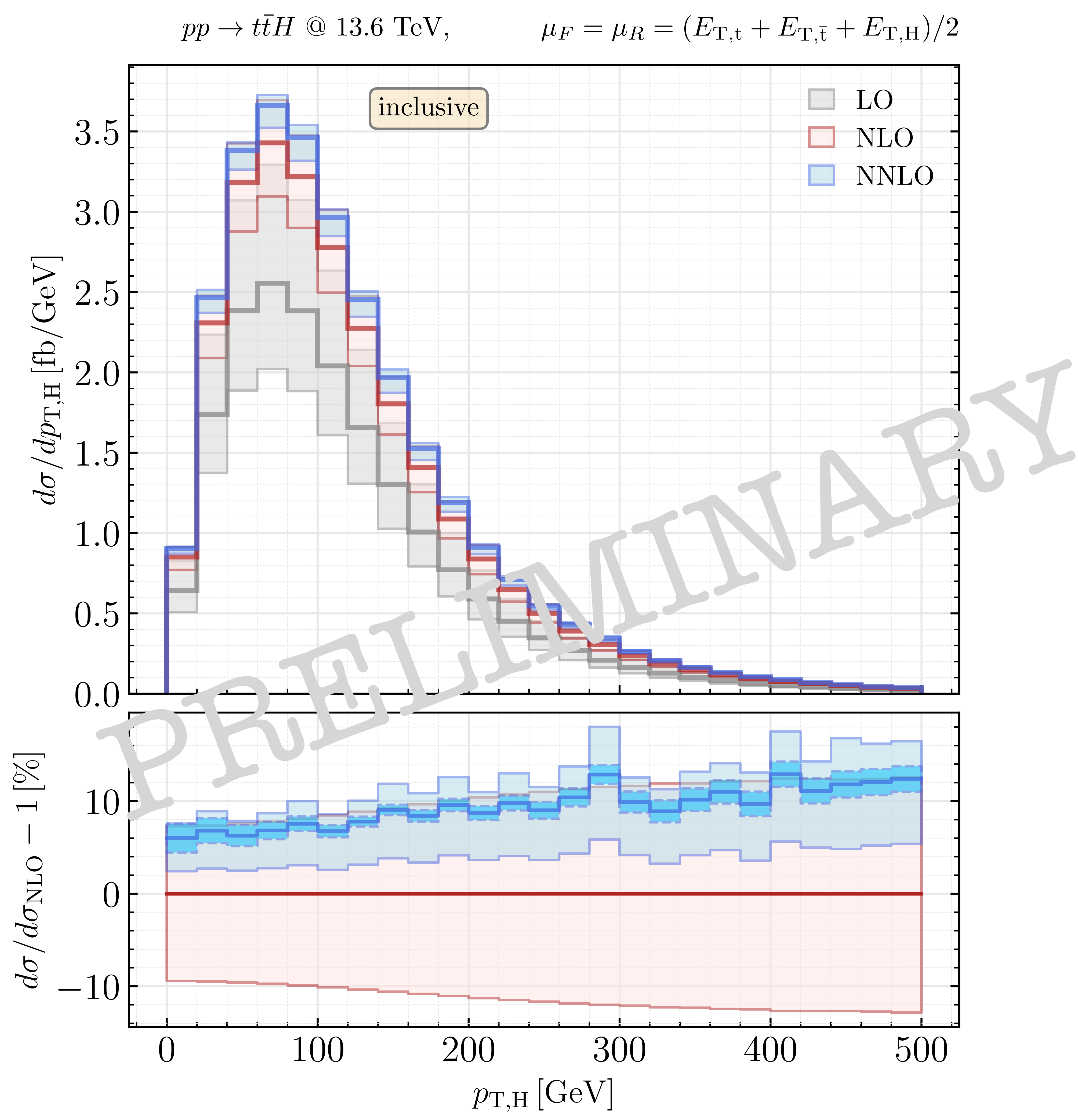
$$\left(\frac{\alpha_s(\mu_R)}{\pi}\right)^n \left(H^{(n)}|_{\mu=\mu_R=Q}\right)|\mathcal{M}^{(0)}|^2 \to \left(\frac{\alpha_s(\mu_R)}{\pi}\right)^n \left(H^{(n)}_{soft}|_{\mu=\xi Q_{proj};\mu_R=Q_{proj}} + (\mu : \xi Q \to Q)\right)|\mathcal{M}^{(0)}|^2$$

where $n = 1,2$ and $\xi = \left\{\frac{1}{2},1,2\right\}$

exact running terms

ttH: Higgs transverse momentum

setup: NNLO NNPDF31, $m_H = 125 GeV$, $m_t = 173.3 GeV$, $\mu_R = \mu_F = (E_t + E_{\bar{t}} + E_H)/2$



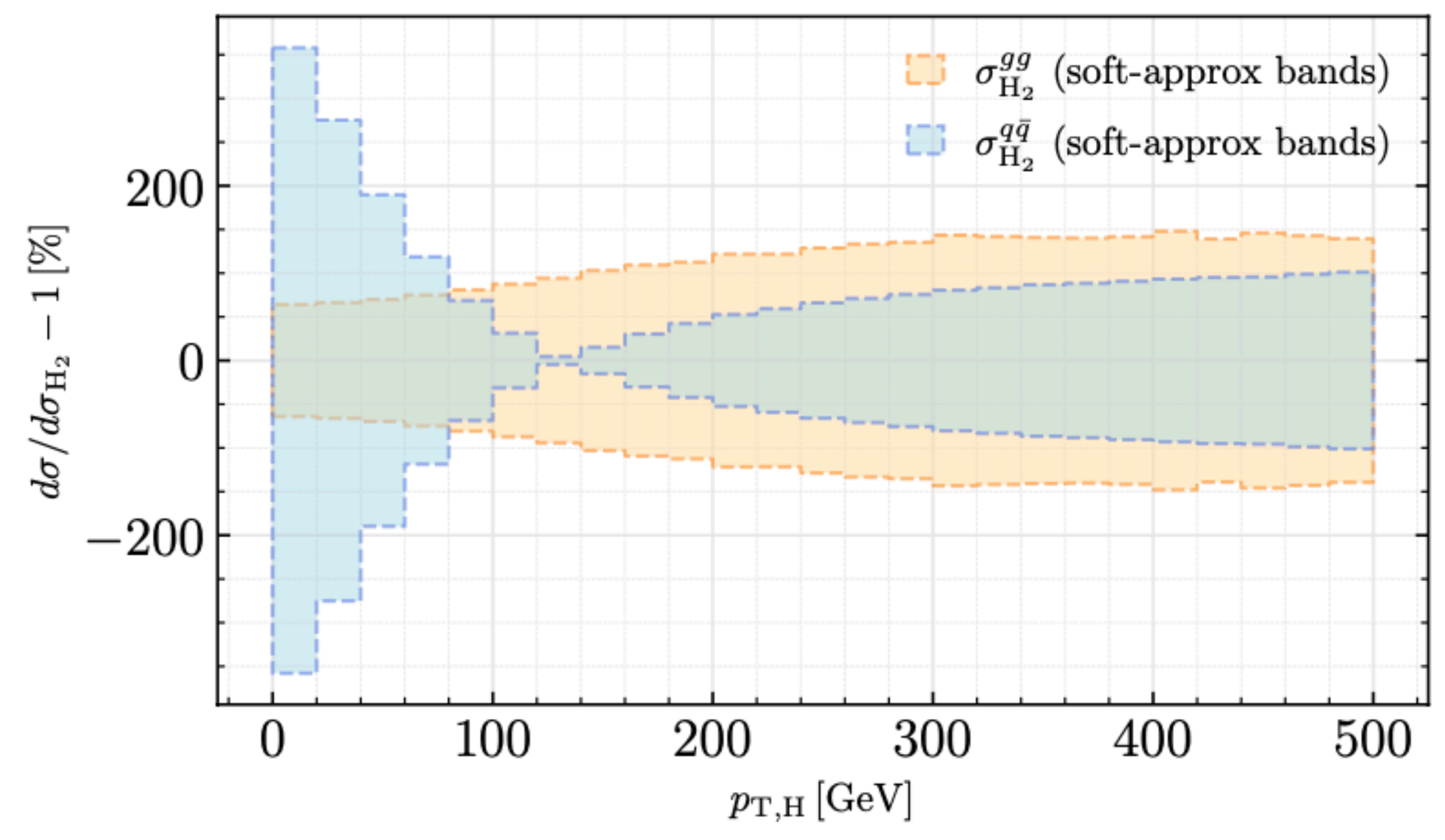
- first results for differential distributions (*inclusive setup*)
- significant reduction of the perturbative uncertainties

$$\sigma_{NNLO} = 565.1^{+2.0\%}_{-4.3\%} \text{ fb}$$

$$\sigma_{NLO} = 524.8^{+8.7\%}_{-10.3\%} \text{ fb}$$

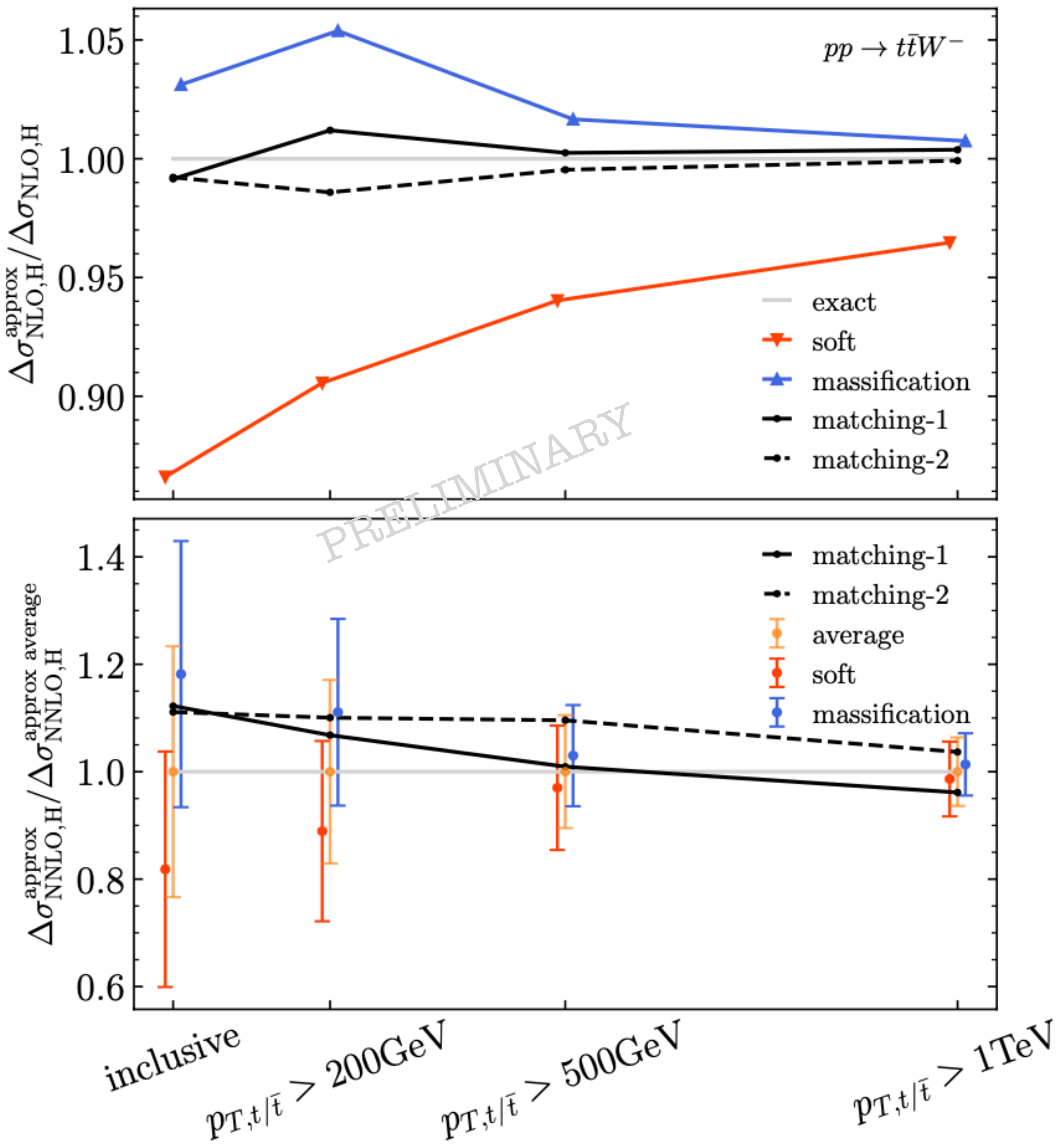
→ $\frac{NNLO}{NLO}$ K-factor: 7.8%

- soft approximation uncertainty computed on a **bin-by-bin basis**
- and of the **same order** over all the $p_{T,H}$ spectrum



Wtt: "matching"

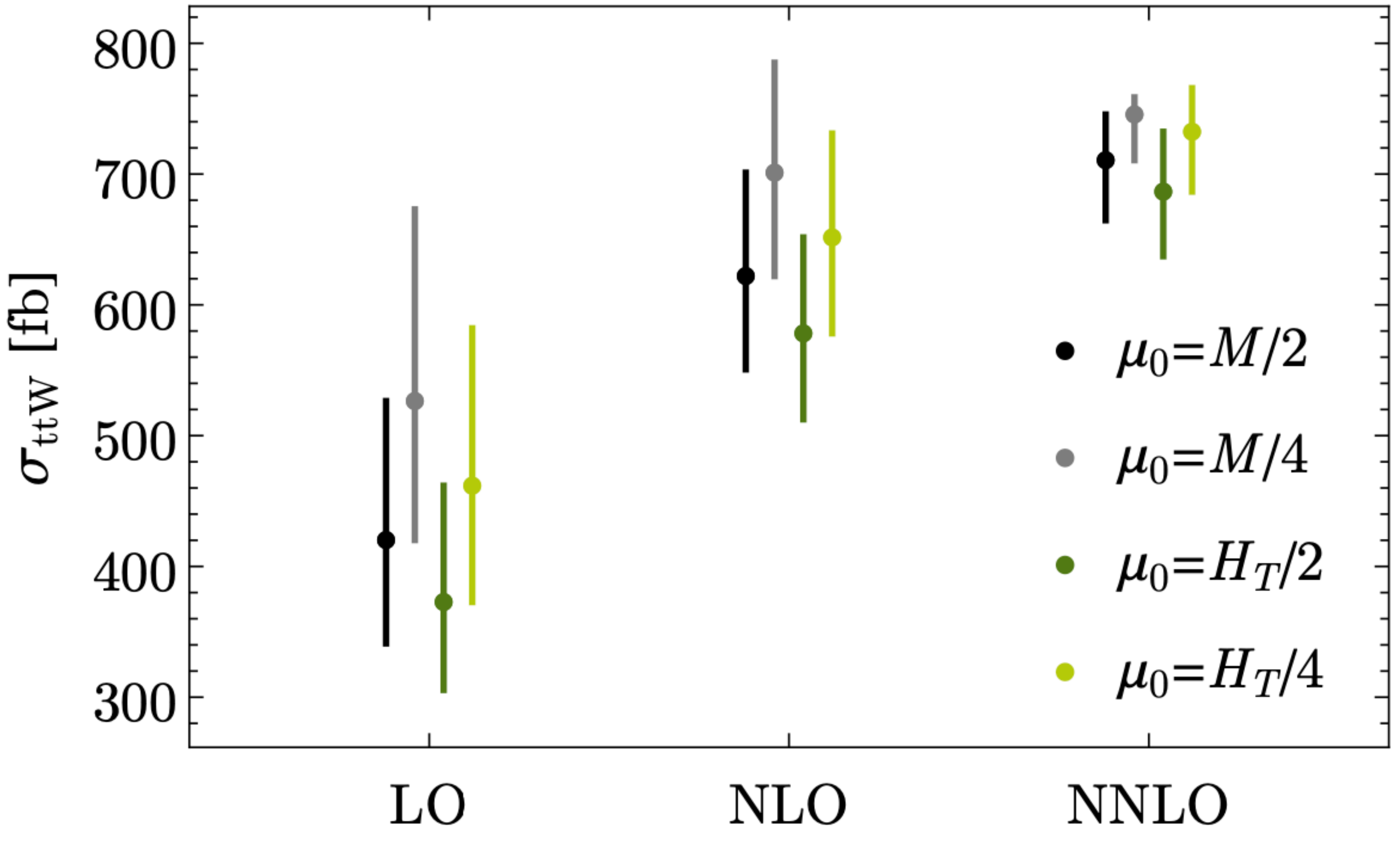
setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385 GeV$, $m_t = 173.2 GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$



- instead of defining our "best" prediction as the arithmetic average of the soft-approximated and massified two-loop finite reminders
  - **matching-1**: $H^{(n)} \sim H^{(n)}_{MA} + H^{(n)}_{SA} - H^{(n)}_{SA\to MA}$
  - matching-2: $H^{(n)} \sim H^{(n),\rm ntl}_{MA} + (H^{(n)}_{SA} - H^{(n)}_{SA\to MA}) + (H^{(n)}_{SA} - H^{(n),\rm ntl}_{SA\to MA})$
- at NLO:
  - the two matching procedures are **almost equivalent**
  - the matched result differs by less than 2% from the exact $H^{(1)}$
- at NNLO:
  - the matched result is **within the uncertainties** of our "best" prediction
  - larger differences between the two matching procedures in the high $p_{T,t}$ region

Wtt: other sources of uncertainties

setup: NNLO NNPDF31 luxqed, $\sqrt{s} = 13\,TeV$, $m_W = 80.385 GeV$, $m_t = 173.2 GeV$, $\mu_R = \mu_F = (2m_t + m_W)/2$



- **perturbative scale uncertainties**:
  - 7-point scale variation around the central scale $\mu_0 = M/2$
  - choice of other possible central scales
  - better convergence for smaller scales (exclude $\mu_0 = H_T/2$)
  - **symmetrisation** of the $M/2$ scale uncertainty

we rely on our perturbative scale uncertainties also because NNLO corrections are not dominated by new opening channels

- **PDF** and α_s **uncertainties**: $\sim 2\,\%$

(computed with the new MATRIX+PineAPPL implementation) [Devoto, Jezo, Kallweit, Schwan (in preparation)]

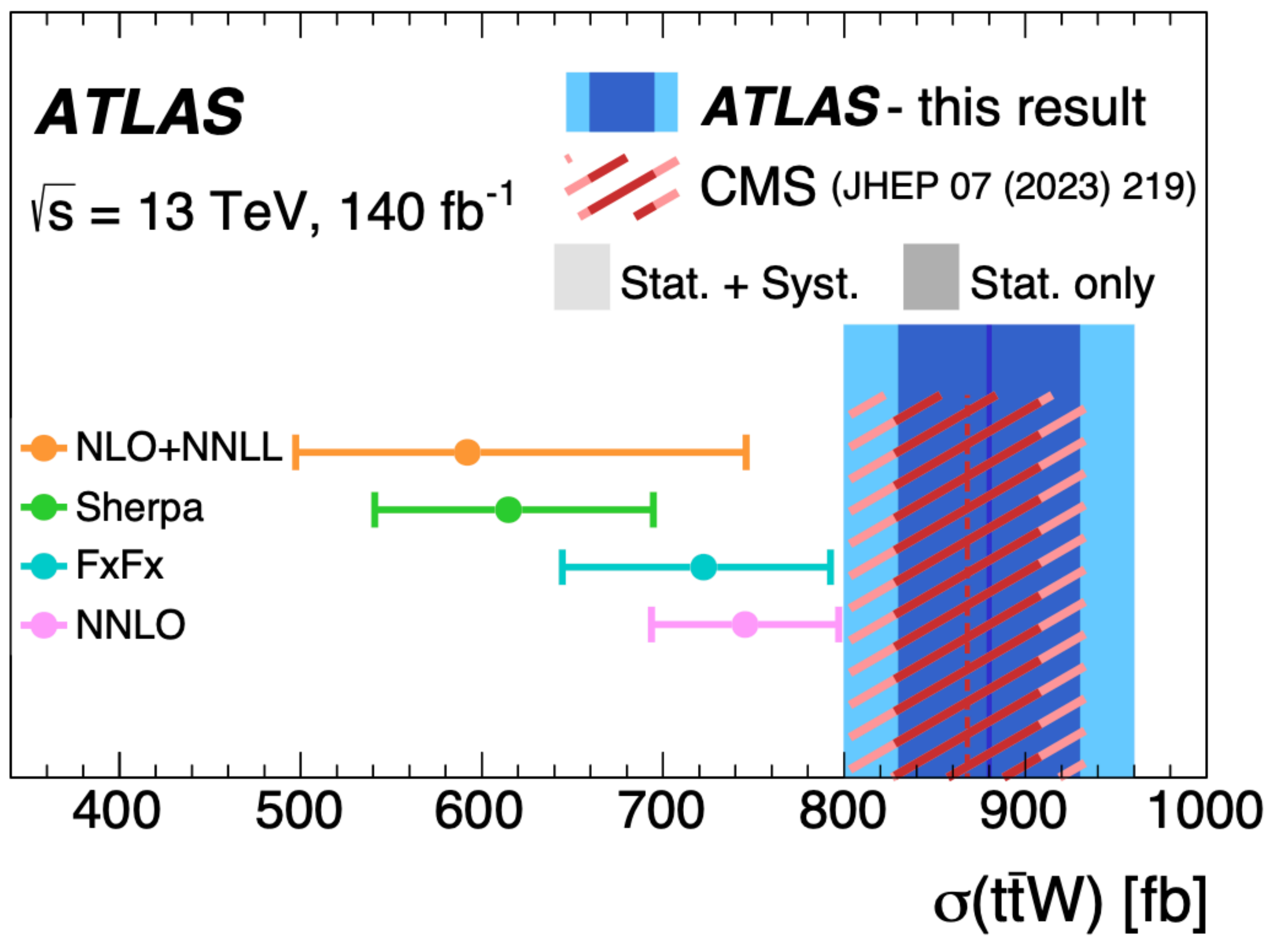
- statistical uncertainties: negligible

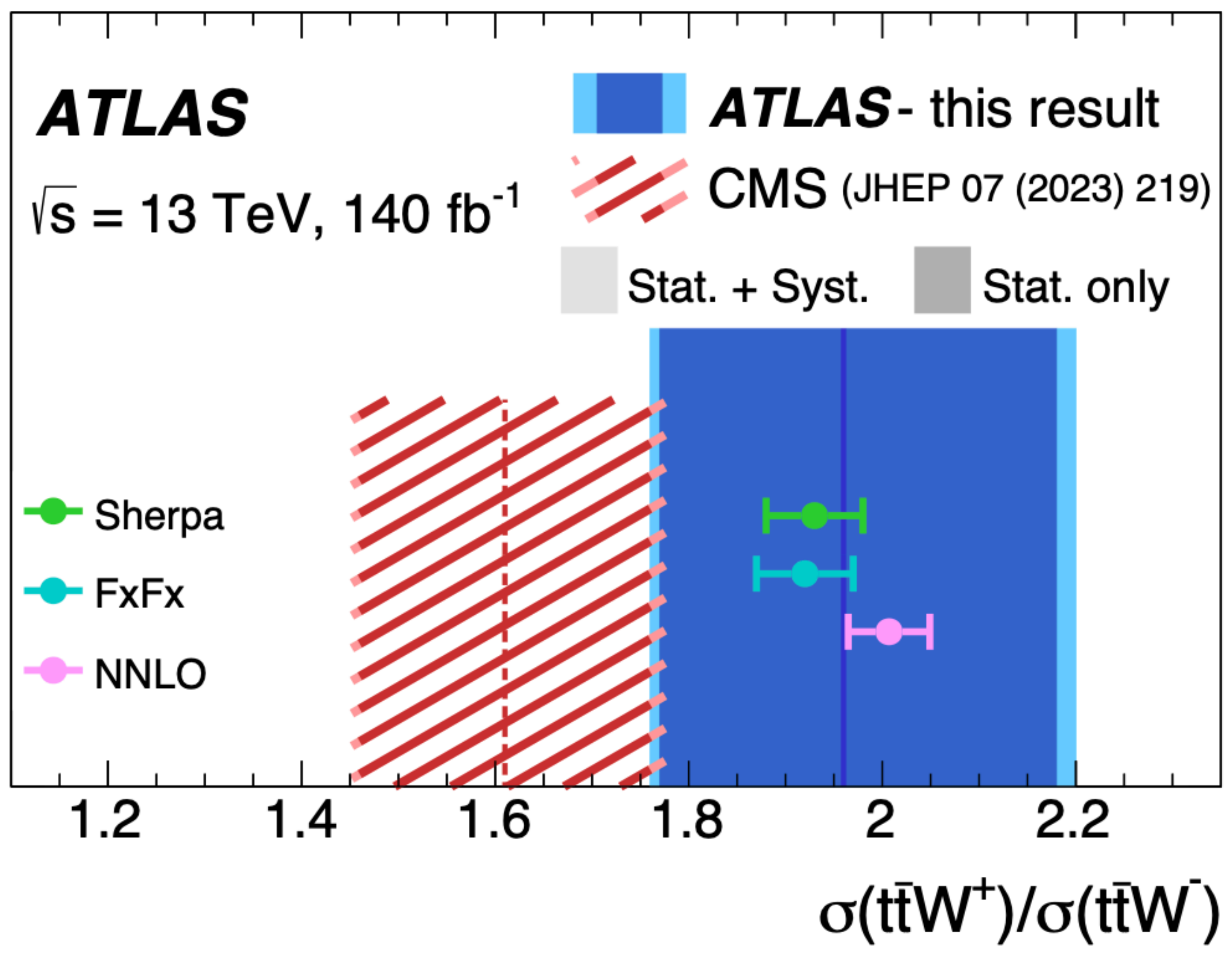
Wtt: updated ATLAS measurement

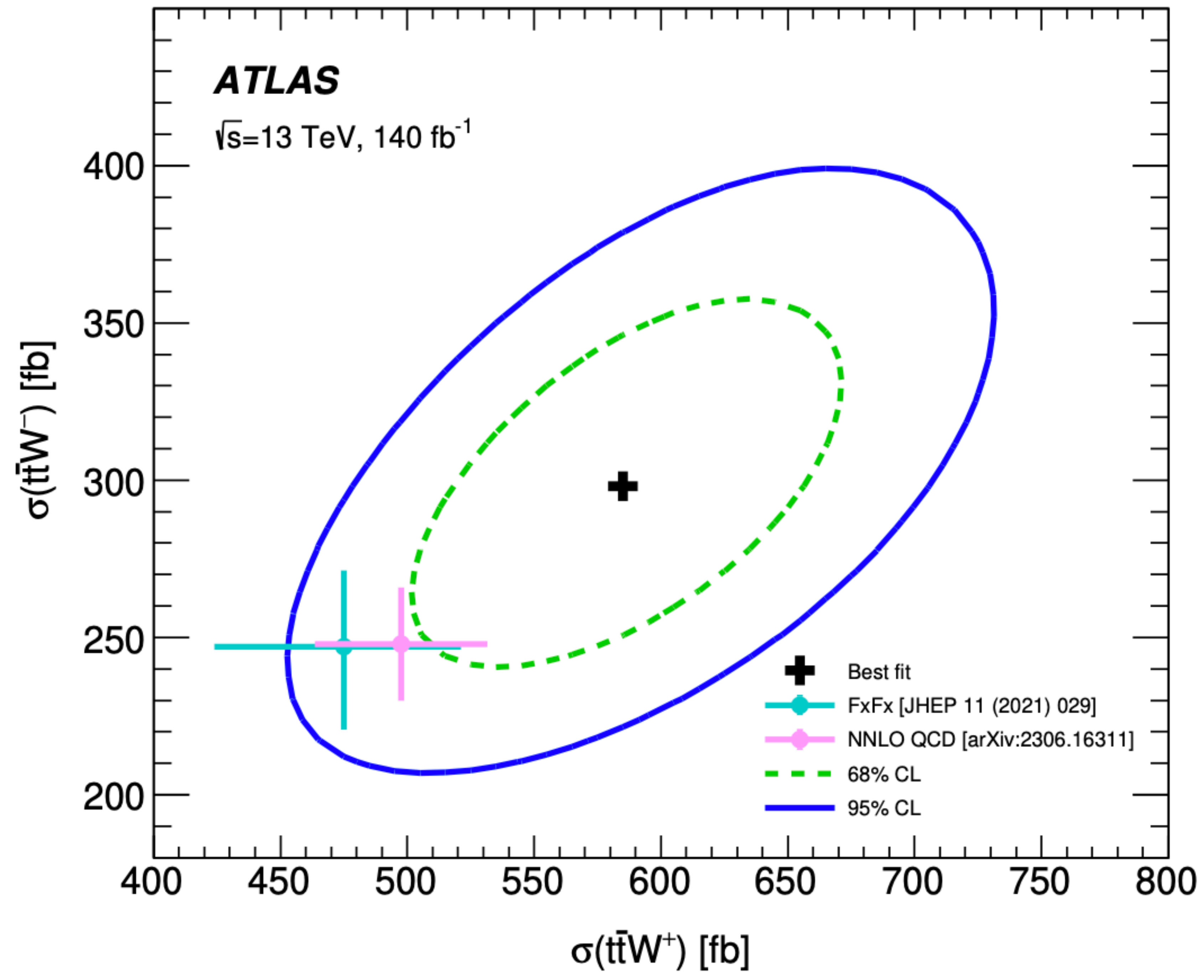
updated ATLAS measurements

[ATLAS: arXiv 2401.05299]

first time in which our NNLO SM prediction is used as theory reference!!







- the updated measurement is **compatible** with our prediction at the level of 1.4σ

$$\sigma_{\rm ATLAS} = 880 \pm 50\,(\text{stat.}) \pm 70\,(\text{syst.}) = 880 \pm 80 \text{ fb}$$

$$\sigma_{\rm theory} = 745 \pm 50\,(\text{scale}) \pm 13\,(\text{2loop approx.}) \pm 19\,(\text{PDF}, \alpha_s) \text{ fb}$$

- good agreement also for the ratio

$$\sigma(t\bar{t}W^+)/\sigma(t\bar{t}W^-) = 1.96 \pm 0.21\,(\text{stat.}) \pm 0.09\,(\text{syst.}) = 1.96 \pm 0.22$$