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N-jettiness soft function at NNLO in QCD

Loops and Legs in Quantum Field Theory (LL2024)

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Talk plan

This talk is mostly based in our work presented in hep-ph/2403.03078

Higher-order QCD corrections (at NNLO)

Subtraction methods
Analytically removes $1/\epsilon^n$ poles by constructing integrable counterterms

- Antenna subtraction
  Gehrmann-De Ridder, Gehrmann, Glover - hep-ph/0505111
- CoLoRFul subtraction
  Somogyi, Trócsányi, Del Duca - hep-ph/0502226
- Sector subtraction
  Czakon - hep-ph/1005.0274, Boughezal et al. - hep-ph/1111.7041
- Projection-to-Born
  Cacciari et al. - hep-ph/1506.02660
- Nested soft-collinear subtraction
  Caola, Melnikov, Röntsch - hep-ph/1702.01352
- Local analytic sector subtraction
  Magnea et al. - hep-ph/1806.09570

Slicing methods
Imposes cuts in some variable to split the phase space. Below the cut a soft-collinear approximation is used

- q_t-subtraction
  Catani, Grazzini - hep-ph/0703012
- N-jettiness subtraction
  Boughezal et al. - hep-ph/1504.02131, Gaunt et al. - hep-ph/1505.04794

N-jettiness subtraction

The N-jettiness variable, defined by

$$\mathcal{T}(\mathcal{R},\mathcal{U}) = \sum_{x\in\mathcal{U}} \min\left\{\frac{2p_x p_{h_1}}{P_{h_1}}, \frac{2p_x p_{h_2}}{P_{h_2}}, \frac{2p_x p_{h_3}}{P_{h_3}}, \dots\right\}$$

Can be used to perform a slicing of the phase space (like in q_T subtraction)

$$\sigma = \int^{\mathcal{T}_0} d\mathcal{T} \frac{d\sigma}{d\mathcal{T}} + \int_{\mathcal{T}_0} d\mathcal{T} \frac{d\sigma}{d\mathcal{T}}$$

and, thanks to the factorization theorem from SCET, we can calculate

$$\int^{\mathcal{T}_0} d\mathcal{T} \frac{d\sigma}{d\mathcal{T}} = \int B \otimes B \otimes S \otimes H \otimes \prod_i^N J_i + \mathcal{O}(\mathcal{T}_0)$$

N-jettiness subtraction

$$\int^{\mathcal{T}_0} d\mathcal{T} \frac{d\sigma}{d\mathcal{T}} = \int B \otimes B \otimes S \otimes H \otimes \prod_i^N J_i + \mathcal{O}(\mathcal{T}_0)$$

- The beam function and jet functions (B and J_i) describe the initial- and final-state collinear radiation, the soft function S describes the soft radiation, and the (process dependent) hard function H encodes the virtual corrections
- At NNLO, all ingredients are known since years ago. The soft function was available for 0-, 1- and 2-jettiness, but only recently for generic N-jettiness (hep-ph/2312.11626, hep-ph/2403.03078)
- At N3LO, only the soft function is missing. There are current efforts to obtain the analytic N3LO 0-jettiness soft function (see Pikelner's talk)

Soft function calculation

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- Previous calculations of the NNLO were mainly based on mapping the available phase space of soft-gluon emissions onto hemispheres and computing the required integrals numerically.
  (Boughezal et al. - hep-ph/1504.02540, Campbell et al. - hep-ph/1711.09984, Bell et al. - hep-ph/2312.11626)
- Here, we use the well established **subtraction methods** to calculate this ingredient of a phase space slicing method, showing the *explicit analytical cancellation of divergences*. Also, in our calculation *N is treated genuinely as a parameter*.
- We show that borrowing ideas from generic NNLO QCD subtraction schemes is beneficial for computing ingredients of modern slicing calculations.

Soft function renormalization

Since loop-corrections are not present, the IR divergences in the soft function turn into UV ones that require renormalization. It is convenient to work in Laplace space

$$S(u) = \int_0^\infty d\mathcal{T}\; S_{\mathcal{T}}(\mathcal{T}) e^{-u\mathcal{T}}$$

Since there the renormalization is multiplicative (with matrices in color space)

$$S = Z\tilde{S}Z^\dagger$$

If we write the expansion in powers of α_s

$$Z = 1 + Z_1 + Z_2,\qquad S = 1 + S_1 + S_2,\qquad \tilde{S} = 1 + \tilde{S}_1 + \tilde{S}_2,$$

$$\tilde{S}_1 = S_1 - Z_1 - Z_1^\dagger, \qquad \left(Z_2 = \tfrac{1}{2} Z_1 Z_1 + Z_{2,r}\right)$$

$$\tilde{S}_2 = S_2 - Z_2 - Z_2^\dagger + Z_1 Z_1 + Z_1^\dagger Z_1^\dagger - Z_1 S_1 - S_1 Z_1^\dagger + Z_1 Z_1^\dagger$$

$$= \frac{1}{2}\tilde{S}_1\tilde{S}_1 + \frac{1}{2}[Z_1, Z_1^\dagger] + \frac{1}{2}\left[S_1, Z_1 - Z_1^\dagger\right] + S_{2,r} - Z_{2,r} - Z_{2,r}^\dagger.$$

Soft function at NLO

If we take $P_{h_i} = E_i$ with and unresolved gluon m, the N-jettiness is given by

$$\mathcal{T}(m) = E_m \psi_m = E_m \min\{\rho_{1m}, \rho_{2m}, \rho_{3m}, \ldots, \rho_{Nm}\},$$

where $\rho_{ij} = 1 - \vec{n}_i \cdot \vec{n}_j$. Then, the soft function is given by

$$S(\tau) = -\sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \, g_s^2 \int \frac{d\Omega_m^{(d-1)}}{2(2\pi)^{d-1}} \frac{dE_m}{E_m^{1+2\epsilon}} E_m^2 \, \delta(\tau - E_m \psi_m) \, \langle S_{ij}(m) \rangle_m , \qquad S_{ij}(m) = \frac{1}{E_m^2} \frac{\rho_{ij}}{\rho_{im}\rho_{jm}}$$

We integrate over energy and use that we know the limit $\lim_{m||i} \psi_m = \rho_{im}$., so we can rewrite

$$\psi_m^{2\epsilon} \frac{\rho_{ij}}{\rho_{im}\rho_{jm}} = \left(\frac{\psi_m \rho_{ij}}{\rho_{im}\rho_{jm}} \right)^{2\epsilon} \frac{\rho_{ij}^{1-2\epsilon}}{\rho_{im}^{1-2\epsilon} \rho_{jm}^{1-2\epsilon}} = \left(1 + 2\epsilon g_{ij,m}^{(2)} \right) \frac{\rho_{ij}^{1-2\epsilon}}{\rho_{im}^{1-2\epsilon} \rho_{jm}^{1-2\epsilon}}$$

Soft function at NLO

Knowing that ($\eta_{ij} = \rho_{ij}/2$)

$$\left\langle \frac{\rho_{ij}^{1-2\epsilon}}{\rho_{im}^{1-2\epsilon}\rho_{jm}^{1-2\epsilon}} \right\rangle_m = \frac{2\eta_{ij}^{\epsilon}}{\epsilon} K_{ij}^{(2)} = \frac{2\eta_{ij}^{\epsilon}}{\epsilon} \frac{\Gamma(1+\epsilon)^2}{\Gamma(1+2\epsilon)} \, {}_2F_1\left(\epsilon, \epsilon, 1-\epsilon, 1-\eta_{ij}\right),$$

in Laplace space we arrive to the following bare soft function

$$S_1 = a_s \left(\mu \bar{u}\right)^{2\epsilon} \frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)e^{\epsilon\gamma_E}} \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \left[\frac{\eta_{ij}^{\epsilon}}{\epsilon^2} K_{ij}^{(2)} + \left\langle g_{ij,m}^{(2)} \frac{\rho_{ij}^{1-2\epsilon}}{\rho_{im}^{1-2\epsilon}\rho_{jm}^{1-2\epsilon}} \right\rangle_m \right].$$

By combining S_1 with the renormalization matrices Z_1 and $Z_1^{\dagger}$, we finally obtain ($L_{ij} = \ln\left(\mu \bar{u} \sqrt{\eta_{ij}}\right)$)

$$\tilde{S}_1 = a_s \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \left[2L_{ij}^2 + \mathrm{Li}_2(1-\eta_{ij}) + \frac{\pi^2}{12} + \left\langle \ln\left(\frac{\psi_m \rho_{ij}}{\rho_{im}\rho_{jm}}\right) \frac{\rho_{ij}}{\rho_{im}\rho_{jm}} \right\rangle_m + \mathcal{O}(\epsilon) \right]$$

Soft function at NNLO

The NNLO contribution to the bare soft function is

$$S_2 = S_{2,RR} + S_{2,RV} - a_s \frac{\beta_0}{\epsilon} S_1$$

We further split the double-real contribution into correlated and uncorrelated pieces

$$S_{2,RR,\tau} = S_{2,RR,T^4} + S_{2,RR,T^2} = \frac{1}{2} \sum_{(ij),(k,l)} \{\mathbf{T}_i \cdot \mathbf{T}_j, \mathbf{T}_k \cdot \mathbf{T}_l\} \; I_{T^4,ij,kl} - \frac{C_A}{2} \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \; I_{T^2,ij}$$

The real-virtual contribution reads

$$S_{2,RV,\tau} = S_{RV,T^2} + S_{RV,\mathrm{tc}} = \frac{[\alpha_s]\, 2^{-\epsilon}}{\epsilon^2} C_A A_K(\epsilon) \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \; I_{RV,ij} + [\alpha_s] \frac{4\pi N_\epsilon}{\epsilon} \sum_{(kij)} \kappa_{ij} F^{kij} I_{kij}$$

where $\kappa_{ij} = \lambda_{ij} - \lambda_{im} - \lambda_{jm}$, with $\lambda_{ij} = 1$ if both i and j refer to incoming/outgoing partons and zero otherwise. We have defined $F^{kij} = f_{abc} T_k^a T_i^b T_j^c$, while $A_K(\epsilon)$ and N_ϵ are normalization factors.

Soft function at NNLO

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The calculation of the renormalized soft function is organized as follows

$$\tilde{S}_2 = \tilde{S}_2^{\text{uncorr}} + \tilde{S}_2^{\text{corr}} + \tilde{S}_2^{\text{tc}}$$

Where the pieces are given by the following contributions

Uncorrelated emission

$$\tilde{S}_2^{\text{uncorr}} = \frac{1}{2} \tilde{S}_1 \tilde{S}_1$$

Correlated emission

$$\tilde{S}_2^{\text{corr}} = S_{2,RR,T^2} + S_{RV,T^2} - Z_{2,r} - Z_{2,r}^{\dagger} - \frac{a_s \beta_0}{\epsilon} S_1 ,$$

Triple color terms

$$\tilde{S}_2^{\text{tc}} = \frac{1}{2}\left[Z_1, Z_1^{\dagger}\right] + \frac{1}{2}\left[S_1, Z_1 - Z_1^{\dagger}\right] + S_{RV,\text{tc}}$$

Uncorrelated emission

The S_2 contains an iterated contribution of the NLO soft function S_1

$$I_{T^4,ij,kl} = \frac{[\alpha_s]^2}{2} \left\langle \int_0^\infty \frac{dE_m}{E_m^{1+2\epsilon}} \frac{dE_n}{E_n^{1+2\epsilon}} \, \delta(\tau - E_m\psi_m - E_n\psi_n) \frac{\rho_{ij}}{\rho_{im}\rho_{jm}} \frac{\rho_{kl}}{\rho_{kn}\rho_{ln}} \right\rangle_{mn}$$

If we integrate over both energies

$$\int_0^\infty \frac{dE_m}{E_m^{1+2\epsilon}} \frac{dE_n}{E_n^{1+2\epsilon}} \, \delta(\tau - E_m\psi_m - E_n\psi_n) = \frac{\tau^{-1-4\epsilon}}{\Gamma(-4\epsilon)} \frac{\psi_m^{2\epsilon}\Gamma(1-2\epsilon)}{2\epsilon} \frac{\psi_n^{2\epsilon}\Gamma(1-2\epsilon)}{2\epsilon}$$

The Laplace transform allows us to identify this iteration

$$S_{2,RR,T^4} = \frac{[\alpha_s]^2}{4} \sum_{(ij),(kl)} \{\mathbf{T}_i \cdot \mathbf{T}_j, \mathbf{T}_k \cdot \mathbf{T}_l\} \left(\frac{u^{2\epsilon}\Gamma(1-2\epsilon)}{2\epsilon} \right)^2 \left\langle \psi_m^{2\epsilon} \frac{\rho_{ij}}{\rho_{im}\rho_{jm}} \right\rangle_m \left\langle \psi_n^{2\epsilon} \frac{\rho_{kl}}{\rho_{kn}\rho_{ln}} \right\rangle_n = \frac{1}{2} S_1 S_1$$

Triple color terms

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This contribution depends on triple products of color charges

$$\tilde{S}_2^{\text{tc}} = \frac{1}{2}\left[Z_1, Z_1^\dagger\right] + \frac{1}{2}\left[S_1, Z_1 - Z_1^\dagger\right] + S_{RV,\text{tc}}$$

The commutators can be computed as shown in (Devoto et al. - hep-ph/2310.17598).

$$\frac{1}{2}[Z_1, Z_1^\dagger] = -\frac{2\pi a_s^2}{\epsilon^2}\sum_{(kij)}\lambda_{kj}L_{ij}F^{kij} = -\frac{\pi a_s^2}{\epsilon^2}\sum_{(kij)}\lambda_{kj}\ln\eta_{ij}\; F^{kij}$$

$$\frac{1}{2}[S_1, Z_1 - Z_1^\dagger] = -\frac{a_s^2\pi(\mu u)^{2\epsilon}}{\epsilon^2}\frac{e^{\gamma_E\epsilon}\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\sum_{(kij)}\kappa_{kj}\left\langle\psi_m^{2\epsilon}\frac{\rho_{ki}}{\rho_{km}\rho_{im}}\right\rangle_m F^{kij}$$

The real-virtual triple-color correlated contribution is

$$S_{RV,\text{tc}} = \frac{a_s^2\pi(\mu\,\bar{u})^{4\epsilon}N_\epsilon 2^{-\epsilon}}{2\epsilon^2}\frac{\Gamma(1-4\epsilon)}{\Gamma^2(1-\epsilon)e^{2\gamma_E\epsilon}}\sum_{(kij)}\kappa_{kj}\left\langle\psi_m^{4\epsilon}\frac{\rho_{ki}}{\rho_{km}\rho_{im}}\left(\frac{\rho_{kj}}{\rho_{km}\rho_{jm}}\right)^{\epsilon}\right\rangle_m F^{kij}$$

Triple color terms

Following the NLO case,

$$\left\langle \psi_m^{2\epsilon} \frac{\rho_{ki}}{\rho_{km}\rho_{im}} \right\rangle_m = \left\langle (1 + 2\epsilon g_{ki,m}^{(2)}) \frac{\rho_{ki}^{1-2\epsilon}}{\rho_{km}^{1-2\epsilon}\rho_{im}^{1-2\epsilon}} \right\rangle_m ,$$

$$\left\langle \psi_m^{4\epsilon} \frac{\rho_{ki}}{\rho_{km}\rho_{im}} \left(\frac{\rho_{kj}}{\rho_{km}\rho_{jm}} \right)^{\epsilon} \right\rangle_m = \left\langle \left(1 + 4\epsilon g_{ki,m}^{(4)}\right) \frac{\rho_{ki}^{1-4\epsilon}}{\rho_{km}^{1-4\epsilon}\rho_{im}^{1-4\epsilon}} \left(\frac{\rho_{kj}}{\rho_{km}\rho_{jm}} \right)^{\epsilon} \right\rangle_m .$$

It is easy to show that the N-jettiness dependent poles cancel

$$\tilde{S}_2^{\mathrm{tc}} \to -\frac{2a_s^2\pi}{\epsilon} \sum_{(kij)} \kappa_{kj} \left\langle \frac{\rho_{ik}}{\rho_{im}\rho_{km}} \left(g_{ki,m}^{(2)} - g_{ki,m}^{(4)} \right) \right\rangle_m F^{kij} = \mathcal{O}(\epsilon^0)$$

While the N-jettiness dependent finite reminder is

$$\tilde{S}_2^{\mathrm{tc}} \to 2a_s^2\pi \sum_{(kij)} \kappa_{kj} \left\langle \frac{\rho_{ki}}{\rho_{im}\rho_{km}} \ln\left(\frac{\psi_m \rho_{ki}}{\rho_{km}\rho_{im}} \right) \ln\left(\frac{(\bar{u}\mu)^2 \psi_m \rho_{im}\rho_{kj}}{2\rho_{jm}\rho_{ki}} \right) \right\rangle_m F^{kij}$$

Triple color terms

What about the rest of the finite part?

The idea is to use the integral of

$$\left\langle \frac{\rho_{ki}}{\rho_{km}\rho_{im}} \left(\frac{\rho_{kj}}{\rho_{km}\rho_{jm}} \right)^{\epsilon} \right\rangle_m ,$$

which was already calculated in Devoto et al. - hep-ph/2310.17598, and take the difference of the two results

Correlated emission

The calculation of the correlated terms are the main bulk of the calculation

$$\tilde{S}_2^{\text{corr}} = S_{2,RR,T^2} + S_{RV,T^2} - Z_{2,r} - Z_{2,r}^{\dagger} - \frac{a_s \beta_0}{\epsilon} S_1 ,$$

The last three renormalization term do not require any integration, and the real-virtual one is simply

$$S_{RV,T^2} \propto -\frac{[\alpha_s]^2}{\epsilon^3} C_A \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \left\langle \psi_m^{4\epsilon} \left(\frac{\rho_{ij}}{\rho_{im}\rho_{jm}} \right)^{1+\epsilon} \right\rangle_m$$

The first term, that involves the correlated emission eikonal term $S_{ij}^{gg}(m, n)$, is the one that requires atention

$$S_{2,RR,T^2,\tau} = -\frac{C_A}{2} \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \, I_{ij,\tau} = -\frac{C_A}{2} \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \, \frac{g_s^4}{2} \int [dp_m][dp_n] \, \delta \left(\tau - E_m \psi_m - E_n \psi_n \right) \tilde{S}_{ij}^{gg}(m, n)$$

(there is also an analogous and simpler quark contribution, but we focus on the gluon case)

Correlated emission

We perform a nested subtraction of all divergent limits in the correlated term as follows

$$I_{ij} = \bar{S}_\omega I_{ij} + S_\omega I_{ij}, \qquad \text{with } \bar{S}_\omega = (1 - S_\omega)$$

$$\bar{S}_\omega I_{ij}^{dc} + \bar{S}_\omega I_{ij}^{tc}$$

(Introduce partitions)

(sectors + subtract)

$$\left\{\theta^{bd} C_{mn} + \left(1 - \theta^{bd} C_{mn}\right) \left[C_{imn} + (1 - C_{imn})\right]\right\} \bar{S}_\omega I_{ij}^{tc}$$

Correlated emission

- We perform a nested subtraction of all divergent limits. The first one is the *strongly ordered* one, the double soft limit with energy ordering

$$S_{2,RR,T^2} = -\frac{C_A}{2}\sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \left[\bar{S}_\omega I_{ij} + S_\omega I_{ij}\right] ,$$

  with S_ω being the operator that enforces the strongly-ordered limit

- Divergent terms of $S_\omega I_{ij}$ can be calculated analytically. The N-jettiness dependent ones cancel against those of the RV term

- There are only collinear divergences remaining in $\bar{S}_\omega I_{ij} = (1 - S_\omega)\, I_{ij}$

Correlated emission

- Now, we introduce partition functions (see hep-ph/2310.17598) to separate double-collinear and triple-collinear singularities

$$\bar{S}_\omega[I_{ij}] = \bar{S}_\omega[I_{ij}^{dc}] + \bar{S}_\omega[I_{ij}^{tc}]$$

- The double-collinear contribution

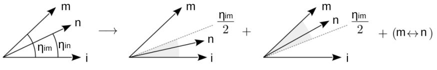
$$\bar{S}_\omega[I_{ij}^{dc}] = \frac{N_u}{\epsilon} \int\limits_0^1 \frac{d\omega}{\omega^{1+2\epsilon}} \left\langle\, \psi_{mn}^{4\epsilon} \left(w^{mi,nj} + w^{ni,mj} \right) \bar{S}_\omega \left[\omega^2 \tilde{S}_{ij}^{gg} \right] \right\rangle_{mn}$$

has poles independent of the N-jettiness, which can be obtained from the calculation done in hep-ph/1807.05835. The jettiness dependent part can be calculated numerically.

Correlated emission

- For the triple-collinear contribution (with $w^{tc} = w^{mi,ni} + w^{mj,nj}$)

$$\bar{S}_\omega[I_{ij}^{tc}] = \frac{N_u}{\epsilon}\int_0^1 \frac{d\omega}{\omega^{1+2\epsilon}} \left\langle \psi_{mn}^{4\epsilon}\, w^{tc}\, \bar{S}_\omega\left[\omega^2 \tilde{S}_{ij}^{gg}\right]\right\rangle_{mn}$$



(sectoring)

we will also introduce sectors to handle the $m||n$ singularity

- Again, the strategy is to identify the terms that correspond to the calculation without the N-jettiness constraint

- This allows us to avoid calculating complicated finite terms, leaving only the jettiness dependent ones for numeric evaluation

The final result

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- The NLO contribution reads

$$\tilde{S}_1 = a_s \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \left[2L_{ij}^2 + \mathrm{Li}_2(1-\eta_{ij}) + \frac{\pi^2}{12} + \left\langle L_{ij,m}^{\psi} \frac{\rho_{ij}}{\rho_{im}\rho_{jm}} \right\rangle_m \right],$$

where $L_{ij} = \ln(\bar{u}\sqrt{\eta_{ij}}\mu)$ and $L_{ij,m}^{\psi} = \ln\left(\frac{\psi_m \rho_{ij}}{\rho_{im}\rho_{jm}}\right)$.

- The NNLO one is

$$\tilde{S}_2 = \frac{1}{2}\tilde{S}_1^2 + a_s^2 C_A \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \, G_{ij} + a_s^2 \, n_f \, T_R \sum_{(ij)} \mathbf{T}_i \cdot \mathbf{T}_j \, Q_{ij} + a_s^2 \pi \sum_{(kij)} F^{kij} \, \kappa_{kj} G_{kij}^{\text{triple}},$$

where G_{ij}, Q_{ij} and G_{kij}^{triple} are finite function with analytical terms along with a *low number numerical integrations* over one- and two-particle phase space in four-dimensions

Numerical checks

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- We compared results for the N-Jettiness soft function obtained in this paper with those of the Ref:

The NNLO soft function for N-jettiness in hadronic collisions

Bell, Dehnadi, Mohrmann, Rahn, arXiv hep-ph/2312.11626.

- Here, we focus in the (new) 3-jettiness case. We consider the configuration with two back-to-back beams. The five directions are

$$n_1 = (0,0,1), \quad n_2 = (0,0,-1), \quad n_3 = (\sin\theta_{13}, 0, \cos\theta_{13}),$$
$$n_4 = (\sin\theta_{14}\cos\phi_4, \sin\theta_{14}\sin\phi_4, \cos\theta_{14}), \quad n_5 = (\sin\theta_{15}\cos\phi_5, \sin\theta_{15}\sin\phi_5, \cos\theta_{15}),$$

  in the following phase space point

$$\theta_{13} = \frac{3\pi}{10}, \quad \theta_{14} = \frac{6\pi}{10}, \quad \theta_{15} = \frac{9\pi}{10}, \quad \phi_4 = \frac{3\pi}{5}, \quad \phi_5 = \frac{6\pi}{5}$$

Numerical checks

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Dipole configurations

Dipoles	Gluons		Quarks	
	G_{ij}^{nl}	Bell et al.	Q_{ij}^{nl}	Bell et al.
12	116.20 ± 0.01	116.20 ± 0.16	-36.249 ± 0.001	-36.244 ± 0.009
13	38.13 ± 0.03	37.63 ± 0.03	-21.717 ± 0.007	-21.732 ± 0.005
14	63.63 ± 0.01	63.66 ± 0.06	-25.189 ± 0.003	-25.192 ± 0.006
15	107.17 ± 0.01	106.99 ± 0.12	-35.268 ± 0.001	-35.256 ± 0.009
23	97.11 ± 0.01	96.97 ± 0.10	-32.875 ± 0.002	-32.872 ± 0.008
24	67.36 ± 0.02	67.51 ± 0.08	-26.821 ± 0.003	-26.815 ± 0.007
25	30.87 ± 0.03	30.73 ± 0.04	-21.561 ± 0.009	-21.561 ± 0.005
34	69.43 ± 0.01	69.24 ± 0.07	-25.854 ± 0.002	-25.861 ± 0.006
35	106.13 ± 0.02	105.97 ± 0.13	-34.799 ± 0.002	-34.796 ± 0.008
45	74.45 ± 0.02	74.36 ± 0.09	-28.247 ± 0.004	-28.251 ± 0.007

Tripole sums

	$\tilde{c}_{\text{tripoles}}$	Bell et al.
$\tilde{c}_{\text{tripoles}}^{(2,124)}$	-683.25 ± 0.01	-683.23 ± 0.04
$\tilde{c}_{\text{tripoles}}^{(2,125)}$	-2203.3 ± 0.2	-2203.5 ± 0.1
$\tilde{c}_{\text{tripoles}}^{(2,145)}$	-6.324 ± 0.004	-6.325 ± 0.04
$\tilde{c}_{\text{tripoles}}^{(2,245)}$	-0.837 ± 0.008	-0.830 ± 0.039

The tripole sums correspond to the four independent color structures as specified by Bell et al.

Conclusions

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In our work

- We calculated the N-jettiness soft function for generic N and demonstrated the **analytical cancellation** of poles against renormalization matrix
- We derived a simple representation for the finite, jettiness-dependent remainder, allowing for faster implementations
- We found excellent agreement between our numerical results for $N = 1, 2$ and $N = 3$ and previous calculations
- We have shown the significant benefits of applying subtraction-inspired methods to derive representations for building blocks of slicing methods

Thank you!

Correlated emission

Introducing sectors, we arrive to

$$\bar{S}_\omega[I_{ij}^{tc}] = \frac{N_u}{\epsilon} \int_0^1 \frac{d\omega}{\omega^{1+2\epsilon}} \left\langle C_{mn} \left[d\Omega_{mn}\right] \theta^{b+d} w^{tc} \psi_{mn}^{4\epsilon} \bar{S}_\omega \left[\tilde{S}_{ij}^{gg}\right] \right\rangle_{mn}$$

$$+ \frac{N_u}{\epsilon} \sum_{x\in\{i,j\}} \int_0^1 \frac{d\omega}{\omega^{1+2\epsilon}} \left\langle \left(1 - \theta^{b+d} C_{mn}\right) \left[d\Omega_{mn}\right] C_{xmn} w^{tc} \psi_{mn}^{4\epsilon} \bar{S}_\omega \left[\tilde{S}_{ij}^{gg}\right] \right\rangle_{mn}$$

$$+ \frac{N_u}{\epsilon} \sum_{x\in\{i,j\}} \int_0^1 \frac{\mathrm{d}\omega}{\omega^{1+2\epsilon}} \left\langle \left(1 - \theta^{b+d} C_{mn}\right) \left[\mathrm{d}\Omega_{mn}\right] \bar{C}_{xmn}\, w^{mx,nx} \psi_{mn}^{4\epsilon} \bar{S}_\omega \left[\tilde{S}_{ij}^{gg}\right] \right\rangle_{mn},$$

where $\bar{C}_{xmn} = I - C_{xmn}$ and $[\mathrm{d}\Omega_{mn}] = [\mathrm{d}\Omega_m][\mathrm{d}\Omega_n]$. The idea is to calculate the first two terms explicitly, and expand the last integrand in ϵ.