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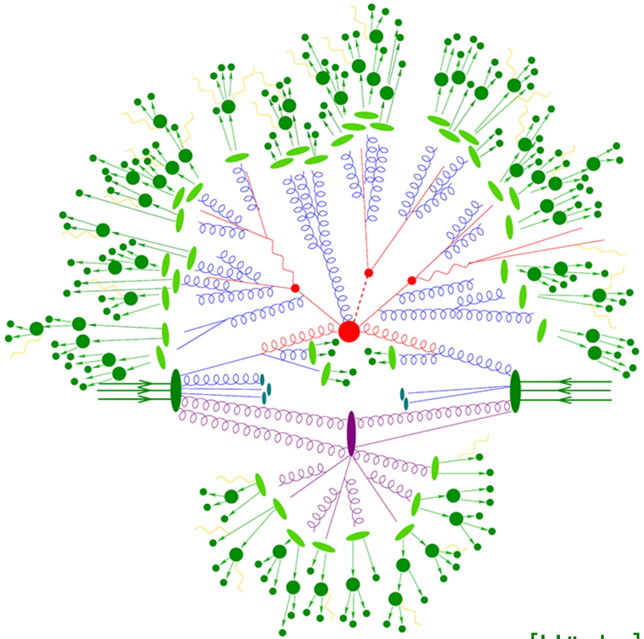
Status of two-loop automation in OpenLoops

M. F. Zoller

Loops and Legs in Quantum Field Theory, 18 April 2024, Wittenberg

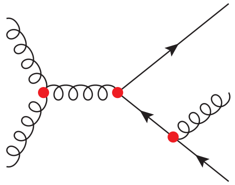
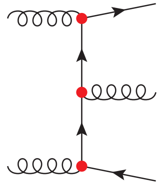
Scattering amplitudes in perturbation theory

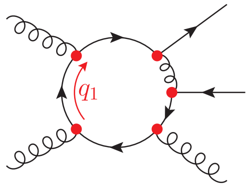


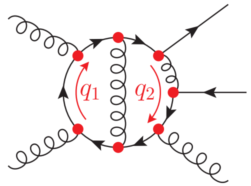
[Höche]

Hard scattering amplitudes for Monte Carlo simulations are computed in perturbation theory from matrix elements

$$\bar{\mathcal{M}} = \bar{\mathcal{M}}_0 + \bar{\mathcal{M}}_1 + \bar{\mathcal{M}}_2 + \dots$$

with $\bar{\mathcal{M}}_0 \;=$  $+$  $+ \dots,$

$\bar{\mathcal{M}}_1 \;=$ 

$+ \dots,\quad \bar{\mathcal{M}}_2 =$ 

$+ \dots$

Partonic cross sections computed from colour- and helicity-summed **scattering probability density**

$$\mathcal{W} \;=\; \underbrace{\bar{\sum_{h,\mathrm{col}}}}_{\text{colour and helicity sum with average and symmetry factor}} \left|\mathbf{R}\bar{\mathcal{M}}\right|^2 \;=\; \bar{\sum_{h,\mathrm{col}}}\left\{\underbrace{|\bar{\mathcal{M}}_0|^2}_{\textbf{LO}} + \underbrace{2\,\mathrm{Re}\Big[\bar{\mathcal{M}}_0^*\,\mathbf{R}\bar{\mathcal{M}}_1\Big]}_{\textbf{NLO virtual}} + \underbrace{|\mathbf{R}\bar{\mathcal{M}}_1|^2 + 2\,\mathrm{Re}\Big[\bar{\mathcal{M}}_0^*\,\mathbf{R}\bar{\mathcal{M}}_2\Big]}_{\textbf{NNLO virtual-virtual}} + \;\dots\right\}$$

with UV divergences subtracted by the renormalisation procedure $\mathbf{R}\,\bar{\mathcal{M}} = \bar{\mathcal{M}}_0 + \mathbf{R}\,\bar{\mathcal{M}}_1 + \mathbf{R}\,\bar{\mathcal{M}}_2 + \dots$

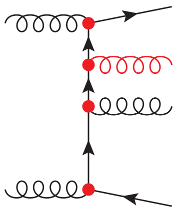
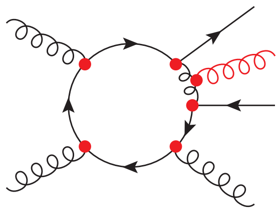
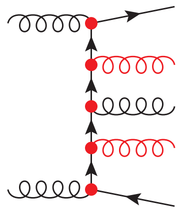
Scattering amplitudes in perturbation theory

Finite partonic cross sections require factorisation of initial-state collinear singularities into PDFs, and addition of **real-emission contributions** to cancel final-state collinear and soft divergences

$$\hat{\sigma} = \underbrace{\int \mathrm{d}\Phi_N}_{\text{N-particle phase space integration, flux factor}} \mathcal{W} + \sum_X \underbrace{\int \mathrm{d}\Phi_{N+X} \, \mathcal{W}^{(X)}}_{\text{contribution with X extra unresolved particles}}$$

with the real-emission **scattering probability densities** up to NNLO

$$\mathcal{W}^{(1)} = \bar{\sum_{h,\mathrm{col}}} \Bigg\{ \underbrace{|\bar{\mathcal{M}}_0^{(1)}|^2}_{\textbf{NLO real}} + \underbrace{2\,\mathrm{Re}\Big[\Big(\bar{\mathcal{M}}_0^{(1)}\Big)^* \, \mathbf{R}\bar{\mathcal{M}}_1^{(1)}\Big]}_{\textbf{NNLO real-virtual}} + \dots \Bigg\}, \qquad \mathcal{W}^{(2)} = \bar{\sum_{h,\mathrm{col}}} \Bigg\{ \underbrace{|\bar{\mathcal{M}}_0^{(2)}|^2}_{\textbf{NNLO real-real}} + \dots \Bigg\}$$

where $\bar{\mathcal{M}}_0^{(1)} =$  $+ \dots,$ $\bar{\mathcal{M}}_1^{(1)} =$  $+ \dots,$ $\bar{\mathcal{M}}_0^{(2)} =$  $+ \dots$

Challenges in automation of numerical NNLO calculations:

▷ **Real-virtual contributions** require **excellent numerical stability** in soft and collinear regions

▷ **Automated calculations of virtual-virtual part** $2\,\mathrm{Re}[\bar{\mathcal{M}}_0^* \, \mathbf{R}\bar{\mathcal{M}}_2]$

Outline

I. OpenLoops (tree-level and one-loop public version)

II. Automated numerical calculation of scattering amplitudes
$\rightarrow$ Strategy at one and two loops

III. Status of two-loop amplitudes in OpenLoops

(i) Tensor coefficients

(ii) Tensor integrals

(iii) Rational terms

IV. Summary and Outlook

I. OpenLoops

OpenLoops [Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, M.Z.] is a fully automated numerical tool for the computation of **scattering probability densities** from tree and one-loop amplitudes

$$\mathcal{W}_{00} = \sum_{h,\mathrm{col}}^{-} |\bar{\mathcal{M}}_0|^2, \qquad \mathcal{W}_{01} = \sum_{h,\mathrm{col}}^{-} 2\,\mathrm{Re}\Big[\bar{\mathcal{M}}_0^{*}\mathbf{R}\bar{\mathcal{M}}_1\Big], \qquad \mathcal{W}_{11} = \sum_{h,\mathrm{col}}^{-} |\mathbf{R}\bar{\mathcal{M}}_1|^2$$

Download from `https://gitlab.com/openloops/OpenLoops.git`

- **Full NLO QCD and NLO EW corrections available**
- **Efficient calculation of colour and helicity sums in squared amplitudes**
- **Excellent CPU performance and numerical stability** due to
  - On-the-fly tensor integral reduction
  - Expansions to any order in critical kinematic variables
  - Hybrid-precision mode (targeted use of quadruple precision, bulk in a double precision)

→ **Real-emission contributions up to NNLO** used e.g. in
  - Matrix [Grazzini, Kallweit, Wiesemann]
  - NNLOJeT [Gauld, Glover, Huss, Majer, Gehrmann-De Ridder]
  - McMule [Banerjee, Engel, Signer, Ulrich] ← NNLO QED

New feature: QED with OpenLoops

in collaboration with J. Lindert

- Separation of electromagnetic and weak contributions for given order in α
- Implementation of three massive lepton generations
- Calculations with variable number of lepton and/or quark generations

Governed by three OpenLoops parameters (dedicated process libraries required)

`qed`	active α-corrections	$=$ 0 (full EW), 1 (pure QED), 3 (pure weak)
`nf`	number of active quarks	$=$ 0,4,5,6
`nfl`	number of active lepton generations	$=$ 0,1,2,3

$\Rightarrow$ **QED calculations with different lepton masses available**

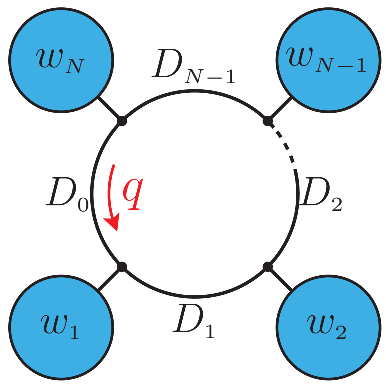
Recent NNLO applications with McMule [Banerjee, Engel, Signer, Ulrich]

- Bhabba and Møller scattering (one lepton mass) [Banerjee, Engel, Schalch, Signer, Ulrich, 2021; 2022]
- Muon-electron scattering at NNLO (two different lepton masses) [Broggio, Engel, Ferroglia, Mandal, Mastrolia, Rocco, Ronca, Signer, Torres Bobadilla, Ulrich, M.Z., 2023]
  $\rightarrow$ **Complete and fully differential NNLO calculation of a $2 \rightarrow 2$ process with two different non-zero masses on the external lines**

II. Automated numerical calculation of scattering amplitudes

Feynman integrals in $D = 4 - 2\varepsilon$ dimensions to regularise divergences, e.g. one-loop diagram Γ:

$$\bar{\mathcal{M}}_{1,\Gamma} = \underbrace{C_{1,\Gamma}}_{\text{colour factor}} \int \mathrm{d}\bar{q}_1 \frac{\bar{\mathcal{N}}(\bar{q}_1)}{\mathcal{D}(\bar{q}_1)} =$$



$$\mathcal{D}(\bar{q}_1) = \prod_{i=0}^{N-1} D_k(\bar{q}_1),$$

$$D_k(\bar{q}_1) = (\bar{q}_1 + p_k)^2 - m_k^2,$$

$$\int \mathrm{d}\bar{q}_1 = \mu^{2\varepsilon} \int \frac{\mathrm{d}^D \bar{q}_1}{(2\pi)^D}$$

- **Numerical tools**, such as OpenLoops [Buccioni et al], Recola [Actis et al], MadLoop [Hirschi et al], Helac [Bevilacqua et al] **construct the numerator in 4 dimensions** $\Rightarrow$ Split numerator

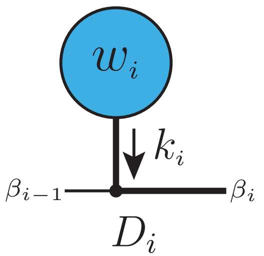
$$\underbrace{\bar{\mathcal{N}}(\bar{q}_1)}_{D-\text{dim}} = \underbrace{\mathcal{N}(q_1)}_{4-\text{dim}} + \underbrace{\tilde{\mathcal{N}}(\bar{q}_1)}_{(D-4)-\text{dim}} \quad \text{with } \mathcal{N}(q_1) = \bar{\mathcal{N}}(\bar{q}_1)\Big|_{\substack{\bar{q}_i \to q_i, \\ \bar{\gamma}^{\bar{\mu}} \to \gamma^{\mu}, \\ \bar{g}^{\bar{\mu}\bar{\nu}} \to g^{\mu\nu}}}$$

(project D-dim $\to$ 4-dim)

- **Requirement for automation: Construct amplitude from process-independent elements**

Exploit **factorisation** into **universal building blocks**: $\mathcal{N}(q_1) = S_1(q_1) \ldots S_N(q_1)$

with loop segments $S_i(q_1) =$



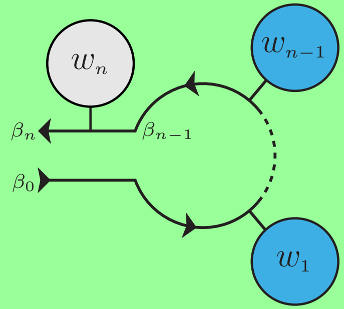
$$= \underbrace{\left\{ Y_\sigma^i + Z_{\nu;\sigma}^i \, q_1^\nu \right\}}_{\text{loop vertex and propagator}} \quad \underbrace{[w_i]^\sigma}_{\text{external subtree}}$$

Automation strategy at one loop in OpenLoops

$$\bar{\mathcal{M}}_{1,\Gamma} = \underbrace{\mathcal{N}_{\mu_1\cdots\mu_N}}_{\text{4-dim coefficient}} \underbrace{\int \mathrm{d}^D q \frac{q^{\mu_1}\cdots q^{\mu_N}}{D_0(q)\cdots D_{N-1}(q)}}_{\text{tensor integral}} + \underbrace{\int \mathrm{d}^D q \frac{\tilde{\mathcal{N}}}{D_0(q)\cdots D_{N-1}(q)}}_{(D-4)\text{-dim numerator}}$$

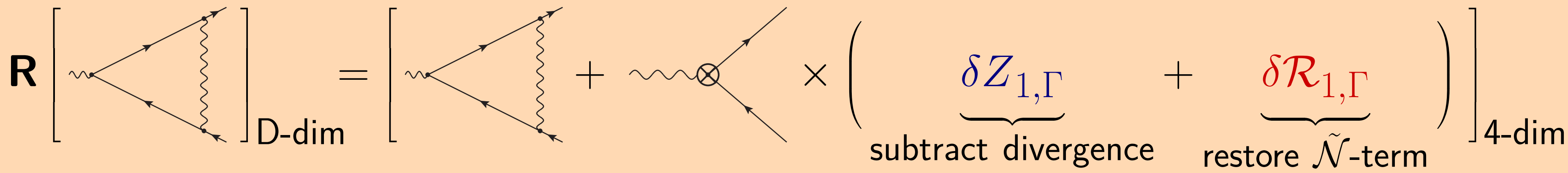
Recursive construction of tensor coefficients from the segments of the cut-opened loop [van Hameren; Cascioli, Maierhöfer, Pozzorini; Buccioni, Lang, Lindert, Maierhöfer, Pozzorini, Zhang, M.Z.]

$$\mathcal{N}_n(q) = \mathcal{N}_{n-1}(q) \cdot S_n(q) =$$

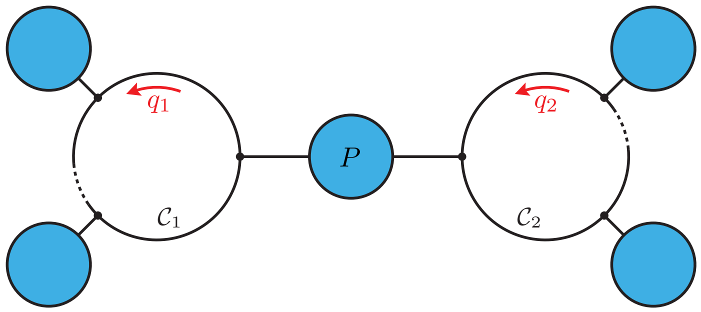


Tensor integrals: On-the-fly reduction [Buccioni, Pozzorini, M.Z] and external tools: Collier [Denner, Dittmaier, Hofer], OneLoop [van Hameren]

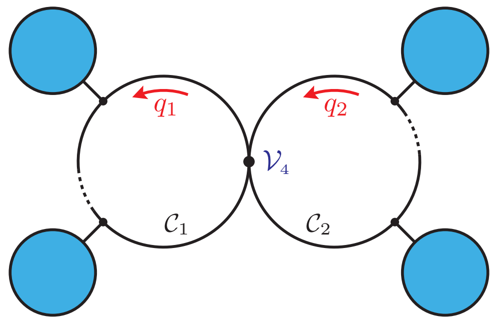
Restoration of $(D-4)$-dim numerator parts together with renormalization procedure **R** through **universal rational counterterms** [Ossola, Papadopoulos, Pittau]



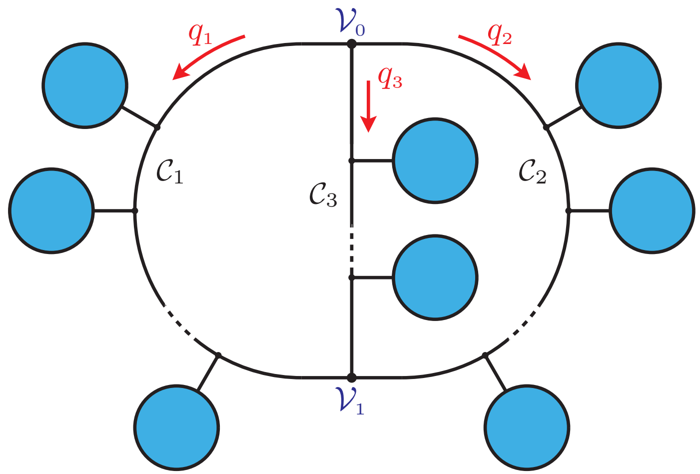
Two-loop diagrams



(Red2)



(Red1)



(Irreducible)

Diagrams consist of **loop chains** $\mathcal{C}_i$, **each depending on a single loop momentum** q_i.

Types of diagrams:

- **Reducible diagrams:** Two factorised loop integrals
  - **Red2:** Two loop chains $\mathcal{C}_1, \mathcal{C}_2$ connected by a tree-like bridge P.
  - **Red1:** Two loop chains $\mathcal{C}_1, \mathcal{C}_2$ connected by a single quartic vertex $\mathcal{V}_4$

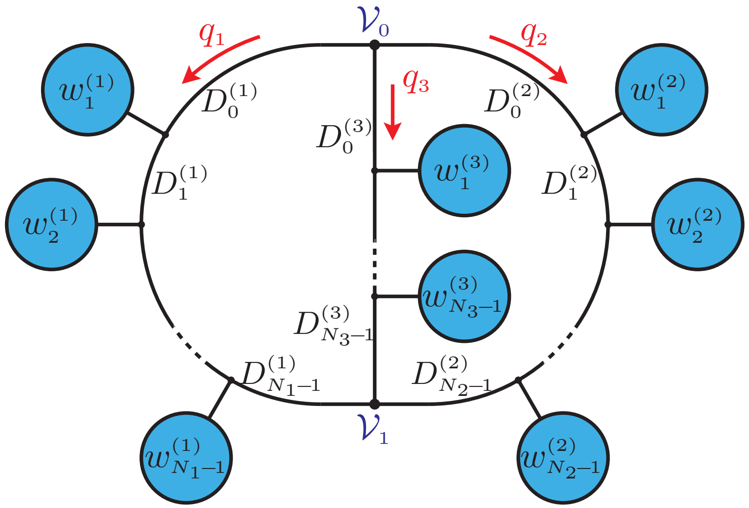
  Extension of one-loop OpenLoops $\rightarrow$ Fully implemented

- **Irreducible diagrams:** Three loop chains $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3$ with loop momenta $q_1, q_2, q_3 = -(q_1 + q_2)$ and two connecting vertices $\mathcal{V}_0, \mathcal{V}_1$

Irreducible two-loop diagrams

Irreducible two-loop diagram Γ (1PI on amputation of all external subtrees):

$\bar{\mathcal{M}}_{2,\Gamma} =$



$$= \mathcal{C}_{2,\Gamma} \int \mathrm{d}^D q_1 \int \mathrm{d}^D q_2 \frac{\bar{\mathcal{N}}(q_1, q_2)}{\prod_{i=1}^{3} \mathcal{D}^{(i)}(q_i)} \Bigg|_{q_3 \to -(q_1+q_2)}$$

- **Numerical calculation in integer dimensions $\Rightarrow$ Split numerator**

$$\underbrace{\bar{\mathcal{N}}(\bar{q}_1, \bar{q}_2)}_{D-\text{dim}} = \underbrace{\mathcal{N}(q_1, q_2)}_{4-\text{dim}} + \underbrace{\tilde{\mathcal{N}}(\bar{q}_1, \bar{q}_2)}_{(D-4)-\text{dim}} \quad \text{with } \mathcal{N}(q_1, q_2) = \bar{\mathcal{N}}(\bar{q}_1, \bar{q}_2)\Bigg|_{\substack{\bar{q}_i \to q_i, \\ \bar{\gamma}^{\bar{\mu}} \to \gamma^{\mu}, \\ \bar{g}^{\bar{\mu}\bar{\nu}} \to g^{\mu\nu}}}$$

- **Exploit factorisation** into **universal building blocks**

▷ **Numerator** $\mathcal{N}(q_1, q_2) = \prod_{i=1}^{3} \mathcal{N}^{(i)}(q_i) \prod_{j=0}^{1} \mathcal{V}_j(q_1, q_2)$ with $\mathcal{N}^{(i)}(q_i) = S_0^{(i)}(q_i) \cdots S_{N_i-1}^{(i)}(q_i)$

▷ **Denominators** $\mathcal{D}^{(i)}(q_i) = D_0^{(i)}(q_i) \cdots D_{N_i-1}^{(i)}(q_i)$ where $D_a^{(i)}(q_i) = (q_i + p_{ia})^2 - m_{ia}^2$

(External momenta p_{ia} and masses m_{ia} along i-th chain)

III. Automation strategy at two loops

$$\bar{\mathcal{M}}_{2,\Gamma} = \mathcal{N}_{\mu_1\cdots\mu_r\nu_1\cdots\nu_s} \int \mathrm{d}^D q_1 \int \mathrm{d}^D q_2 \, \frac{q_1^{\mu_1}\cdots q_1^{\mu_r} q_2^{\nu_1}\cdots q_2^{\nu_s}}{\prod_{i,j} D_j^{(i)}(q_i)} + \int \mathrm{d}^D q_1 \int \mathrm{d}^D q_2 \, \frac{\tilde{\mathcal{N}}}{\prod_{i,j} D_j^{(i)}(q_i)} \Bigg|_{q_3=-(q_1+q_2)}$$

Numerical construction of 4-dim tensor coefficients

Completely general recursive algorithm [Pozzorini, Schär, M.Z.] with steps

$\mathcal{N}_n(q_1, q_2) \;=\; \mathcal{N}_{n-1}(q_1, q_2) \cdot \mathcal{K}_n$ where $\mathcal{K}_n \in \{S_n^{(i)}(q_i), \mathcal{V}_{0,1}(q_1, q_2)\}$

Reduction of tensor integrals $\longrightarrow$ scalar integrals $\mathcal{I}_k$ $\longrightarrow$ master integrals $\mathcal{I}_l^{\mathrm{M}}$

$\rightarrow$ Evaluation of master integrals with external tools

Bottle neck of NNLO automation $\rightarrow$ Main focus of our current projects

Restoration of $(D-4)$-dim numerator parts from **universal two-loop rational terms**

[Lang, Pozzorini, Zhang, M.Z.] stemming from the interplay of $\tilde{\mathcal{N}}$ with UV and IR divergences.

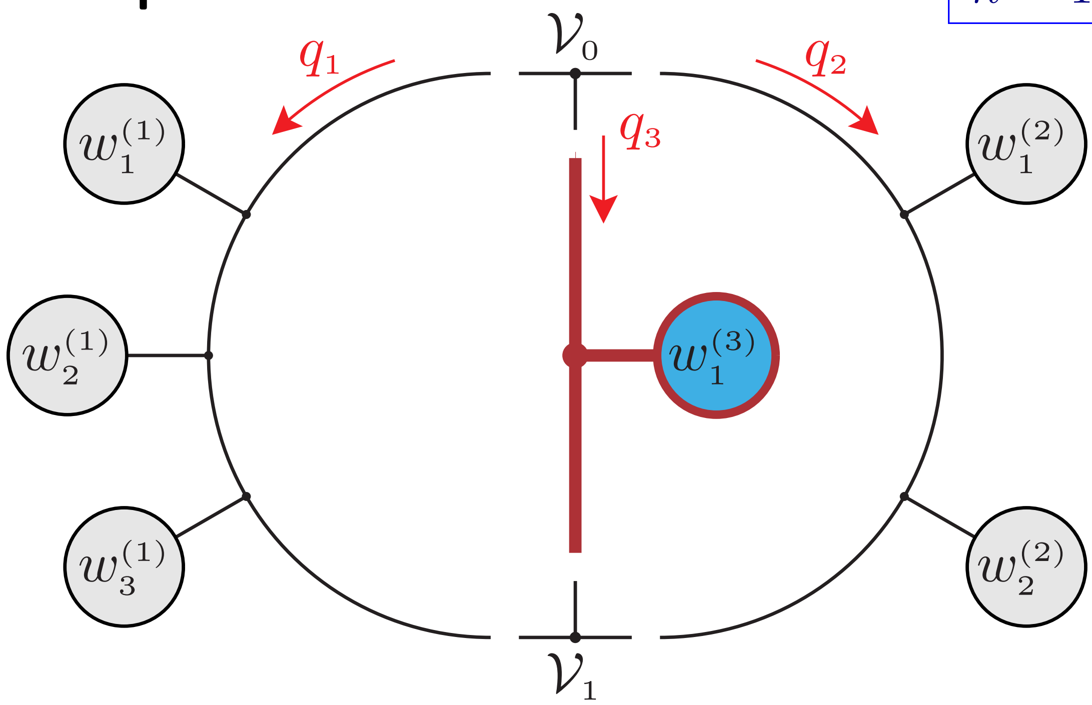
$\rightarrow$ together with renormalisation procedure via counterterm insertions in lower-loop diagrams

Two-loop tensor coefficients (irreducible diagrams)

- Sort chains by length: $N_1 \geq N_2 \geq N_3$
  Choose order of $\mathcal{V}_0, \mathcal{V}_1$ by vertex type
- Dress $\mathcal{N}^{(3)}$ (shortest chain)

Example:

$n = 1$



$$\mathcal{N}_n^{(3)}(q_3) = \mathcal{N}_{n-1}^{(3)}(q_3) \cdot S_n^{(3)}(q_3) \qquad \text{with initial condition } \mathcal{N}_{-1}^{(3)} = \mathbb{1}$$

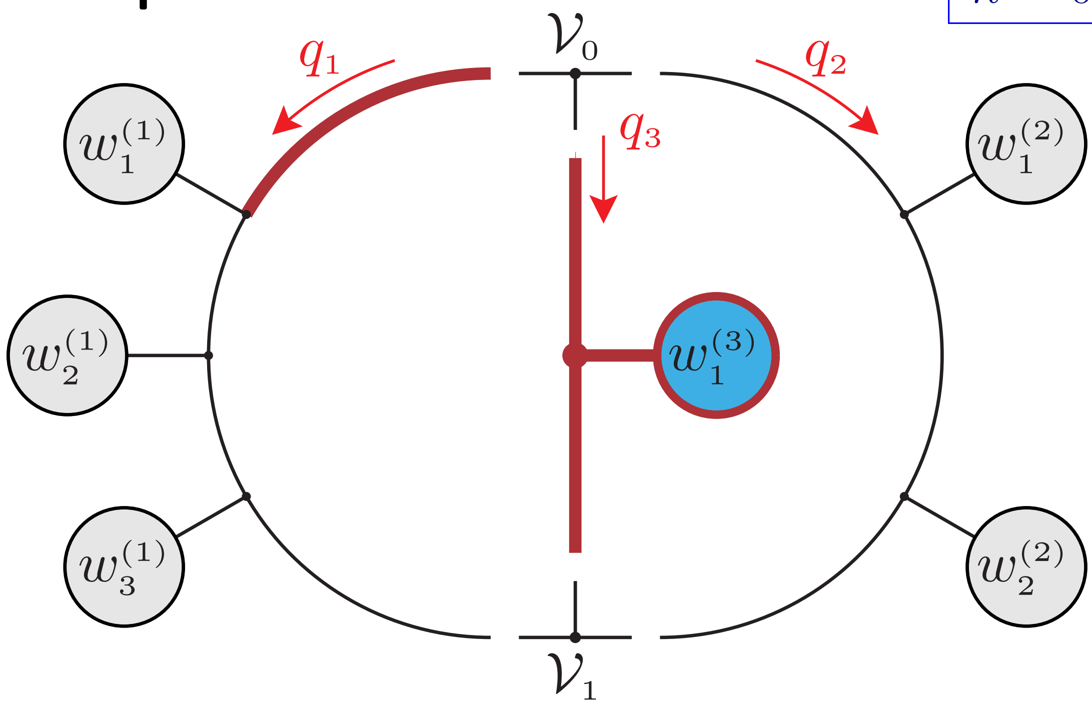
Partial chains $\mathcal{N}_n^{(3)}$ computed only once for multiple diagrams

Two-loop tensor coefficients (irreducible diagrams)

- Sort chains by length: $N_1 \geq N_2 \geq N_3$
  Choose order of $\mathcal{V}_0, \mathcal{V}_1$ by vertex type
- Dress $\mathcal{N}^{(3)}$ (shortest chain)
- Dress $\mathcal{U}^{(1)} \propto \mathcal{M}_0^* \mathcal{N}^{(1)}$ (longest chain)

Example:

$n = 0$



$$\mathcal{U}_n^{(1)}(q_1, \check{h}_n^{(1)}) = \sum_{h_n^{(1)}} \mathcal{U}_{n-1}^{(1)}(q_1, \check{h}_{n-1}^{(1)}) \cdot S_n^{(1)}(q_1, h_n^{(1)}) \qquad \text{with} \quad \mathcal{U}_{-1}^{(1)}(h) = 2\Big(\sum_{\text{col}} \underbrace{\mathcal{M}_0^*(h)}_{\text{Born}} \underbrace{C_{2,\Gamma}}_{\text{colour}}\Big)$$

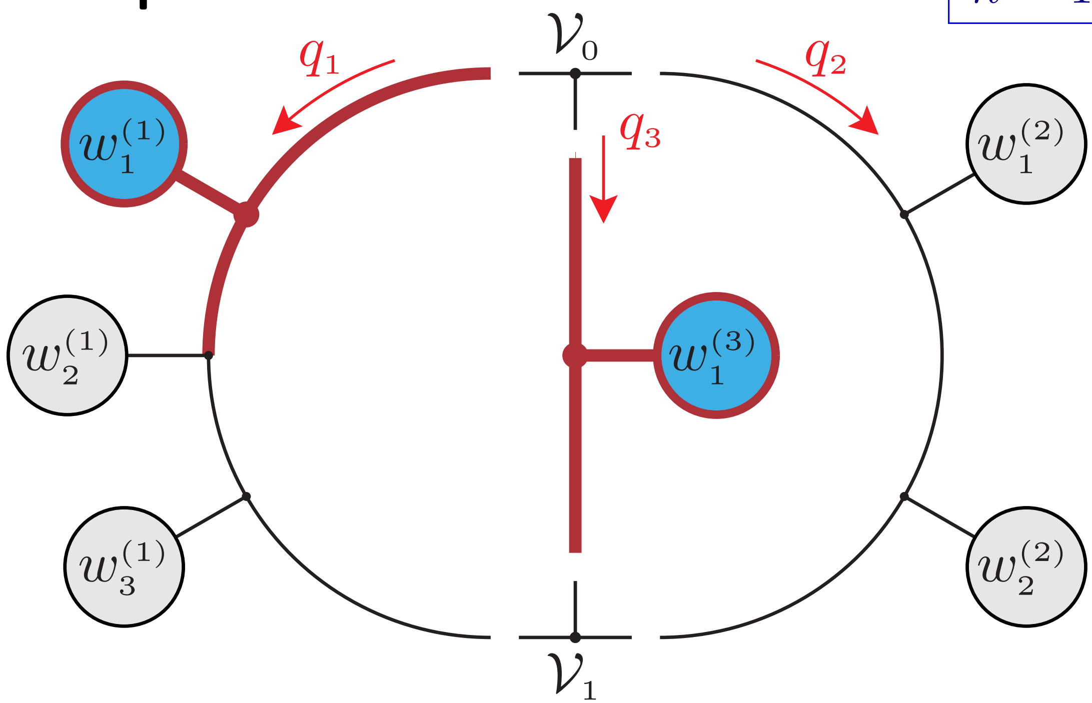
Initial building block: Born-colour interference depending on helicity h of all external particles

Two-loop tensor coefficients (irreducible diagrams)

- Sort chains by length: $N_1 \geq N_2 \geq N_3$
  Choose order of $\mathcal{V}_0, \mathcal{V}_1$ by vertex type
- Dress $\mathcal{N}^{(3)}$ (shortest chain)
- Dress $\mathcal{U}^{(1)} \propto \mathcal{M}_0^* \mathcal{N}^{(1)}$ (longest chain)

Example:

$n = 1$



$$\mathcal{U}_n^{(1)}(q_1, \check{h}_n^{(1)}) = \sum_{h_n^{(1)}} \mathcal{U}_{n-1}^{(1)}(q_1, \check{h}_{n-1}^{(1)}) \cdot S_n^{(1)}(q_1, h_n^{(1)}) \qquad \text{with} \quad \mathcal{U}_{-1}^{(1)}(h) = 2\Big(\sum_{\text{col}} \underbrace{\mathcal{M}_0^*(h)}_{\text{Born}} \underbrace{C_{2,\Gamma}}_{\text{colour}}\Big)$$

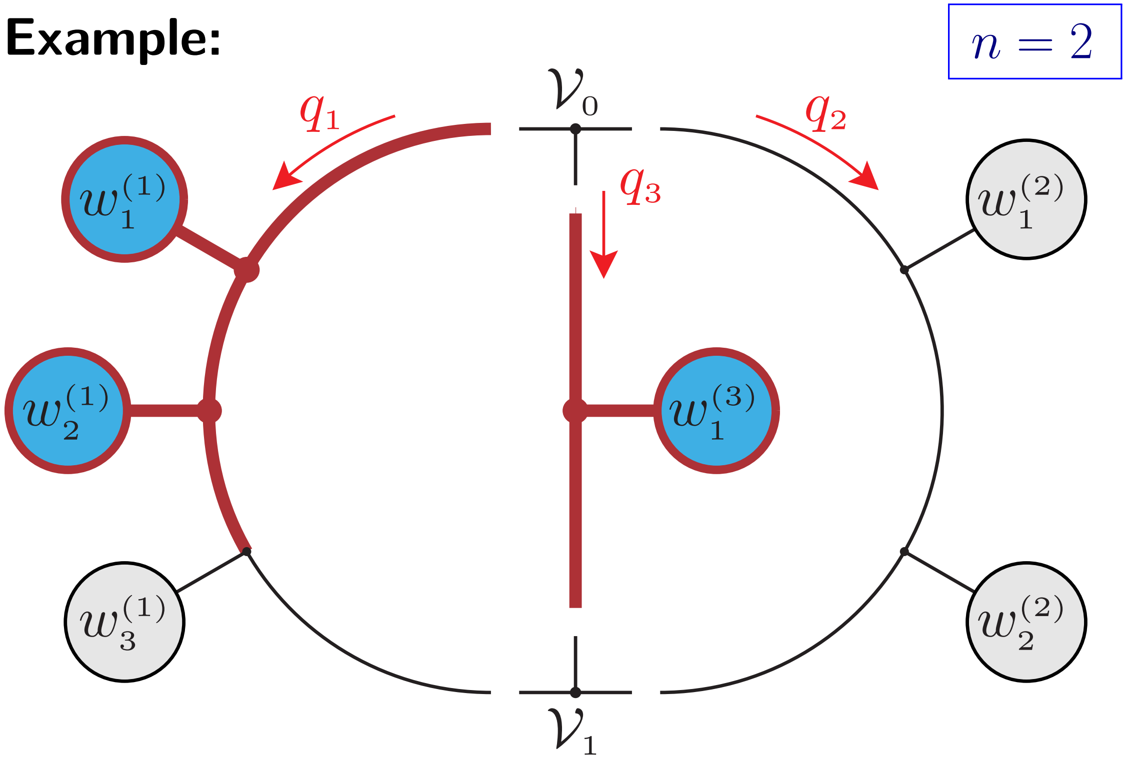
On-the-fly summation of segment helicities $h_n^{(1)}$

$\rightarrow$ Constructed object depends on helicities of remaining (undressed) segments of the diagram $\check{h}_n^{(1)}$

Two-loop tensor coefficients (irreducible diagrams)

- Sort chains by length: $N_1 \geq N_2 \geq N_3$
  Choose order of $\mathcal{V}_0, \mathcal{V}_1$ by vertex type
- Dress $\mathcal{N}^{(3)}$ (shortest chain)
- Dress $\mathcal{U}^{(1)} \propto \mathcal{M}_0^* \mathcal{N}^{(1)}$ (longest chain)

Example:



$$\mathcal{U}_n^{(1)}(q_1, \check{h}_n^{(1)}) = \sum_{h_n^{(1)}} \mathcal{U}_{n-1}^{(1)}(q_1, \check{h}_{n-1}^{(1)}) \cdot S_n^{(1)}(q_1, h_n^{(1)}) \qquad \text{with} \quad \mathcal{U}_{-1}^{(1)}(h) = 2\Big(\sum_{\text{col}} \underbrace{\mathcal{M}_0^*(h)}_{\text{Born}} \underbrace{C_{2,\Gamma}}_{\text{colour}}\Big)$$

On-the-fly summation of segment helicities $h_n^{(1)}$

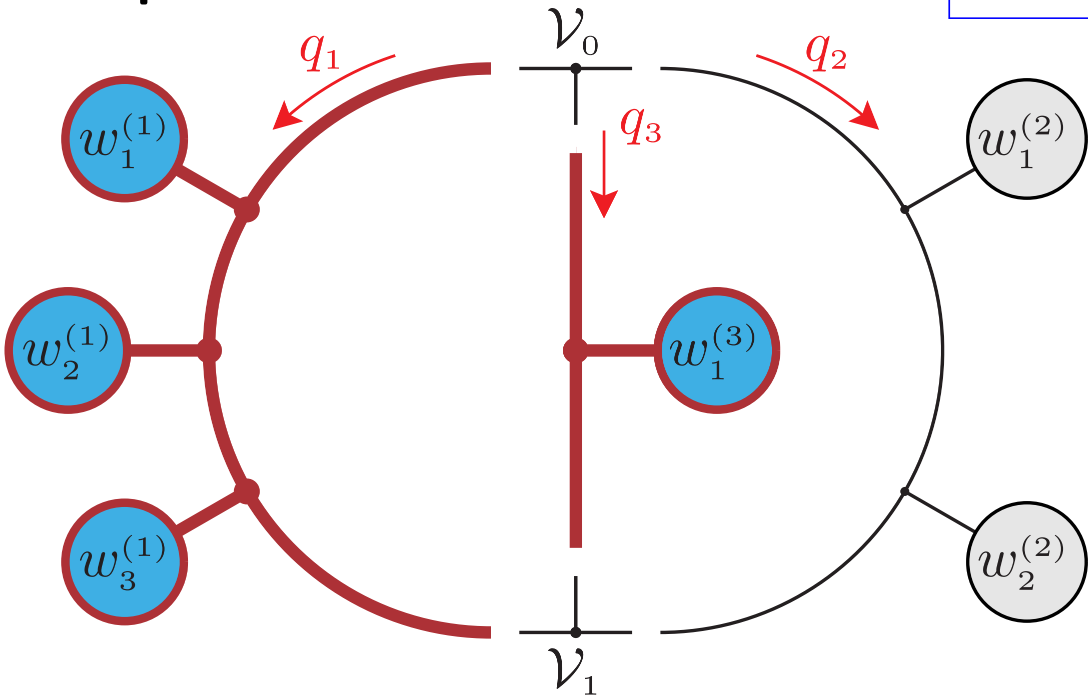
→ Constructed object depends on helicities of remaining (undressed) segments of the diagram $\check{h}_n^{(1)}$

Two-loop tensor coefficients (irreducible diagrams)

- Sort chains by length: $N_1 \geq N_2 \geq N_3$
  Choose order of $\mathcal{V}_0, \mathcal{V}_1$ by vertex type
- Dress $\mathcal{N}^{(3)}$ (shortest chain)
- Dress $\mathcal{U}^{(1)} \propto \mathcal{M}_0^* \mathcal{N}^{(1)}$ (longest chain)

Example:

$n = 3$



$$\mathcal{U}_n^{(1)}(q_1, \check{h}_n^{(1)}) = \sum_{h_n^{(1)}} \mathcal{U}_{n-1}^{(1)}(q_1, \check{h}_{n-1}^{(1)}) \cdot S_n^{(1)}(q_1, h_n^{(1)}) \qquad \text{with} \quad \mathcal{U}_{-1}^{(1)}(h) = 2\Big(\sum_{\text{col}} \underbrace{\mathcal{M}_0^*(h)}_{\text{Born}} \underbrace{C_{2,\Gamma}}_{\text{colour}}\Big)$$

On-the-fly summation of segment helicities $h_n^{(1)}$

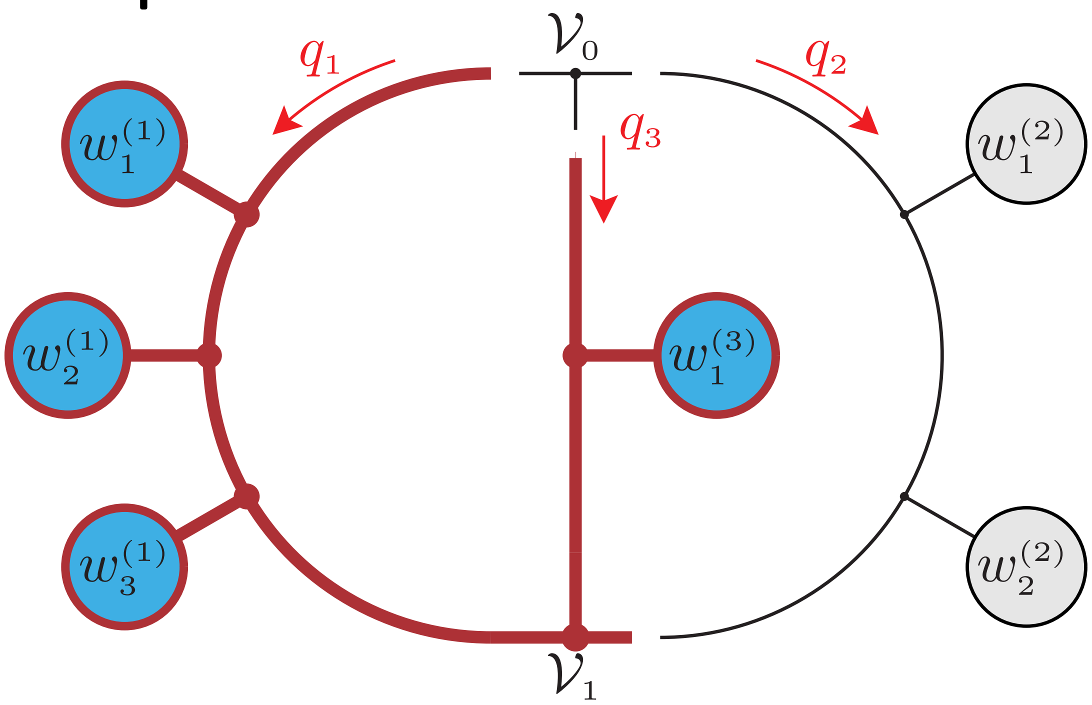
$\rightarrow$ Constructed object depends on helicities of remaining (undressed) segments of the diagram $\check{h}_n^{(1)}$

$\Rightarrow$ Most helicity d.o.f already summed at stage with low tensor rank complexity

Two-loop tensor coefficients (irreducible diagrams)

- Sort chains by length: $N_1 \geq N_2 \geq N_3$
  Choose order of $\mathcal{V}_0, \mathcal{V}_1$ by vertex type
- Dress $\mathcal{N}^{(3)}$ (shortest chain)
- Dress $\mathcal{U}^{(1)} \propto \mathcal{M}_0^* \mathcal{N}^{(1)}$ (longest chain)
- Connect $\mathcal{V}_1$ with $\mathcal{U}^{(1)}$ and $\mathcal{N}^{(3)}$

Example:



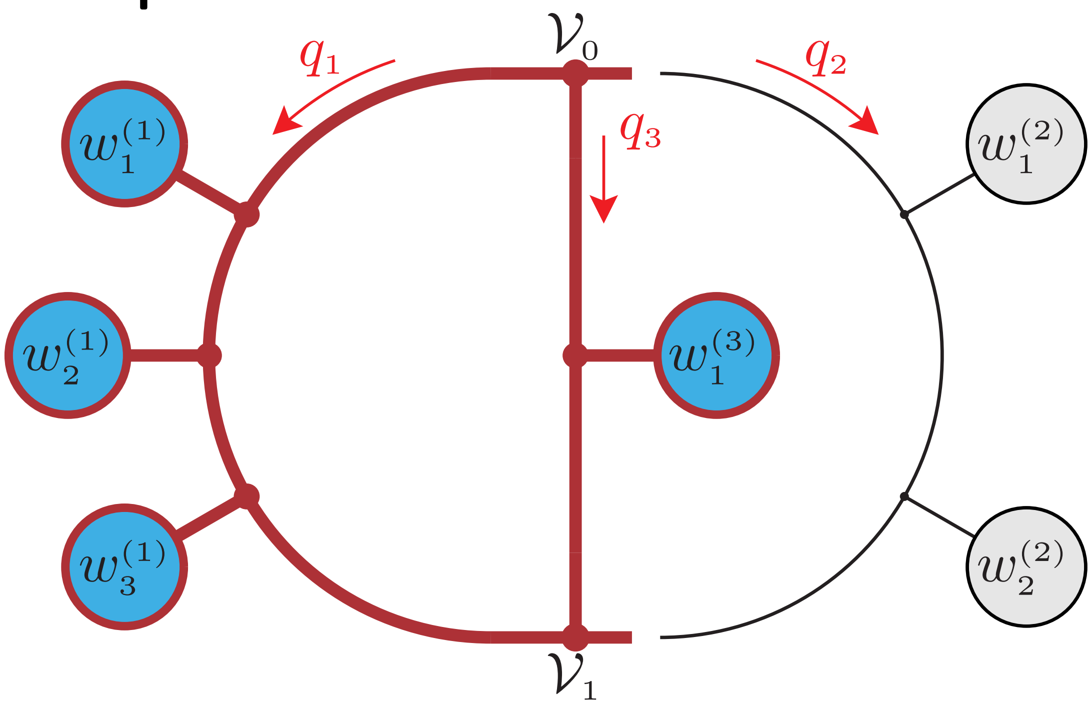
$$\mathcal{U}_1^{(13)}(q_1, q_3, h^{(2)}) = \sum_{h^{(3)}} \mathcal{U}^{(1)}(q_1, \check{h}^{(1)}_{N_1-1})\, \mathcal{N}^{(3)}(q_3, h^{(3)})\, \mathcal{V}_1(q_1, q_3)$$

Highest complexity step due to dependence on 3 open indices and 2 loop momenta
$\rightarrow$ performed at lowest rank in q_2 and for only a few unsummed helicity configurations

Two-loop tensor coefficients (irreducible diagrams)

- Sort chains by length: $N_1 \geq N_2 \geq N_3$
  Choose order of $\mathcal{V}_0, \mathcal{V}_1$ by vertex type
- Dress $\mathcal{N}^{(3)}$ (shortest chain)
- Dress $\mathcal{U}^{(1)} \propto \mathcal{M}_0^* \mathcal{N}^{(1)}$ (longest chain)
- Connect $\mathcal{V}_1$ with $\mathcal{U}^{(1)}$ and $\mathcal{N}^{(3)}$
- Connect $\mathcal{V}_0$ and map $q_3 \to -(q_1 + q_2)$

Example:



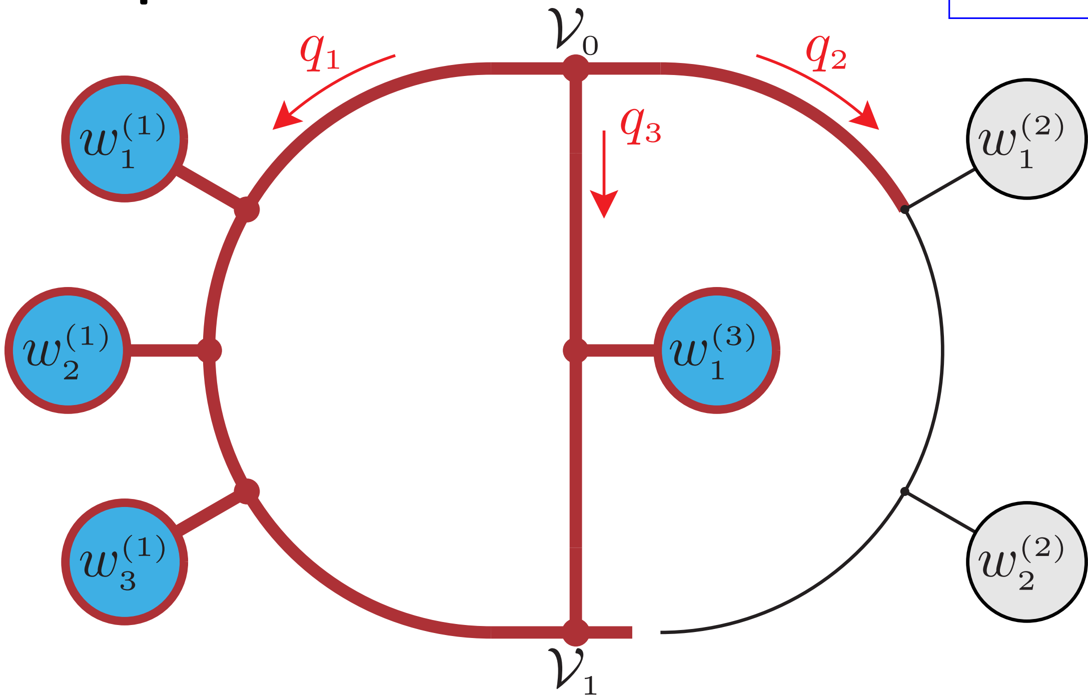
$$\mathcal{U}_{-1}^{(123)}(q_1, q_2, h^{(2)}) = \mathcal{U}_1^{(13)}(q_1, q_3, h^{(2)})\ \mathcal{V}_0(q_1, q_2)\Big|_{q_3 \to -(q_1+q_2)}$$

Two-loop tensor coefficients (irreducible diagrams)

- Sort chains by length: $N_1 \geq N_2 \geq N_3$
  Choose order of $\mathcal{V}_0, \mathcal{V}_1$ by vertex type
- Dress $\mathcal{N}^{(3)}$ (shortest chain)
- Dress $\mathcal{U}^{(1)} \propto \mathcal{M}_0^* \mathcal{N}^{(1)}$ (longest chain)
- Connect $\mathcal{V}_1$ with $\mathcal{U}^{(1)}$ and $\mathcal{N}^{(3)}$
- Connect $\mathcal{V}_0$ and map $q_3 \to -(q_1 + q_2)$
- Connect segments of $\mathcal{N}^{(2)}$

Example:

$n = 0$



$$\mathcal{U}_n^{(123)}(q_1, q_2, \tilde{h}_n^{(2)}) = \sum_{h_n^{(2)}} \mathcal{U}_{n-1}^{(123)}(q_1, q_2, \tilde{h}_{n-1}^{(2)})\, S_n^{(2)}(q_2, h_n^{(2)})$$

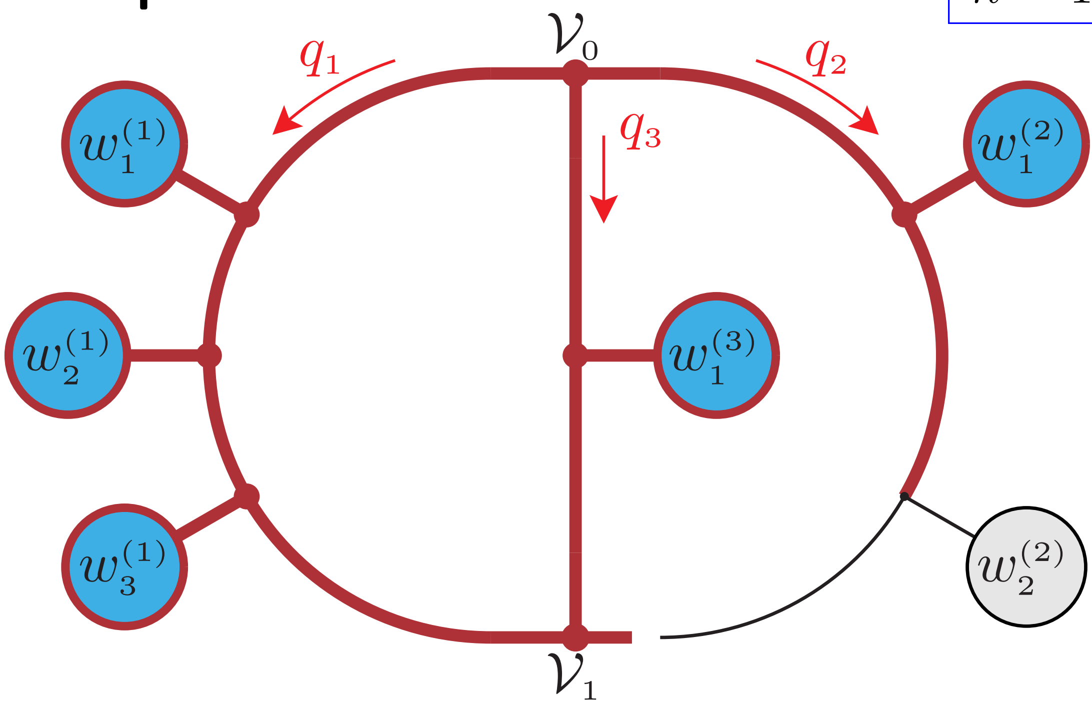
On-the-fly summation of segment helicities $h_n^{(2)}$
$\to$ Constructed object depends on helicities of remaining (undressed) segments of the diagram $\tilde{h}_n^{(2)}$

Two-loop tensor coefficients (irreducible diagrams)

- Sort chains by length: $N_1 \geq N_2 \geq N_3$
  Choose order of $\mathcal{V}_0, \mathcal{V}_1$ by vertex type
- Dress $\mathcal{N}^{(3)}$ (shortest chain)
- Dress $\mathcal{U}^{(1)} \propto \mathcal{M}_0^* \mathcal{N}^{(1)}$ (longest chain)
- Connect $\mathcal{V}_1$ with $\mathcal{U}^{(1)}$ and $\mathcal{N}^{(3)}$
- Connect $\mathcal{V}_0$ and map $q_3 \to -(q_1 + q_2)$
- Connect segments of $\mathcal{N}^{(2)}$

Example:

$n = 1$



$$\mathcal{U}_n^{(123)}(q_1, q_2, \tilde{h}_n^{(2)}) = \sum_{h_n^{(2)}} \mathcal{U}_{n-1}^{(123)}(q_1, q_2, \tilde{h}_{n-1}^{(2)})\, S_n^{(2)}(q_2, h_n^{(2)})$$

On-the-fly summation of segment helicities $h_n^{(2)}$

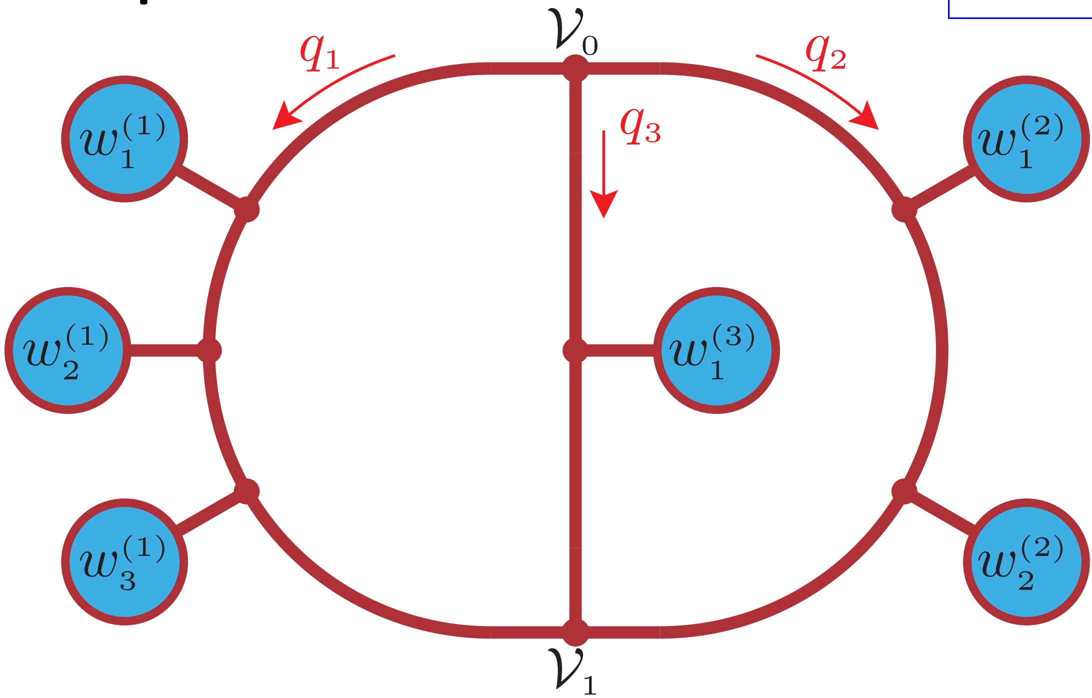
$\rightarrow$ Constructed object depends on helicities of remaining (undressed) segments of the diagram $\tilde{h}_n^{(2)}$

Two-loop tensor coefficients (irreducible diagrams)

- Sort chains by length: $N_1 \geq N_2 \geq N_3$
  Choose order of $\mathcal{V}_0, \mathcal{V}_1$ by vertex type
- Dress $\mathcal{N}^{(3)}$ (shortest chain)
- Dress $\mathcal{U}^{(1)} \propto \mathcal{M}_0^* \mathcal{N}^{(1)}$ (longest chain)
- Connect $\mathcal{V}_1$ with $\mathcal{U}^{(1)}$ and $\mathcal{N}^{(3)}$
- Connect $\mathcal{V}_0$ and map $q_3 \to -(q_1 + q_2)$
- Connect segments of $\mathcal{N}^{(2)}$

Example:

$n = 2$



$$\mathcal{U}_n^{(123)}(q_1, q_2, \tilde{h}_n^{(2)}) = \sum_{h_n^{(2)}} \mathcal{U}_{n-1}^{(123)}(q_1, q_2, \tilde{h}_{n-1}^{(2)})\; S_n^{(2)}(q_2, h_n^{(2)})$$

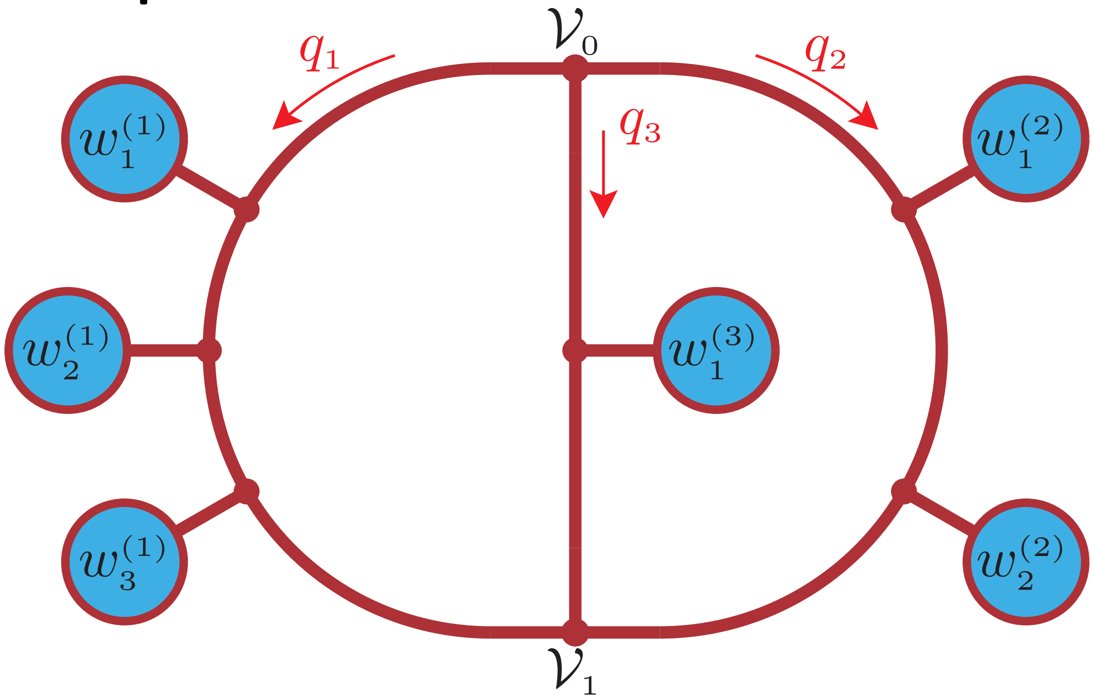
On-the-fly summation of segment helicities $h_n^{(2)}$
$\to$ Constructed object depends on helicities of remaining (undressed) segments of the diagram $\tilde{h}_n^{(2)}$

$\Rightarrow$ Lowest complexity in helicities for steps with highest rank in loop momenta

Two-loop tensor coefficients (irreducible diagrams)

- Sort chains by length: $N_1 \geq N_2 \geq N_3$
  Choose order of $\mathcal{V}_0, \mathcal{V}_1$ by vertex type
- Dress $\mathcal{N}^{(3)}$ (shortest chain)
- Dress $\mathcal{U}^{(1)} \propto \mathcal{M}_0^* \mathcal{N}^{(1)}$ (longest chain)
- Connect $\mathcal{V}_1$ with $\mathcal{U}^{(1)}$ and $\mathcal{N}^{(3)}$
- Connect $\mathcal{V}_0$ and map $q_3 \to -(q_1 + q_2)$
- Connect segments of $\mathcal{N}^{(2)}$

Example:

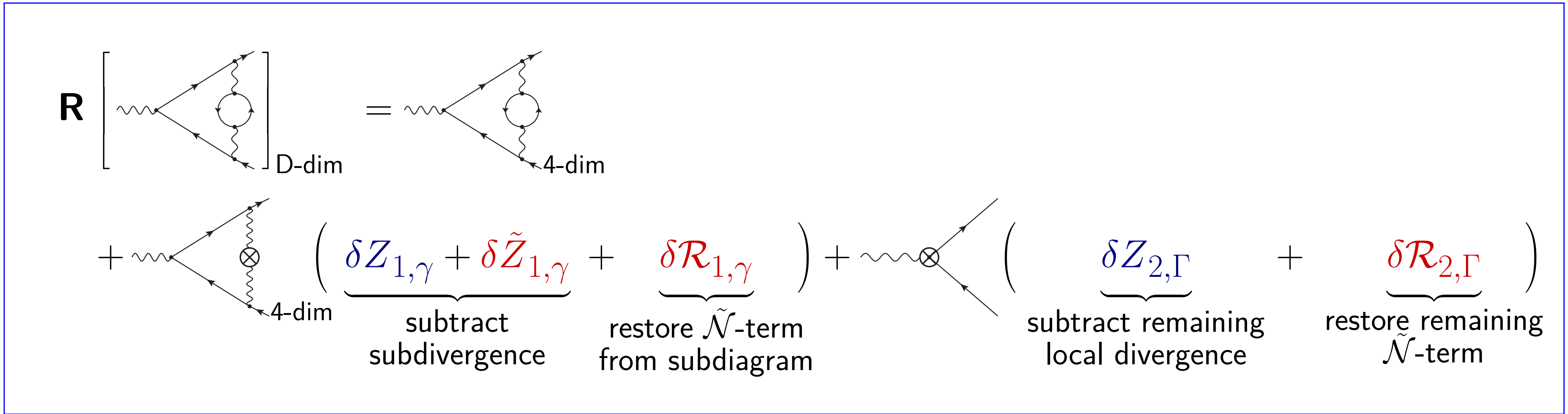


Highly efficient and completely general algorithm for two-loop tensor coefficients

Fully implemented for QED and QCD corrections to the SM

Two-loop rational terms

Renormalised D-dim amplitudes from amplitudes with 4-dim numerator [Pozzorini, Zhang, M.Z.]



Consider UV poles:

- Divergence from subdiagram γ and remaining global one subtracted by usual UV counterterm $\delta Z_{1,\gamma}, \delta Z_{2,\Gamma}$. Additional UV counterterm $\delta\tilde{Z}_{1,\gamma} \propto \frac{\tilde{q}_1^{\,2}}{\varepsilon}$ for subdiagrams with mass dimension 2.

- $\delta\mathcal{R}_{2,\Gamma}$ is a **two-loop rational term** stemming from the **interplay of $\tilde{N}$ with poles**

  $\Rightarrow$ **Finite set of process-independent rational terms for UV divergent vertex functions**

Status of two-loop rational terms

Renormalised D-dim amplitudes can be computed from amplitudes with 4-dim numerators and a **finite set of universal UV and rational counterterms** inserted lower-loop amplitudes

$$\mathbf{R}\,\bar{\mathcal{M}}_{2,\Gamma} \;=\; \mathcal{M}_{2,\Gamma} + \sum_{\gamma}\left(\delta Z_{1,\gamma} + \delta\tilde{Z}_{1,\gamma} + \delta\mathcal{R}_{1,\gamma}\right)\cdot\mathcal{M}_{1,\Gamma/\gamma} + \left(\delta Z_{2,\Gamma} + \delta\mathcal{R}_{2,\Gamma}\right)$$

Rational terms of UV origin

- **General method for the computation** of rational counterterms of UV origin from simple tadpole integrals in any renormalisable model [Pozzorini, Zhang, M.Z.,2020]
- **Complete renormalisation scheme dependence** [Lang, Pozzorini, Zhang, M.Z.,2020]
- **Rational Terms for Spontaneously Broken Theories** [Lang, Pozzorini, Zhang, M.Z.,2021]
- **Full set of two-loop rational terms** computed for
  - QED with full dependence on the gauge parameter [Pozzorini, Zhang, M.Z.,2020]
  - $SU(N)$ and $U(1)$ in any renormalisation scheme [Lang, Pozzorini, Zhang, M.Z.,2020]
  - **QED and QCD corrections to the full SM** [Lang, Pozzorini, Zhang, M.Z.,2021] ✓

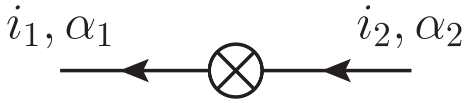
Rational terms of IR origin (ongoing projects): Treat IR subtracted full amplitude through modification of rational terms $\left[\delta\mathcal{R}_{1,\gamma} \text{ at } \mathcal{O}(\varepsilon) \to \delta\mathcal{R}_{2,\gamma} \text{ at } \mathcal{O}(1)\right]$ or of Catani-Seymour **I**-operator

$\rightarrow$ to be published soon for QED [Pozzorini, Zhang]

Sructure of two-loop rational terms of UV origin

Example: Two-point function of a fermion f in SU(N) or U(1) model

with Casimirs $C_\mathrm{F}, C_\mathrm{A}$ and fundamental trace T_F and dimension N in Feynman gauge ($\lambda = 1$, $\mathcal{Z}_\mathrm{gp} = \frac{\mathcal{Z}_A}{\mathcal{Z}_\lambda}$)



$$= \mathrm{i} \underbrace{\delta_{i_1 i_2}}_{\text{gauge group structure}} \left\{ \sum_{k=1}^{2} \left(\frac{\alpha_s t^\varepsilon}{4\pi} \right)^k \left[\delta\hat{\mathcal{R}}^{(\mathrm{P})}_{k,\mathrm{ff}} \not{p}_{\alpha_1\alpha_2} + \delta\hat{\mathcal{R}}^{(\mathrm{m})}_{k,\mathrm{ff}} m_f \delta_{\alpha_1\alpha_2} \right] \right\},$$

$$\delta\hat{\mathcal{R}}^{(\mathrm{P})}_{1,\mathrm{ff}} = -C_\mathrm{F},$$

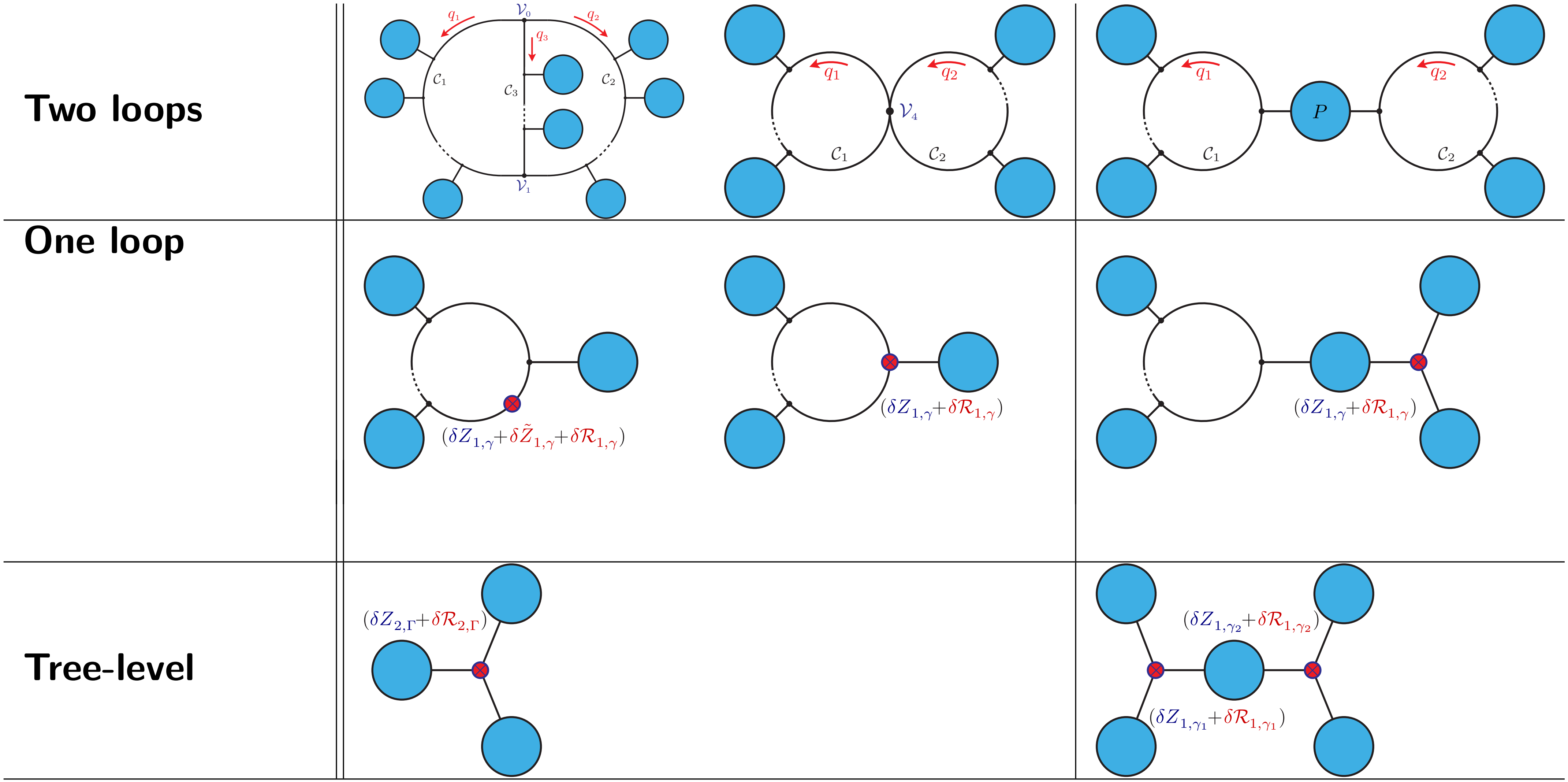
$$\delta\hat{\mathcal{R}}^{(\mathrm{P})}_{2,\mathrm{ff}} = \left(\frac{7}{6} C_\mathrm{F}^2 - \frac{61}{36} C_\mathrm{A} C_\mathrm{F} + \frac{5}{9} T_\mathrm{F} n_\mathrm{f} C_\mathrm{F} \right) \frac{1}{\varepsilon} + \left(\frac{43}{36} C_\mathrm{F}^2 - \frac{1087}{216} C_\mathrm{A} C_\mathrm{F} + \frac{59}{54} T_\mathrm{F} n_\mathrm{f} C_\mathrm{F} \right)$$
$$- C_\mathrm{F} \left(\underbrace{\delta\hat{\mathcal{Z}}_{1,\alpha_s} + \frac{2}{3} \delta\hat{\mathcal{Z}}_{1,f} - \frac{2}{3} \delta\hat{\mathcal{Z}}_{1,\mathrm{gp}}}_{\text{Renormalisation scheme dependendent}} \right)$$

Similarly for $\delta\hat{\mathcal{R}}^{(\mathrm{m})}_{1,\mathrm{ff}}, \mathcal{R}^{(\mathrm{m})}_{2,\mathrm{ff}}$

- **Interaction of $\tilde{\mathcal{N}}$ with $\frac{1}{\varepsilon^2}$ poles leads to rational terms $\propto \frac{1}{\varepsilon}$**
- Rational terms **depend trivially on the scale factor** t^ε of the renormalisation scheme
- At two loops: Non-trivial **dependence on the renormalisation scheme** can be fully expressed in terms of the **one-loop UV counterterms** $\mathcal{Z}_{1,\chi} = \left(\frac{\alpha t^\varepsilon}{4\pi}\right) \delta\hat{\mathcal{Z}}_{1,\chi}$

Two-loop renormalisation and UV rational terms

Ingredients for full two-loop calculation: in collaboration with N. Schär

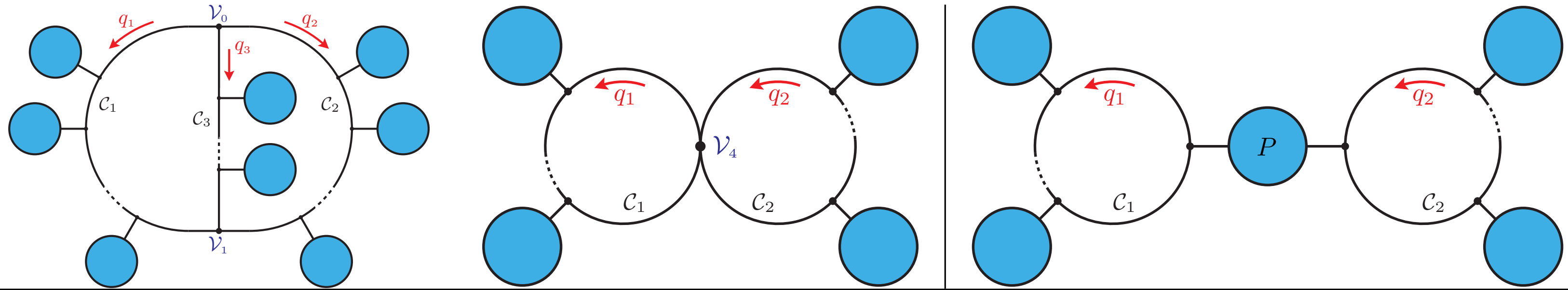


- Poles numerically implemented as parameter $\Delta = \frac{1}{\varepsilon}$ with default $\Delta = 0$ $\quad\rightarrow$ Finite part
- Pole parts computed and pole cancellation checked through variation of $\Delta = 0, 1, -1, 2, -2$.

Two-loop renormalisation and UV rational terms

Ingredients for full two-loop calculation: in collaboration with N. Schär

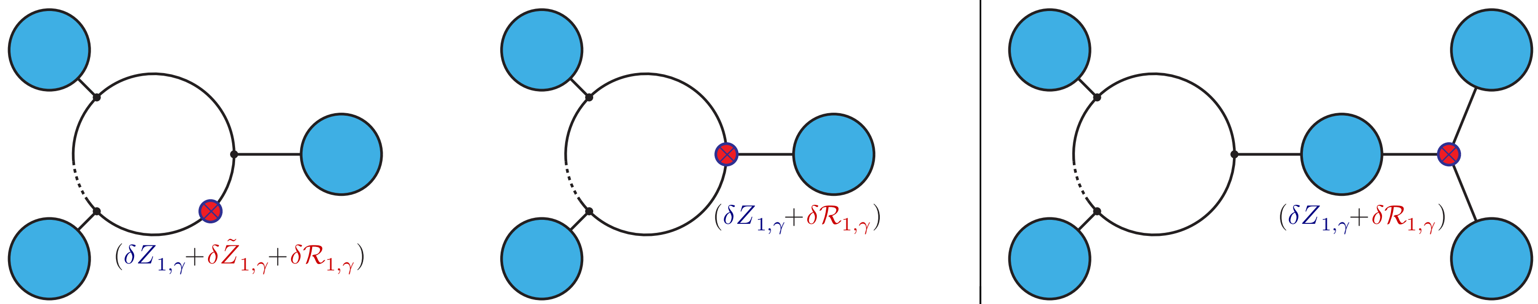
Two loops



One loop

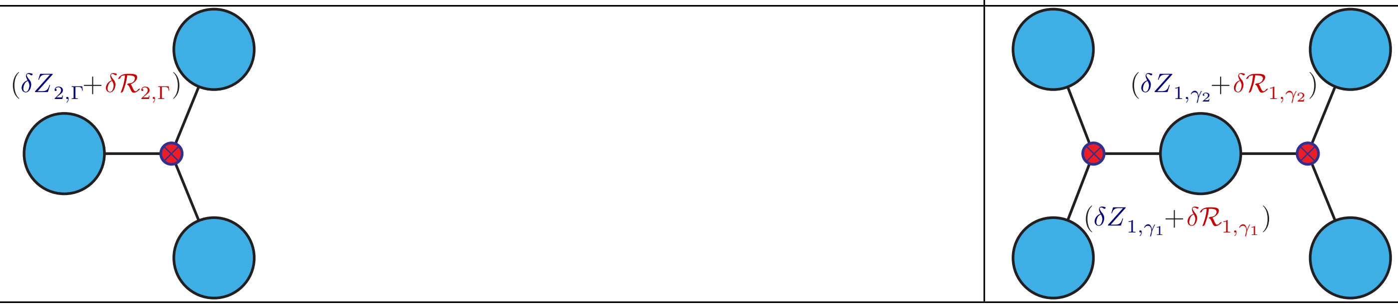
- $\delta\mathcal{Z} = \frac{A}{\varepsilon} + B$ interplay with $\mathcal{O}(\varepsilon)$ of integrals
- $\delta\mathcal{Z}, \delta\tilde{\mathcal{Z}}$ double tensor coefficient complexity

⇒ Insertion in last OpenLoops step



- squared scalar propagator D_0
- integrals $\propto \frac{\tilde{q}^2}{\varepsilon}$

Tree-level



- **Generation and combination of all ingredients automated in OpenLoops framework**
- **Implemented for QED and QCD counterterms** (currently $\overline{\text{MS}}$, but easily extendable)

Validation of two-loop renormalisation and UV rational terms

in collaboration with N. Schär

Validation requires full amplitude calculation and hence tensor integrals

- Compute off-shell amplitudes to avoid IR divergences
- In-house library for simple tensor integrals (currently 2 independent external momenta, massless)
- **Validation** of implementation + **first application of two-loop rational terms** in two steps:

1. **Check cancellation of UV poles** $\rightarrow$ non-trivial since $\delta\mathcal{R}_{2,\Gamma}$ has $\frac{1}{\varepsilon}$ pole
   $\rightarrow$ Successfully completed for several processes
2. **Computation of finite parts of amplitudes** (in progress)
   $\rightarrow$ Computation of off-shell two-loop QCD vertex functions with two-loop OpenLoops
   $\rightarrow$ Comparison against literature [Gracey]

Two-loop tensor integral reduction

In-house tool for validation purposes and simple processes in collaboration with N. Schär

- Covariant decomposition of final result, e.g.

$$I^{\mu;\nu} = \int \mathrm{d}\bar{q}_1 \int \mathrm{d}\bar{q}_2 \frac{q_1^\mu q_2^\nu}{\mathcal{D}(q_1, q_2, \{k_i\})} = A\, g^{\mu\nu} + \sum_i B_{ij}\, k_i^\mu k_j^\nu$$

- Define projector for each tensor structure, here $P_a^{\mu\nu} \in \{g_{\mu\nu}, k_i^\mu k_j^\nu\}$
  $\Rightarrow$ System of equations from $P_a^{\mu\nu}$ applied to both sides of $I_{\mu;\nu}$
  $\Rightarrow$ Solve for $A, B_{ij}, \ldots$ (expressed in terms of scalar integrals)

- FIRE [Smirnov, Chukharev] for IBP reduction [Chetyrkin, Tkachov; Laporta] of scalar to Master integrals

- Perform ε-expansion and store expressions in FORTRAN library

- Analytical expressions for Master integrals [Birthwright, Glover, Marquard] implemented or computed with FIESTA [Smirnov]

Largely automated and easy to extend for more topologies (more legs, masses)
In practice limited due to large systems of equations in matrix inversion and IBP reduction.

$\rightarrow$ **More efficient method and tool for higher-point topologies and higher tensor ranks being developed** $\rightarrow$ current project with Fabian Lange

V. Summary and Outlook

Challenges in automation of numerical NNLO calculations

▷ **Real-virtual part** → OpenLoops offers excellent numerical stability, efficiency and flexibility

▷ **Two-loop amplitudes**

Numerical construction of 4-dim tensor coefficients

- Completely general recursive algorithm
- **Highly efficient and fully implemented for QED and QCD corrections to SM** ✓

Reduction of tensor integrals to master integrals

- In-house tool for simple topologies → validation of renormalisation and rational terms
- New algorithm and tool for higher-point and higher-rank integrals under development

Renormalisation and restoration of $(D-4)$-dim numerator parts

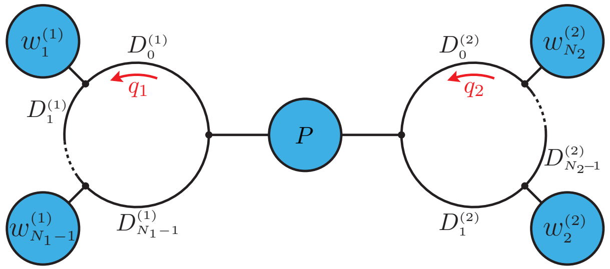
- Rational terms of UV origin:
  - **General method proven**
  - **Computed for QED and QCD corrections to SM**
- UV and rational counterterms **implemented in OpenLoops framework** for QED and QCD corrections to SM
- Rational terms of IR origin: Ongoing project (✓)

Backup

Reducible two-loop diagrams

Reducible diagram Γ **factorises** into one-loop diagrams and a tree-like bridge P (or quartic vertex)

$$\bar{\mathcal{M}}_{2,\Gamma} \quad = $$

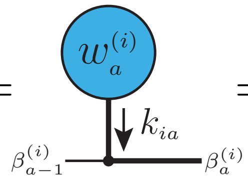


$$= C_{2,\Gamma}\, P_{\alpha_1\alpha_2} \prod_{i=1}^{2} \int \mathrm{d}\bar{q}_i \, \frac{\left[\bar{\mathcal{N}}^{(i)}(q_i)\right]^{\alpha_i}}{\mathcal{D}^{(i)}(\bar{q}_i)}$$

with $\mathcal{D}^{(i)}(\bar{q}_i) = D_0^{(i)}(\bar{q}_i) \cdots D_{N_i-1}^{(i)}(\bar{q}_i)\,, \qquad D_a^{(i)}(\bar{q}_i) \;=\; (\bar{q}_i + p_{ia})^2 - m_{ia}^2$

Loop numerators factorise into segments

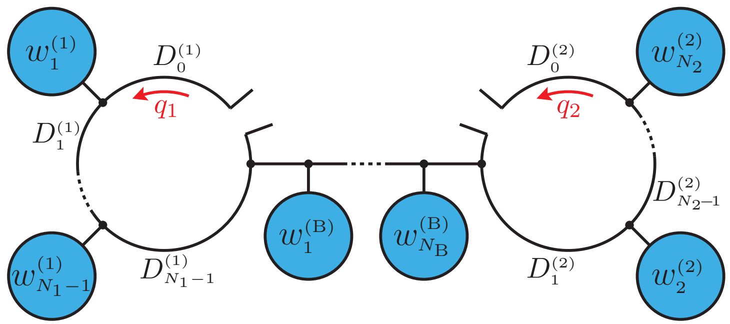
$$S_a^{(i)}(q_i, h_a^{(i)}) = $$



$$= \underbrace{\left\{Y_\sigma^a(k_{ia}, p_{ia}) + Z_{\nu;\sigma}^i \, q_i^\nu\right\}}_{\text{Feynman rule of loop vertex and propagator}} \quad \underbrace{\left[w_a^{(i)}(h_a^{(i)})\right]^\sigma}_{\text{external subtree with helicity configuration } h_a^{(i)}}$$

- Cut-open both loops and dress first one
- Close and integrate first loop, attach bridge
- Use first loop + bridge as "subtree" for second loop

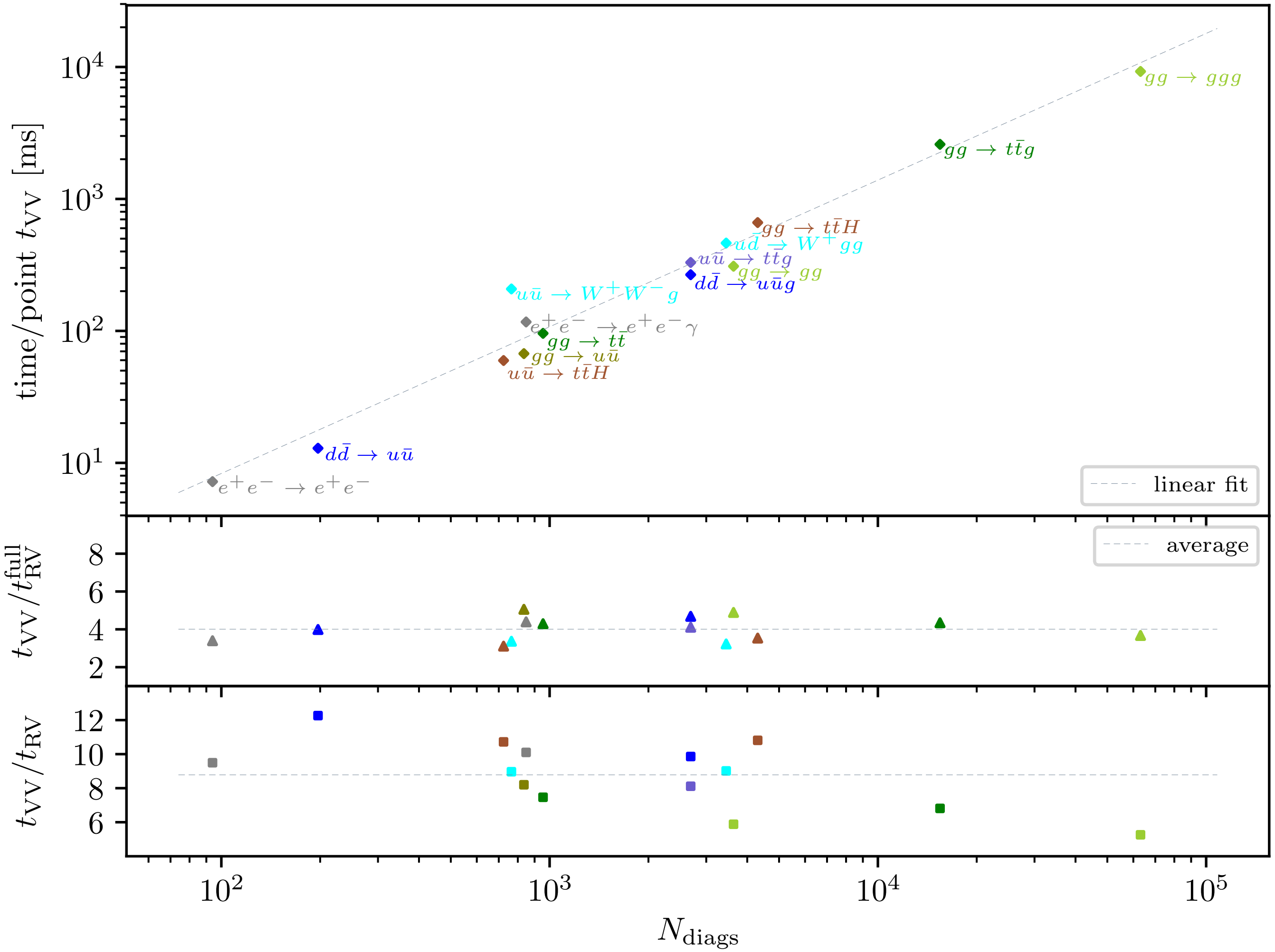
$\Rightarrow$ **Extension of the tree and one-loop algorithm**



Fully implemented for QED and QCD corrections to the SM

Timings for two-loop tensor coefficients

QED, QCD and SM (NNLO QCD) processes (single Intel i7-6600U @ 2.6 GHz, 16GB RAM, 10^3 points)



$2 \to 2$ process: $10 - 300$ ms/psp

$2 \to 3$ process: $65 - 9200$ ms/psp (on a laptop)

Runtime $\propto$ number of diagrams

time/psp/diagram $\sim 150\mu s$

Constant ratios between virtual–virtual (VV) and real-virtual (RV) with and without 1-loop integrals

- tensor coefficients: $\frac{t_{VV}}{t_{RV}} \sim 9$
- full RV: $\frac{t_{VV}}{t_{RV}^{full}} \sim 4$

Strong CPU performance, comparable to real-virtual corrections in OpenLoops

Processes considered in performance tests

corrections	process type	massless fermions	massive fermions	process
QED	$2 \to 2$	e	–	$e^+e^- \to e^+e^-$
	$2 \to 3$	e	–	$e^+e^- \to e^+e^-\gamma$
QCD	$2 \to 2$	u	–	$gg \to u\bar{u}$
		u, d	–	$d\bar{d} \to u\bar{u}$
		u	–	$gg \to gg$
		u	t	$u\bar{u} \to t\bar{t}g$
		u	t	$gg \to t\bar{t}$
		u	t	$gg \to t\bar{t}g$
	$2 \to 3$	u, d	–	$d\bar{d} \to u\bar{u}g$
		u	–	$gg \to ggg$
		u, d	–	$u\bar{d} \to W^+gg$
		u, d	–	$u\bar{u} \to W^+W^-g$
		u	t	$u\bar{u} \to t\bar{t}H$
		u	t	$gg \to t\bar{t}H$

Memory usage of the algorithm for two-loop tensor coefficients

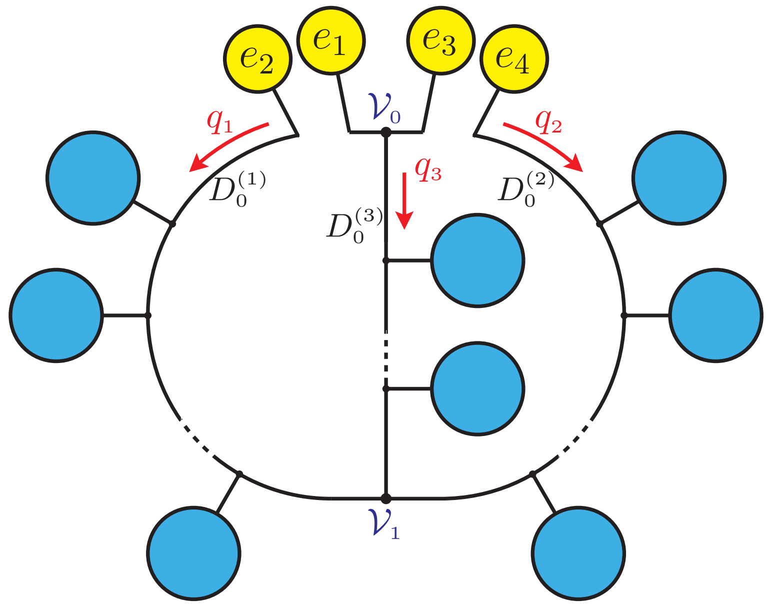
	virtual–virtual memory [MB]		real–virtual [MB]	
hard process	segment-by-segment	diagram-by-diagram	coefficients	full
$e^+e^- \to e^+e^-$	18	8	6	23
$e^+e^- \to e^+e^-\gamma$	154	25	22	54
$gg \to u\bar{u}$	75	31	10	26
$gg \to t\bar{t}$	94	35	15	34
$gg \to t\bar{t}g$	2000	441	152	213
$u\bar{d} \to W^+gg$	563	143	54	90
$u\bar{u} \to W^+W^-g$	264	67	36	67
$u\bar{u} \to t\bar{t}H$	82	28	14	40
$gg \to t\bar{t}H$	604	145	50	90
$u\bar{u} \to t\bar{t}g$	323	83	41	74
$gg \to gg$	271	94	41	55
$d\bar{d} \to u\bar{u}$	18	10	9	20
$d\bar{d} \to u\bar{u}g$	288	85	39	68
$gg \to ggg$	6299	1597	623	683

Numerical stability of two-loop tensor coefficients

Pseudo-tree test

- Cut-open diagram at two propagators
- Saturate indices with random wavefunctions $e_1, \ldots, e_4$
- Evaluate integrand constructed with new two-loop algorithm at fixed values for q_1, q_2

  $\Rightarrow \widehat{\mathcal{W}}^{(2L)}_{02,\Gamma} = \frac{U(q_1,q_2)}{\mathcal{D}(q_1,q_2)} \quad \Rightarrow \quad \widehat{\mathcal{W}}^{(2L)}_{02} = \sum_\Gamma \widehat{\mathcal{W}}^{(2L)}_{02,\Gamma}$



- Compute the same object with the OpenLoops tree-level algorithm for fixed $q_1, q_2 \Rightarrow \widehat{\mathcal{W}}^{(t)}_{02}$
  Compute relative numerical uncertainty in double (DP) and quadruple (QP) precision

$$\mathcal{A}^{(t)} := \log_{10}\left(\frac{|\widehat{\mathcal{W}}^{(t)}_{02} - \widehat{\mathcal{W}}^{(2L)}_{02}|}{\text{Min}(|\widehat{\mathcal{W}}^{(t)}_{02}|, |\widehat{\mathcal{W}}^{(2L)}_{02}|)}\right)$$

$\Rightarrow$ **Implementation validated** for wide range of processes (10^5 uniform random points)

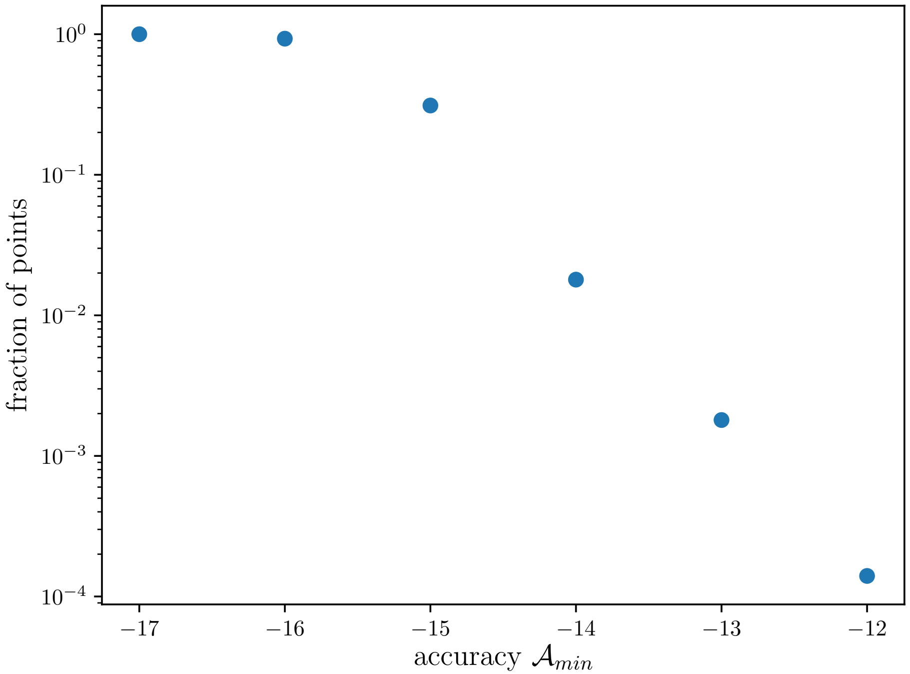
Typical accuracy around 10^{-15} in DP and 10^{-30} in QP, and always much better than 10^{-17} in QP
$\Rightarrow$ **QP calculation as benchmark for numerical accuracy of DP calculation**

Numerical stability of two-loop tensor coefficients

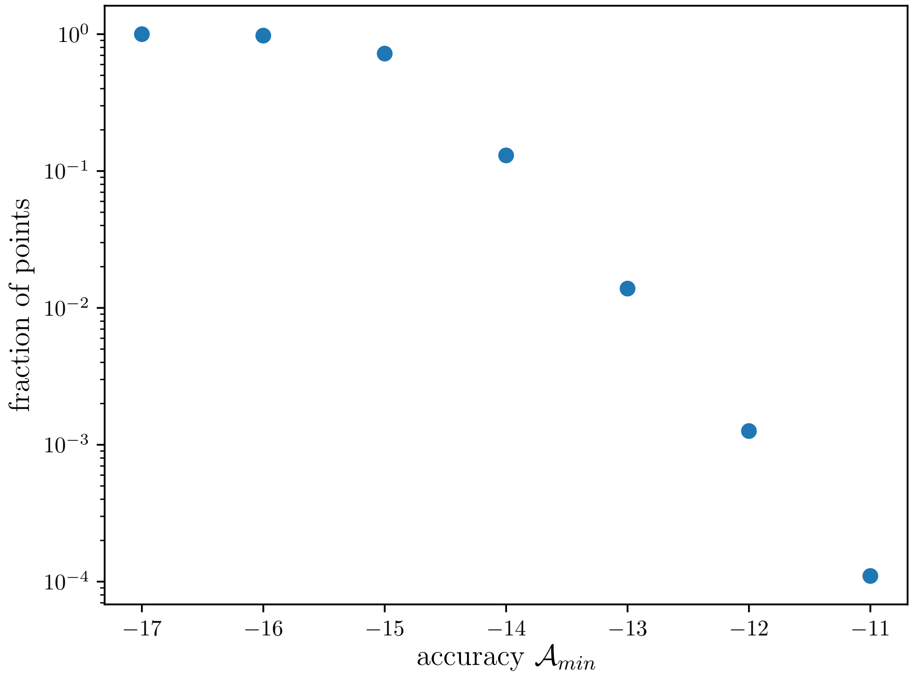
Numerical instability of double (DP) wrt quad precision (QP) calculation:

$$\mathcal{A}_{\text{DP}} = \log_{10}\left(\frac{|\widehat{\mathcal{W}}_{02}^{(2\text{L},\text{DP})} - \widehat{\mathcal{W}}_{02}^{(2\text{L},\text{QP})}|}{\text{Min}(|\widehat{\mathcal{W}}_{02}^{(2\text{L},\text{DP})}|, |\widehat{\mathcal{W}}_{02}^{(2\text{L},\text{QP})}|)}\right)$$

Fraction of points with $\mathcal{A}_{\text{DP}} > A_{\text{min}}$ as a function of A_{min} for 10^5 uniform random points



$gg \to \bar{t}t$



$d\bar{d} \to u\bar{u}g$

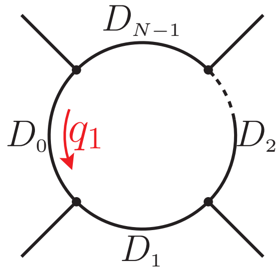
Excellent numerical stability

$\Rightarrow$ **Important for full calculation** (tensor integral reduction will be main source of instabilities)

One-loop rational terms

Amputated one-loop diagram γ (1PI)

$$\bar{\mathcal{M}}_{1,\gamma} = \underbrace{C_{1,\gamma}}_{\text{colour factor}} \int \mathrm{d}\bar{q}_1 \frac{\mathcal{N}(q_1) + \tilde{\mathcal{N}}(\bar{q}_1)}{\mathcal{D}(\bar{q}_1)} =$$



$$\Rightarrow \quad \delta\mathcal{R}_{1,\gamma} = C_{1,\gamma} \int \mathrm{d}\bar{q}_1 \frac{\tilde{\mathcal{N}}(\bar{q}_1)}{\mathcal{D}(\bar{q}_1)}$$

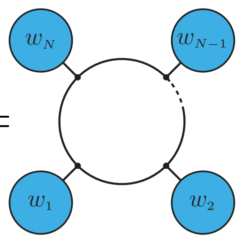
The ε-dim numerator parts $\tilde{\mathcal{N}}(\bar{q}_1) = \bar{\mathcal{N}}(\bar{q}_1) - \mathcal{N}(q_1)$ contribute only via interaction with $\frac{1}{\varepsilon}$ UV poles

$\Rightarrow$ Can be restored through **rational counterterm** $\delta\mathcal{R}_{1,\gamma}$ [Ossola, Papadopoulos, Pittau]

$$\Rightarrow \quad \underbrace{\mathbf{R}\,\bar{\mathcal{M}}_{1,\gamma}}_{D-\text{dim, renormalised}} = \underbrace{\mathcal{M}_{1,\gamma}}_{4-\text{dim numerator}} + \underbrace{\delta Z_{1,\gamma} + \delta\mathcal{R}_{1,\gamma}}_{\text{UV and rational counterterm}}$$

Generic one-loop diagram Γ factorises into 1PI subdiagram γ and external subtrees w_i (4-dim):

$$\bar{\mathcal{M}}_{1,\Gamma} =$$



$$= \left[\bar{\mathcal{M}}_{1,\gamma}\right]^{\sigma_1 \ldots \sigma_N} \prod_{i=1}^{N} [w_i]_{\sigma_i} \quad \Rightarrow$$

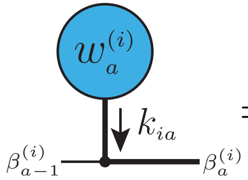
$$\mathbf{R}\,\bar{\mathcal{M}}_{1,\Gamma} = \mathcal{M}_{1,\Gamma} + \left(\delta Z_{1,\gamma} + \delta\mathcal{R}_{1,\gamma}\right) \underbrace{\prod_{i=1}^{N} w_i}_{\text{tree diagram}}$$

Finite set of process-independent rational terms in renormalisable models
computed from UV divergent vertex functions

Explicit recursion steps for tensor coefficients

Triple vertex loop segment:

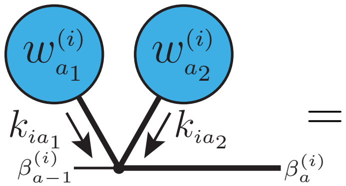
$$\left[S_a^{(i)}(q_i,h_a^{(i)})\right]_{\beta_{a-1}^{(i)}}^{\beta_a^{(i)}} =$$



$$= \left\{\left[Y_{ia}^{\sigma}\right]_{\beta_{a-1}^{(i)}}^{\beta_a^{(i)}} + \left[Z_{ia,\nu}^{\sigma}\right]_{\beta_{a-1}^{(i)}}^{\beta_a^{(i)}} q_i^{\nu}\right\} w_{a\sigma}^{(i)}(k_{ia},h_a^{(i)})$$

Quartic vertex segments:

$$\left[S_a^{(i)}(q_i,h_a^{(i)})\right]_{\beta_{a-1}^{(i)}}^{\beta_a^{(i)}} =$$



$$= \left[Y_{ia}^{\sigma_1\sigma_2}\right]_{\beta_{a-1}^{(i)}}^{\beta_a^{(i)}} w_{a_1\sigma_1}^{(i)}(k_{ia_1},h_{a_1}^{(i)})\, w_{a_2\sigma_2}^{(i)}(k_{ia_2},h_{a_2}^{(i)})$$

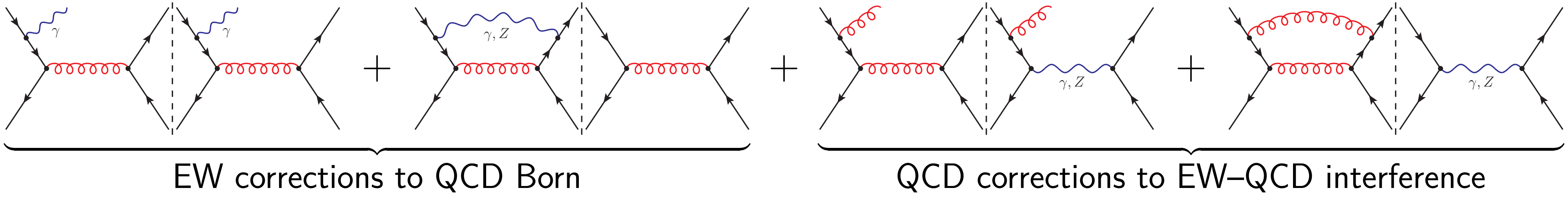
with $h_a^{(i)} = h_{a_1}^{(i)} + h_{a_2}^{(i)}$ and $k_{ia} = k_{ia_1} + k_{ia_2}$.

Dressing step for a segment with a triple vertex:

$$\left[\mathcal{N}_{n;\,\mu_1\ldots\mu_r}^{(1)}(\hat{h}_n^{(1)})\right]_{\beta_0^{(1)}}^{\beta_n^{(1)}} = \Bigg\{\left[\mathcal{N}_{n-1;\,\mu_1\ldots\mu_r}^{(1)}(\hat{h}_{n-1}^{(1)})\right]_{\beta_0^{(1)}}^{\beta_{n-1}^{(1)}}\left[Y_{1n}^{\sigma}\right]_{\beta_{n-1}^{(1)}}^{\beta_n^{(1)}}$$
$$+\left[\mathcal{N}_{n-1;\,\mu_2\ldots\mu_r}^{(1)}(\hat{h}_{n-1}^{(1)})\right]_{\beta_0^{(1)}}^{\beta_{n-1}^{(1)}}\left[Z_{1n,\mu_1}^{\sigma}\right]_{\beta_{n-1}^{(1)}}^{\beta_n^{(1)}}\Bigg\} w_{n\sigma}^{(1)}(k_n,h_n^{(1)}).$$

OpenLoops features

- OpenLoops provides **all contributions of a given power in α** (electroweak) and α_s (strong) to $\mathcal{W}$ in a **fully automated** way, e.g. NLO EW corrections of $\mathcal{O}(\alpha_S^2\alpha^1)$ for $q\bar{q} \to q\bar{q}$:



- Different **EW schemes** implemented: $\alpha(0)$-scheme, G_μ-scheme, $\alpha(M_Z)$-scheme
- Consistent treatment of resonances with **complex mass scheme** at 1-loop [Denner, Dittmaier] $\to$ complex mass $\mu_p^2 = M_p^2 - i\,M_p\Gamma_p$ from real physical mass M_p and width Γ_p as input
- **Different Renormalisation schemes** implemented, e.g. on-shell or $\overline{MS}$ for quark masses; different flavour schemes for α_S
- **Colour and charge correlators; Spin and Spin-colour correlators**
- **Catani-Seymour I-operator**
- **Selection of helicity states** $\to$ polarised initial or final states
- $\cdots$

$\Rightarrow$ **Ingredients for a wide range of applications available**

The structure of OpenLoops

- OpenLoops **program (public):** User interfaces and process-independent routines.

- **Process generator (not public):** Perform analytical steps (e.g. colour factors) and generate process-dependent code for numerical calculation $\to$ stored in process libraries

- **Process libraries (public)**: Collection of partonic channels for a process class, e.g. $pp \to jj$, automatically downloaded by the user.

- **Third party tools** for integral evaluation (included): Collier [Denner, Dittmaier, Hofer], OneLoop [van Hameren]

Same structure at two loops. Minimal extension of widely-used interfaces