Analysis of $(n+1)$ and n-parton contributions for computing QCD jet cross sections in the LASS scheme

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- The general structure of the NNLO correction:

$$\frac{d\sigma_{\text{NNLO}}}{dX} = \int d\Phi_n \, VV \, \delta_n(X) + \int d\Phi_{n+1} \, RV \, \delta_{n+1}(X) + \int d\Phi_{n+2} \, RR \, \delta_{n+2}(X)$$

- In the Local Analytic Sector Subtraction scheme this is rewritten as:

$$\frac{d\sigma_{\text{NNLO}}}{dX} = \int d\Phi_n \, VV_{\rm sub}(X) + \int d\Phi_{n+1} \, RV_{\rm sub}(X) + \int d\Phi_{n+2} \, RR_{\rm sub}(X) \, ,$$

where

$$\begin{aligned} RR_{\rm sub}(X) &\equiv RR \, \delta_{n+2}(X) - K^{(1)} \, \delta_{n+1}(X) - \left(K^{(2)} - K^{(12)}\right) \delta_n(X) \, , \\ RV_{\rm sub}(X) &\equiv \left(RV + I^{(1)}\right) \delta_{n+1}(X) - \left(K^{(\mathbf{RV})} + I^{(12)}\right) \delta_n(X) \, , \\ VV_{\rm sub}(X) &\equiv \left(VV + I^{(2)} + I^{(\mathbf{RV})}\right) \delta_n(X) \, . \end{aligned}$$

- See Adam's talk for the discussion regarding $RR_{\rm sub}$
- Focus of this talk: $RV_{\rm sub}$ and $VV_{\rm sub}$

Subtraction terms

$$RR_{\rm sub}(X) \equiv RR\,\delta_{n+2}(X) - K^{(1)}\,\delta_{n+1}(X) - \left(K^{(2)} - K^{(12)}\right)\delta_n(X)\,,$$
$$RV_{\rm sub}(X) \equiv \left(RV + I^{(1)}\right)\delta_{n+1}(X) - \left(K^{(\mathbf{RV})} + I^{(12)}\right)\delta_n(X)\,,$$
$$VV_{\rm sub}(X) \equiv \left(VV + I^{(2)} + I^{(\mathbf{RV})}\right)\delta_n(X)\,.$$

- $K^{(1)}$ - captures the single unresolved singularities
- $K^{(2)}$ - captures the double unresolved singularities
- $K^{(12)}$ - accounts for the overlap of $K^{(1)}$ and $K^{(12)}$
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- $K^{(\mathbf{RV})}$ - captures the phase-space singularities of RV
- I-terms - the integrated versions of the above terms

The beauty of the LASS scheme: *The clever design of K-terms*

- Every contribution is free of explicit ϵ-poles and phase-space singularities
- All the subtraction terms are analytical expressions, requiring only numerical evaluation for each phase-space point during the integration process

The strategy of an implementation of LASS

- All the building blocks of the scheme are given in the references [arXiv:1806.09570], [arXiv:2010.14493], [arXiv:2212.11190] (Magnea et al. 2019, 2020, 2022)

Implementation strategy:

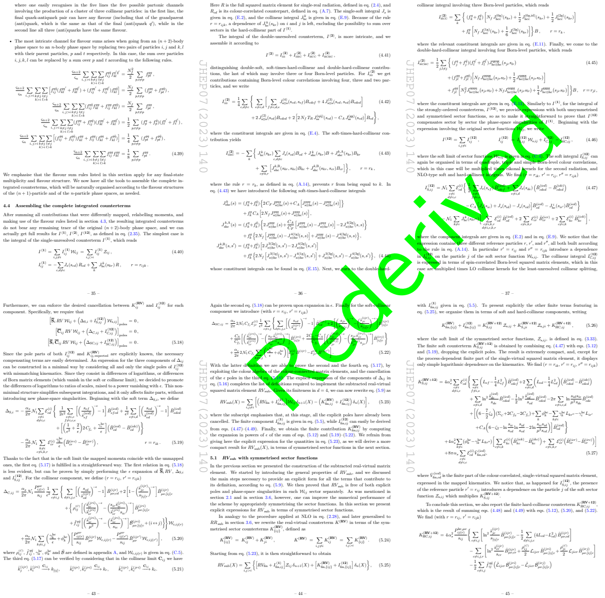
1. Rederive every analytical expression given in these papers
   - Protects against accidental typos in lengthy equations
   - Confirmation that one has a correct understanding of the procedure
2. Check the explicit ϵ-pole cancellation
3. Check the singular behavior during the phase-space integration
4. Design an automatic generator of the subtraction terms for any process
5. Implement an efficient MC generator
6. Study the physical processes $e^+e^- \to 3\,\text{jets}, 4\,\text{jets}, \ldots$

Derivation of the integrated subtraction terms

- In some cases the analytical integration of the subtraction terms can be quite involved
- Following the strategy of [arXiv:2010.14493] every integral can be reduced to the following form

$$I_{a,A,B,C,D,E,F,G} = \int_0^1 dy' \int_0^1 dz \int_0^1 dz' \int_0^1 dw' \times$$
$$\times \frac{\left(w'\left(1-w'\right)\right)^{-1/2+\epsilon}\left(1-y'\right)^{A}\left(y'\right)^{B}(1-z)^{C} z^{D}\left(1-z'\right)^{E}\left(z'\right)^{F}}{\left[1-\left(1-y'\right)z'\right]^{G-1}\left[z\left(1-z'\right)+y'(1-z)z'+2\left(1-2w'\right)\sqrt{y'}\sqrt{(1-z)z}\sqrt{\left(1-z'\right)z'}\right]^{a}}.$$

- For the specific values of (a, A, B, C, D, E, F, G) which appear in the problem, this was possible to integrate analytically.
- Some of the master integrals were double-checked using `pySecDec` (Heinrich et al.).
- The procedure was automatized in `FORM` and `Mathematica` packages.
- `FORM` package can rederive all the integrated subtraction terms from the LASS papers within ~ 5 seconds.


Rederived

Rederived expressions

Cancellation of the poles

- The cancellation of the explicit poles of RV by $\boldsymbol{I}^{(1)}$ is manifest

- In case of VV, upon expansion of $\boldsymbol{I}^{(2)} + \boldsymbol{I}^{(\mathbf{RV})}$ we could obtain the general pole structure of VV [Catani 1998] with the opposite sign (as advertised by the LASS papers):

$$
\begin{aligned}
VV_{\text{poles}} = 2\,\text{Re}\Bigg[& -\frac{1}{2}\left\langle \mathcal{M}^{(0)} \middle| \boldsymbol{I}^{(1)}(\epsilon)\boldsymbol{I}^{(1)}(\epsilon) \middle| \mathcal{M}^{(0)} \right\rangle - \frac{\beta_0}{\epsilon}\left\langle \mathcal{M}^{(0)} \middle| \boldsymbol{I}^{(1)}(\epsilon) \middle| \mathcal{M}^{(0)} \right\rangle \\
& + \left\langle \mathcal{M}^{(0)} \middle| \boldsymbol{I}^{(1)}(\epsilon) \middle| \mathcal{M}^{(1)} \right\rangle + e^{-\epsilon\gamma}\frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)}\left(\frac{\beta_0}{\epsilon} + K\right)\left\langle \mathcal{M}^{(0)} \middle| \boldsymbol{I}^{(1)}(2\epsilon) \middle| \mathcal{M}^{(0)} \right\rangle \\
& + \left\langle \mathcal{M}^{(0)} \middle| \boldsymbol{H}^{(2)}(\epsilon) \middle| \mathcal{M}^{(0)} \right\rangle \Bigg] \\
+\,\text{Re}\Bigg[& 2\left\langle \mathcal{M}^{(1)} \middle| \boldsymbol{I}^{(1)}(\epsilon) \middle| \mathcal{M}^{(0)} \right\rangle - \left\langle \mathcal{M}^{(0)} \middle| \boldsymbol{I}^{(1)\dagger}(\epsilon)\boldsymbol{I}^{(1)}(\epsilon) \middle| \mathcal{M}^{(0)} \right\rangle \Bigg]
\end{aligned}
$$

Explicit check of the cancellation for $e^+e^- \to 3$ jets

$$
\begin{aligned}
\left(I^{(2)} + I^{(\mathrm{RV})}\right)_{\mathrm{poles}} = B\left(\frac{\alpha_s}{2\pi}\right)^2 \Bigg[& \frac{1}{\epsilon^4}\left(\frac{\mathrm{CA}^2}{2} + \left(2\mathrm{CF} - \frac{17}{3}\right)\mathrm{CA} + 2\mathrm{CF}^2 - \frac{34\mathrm{CF}}{3}\right) \\
& + \frac{1}{\epsilon^3}\Bigg(\left(\log\left(\mu^2\right) + \log(s12) - \log(s13) - \log(s23) + \frac{11}{24}\right)\mathrm{CA}^2 \\
& + \Bigg(-\frac{\mathrm{nfTr}}{6} - \frac{34\log\left(\mu^2\right)}{3} - 6\log(s12) + \frac{26\log(s13)}{3} \\
& + \mathrm{CF}\left(4\log\left(\mu^2\right) - 2\log(s13) - 2\log(s23) + \frac{47}{12}\right) + \frac{26\log(s23)}{3} - \frac{164}{9}\Bigg)\mathrm{CA} \\
& + \frac{34\mathrm{nfTr}}{9} + \mathrm{CF}^2\left(4\log\left(\mu^2\right) - 4\log(s12) + 6\right) \\
& + \mathrm{CF}\left(-\frac{\mathrm{nfTr}}{3} - \frac{68\log\left(\mu^2\right)}{3} + \frac{32\log(s12)}{3} + 6\log(s13) + 6\log(s23) - \frac{98}{3}\right)\Bigg) \\
& + \frac{1}{\epsilon^2}(\ldots) + \frac{1}{\epsilon}(\ldots)\Bigg]
\end{aligned}
$$

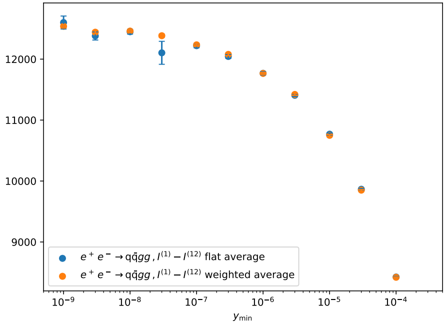
- With the opposite sign this exactly agrees with the standard $e^+e^- \to 3$ jet references [Garland, Gehrmann et al. 2001], [Gehrmann-De Ridder, Gehrmann et al. 2007].

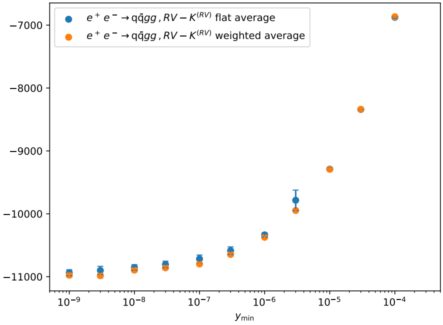
RV contribution

$$RV_{\mathrm{sub}}(X) \equiv \left(RV + I^{(\mathbf{1})} \right) \delta_{n+1}(X) - \left(K^{(\mathbf{RV})} + I^{(\mathbf{12})} \right) \delta_n(X) \tag{1}$$

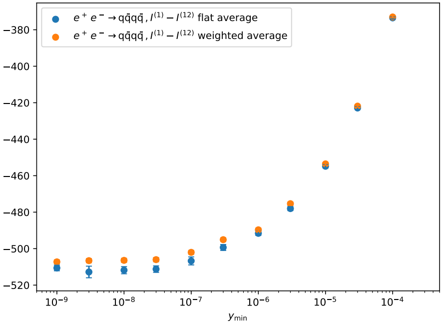
- Automatic generation of $I^{(1)}$, $K^{(\mathbf{RV})}$ and $I^{(\mathbf{12})}$ using FORM.
- The RV matrix element is extracted from [Bern, Dixon, Kosower 1997].
- Everything is fed to the MC integrator (see Adam's talk).
- Time to perform $y_{\min}$ study (see Adam's talk for the definition).

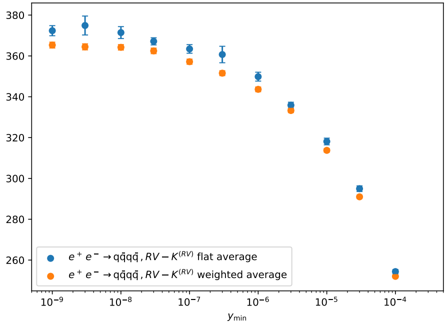
Saturation plots for $e^+e^- \to q\bar{q}gg$



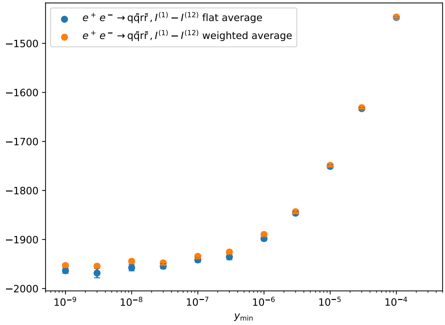


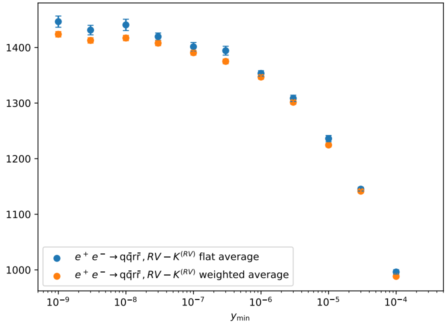
Saturation plots for $e^+e^- \to q\bar{q}q\bar{q}$





Saturation plots for $e^+e^- \to q\bar{q}r\bar{r}$





- After reassuring in the stability of the integration procedure, one can fix the $y_{\min}$ value and produce the distributions of some event shape observables.
- In the following we will examine following event shape observables:
  1. τ-parameter (thrust):

$$T = \max_{\vec{n}} \left(\frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \right), \quad \tau \equiv 1 - T.$$

  2. C-parameter:

$$C_{\rm par} = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i||\vec{p}_j| \sin^2 \theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}.$$

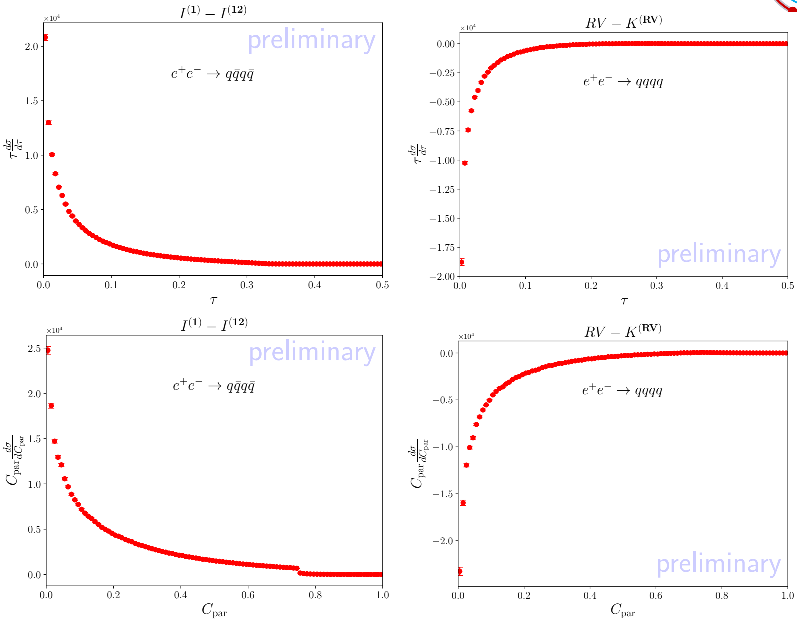
  3. Energy-energy correlation:

$$\mathrm{EEC}(\chi) = \frac{1}{\sigma_{\rm had}} \sum_{i,j} \int \frac{E_i E_j}{Q^2} \mathrm{d}\sigma_{e^+e^- \to i\, j + X} \delta(\cos\chi + \cos\theta_{ij}).$$

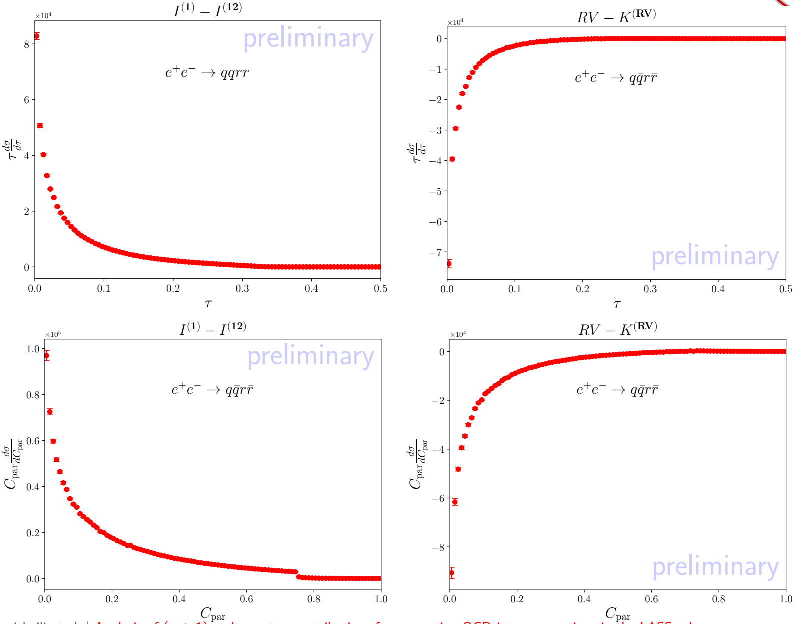
  4. Jet-cone energy fraction:

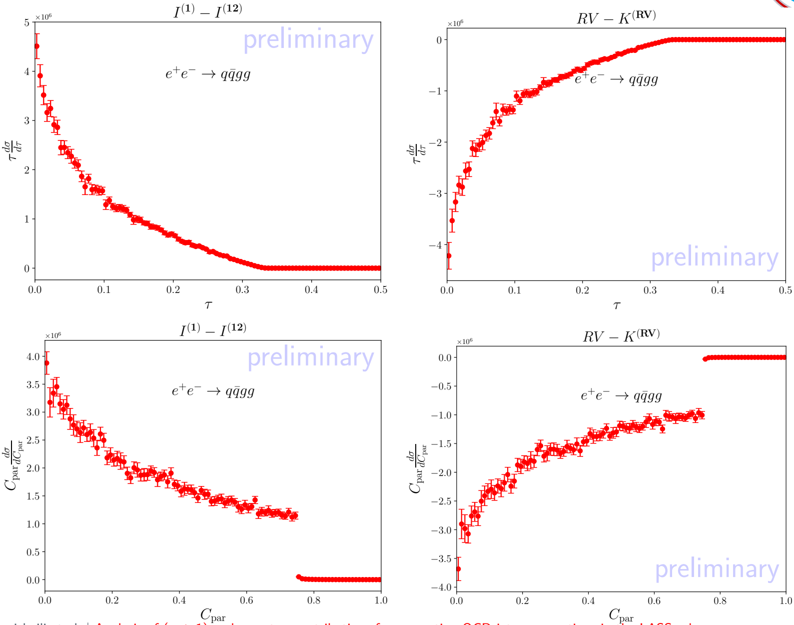
$$\frac{\mathrm{d}\Sigma_{\rm JCEF}}{\mathrm{d}\cos\chi} = \sum_i \int \frac{E_i}{Q} \, \mathrm{d}\sigma_{e^+e^- \to i + X} \delta\left(\cos\chi - \frac{\vec{p}_i \cdot \vec{n}_T}{|\vec{p}_i|}\right).$$

τ-parameter and C-parameter: $q\bar{q}q\bar{q}$-channel



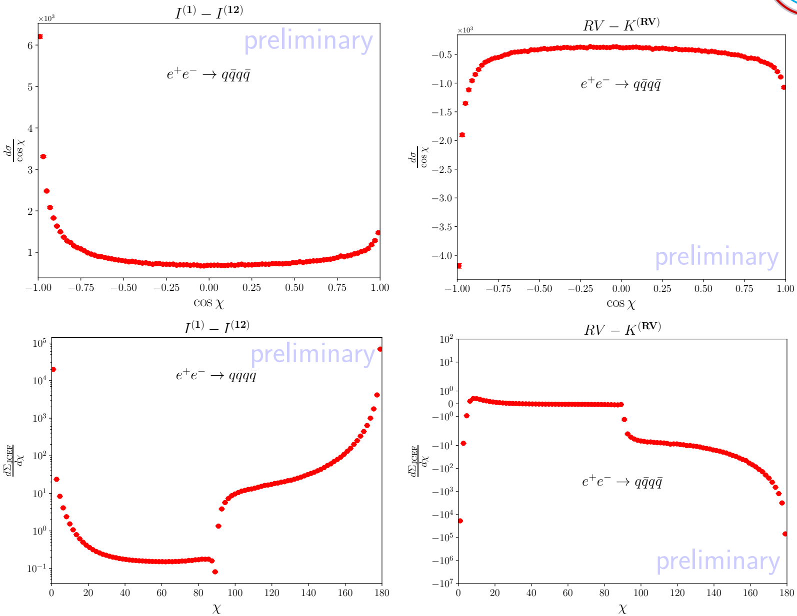
τ-parameter and C-parameter: $q\bar{q}r\bar{r}$-channel



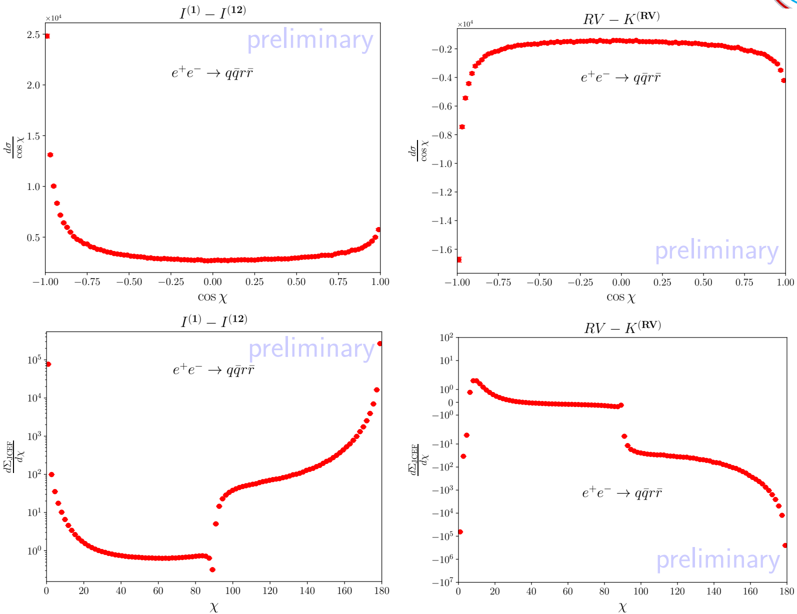


$I^{(\mathbf{1})} - I^{(\mathbf{12})}$
preliminary
$e^+e^- \to q\bar{q}gg$
$\tau \frac{d\sigma}{d\tau}$
τ
$\times 10^6$
$RV - K^{(\mathbf{RV})}$
preliminary
$e^+e^- \to q\bar{q}gg$
$\tau \frac{d\sigma}{d\tau}$
τ
$\times 10^6$
$I^{(\mathbf{1})} - I^{(\mathbf{12})}$
preliminary
$e^+e^- \to q\bar{q}gg$
$C_{\mathrm{par}} \frac{d\sigma}{dC_{\mathrm{par}}}$
C_{par}
$\times 10^6$
$RV - K^{(\mathbf{RV})}$
preliminary
$e^+e^- \to q\bar{q}gg$
$C_{\mathrm{par}} \frac{d\sigma}{dC_{\mathrm{par}}}$
C_{par}
$\times 10^6$

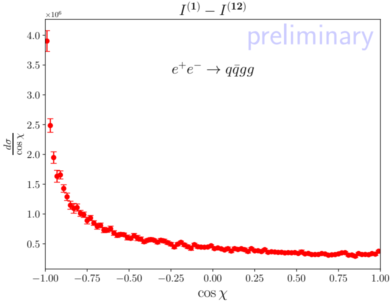
EEC and JCEF: $q\bar{q}q\bar{q}$-channel

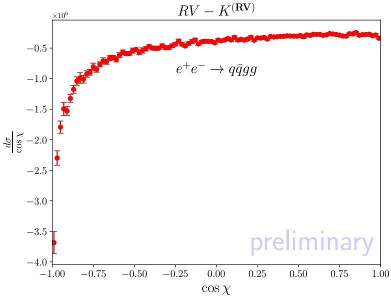


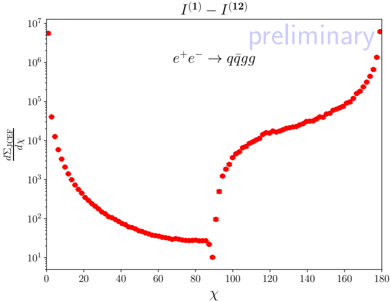
EEC and JCEF: $q\bar{q}r\bar{r}$-channel

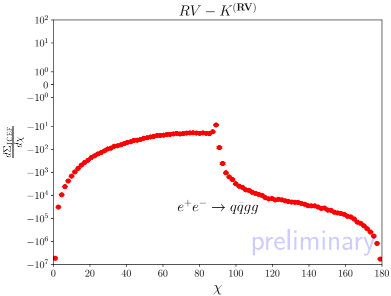


EEC and JCEF: $q\bar{q}gg$-channel









Conclusions

- We demonstrated a proof-of-concept implementation of the LASS scheme.
- Numerous expressions from the LASS papers were successfully cross-checked.
- We have full analytic and numeric control of RR, RV, and VV contributions.
- Efficient automatic generation and integration of the subtraction terms were achieved.
- First distributions of the event shape observables using the LASS scheme were obtained.
- Significant progress was made towards the full automation of the current LASS scheme.

References to the LASS papers

[1] L. Magnea, E. Maina, G. Pelliccioli, C. Signorile-Signorile, P. Torrielli and S. Uccirati, "Local analytic sector subtraction at NNLO," JHEP **12** (2018), 107 [erratum: JHEP **06** (2019), 013] doi:10.1007/JHEP12(2018)107 [arXiv:1806.09570 [hep-ph]].

[2] L. Magnea, G. Pelliccioli, C. Signorile-Signorile, P. Torrielli and S. Uccirati, "Analytic integration of soft and collinear radiation in factorised QCD cross sections at NNLO," JHEP **02** (2021), 037 doi:10.1007/JHEP02(2021)037 [arXiv:2010.14493 [hep-ph]].

[3] G. Bertolotti, L. Magnea, G. Pelliccioli, A. Ratti, C. Signorile-Signorile, P. Torrielli and S. Uccirati, "NNLO subtraction for any massless final state: a complete analytic expression," JHEP **07** (2023), 140 doi:10.1007/JHEP07(2023)140 [arXiv:2212.11190 [hep-ph]].

Thanks for your attention!

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