# Analysis of (n + 1) and *n*-parton contributions for computing QCD jet cross sections in the LASS scheme

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#### Introduction



► The general structure of the NNLO correction:

$$\frac{d\sigma_{\text{NNLO}}}{dX} = \int d\Phi_n \, VV \, \delta_n(X) + \int d\Phi_{n+1} \, RV \, \delta_{n+1}(X) + \int d\Phi_{n+2} \, RR \, \delta_{n+2}(X)$$

▶ In the Local Analytic Sector Subtraction scheme this is rewritten as:

$$\frac{d\sigma_{\text{NNLO}}}{dX} = \int d\Phi_n \, VV_{\text{sub}}(X) + \int d\Phi_{n+1} \, RV_{\text{sub}}(X) + \int d\Phi_{n+2} \, RR_{\text{sub}}(X) \,,$$

where

$$\begin{aligned} RR_{\rm sub}(X) &\equiv RR\,\delta_{n+2}(X) - K^{(1)}\,\delta_{n+1}(X) - \left(K^{(2)} - K^{(12)}\right)\delta_n(X)\,, \\ RV_{\rm sub}(X) &\equiv \left(RV + I^{(1)}\right)\delta_{n+1}(X) - \left(K^{(\rm RV)} + I^{(12)}\right)\delta_n(X)\,, \\ VV_{\rm sub}(X) &\equiv \left(VV + I^{(2)} + I^{(\rm RV)}\right)\delta_n(X)\,. \end{aligned}$$

 $\blacktriangleright$  See Adam's talk for the discussion regarding  $R\!R_{\rm sub}$ 

Focus of this talk:  $RV_{sub}$  and  $VV_{sub}$ 

#### Subtraction terms



$$\begin{aligned} RR_{\rm sub}(X) &\equiv RR\,\delta_{n+2}(X) - K^{(1)}\,\delta_{n+1}(X) - \left(K^{(2)} - K^{(12)}\right)\delta_n(X) \\ RV_{\rm sub}(X) &\equiv \left(RV + I^{(1)}\right)\delta_{n+1}(X) - \left(K^{(\mathsf{RV})} + I^{(12)}\right)\delta_n(X) , \\ VV_{\rm sub}(X) &\equiv \left(VV + I^{(2)} + I^{(\mathsf{RV})}\right)\delta_n(X) . \end{aligned}$$

• 
$$K^{(12)}$$
 - accounts for the overlap of  $K^{(1)}$  and  $K^{(12)}$ 

- $\mathcal{K}^{(\mathrm{RV})}$  captures the phase-space singularities of RV
- I-terms the integrated versions of the above terms

#### The beauty of the LASS scheme: The clever design of K-terms

- Every contribution is free of explicit ε-poles and phase-space singularities
- All the subtraction terms are analytical expressions, requiring only numerical evaluation for each phase-space point during the integration process



 All the building blocks of the scheme are given in the references [arXiv:1806.09570], [arXiv:2010.14493], [arXiv:2212.11190] (Magnea et al. 2019, 2020, 2022)

Implementation strategy:

- 1. Rederive every analytical expression given in these papers
  - Protects against accidental typos in lengthy equations
  - Confirmation that one has a correct understanding of the procedure
- 2. Check the explicit  $\epsilon$ -pole cancellation
- 3. Check the singular behavior during the phase-space integration
- 4. Design an automatic generator of the subtraction terms for any process
- 5. Implement an efficient MC generator
- 6. Study the physical processes  $e^+e^- 
  ightarrow$  3 jets, 4 jets,  $\ldots$



$$I_{a,A,B,C,D,E,F,G} = \int_{0}^{1} dy' \int_{0}^{1} dz \int_{0}^{1} dz' \int_{0}^{1} dw' \times \frac{\left(w'\left(1-w'\right)\right)^{-1/2+\epsilon} \left(1-y'\right)^{A} \left(y'\right)^{B} (1-z)^{C} z^{D} \left(1-z'\right)^{E} \left(z'\right)^{F}}{\left[1-\left(1-y'\right)z'\right]^{G-1} \left[z \left(1-z'\right)+y'(1-z)z'+2 \left(1-2w'\right) \sqrt{y'} \sqrt{(1-z)z} \sqrt{\left(1-z'\right)z'}\right]^{a}}.$$

- ▶ For the specific values of (*a*, *A*, *B*, *C*, *D*, *E*, *F*, *G*) which appear in the problem, this was possible to integrate analytically.
- Some of the master integrals were double-checked using pySecDec (Heinrich et al.).
- ▶ The procedure was automatized in FORM and Mathematica packages.
- FORM package can rederive all the integrated subtraction terms from the LASS papers within ~ 5 seconds.

#### Rederived expressions

$$\begin{split} & \underset{m}{\overset{\text{der}}{\longrightarrow}} \sum_{\substack{m \in \mathcal{M}(\mathcal{M}) \\ m \in \mathcal{M}(\mathcal{M})}} \mathcal{M}^{m} - \mathcal{$$

After manufact all contributions that were differently manuel, relabeling momenta, and

$$I^{(k)} = \sum_{i \neq 0} \ell_{ij}^{(k)} H_{ij} = \sum_{i \neq 0} \ell_{ij}^{(k)} Z_{ij},$$
 (4.8  
 $\ell_{ij}^{(k)} = -\sum_{i} J_i(v_{ij}) R_{ij} + \sum_{i} J_{ii}^2(v_{ij}) R, \quad r = v_{ijk}.$ 

Furthermore, we can endowe the denierd convolution between  $K_{\pm}^{(\rm BN)}$  and  $I_{\pm}^{(\rm BD)}$  for each

$$\begin{split} \left\| \overline{\mathbf{R}}_i \, \delta S' \, W_{ij} + \left( \Delta c_{i,j} + \overline{\delta}_{i,j}^{(M)} \right) W_{i,j} \right\|_{p,dm} &= 0, \\ \left[ \overline{\mathbf{C}}_{ij} \, \delta S' \, W_{ij} + \left( \Delta c_{i,j} + \overline{\delta}_{i,j}^{(M)} \right) \right]_{p,dm} &= 0, \\ \left[ \overline{\mathbf{R}}_{ij} \, \overline{\Delta} S' \, W_{ij} + \left( \Delta u_{i,j} + \overline{\delta}_{i,j}^{(M)} \right) \right]_{p,dm} &= 0. \end{split}$$

Since the pole parts of both  $I_{ij}^{(40)}$  and  $K_{ij,inputual}^{(40)}$  are replicitly known, the accounty

where  $q_{i}$  (11) is manifed in a straight performing the straight relation of  $\overline{s}_{i}$  (27),  $\Delta_{i,i}$ and  $J_{1,1}^{(14)}$ . For the collinear component, we define  $(r = r_{11}, r' = r_{12})$ 

$$\begin{split} \Delta (z) &= \frac{1}{W} (z) \frac{d z}{d z} \sum_{i \in \mathcal{I}} \left( \sum_{i \in \mathcal{I}} \left( \frac{d z}{d z} \right) - \frac{1}{2} \left( \frac{d z}{d z} \right) -$$

 $I^{(2)} = L_{1}^{(2)} + L_{2}^{(2)} + L_{2}^{(2)} + L_{2}^{(2)} + L_{2}^{(2)} , \label{eq:Interm}$ 

 $I_{ii}^{(0)} = \frac{1}{4} \sum_{i} \left\{ \sum_{j} \left[ \sum_{i,j} J_{ijj}^{(0)}(s_{ij}, s_{ij}) R_{ikij} + i J_{ijj}^{(0)}(s_{ij}, s_{ij}) R_{ikki} \right]$  (4.67)  $+2J_{2,0}^{(2)}(s_{4})R_{44d}+2\left[2N_{1}T_{2}J_{2}^{(4)}(s_{4d})-C_{4}J_{2}^{(4)}(s_{4d})\right]R_{4d}\Big\}$  $I_{abc}^{(0)} = -\sum \left\{ J_{bc}^{\pm}(ac) \sum J_{c}(ac) B_{cc} + J_{abc}^{\pm}(ac) B + J_{abc}^{\pm,0}(ac) B_{cc} - (4.42) \right\}$ 

#### where the cube r = ri, as defined in eq. (A14), presents r from being regal to k. In

 $J_{her}^{0}(s) = (f_{1}^{0} + f_{2}^{0}) \left\{ 2Cr J_{her}^{00}(s) + C_{4} \left[ J_{her}^{00}(s) - J_{her}^{00}(s) \right] \right\}$  $+ \int_{0}^{0} C_{\lambda} \left[ 2N (J_{\mu\nu\mu\nu}^{DH}(s) + J_{\mu\nu\mu\nu}^{DH}(s) \right],$ 

$$\begin{split} J^{\text{int}}_{2n}(s) &= (f_{n}^{\text{int}} + f_{n}^{\text{int}}) \Big\{ 2A^{\text{int}}_{2n}(s) + \frac{1}{C_{F}} \Big[ A^{\text{int}}_{2n}(s) - A^{\text{int}}_{2n}(s) \Big] - 2A^{\text{int}}_{2n}(s,s) \Big\} \\ &+ f_{n}^{\text{int}} \Big\{ 2N_{f} \Big[ A^{\text{int}}_{2n}(s) - A^{\text{int}}_{2n}(s,s) \Big] + A^{\text{int}}_{2n}(s) - A^{\text{int}}_{2n}(s,s) \Big\}, \end{split}$$

+  $ff \left\{ 2N_{\ell} \left[ J_{\ell}^{(0)}(s,s') - J_{\ell}^{(0)}(s,s') \right] + J_{\ell}^{(0)}(s,s') - J_{\ell}^{(0)}(s,s') \right\}, \ ($ 

$$\begin{split} \mathbf{A}_{m,q} &= \frac{2}{2} \mathrm{M}(\mathbf{C}_{\mathbf{r}}, \mathbf{r}')^{\frac{1}{2}} \frac{1}{2m} \sum_{m=1}^{m} \left[ \frac{1}{2m} \left[ \left( \frac{d^2}{dm} \right)^{-1} \left[ \frac{d^2}{dm} \left[ \left( \frac{d^2}{dm} \right)^{-1} \left( \frac{d^2}{dm} \left[ \frac{d^2}{dm} \right]^{-1} \right] \right] d \mathbf{r} \right] \right] \right] d \mathbf{r} \end{split}$$

and the summaries of A. In  $_{11}(X) = \left\{K_{Bar}^{(BB)} + I_{Bar}^{(BB)}\right\}\delta_{2}(X)$ , (5.20)

where the solver approximates that, at this stage, all the region poles have already here rangeful. The finite rangement  $I_{(n)}^{(n)}$  is given in eq. (5.3), while  $I_{(n)}^{(n)}$  can easily be derived from res. (L17) (L19). Finally, we obtain the finite contribution  $K_{here}^{(W)}$  by concention

- 2.1 RV<sub>100</sub> with symmetrized sector functions

In analogy in the prevoluer applied at NLO in eq. (2.29), and later generalized to  $\tilde{K}^2_{\rm chb}$  in seriim 3.6, we rewrite the real-vietual construction  $\tilde{K}^{(00)}$  in terms of the spin metrical series constructors  $\tilde{K}^{(00)}_{\rm chb}$ , defined as

$$\bar{K}_{[\pm]}^{(20)} = \bar{K}_{[}^{(20)} + \bar{K}_{[+]}^{(20)}, \qquad \bar{K}^{(20)} = \sum_{i,j\neq i} \bar{K}_{[+]}^{(20)} = \sum_{i,j\neq i} \bar{K}_{[+]}^{(20)}.$$
 (5.20)  
Intring from eq. (5.22), it is then straightforward to obtain

$$\delta V_{abb}^{i}(X) = \sum_{k} \left\{ \left[ \delta V_{bb}^{i} + \delta_{bb}^{(k)} \right] Z_{ij} \delta_{acj} \delta_{acj}(X) + \left[ K_{bb}^{(200)} + \delta_{bb}^{(10)} \right] \delta_{a}(X) \right\},$$
 (5.

$$\mathcal{L}_{acc}^{(0)} = \sum_{2} \left\{ (l_{1}^{0} + l_{1}^{0}) \left[ N(J_{acc}^{(0)}(ac)) + \frac{1}{2} J_{acc}^{(0)}(ac) + \frac{1}{2} J_{acc}^{(0)}(ac) \right] + J_{1}^{0} \left[ N(J_{acc}^{(0)}(ac)) + \frac{1}{2} J_{acc}^{(0)}(ac) \right] \right\} R, \quad r = ri,$$

where the relevant constituent integrals are given in eq. (E.11). Finally, we come to the

$$\begin{split} & (G_{n-1}+\sum_{i=1}^{n} (\int_{-\infty}^{\infty} (G_{n-1}^{-1}(G_{n$$

Were different reference particles r, r', and r\*, all both built according where  $r_{ij}$  (A.14). In particular  $r_{j}$  =  $r_{ij}$  and  $r''_{j}$  =  $r_{ij}$  introduce a dependence of  $r_{ij}$  (A.14). In particular  $r_{j}$  =  $r_{ij}$  and  $r''_{j}$  =  $r_{ij}$  introduce a dependence  $r_{ij}$  on the particle j of the suft series function  $W_{ijj}$ . The collinear integral  $I_{ij}^{(m)}$ 

$$\hat{r}_{hn,(r)}^{(BN)} + \hat{r}_{hn,(r)}^{(BN)} = \hat{K}_{h,(r)}^{(BN-14)} Z_{n,(r)} + \hat{K}_{h,(r)}^{(BN-14)} Z_{n,(r)} + \hat{K}_{(BC,(r))}^{(BN-14)}$$
, (5.26)

where the soft limit of the symmetrized vector functions,  $Z_{i,ij}$ , is defined in eq. (3.33). The finite soft constructions  $\hat{K}_{i,ij}^{(M)}$  is obtained by combining eq. (4.37) with eqs. (3.32)

$$\begin{split} & \sup_{0} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{j=1$$

$$\begin{split} K^{\text{(Weyes)}}_{\text{(Weyes)}} &= a_{ij} \frac{P^{\text{(Weyes)}}_{ij}}{v_{ij}} \left\{ \sum_{m=1}^{ij} \left[ a_{ij}^{ij} \frac{e_{ij}^{ij}}{e_{ij}^{ij}} \delta^{(ij)}_{ij} a_{ij}^{ij} - \frac{1}{2} \sum_{m=1}^{ij} (a_{ij} - a_{ij}^{ij} a_{ij}^{ij}) + \frac{1}{2} \delta_{ij} a_{ij} a_{ij}^{ij} + \frac{1}{2} \delta_{ij} a_{ij}^{ij} + \frac{1}{2} \delta_{ij} a_{ij}^{ij} + \delta_{ij}^{ij} a_{i$$

 $\mathcal{L}_{min}^{(m)} = -\delta \hat{c} \frac{P_{min}^{(m)}}{2} \int \nabla \nabla \nabla A(m) R_{min}^{(m)} + C_{1,m} \hat{c}^{(1)} A(m) R_{min}^{(m)}$ 

$$\begin{split} & -38 - \\ & - \left[ \left( + \frac{2}{3} G_{1} \right) \left( \xi_{1} - \xi_{1} \xi_{2} \right) + \left( \xi_{2} - \xi_{2} \right) \left( \xi_{1} - \xi_{2} \right) \right] \\ & - \xi_{2} \left( \frac{2}{3} \xi_{1} - \xi_{2} \right) - \left( \xi_{2} - \xi_{2} \right) \left( \xi_{2} - \xi_{2} \right) + \xi_{2} - \left[ \xi_{2} \right] \left( \xi_{2} - \xi_{2} \right) \\ & - 4 \xi_{2} \left[ \xi_{2} \xi_{1} \xi_{2} \right] \left( \xi_{2} - \xi_{2} \right) - \left( \xi_{2} - \xi_{2} \right) \left( \xi_{2} - \xi_{2} \right) \right] \\ & + 4 \xi_{2} \left[ \frac{\xi_{2} \xi_{2} - \xi_{2} - \xi_{2} - \xi_{2} - \xi_{2} \right] \left( \xi_{2} - \xi_{2} - \xi_{2} \right) \left( \xi_{2} - \xi_{2} - \xi_{2} \right) \left( \xi_{2} - \xi_{2} \right) \right] \\ & + 4 \xi_{2} \left[ \frac{\xi_{2} - \xi_{2} \right] \\ & - \xi_{2} \left[ \xi_{2} - \xi_{2} - \xi_{2} - \xi_{2} \right] \left( \xi_{2} - \xi_{2} - \xi_{2} - \xi_{2} - \xi_{2} \right) \left( \xi_{2} - \xi_{2} - \xi_{2} - \xi_{2} \right) \right] \\ & - \xi_{2} \left[ \xi_{2} - \xi_{2} - \xi_{2} - \xi_{2} \right] \\ & - \xi_{2} \left[ \xi_{2} - \xi_{2} - \xi_{2} - \xi_{2} - \xi_{2} \right] \\ & - \xi_{2} \left[ \xi_{2} - \xi_{2} \right] \\ & - \xi_{2} \left[ \xi_{2} - \xi_$$

 $\frac{P_{\mu\nu\nu\gamma}^{(i)}}{r}\left[J_{\mu}^{i}(u_{\mu}) + J_{\mu}^{j}(u_{\mu\nu})\right]B_{\mu\nu}^{(i)\nu\gamma} + N_{u}^{i}\sum \frac{P_{\mu\nu\nu\gamma}^{(i)\nu\gamma}}{r}J_{\mu}^{k}(u_{\mu\nu})B_{\mu\nu}^{(i)\nu\gamma}. \quad (4.51)$ 

 $-1 + \frac{2}{n} = \frac{p_{m,p,p}}{p_{m,p}} B_{m,p}^{((p))} - g_{m,m} = \frac{p_{m,p,m}}{p_{m,p}} \nabla_{m,m}^{(n)}$ 

$$\mathcal{L}_{ijer} = 2\ln\frac{a_{ij}}{a_{ir}} \left[ 2 - L_{ir} + \ln\frac{Q_{i}^{((r))}}{Q_{ir}^{(r)}} \right], \qquad \mathcal{L}_{ijer} = 2L_{ir} \left[ 2 - L_{ir} + \ln\frac{Q_{ir}^{((r))}}{p^2} \right]. \quad (5.2)$$

#### ( ) 6 Integration of the real-virtual counterterm

In eqs. (5.14), (5.26) we have defined the constructions  $K^{(BB)}$ , that enabled us to build

$$\int d\Phi_{n+1}(|\bar{a}|) = \frac{m+1}{\eta_0} \int d\Phi_n^{(mn)} \int d\Phi_{n-1}^{(mn)}, \quad d\Phi_n^{(mn)} = d\Phi_n(|\bar{a}|)^{(mn)}, \quad (6)$$

where  $\partial \Phi_{ext}^{(ad)}$  is defined in eq. (43). The integration of  $E^{(60)}$  is careted out following

$$\int d\Phi_{n+1} K^{(BN)} = \int d\Phi_{n+1} \left[ \sum_{i} \left( \overline{\mathbf{R}}_{i} BV + \Delta_{0,i} \right) + \sum_{i,i=1} \left( \overline{\mathbf{RC}}_{ij} \delta S' + \Delta_{BC,i} \right) \right], \quad (6.2)$$

#### Rederived expressions



and the integrals of  $Q^{\mu\nu}_{(1)(1)}, \bar{Q}^{\mu\nu}_{(2)(1)}$  and  $\bar{Q}^{\mu\nu}_{(2)(1)}$  vanish in all rematerized  $d_{1}^{(2)}(x) = \frac{2\pi}{2} \left( \frac{x_{1}}{2} \right)^{-1} \left[ \left( 2 - \frac{x^{2}}{2} \right) \frac{1}{2} + \left( 12 - \frac{2}{2} x^{2} - 10 \phi \right) \frac{1}{2} \right]$  $J_{(0)}^{(4)}(s,s') = \left(\frac{\pi s}{2}\right)^{2} \left(\frac{\pi s}{2}\right)^{-1} \left[\frac{1}{2} + \frac{4}{2} + \left(16 - \frac{7}{6}s^{2}\right)\frac{1}{2} + \left(66 - \frac{14}{7}s^{2} - \frac{26}{7}\zeta_{2}\right)\frac{1}{2}$  $+71 - \frac{23}{49}\pi^2 - 20\zeta_5 - \frac{7}{49}\pi^4 + O(r)$  $J_{int}^{(N)}(s) = \left(\frac{\pi s}{4\pi}\right)^2 \left(\frac{s}{2\pi}\right)^{-2\epsilon} C_2 T_B \left[\frac{1}{2} \frac{1}{2^2} + \left(\frac{13}{4\pi} + \frac{1}{3} s^2\right) \frac{1}{2} - \frac{128}{4\pi s} + \frac{17}{4\pi s} s^2 + \frac{14}{3} \tau_{is} + O(s)\right]$  $\int d\Phi_{ind}^{(r)} \frac{Q_{ind}^{(r)}}{2\pi} = \int d\Phi_{ind}^{(r)} \frac{Q_{ind}}{2\pi} \left[ -g^{\mu\nu} + (d-2) \frac{k^{\mu} k c}{2\pi} \right] \rightarrow 0,$  $\sum_{(i,j,k)\in \mathcal{I}} f_{i,k}^{i,k} R_{i,k} = -\beta^{i} \frac{a_{i}}{2\sigma} \sum_{(i,j,k)\in \mathcal{I}} R_{i,k} \left[ \frac{1}{2} \ln \frac{b_{i,k}}{b_{i,k}} \ln^{k} \frac{b_{i,k}}{\sigma^{2}} + \frac{1}{6} \ln^{k} \frac{b_{i,k}}{b_{i,k}} + \log \left( -\frac{b_{i,k}}{\delta_{i,k}} \right) + \mathcal{O}(r) \right]$  $+216 - \frac{36}{21}\pi^2 - \frac{203}{21}\zeta_4 + \frac{29}{120}\pi^4 + O(r)$  $J_{har}^{(0_0(s))}(s) = \left(\frac{a_s}{c_s}\right)^2 \left(\frac{s}{c_s}\right)^{-h} C_F \left(2C_F - C_A\right)$  $\int d\Phi_{1,1}^{(r)} \frac{Q_{1,1,1}^{(r)}}{n_{r-}} = \int d\Phi_{1,1}^{(r)} \frac{\dot{Q}_{1,1,1}}{n_{r-}} \left[ -g^{\mu\nu} + (d-2) \frac{\dot{Q}_{1,1}^{\mu} U_{1}^{\nu}}{L^{2}} \right] \rightarrow 0,$  $J_{\alpha\beta}^{(0)}(s,s') = \left(\frac{\pi s}{2\pi}\right)^{2} \left(\frac{\pi s'}{2\pi}\right)^{-1} \left[\frac{1}{2\pi} + \frac{4}{2\pi} + \left(17 - \frac{4}{2}s^{2}\right)\frac{1}{2} + \left(79 - \frac{26}{2}s^{2} - \frac{68}{2}\zeta_{1}\right)\frac{1}{2}$  $\times \left[ - \left( \frac{11}{2} - \frac{1}{2} \, s^2 + \zeta_0 \right) \frac{1}{2} - \frac{227}{10} + s^2 + \frac{17}{47} \zeta_0 - \frac{11}{110} \, s^4 + \mathcal{O}(s) \right]$  $\int d\Phi_{ind,2}^{(r)} \frac{Q_{ind,2}^{r}}{d_{1}} = \sum_{\nu} \int d\Phi_{ind,2}^{(r)} \frac{Q_{ind,2}^{(r)}}{d_{1}} \left[ -g^{\mu\nu} + (d-2)\frac{\tilde{k}^{\mu}\tilde{k}^{\nu}}{d_{1}} \right] \rightarrow 0, \quad r = ijkr. \quad (D.7)$  $+281 - \frac{68}{2}\pi^2 - \frac{272}{2}(1 + \frac{13}{2}\pi^4 + O(r))$  $J_{\text{curr}}^{(N)}(x) = \left(\frac{2\pi}{n}\right)^2 \left(\frac{x}{n}\right)^{-1}$  $Z_{i,i}^{(p)} = N_i \int d\Phi_{ijkl}^{(p)+} \frac{P_{ijkl}^{(p)}}{2}$  $J^{(1)}_{2,1}(z) = \left(\frac{z_{2}}{z_{2}}\right)^{2} \left(\frac{z_{1}}{z_{2}}\right)^{-2z} \left[\frac{1}{z_{1}} + \frac{z_{1}}{z_{1}} + \left(1 + \frac{z_{1}}{z_{1}}z^{2}\right)\frac{1}{z_{1}} + \left(7 - 6z^{2} - \frac{74}{2}\zeta_{1}\right)\frac{1}{z_{1}}$  $\times \left\{ CrTn \left[ -\frac{2}{4} \frac{1}{c^2} - \frac{31}{n} \frac{1}{c^2} - \left( \frac{800}{3 c^2} - s^2 \right) \frac{1}{c} - \frac{20003}{970} + \frac{31}{c} s^2 + \frac{100}{n} \zeta_2 + O(c) \right] \right\}$ E Constituent integrale  $+312 - 27\pi^2 - \frac{324}{\pi}(z + \frac{25}{12\pi}\pi^4 + O(z))$  $+C_{3}T_{0}\left[-\frac{1}{r^{2}}-\frac{80}{14}\frac{1}{r^{2}}-\left(\frac{1211}{34}-\frac{3}{2}r^{2}\right)\frac{1}{r}-\frac{3620}{27}+\frac{80}{12}r^{2}+\frac{80}{3}c_{3}+\mathcal{O}(r)\right]\right\}$  $J_{\mathrm{eff}}^{\mathrm{eff}}(s) = \frac{m_{\mathrm{eff}}}{m_{\mathrm{eff}}} \left( \frac{s}{s^{1/2} s^{1/2}} \right)^{-1} \frac{\Gamma(1)}{s}$  $J_{\alpha}^{(q)}(s) = \left(\frac{\alpha_{0}}{2\pi}\right)^{2} \left(\frac{s}{2\pi}\right)^{-2s} \left[\frac{1}{2\pi}\frac{1}{2\pi} + \frac{17}{2\pi}\frac{1}{2\pi} + \left(\frac{128}{2\pi} - \frac{7}{2\pi}s^{2}\right)\frac{1}{2} + \frac{1274}{2\pi} - \frac{12}{2\pi}s^{2} - \frac{19}{2\pi}c_{\alpha} + O(r)\right]$  $J_{122}^{(2q)}(s) = \left(\frac{2\pi i}{2}\right)^{2} \left(\frac{1}{2q}\right)^{-2q} \left\{C_{2}^{2q}\left[-\frac{2}{2}-\frac{2\pi}{2}\frac{1}{2}-\left(\frac{2\pi i}{2}-2\pi s^{2}+4q^{2}\right)\frac{1}{2}\right]\right\}$  $\frac{1}{2} \left(\frac{1}{2}\right)^{-1} T_{0} \left[-\frac{2}{2}\frac{1}{2}-\frac{10}{2}-\left(\frac{10}{2}\right)\right]$  $J_{2}^{(q)}(z) = \left(\frac{m_{1}}{m_{2}}\right)^{2} \left(\frac{z}{m_{1}}\right)^{-2c} \left[\frac{1}{2}\frac{1}{m_{1}} + \frac{m_{1}}{m_{1}} + \left(\frac{4m_{1}}{2} - \frac{z}{2}z^{2}\right)\frac{1}{2} + \left(\frac{2m_{1}}{2} - \frac{2m_{1}}{m_{1}}z^{2} - \frac{m_{1}}{2}z_{1}\right)\frac{1}{2}$  $\left(\frac{1210}{11} - \frac{24}{10}e^2 - \frac{24}{10}e^2\right)$  $\frac{2361}{12r} + \frac{111}{2r} e^2 + \frac{136}{2r} \zeta_4 - \frac{e^4}{2r} + O(r)$  $+\frac{18351}{24} - \frac{3829}{24} e^4 - \frac{1825}{24} e^4 - \frac{21}{24} e^4 + O(r)$  $J_{he}^{(10)}(s) = \frac{n_{\mu}}{2\pi} \left(\frac{s}{s^{10}\mu^2}\right)^{-1}$  $+ CrC_{4} \left[ -\frac{1}{2} \frac{1}{24} - \frac{21}{14} \frac{1}{24} - \left( \frac{241}{26} - \frac{1}{24} x^{2} - 4_{12} \right) \frac{1}{2} \right]$  $J^{km} = N_{1} \int d\Phi_{i+1}^{(km)} S_{i+1}^{(i)} = \delta_{I,k} J_{i}(\delta_{i+1}^{(km)})$ ≘(±)"cr[  $\frac{2600}{700} + \frac{53}{910} x^2 - \frac{47}{9} (x + \frac{7}{90} x^4 + O(r))$  $J_{i}^{i,j} = N_i \int d\Phi_{i,j}^{(i,j)} J_{i,j}^{(i)} = \delta_{f,i} C_k J_i \{\delta_{i,j}^{(i,j)}\}$  $J_{12}^{(k)\ell}(s) = \frac{m_{\ell}}{2^{k}} \left( \frac{1}{1+1-2} \right)$  $J_{2n}^{(-d_1)} = N_1 \frac{2}{r^2} \int d\Phi_{1n}^{(-d_1)} J_{1n}^{(d)} \left[ \left( \frac{n_{1d}}{r^{(-d_1)}} \right)^{-1} - 1 \right] \\ = f_1^d J_{2n}^{(-d)} \left( r_{2n}^{(-d_1)} \right),$  $J_{abc}^{(0)}(s) = \left(\frac{2\pi}{3}\right)^{0} \left(\frac{s}{2}\right)^{-2s} C_{A}^{2} \left[-\frac{3}{2}\frac{1}{2} - \frac{77}{2}\frac{1}{2} - \left(\frac{s}{2}s^{2} + 3(s)\right)\frac{1}{2}\right]$ - 윤( <del>(</del> )  $J_{1}(s) = \frac{m_{1}}{s} \left( \frac{s}{1-s} \right)^{-1} \frac{\Gamma(1-s)}{\Gamma(s-s)}$  $J_{n+1}^{(n,d)} = N_1 \frac{1}{n!} \int d\theta_{n+1}^{(n,d)} g_{n+1}^{(n)} \left[ \left( \frac{1}{1+n!} \right)^{-1} - 1 \right] = f_1^n J_{n+1}^{(n)} \left( g_{n+1}^{(n,d)} \right)$  $\frac{149421}{149} + \frac{61}{\pi}r^2 + \frac{26}{\tau_A} - \frac{9}{\pi}r^4 + O(r)$  $=\frac{1}{2}\left(\frac{1}{2}\right)^{-1}\left[-\frac{1}{2}+\frac{2}{2}+6-\frac{7}{22}\pi^{2}+\left(28-\frac{7}{2}\pi^{2}-\frac{23}{27}\zeta_{1}\right)\right]$  $Z_{1}^{(\alpha,0)}=N_{1}^{\prime}\int d\Phi_{\alpha,\alpha}^{(\alpha,0)}Z_{\alpha,\alpha}^{(1)}$  $+\left(54 - \frac{7}{4}\sigma^2 - \frac{53}{24}\zeta_5 - \frac{71}{1100}\sigma^4\right)c^2 + O(c^2)\right]$  $J_{n+n}^{(2)} = N_n^2 \int d\Phi_{n+n}^{(2)} \frac{P_{n+n}^{(2)}(a_n, a_p)}{n} \frac{P_{n+n}^{(2)}(a_n, a_p)}{n}$  $\tilde{J}_{0}(s) = \frac{n_{0}}{n_{0}} \left( \frac{s}{s^{2} r_{0}^{2}} \right)^{-2\epsilon} \frac{\Gamma^{6}(1 + s)\Gamma^{6}(1 - s)}{4s^{2}\Gamma(1 + 2s)\Gamma(1 - s)}$ = 2000 (distant distant) (diffe [2-++++ ov]  $= \frac{m_{1}}{m_{2}} \left( \frac{m}{2^{2}} \right)^{-2} \left[ -\frac{1}{2^{2}} + \frac{1}{2^{2}} + \left( 1 - \frac{T}{2^{2}} \pi^{2} \right) \frac{1}{2^{2}} + \left( 11 - \frac{T}{2^{2}} \pi^{2} - \frac{14}{2^{2}} \zeta_{1} \right) \frac{1}{2} \right]$  $+ J_{n+1}^{\text{res}} \left\{ k_{n}^{((r),hh)} k_{n}^{((r),hh)} \right\} \left[ J_{n}^{\text{res}} (J_{n}^{\text{res}} + J_{n}^{\text{res}}) + (J_{n}^{\text{res}} + J_{n}^{\text{res}}) J_{n}^{\text{res}} \right]$  $+ J^{\mathrm{HEII}}_{\mathrm{inverse}} \left( a^{((p,kh))}_{p} a^{((p,kh))}_{\mu} \right) \left( I^{\mathrm{HI}}_{0} I^{\mathrm{HI}}_{\mathrm{HI}} + I^{\mathrm{HI}}_{1} I^{\mathrm{HI}}_{\mathrm{HI}} \right)$  $+64 - \frac{14}{7}\sigma^2 - \frac{24}{7}c_5 - \frac{7}{700}\sigma^4 + O(r)$  $J_{\mu\nu\mu}^{(ab)} = N_{1}^{(a)} \int d\Phi_{(ab)}^{(ab),ab} X_{ab}^{(c)} X_{ab}^{(c)} = J_{\mu\nu\mu}^{(b)} \left( \hat{\kappa}_{ab}^{(ab),ab}, \hat{\kappa}_{ab}^{(ab),ab} \right) f_{1}^{(b)}$  $+ J_{\mu\nu\mu}^{aug} \left\{ a_{\nu\nu}^{a(\mu,\mu\nu)} a_{\nu\nu}^{a(\mu,\mu\nu)} \right\} \left( f_{\nu}^{a\mu} + f_{\nu}^{a\mu} \right) \left( f_{\nu}^{a\mu} + f_{\nu}^{a\mu} \right)$  $\mathcal{L}^{(0)}(x) = \frac{2\pi}{2} \left(\frac{x}{2\pi}\right)^{-1} \left[ \left(2 - \frac{x^2}{2\pi}\right) \frac{1}{2\pi} + \left(16 - \frac{2}{2\pi}x^2 - 12\chi_0\right) \frac{1}{2\pi} \right]$  $J_{area}^{(1)ad} = N_{a}^{(2)} \int d\Phi_{aab,2}^{(1)ad} F_{ad}^{(1)} F_{ad}^{(1)} = J_{area}^{(0)} \left( F_{ad}^{(1)ad} \right) f_{a}^{(2)},$  $+ J_{max}^{max} \left\{ a_{1}^{(\mu,\lambda h)} a_{2}^{(\mu,\lambda h)} \right\} \left[ (f_{1}^{\mu\nu} + f_{2}^{\mu\nu}) f_{1}^{\mu\nu} + f_{2}^{\mu\nu} (f_{1}^{\mu\nu} + f_{2}^{\mu\nu}) \right]$  $J_{21}^{(1)d} = N_1^{(2)} \int d\Phi_{1212}^{(1)d} J_{121}^{(1)} = 2 T \pi J_{21}^{(q)} \left( I_{12}^{(1)d} \right) f_{11}^{(1)} - 2 C \pi J_{21}^{(q)} \left( I_{12}^{(1)d} \right) f_{11}^{(1)}, \quad (8.2)$  $+92 - \frac{7}{4}\pi^2 - 24\zeta_1 - \frac{7}{14}\pi^4 + O($ +  $J_{\mu\nu\mu\nu}^{\text{HIM}}\left(\epsilon_{\mu\nu}^{((\mu,\lambda\lambda\nu)}\epsilon_{\mu\nu}^{((\mu,\lambda\lambda\nu)}\right)f_{\mu\nu}^{\text{HI}}f_{\mu\nu}^{\text{H}}$ ,  $\tilde{J}_{4e}^{(2q)}(s) = \frac{\alpha_{1}}{2\pi} \left( \frac{s}{s^{2}} \right)^{-2\epsilon} C_{2} \left[ N/T_{2} \left[ \frac{1}{2} + \frac{25}{n} + O(s) \right] \right]$ ÷....  $J_{22,n}^{22}(x) = \frac{\alpha_2}{2} \left(\frac{x}{22}\right)^{-1} C_2 \left[\frac{1}{2\pi^2} + \frac{8}{2\pi^2} + \left(\frac{88}{2\pi} + \frac{11}{2\pi}x^2\right)^2 + \frac{714}{2\pi^2} + \frac{17}{2\pi}x^2 - \frac{64}{2}(x+O(x))\right]$ 290. A. (2)(2)(2). 11. 11.  $J_{n+1,n}^{max}(ss^{2}) = \left(\frac{\alpha_{n}}{2\pi}\right)^{2} \left(\frac{ss^{2}}{2\pi}\right)^{-1} T_{0}^{2} \left[\frac{4}{2\pi}\frac{1}{2} + \frac{64}{2\pi}\frac{1}{2} + \frac{284}{2\pi} - \frac{16}{2\pi}\pi^{2} + O(s)\right]$  $J_{1}^{(1)}(z) = \left(\frac{z_{1}}{2z}\right)^{2} \left(\frac{z}{2z}\right)^{-\frac{z_{1}}{2}} \left[-\frac{2}{3}\frac{1}{z^{2}} - \frac{2z}{3}\frac{1}{z^{2}} - \left(\frac{2z}{3z} - \frac{1}{12}\right)^{-\frac{z_{1}}{2}} + \frac{1}{12}\right]$  $+C_{4}\left[\frac{1}{2}\frac{1}{4}-\left(\frac{24}{4}-\frac{2}{4}\pi^{2}\right)\frac{1}{4}-\left(\frac{44}{4}-\frac{7}{4}\pi^{2}-13\zeta_{4}\right)\frac{1}{4}\right]$  $J_{\underline{M},n}(s) = \frac{\alpha_0}{2\pi} \left( \frac{s}{\sqrt{2}} \right)^{-1} C_0 \left[ -\frac{1}{2\sqrt{2}} + \frac{8}{2\pi} \frac{1}{\sqrt{2}} - \left( \frac{73}{2\pi} - \frac{7}{2\pi} \pi^2 \right) \frac{1}{2} - \frac{626}{24} + \frac{14}{2\pi} \pi^2 + \frac{25}{2\pi} \hat{c}_0 + O(r) \right]$  $J^{\rm eff}_{\rm eff}(\mu,\mu') = \left(\frac{\alpha_{\rm e}}{2\pi}\right)^2 \left(\frac{\mu^2}{\mu^2}\right)^{-1} T_{\rm e} C_F \left[\frac{1}{2}\frac{1}{\mu^2} + \frac{14}{9}\frac{1}{\mu} + \frac{141}{2^2} - \frac{4}{4}\mu^2 + \mathcal{O}(\nu)\right]$  $\frac{15805}{507} + \frac{38}{5}\sigma^2 + \frac{221}{5}c_4 - \frac{5}{5}\sigma^4 + O(r)$  $J_{kn+kr}^{\text{NMS}}(nr) = \left(\frac{\alpha_{n}}{2}\right)^{2} \left(\frac{nr'}{2}\right)^{2} T_{R}C_{A}\left[\frac{2}{n}\frac{1}{2} + \frac{32}{nr}\frac{1}{2} + \frac{12}{nr} - \frac{8}{n}\pi^{2} + O(r)\right]$  $J_{++}^{(b)}(x) = \frac{m_b}{m^b} \left(\frac{x}{m_b}\right)^{-1} T_{0} \left[ -\left(\frac{4}{\pi} - \frac{2}{\pi}x^2\right) \frac{1}{\pi} - \frac{2m_b}{m^{1+1}} + \frac{16}{m^{1+1}}x^2 + 8c_{ab} + O(x) \right]$  $J_{m}^{m}(\mathbf{L}_{n}(\mathbf{r}^{2})) = \left(\frac{a_{n}}{a_{m}^{2}}\right)^{n} \left(\frac{a_{n}}{a_{m}^{2}}\right)^{n} C_{p}^{2} \left[\frac{1}{2}\frac{1}{a_{m}^{2}} + \frac{1}{a} + \frac{17}{a} - \frac{1}{4}u^{2} + O(\nu)\right],$ (l.# 172.00 - (P)  $J_{\pm i \nu}^{(\eta)}(s) = \frac{m_0}{m_0} \left( \frac{s}{-1} \right)^{-1} C_F \left[ - \left( 1 - \frac{s^2}{4 \epsilon} \right) \frac{1}{2} - 8 + \frac{s^2}{4} + 6 c_0 + O(s) \right].$ 12 A. Bun, J. Burton, K. Jones and M. Pollen, Les Houches 2021 - schools at 2021 collider  $J_{m+1,c}^{HH}(ss^2) = \left(\frac{\pi_0}{2\pi}\right)^2 \left(\frac{ss^2}{2\pi}\right)^{-2} C_A C_F \left[\frac{1}{2}\frac{1}{2\pi} + \frac{7}{2\pi}\frac{1}{2} + \frac{180}{2\pi} - \frac{2}{8}s^2 + O(s)\right]$  $J_{44}^{(0)}(s) = \frac{n_4}{2\pi} \left(\frac{s}{s^2}\right)^{-1} C_4 \left[ -\left(\frac{2}{3} - \frac{s^2}{9}\right) \frac{1}{s} - \frac{N_2}{9} + \frac{3}{22} s^2 + 4\varepsilon_4 + O(r) \right]$ 1) et + 20 (and ) et + ett + 20 (and ) et  $J_{(1,1)}^{(0,1)}(\alpha^2) = \begin{pmatrix} \alpha_1 \\ \beta_2 \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \beta_2 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \beta_2 \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \beta_2 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \beta_2 \end{pmatrix} + \frac{\beta_2}{\beta_2} \frac{1}{\beta_1} + \frac{\beta_2}{\beta_2} + \frac{\beta_2}{\beta_2} - \frac{\beta_2}{\beta_2} \frac{\alpha^2}{\beta_1} + O(p) \end{pmatrix}$  $J_{\rm dist,h}^{(0)}(s) = \frac{\alpha_{\rm f}}{\alpha_{\rm c}} \left(\frac{s}{-1}\right)^{-1} T_{\rm fb} \left[ -\frac{4}{2} \frac{1}{-1} - \frac{32}{6} \frac{1}{-1} - \left(\frac{280}{27} - \frac{7}{6} s^2\right) \frac{1}{s} - \frac{2364}{27} + \frac{56}{6} s^2 + \frac{900}{6} \zeta_h + O(s) \right]$  $\mathcal{L}_{i}^{\mu} = N_{i} \frac{2}{\pi} \int dA_{i}^{\mu} g^{\mu} \frac{P_{i}^{\mu}}{2\pi}$  $J_{-10}^{(0)}(s) = \frac{\alpha_0}{2} \left(\frac{s}{s_1}\right)^{-1} T_0 \left[\frac{1}{2} \frac{1}{s_1} + \frac{32}{2} \frac{1}{s_1} + \left(\frac{244}{2\alpha} - \frac{3}{2} s^2\right) \frac{1}{s_1} + \frac{1764}{2\alpha} - \frac{30}{2\alpha} s^2 - \frac{32}{2} (s + O(s))$  $= \mathcal{L}^{0}(\hat{c}_{i}^{(r)})f_{i}^{0} + \mathcal{L}^{0}(\hat{c}_{i}^{(r)})$ e) + e<sup>(i</sup>(i)<sup>2</sup>),<sup>2</sup> [2] B.B. Bartanio, E. Pancelei, A. Paparez and S. Zoin, Nucl. in cont. in neuroisma in Will prediction at the LBC, Phys. Rev. D 106 (2022). start = N ( a start Print )  $E_{n}^{(1)} = N_{n} \frac{2}{n} \left[ d\theta_{n}^{(2)} \left\{ \frac{P_{n+1}}{m} \left[ 1 + \left( \frac{L_{n}^{(1)}}{m} \right)^{-1} \right] + N_{n} L_{n}^{(1)} \left[ 1 + \left( \frac{L_{n}}{m} \right)^{-1} \right] + N_{n} L_{n}^{(1)} \left[ 1 + \left( \frac{L_{n}}{m} \right)^{-1} \right] \right\}$  $J_{\rm hells}^{(k)}(s) = \frac{m_{\rm e}}{m_{\rm e}} \left(\frac{s}{-s}\right)^{-1} Cr \left[-\frac{1}{s^4} - \left(0 - \frac{2}{s} \sigma^2\right) \frac{1}{-s} - \left(30 - \frac{21}{s^4} \sigma^2 - 16\zeta_1\right) \frac{1}{s}\right]$ - x 22(0.0)x+22(0.0)x+x2(0.0)x]  $-126 + \frac{43}{r} e^2 + \frac{121}{r} (r + \frac{e^4}{r} + O(r))$ = [25(07)+25(07)](7+[25(07)+25(07)](07+85  $J_{ijkl}^{(0)er} = N_i^0 \int d\Phi_{ijkl}^{(0)er} \frac{P_{\mu_{kl}}^{\mu_{kl}}}{E^2} E^2$  $J_{2n}^{(k)}(x) = \frac{m_{0}}{m_{0}} \left( \frac{x}{\sqrt{2}} \right)^{-1} Or \left[ \frac{1}{\sqrt{2}} + \frac{2}{\sqrt{2}} + \left( 5 - \frac{5}{\sqrt{2}} x^{2} \right) \frac{1}{2} + 12 - \frac{5}{m} x^{2} - \frac{12}{m} \zeta (x + O(x)) \right].$ = x [222 (42.47) x3+ 222 (42.47) x3+ 222 (42.47) x3 ] (3) [3] X. Catani et al., Higgs hours production in association with a top-antilipy part part of the avoid dealership of the OCD. Phys. Rev. Lett. 139 (1011) 10100 (ardire 2010).  $d_{m}^{(1)} = N_{1}^{2} \frac{d_{m}^{(1)}}{d_{m}} \int d\theta d_{m}^{(2)} \frac{d_{m}^{(2)}}{d_{m}} \left[ \left( \frac{d_{m}^{(1)}}{d_{m}} \right)^{2} - \left( \frac{d_{m}^{(2)}}{d_{m}^{(2)}} \right)^{2} \right]$  $J_{221,1}^{(0)}(s) = \frac{\alpha_0}{2} \left( \frac{s}{24} \right)^{-1} C_4 \left[ -\frac{2}{3} \frac{1}{2} - \left( \frac{88}{2} - \frac{4}{3} s^2 \right) \frac{1}{2} - \left( \frac{1238}{20} - \frac{35}{20} s^2 - 335 \right) \frac{1}{2}$  $\mathcal{L}_{n}^{(1)} = N^{2} \int dt^{(0)}_{n+1} \frac{P_{n+1}^{(1)}}{P_{n+1}} \mathcal{L}_{n}^{(1)}$  $= \mathcal{K} \left[ \mathcal{L}_{12}^{(2)} \left( \mathcal{L}_{1}^{(1)} \mathcal{L}_{2}^{(2)} \right) \mathcal{L}_{1}^{(2)} + \mathcal{L}_{12}^{(2)} \left( \mathcal{L}_{1}^{(1)} \mathcal{L}_{2}^{(2)} \right) \mathcal{L}_{2}^{(2)} + \mathcal{L}_{12}^{(2)} \left( \mathcal{L}_{1}^{(1)} \mathcal{L}_{2}^{(2)} \right) \mathcal{L}_{2}^{(2)} \right]_{\mu = 0, (n+1)}$  $\frac{19708}{100} + \frac{496}{100} r^2 + \frac{626}{10} c_4 + \frac{2}{9} r^4 + O(r)$ + (c.(m)+c.(m), m+c+(c.(m)+c.(m)),  $J_{n(n)}^{(i)} = N_i^0 \int d\Phi_{nd,0}^{(i)} \frac{P_{p+1}^{(i)}}{1} Z_{j_i}^{(i)}$  $J_{(0,0)}^{(0)}(s) = \frac{\alpha_0}{2\pi} \left(\frac{s}{2\pi}\right)^{-1} C_4 \left[\frac{2}{2\pi} \frac{1}{2\pi} + \frac{16}{2\pi} \frac{1}{2\pi} + \left(\frac{122}{2\pi} - \frac{5}{2\pi} s^2\right) \frac{1}{2} + \frac{862}{2\pi} - \frac{23}{2\pi} s^2 - \frac{26}{2\pi} c_0 + O(r)\right],$  $F_{1n}^{n+n} = \frac{N_{1}}{2} \int d\theta \frac{||x|^{n}}{||x|} \frac{T_{2n}}{2} \left(1 - \frac{2}{1+n} \frac{N_{1} N_{2n}}{2}\right) \left[ \left(\frac{||x|^{n}}{||x|^{n}}\right)^{-1} - \left(\frac{||x|^{n}}{2}\right)^{-1} \right] = F_{1n} \left(Q^{(n)}, Q^{(n)}\right), \quad (R.16)$  $J_{-1}^{(0)}(s) = \frac{m_0}{m_0} \left(\frac{s}{m_0}\right)^{-1} T_0 \left[\frac{2}{2} \frac{1}{m_0} + \frac{14}{m_0} \frac{1}{m_0} + \left(\frac{122}{2m_0} - \frac{1}{2} s^2\right) \frac{1}{m_0} + \frac{140s}{m_0} - \frac{44}{m_0} s^2 - \frac{47}{m_0} t_0 + O(s)$  $J_{c}^{(0)}(s) = \frac{n_0}{4} \left(\frac{s}{m_0}\right)^{-1} T_0 \left\{ N_1 T_0 \left[ \frac{4}{8} \frac{1}{m_1} + \frac{64}{8m_1} + \frac{284}{8m_1} - \frac{2}{8} s^2 + O(s) \right] \right\}$  $J_{2k,n}^{(0)}(s) = \frac{\alpha_k}{\alpha_k} \left(\frac{s}{-1}\right)^{-1} T_k \left[-\frac{2}{3} \frac{1}{1-1} - \frac{16}{3} \frac{1}{-1} - \left(\frac{140}{37} - \frac{7}{14}s^2\right) \frac{1}{2} - \frac{1232}{34} + \frac{26}{34}s^2 + \frac{36}{34}\zeta_k + O(s)\right]$  $+C_F \left[\frac{2}{4}\frac{1}{n!} + \frac{31}{n!}\frac{1}{n!} + \left(\frac{231}{2!!} - x^2\right)\frac{1}{n!} + \frac{5500}{2!!} - \frac{31}{n!}x^2 - \frac{124}{n!}\zeta_4 + O(z)\right]$  $J_{m_{1}}^{q_{1}}(s) = \frac{m_{1}}{2s} \left(\frac{s}{s_{1}}\right)^{-1} (2C_{F} - C_{A}) \left[\frac{1}{2}\frac{1}{s_{1}} + \frac{1}{s_{1}} + \left(\frac{7}{2s} - \frac{11}{2s}s^{2}\right)\frac{1}{s} + \frac{21}{2s} - \frac{7}{2s}s^{2} - \frac{22}{3s}c_{1} + O(s)\right]$  $\mathcal{L}_{0}^{(0)}(x,x') = \left(\frac{\alpha_{0}}{2\pi}\right)^{0} \left(\frac{\alpha_{0}}{2\pi}\right)^{0} T_{0} \left[-\frac{2}{2}\frac{1}{2} - \frac{2\alpha_{0}}{2}\frac{1}{2} - \left(\frac{2\alpha_{0}}{2\pi} - \frac{2}{2}x'\right)\frac{1}{2} - \frac{2\alpha_{0}\alpha_{0}}{2\pi} + \frac{\alpha_{0}}{2\pi}x' + \frac{\alpha_{0}}{2}(2\pi + O(x))\right]$  $+C_{4}\left[-\frac{1}{2r^{2}}-\frac{31}{24}\frac{1}{r}-\left(\frac{211}{2r}-\frac{1}{2}r^{2}\right)\frac{1}{r}-\frac{3281}{142}+\frac{31}{2}r^{2}+\frac{62}{9}q_{2}+\mathcal{O}(r)\right]\right\}$  $J^{H}_{2h,c,0}(s) = \frac{\alpha_{1}}{\alpha_{2}} \left( \frac{s}{-s} \right)^{-1} (2C_{F} - C_{A}) \left[ -\frac{1}{\alpha_{1} s} - \frac{1}{s^{2}} - \left( 3 - \frac{2}{\alpha_{1}} s^{2} \right) \frac{1}{s} - 9 + \frac{2}{15} s^{2} + \frac{20}{\alpha_{1}} \zeta_{A} + O(s) \right]$  $F_{mm}^{(0)}(s,s') = \left(\frac{m}{2}\right)^2 \left(\frac{m'}{2}\right)^{-1} C_{\mu} \left[-\frac{1}{2}\frac{1}{2} - \frac{2}{2} - \left(8 - \frac{7}{12}s^{\mu}\right)\frac{1}{2} - 30 + \frac{7}{4}s^{\mu} + \frac{30}{2}((2) + O(s))\right]$  $\mathcal{L}_{2,2}^{(N)}(u,v') = \left(\frac{u}{2v}\right)^2 \left(\frac{u'}{2v}\right)^{-1} C_{\alpha} \left[-\frac{1}{2}\frac{1}{2}^2 - \frac{14}{8}\frac{1}{2} - \left(\frac{172}{27} - \frac{7}{18}v^2\right)\frac{1}{v} - \frac{1984}{81} + \frac{18}{27}v^2 + \frac{30}{8}(2) + O(v)\right]$  $J_{m}^{(1)}(s) = \frac{m_{0}}{m_{0}} \left(\frac{s}{m_{0}}\right)^{-10} C_{12} \left\{C_{12}\left[-\left(\frac{3}{2}-\frac{s^{2}}{m_{0}}\right)\frac{1}{m}-\left(\frac{13}{2}-\frac{2}{3}s^{2}-10\zeta_{1}\right)\frac{1}{m}\right]$  $J^{H}_{\Delta_{m,n}}(z) = \frac{m}{2\pi} \left( \frac{z}{z^2} \right)^{-1} C_{2} \left[ \frac{1}{2z^2} + \frac{1}{z^2} + \left( \frac{1}{2} - \frac{T}{2z} z^2 \right) \frac{1}{z} + \frac{H}{2} - \frac{3}{4} z^2 - \frac{5}{2} (z + O(z)) \right].$  $-\frac{141}{2} + \frac{100}{210} s^2 + 20(s - \frac{7}{10} s^4 + O(s))$  $J^{\rm th}_{2n-n}(x) = \frac{m}{2n} \left( \frac{x}{2n} \right)^{-1} C_{2n} \left[ -\frac{1}{4n^2} + \frac{1}{2n} - \left( 2 - \frac{7}{4n^2} x^2 \right) \frac{1}{n} - 9 + \frac{7}{2n} x^2 + \frac{20}{n} \zeta x + \mathcal{O}(x) \right)$  $L_{(1)n}^{(0)0}(s,s') = \left(\frac{m}{2s}\right)^{4} \left(\frac{m}{s^{2}}\right)^{-1} Te \left[-\frac{2}{3}\frac{1}{s^{2}} - \frac{28}{3}\frac{1}{s^{2}} - \left(\frac{2m}{3s} + \frac{8}{s^{2}}\right)^{\frac{1}{2}} - \frac{284}{3s} + \frac{12}{2s} + \frac{28}{3s}((2) + O(s))\right]$  $\mathcal{L}_{0}^{(0)}(x,x') = \left(\frac{2\pi}{2\pi}\right)^{2} \left(\frac{2\pi}{2\pi}\right)^{-1} C_{F} \left[-\frac{1}{2\pi}\frac{1}{2} - \frac{2}{2\pi} - \left(\frac{2\pi}{2\pi} - \frac{2}{2\pi}x^{2}\right)\frac{1}{2} - 32 + \frac{2}{2\pi}x^{2} + \frac{24}{2\pi}\zeta(2) + O(x)\right]$  $+C_4 \left[ \frac{1}{1+1} + \frac{1}{4+1} + \left( 1 - \frac{\pi^2}{44} - 4 \zeta_1 \right) \frac{1}{4} + \frac{\pi}{4} + \frac{\pi}{4+1} \sigma^2 - \frac{47}{4} \zeta_2 - \frac{11}{16} \sigma^4 + \mathcal{O}(\rho) \right] \right\},$ 



• The cancellation of the explicit poles of RV by  $I^{(1)}$  is manifest

In case of VV, upon expansion of I<sup>(2)</sup> + I<sup>(RV)</sup> we could obtain the general pole structure of VV [Catani 1998] with the opposite sign (as advertised by the LASS papers):

$$\begin{split} VV_{\text{poles}} &= 2 \operatorname{Re} \left[ -\frac{1}{2} \left\langle \mathcal{M}^{(0)} \left| \mathbf{I}^{(1)}(\epsilon) \mathbf{I}^{(1)}(\epsilon) \right| \mathcal{M}^{(0)} \right\rangle - \frac{\beta_{0}}{\epsilon} \left\langle \mathcal{M}^{(0)} \left| \mathbf{I}^{(1)}(\epsilon) \right| \mathcal{M}^{(0)} \right\rangle \right. \\ &+ \left\langle \mathcal{M}^{(0)} \left| \mathbf{I}^{(1)}(\epsilon) \right| \mathcal{M}^{(1)} \right\rangle + e^{-\epsilon\gamma} \frac{\Gamma(1-2\epsilon)}{\Gamma(1-\epsilon)} \left( \frac{\beta_{0}}{\epsilon} + \mathcal{K} \right) \left\langle \mathcal{M}^{(0)} \left| \mathbf{I}^{(1)}(2\epsilon) \right| \mathcal{M}^{(0)} \right\rangle \\ &+ \left\langle \mathcal{M}^{(0)} \left| \mathbf{H}^{(2)}(\epsilon) \right| \mathcal{M}^{(0)} \right\rangle \right] \\ &+ \operatorname{Re} \left[ 2 \left\langle \mathcal{M}^{(1)} \left| \mathbf{I}^{(1)}(\epsilon) \right| \mathcal{M}^{(0)} \right\rangle - \left\langle \mathcal{M}^{(0)} \left| \mathbf{I}^{(1)\dagger}(\epsilon) \right| \mathcal{M}^{(0)} \right\rangle \right] \end{split}$$

## Explicit check of the cancellation for $e^+e^- ightarrow 3$ jets



$$\begin{pmatrix} l^{(2)} + l^{(\mathrm{RV})} \end{pmatrix}_{\mathrm{poles}} = B\left(\frac{\alpha_{s}}{2\pi}\right)^{2} \left[ \frac{1}{\epsilon^{4}} \left( \frac{\mathrm{CA}^{2}}{2} + \left( 2\mathrm{CF} - \frac{17}{3} \right) \mathrm{CA} + 2\mathrm{CF}^{2} - \frac{34\mathrm{CF}}{3} \right) \\ + \frac{1}{\epsilon^{3}} \left( \left( \log\left(\mu^{2}\right) + \log(\mathrm{s12}) - \log(\mathrm{s13}) - \log(\mathrm{s23}) + \frac{11}{24} \right) \mathrm{CA}^{2} \\ + \left( -\frac{\mathrm{nfTr}}{6} - \frac{34\log\left(\mu^{2}\right)}{3} - 6\log(\mathrm{s12}) + \frac{26\log(\mathrm{s13})}{3} \right) \\ + \mathrm{CF}\left( 4\log\left(\mu^{2}\right) - 2\log(\mathrm{s13}) - 2\log(\mathrm{s23}) + \frac{47}{12} \right) + \frac{26\log(\mathrm{s23})}{3} - \frac{164}{9} \right) \mathrm{CA} \\ + \frac{34\mathrm{nfTr}}{9} + \mathrm{CF}^{2}\left( 4\log\left(\mu^{2}\right) - 4\log(\mathrm{s12}) + 6 \right) \\ + \mathrm{CF}\left( -\frac{\mathrm{nfTr}}{3} - \frac{68\log\left(\mu^{2}\right)}{3} + \frac{32\log(\mathrm{s12})}{3} + 6\log(\mathrm{s13}) + 6\log(\mathrm{s23}) - \frac{98}{3} \right) \right) \\ + \frac{1}{\epsilon^{2}} \left( \ldots \right) + \frac{1}{\epsilon} \left( \ldots \right) \right]$$

▶ With the opposite sign this exactly agrees with the standard  $e^+e^- \rightarrow 3$  jet references [Garland, Gehrmann et al. 2001], [Gehrmann-De Ridder, Gehrmann et al. 2007].



$$RV_{\rm sub}(X) \equiv \left( RV + I^{(1)} \right) \delta_{n+1}(X) - \left( K^{(RV)} + I^{(12)} \right) \delta_n(X)$$
(1)

• Automatic generation of  $I^{(1)}$ ,  $K^{(RV)}$  and  $I^{(12)}$  using FORM.

- ▶ The RV matrix element is extracted from [Bern, Dixon, Kosower 1997].
- Everything is fed to the MC integrator (see Adam's talk).
- Time to perform  $y_{\min}$  study (see Adam's talk for the definition).













#### Event shapes

- After reassuring in the stability of the integration procedure, one can fix the y<sub>min</sub> value and produce the distributions of some event shape observables.
- In the following we will examine following event shape observables:
  - 1.  $\tau$ -parameter (thrust):

$$T = \max_{\vec{n}} \left( \frac{\sum_{i} |\vec{n} \cdot \vec{p}_{i}|}{\sum_{i} |\vec{p}_{i}|} \right), \quad \tau \equiv 1 - T.$$

2. C-parameter:

$$C_{\text{par}} = \frac{3}{2} \frac{\sum_{i,j} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{\left(\sum_i |\vec{p}_i|\right)^2}.$$

3. Energy-energy correlation:

$$\operatorname{EEC}(\chi) = \frac{1}{\sigma_{\operatorname{had}}} \sum_{i,j} \int \frac{E_i E_j}{Q^2} \mathrm{d}\sigma_{e^+e^- \to i\, j+X} \delta(\cos \chi + \cos \theta_{ij}).$$

4. Jet-cone energy fraction:

$$\frac{\mathrm{d}\Sigma_{\mathrm{JCEF}}}{\mathrm{d}\cos\chi} = \sum_{i} \int \frac{E_{i}}{Q} \,\mathrm{d}\sigma_{e^{+}e^{-} \to i+X} \delta\left(\cos\chi - \frac{\vec{p}_{i}\cdot\vec{n}_{T}}{|\vec{p}_{i}|}\right).$$



## $\tau$ -parameter and C-parameter: $q\bar{q}q\bar{q}$ -channel



DESY.(15)

#### $\tau$ -parameter and C-parameter: $q\bar{q}r\bar{r}$ -channel



**Y.**(16)

## $\tau$ -parameter and C-parameter: $q\bar{q}gg$ -channel



17

### EEC and JCEF: $q\bar{q}q\bar{q}$ -channel



SY.(18)

#### EEC and JCEF: qqrr-channel



19

Bakar Chargeishvili et al. | Analysis of (n + 1) and n-parton contributions for computing QCD jet cross sections in the LASS scheme

## EEC and JCEF: qqgg-channel



20



- ▶ We demonstrated a proof-of-concept implementation of the LASS scheme.
- ▶ Numerous expressions from the LASS papers were successfully cross-checked.
- ▶ We have full analytic and numeric control of *RR*, *RV*, and *VV* contributions.
- Efficient automatic generation and integration of the subtraction terms were achieved.
- First distributions of the event shape observables using the LASS scheme were obtained.
- Significant progress was made towards the full automation of the current LASS scheme.



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Thanks for your attention!

