Renormalization of chiral Gauge Theories with non-anticommuting γ_5 in the BMHV Scheme at the Multi-Loop Level

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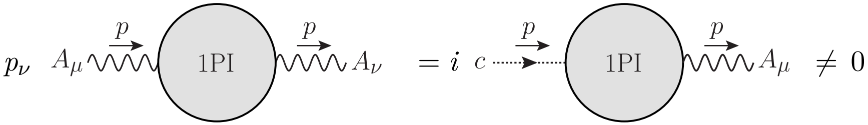
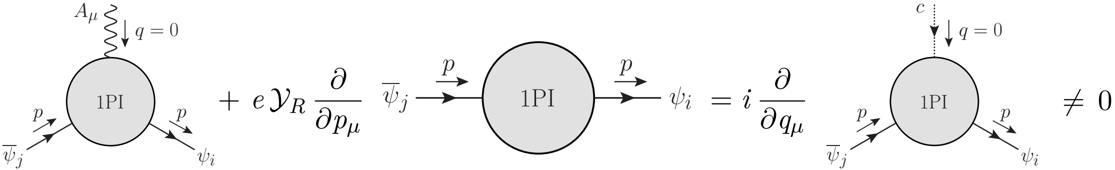
Preview — Breaking of Gauge Invariance in the BMHV Scheme

Spurious symmetry breakings induced by the BMHV algebra, e.g.:

- Violation of the transversality of the gauge boson self energy



- Violation of the fermion self energy to fermion gauge boson interaction current relation



Outline

The γ_5-Problem

Chiral Gauge Theories and the γ_5-Problem

- Electroweak interactions act on chiral fermions
  - Left-handed and right-handed fermions interact differently with gauge bosons
  - SM and all its extensions for potential new physics are chiral gauge theories
- γ_5 is manifestly 4-dimensional
- Cannot simultaneously retain the following properties in D dimensions

$$\{\gamma_5, \gamma^\mu\} = 0 \tag{1}$$

$$\mathrm{Tr}(\Gamma_1\Gamma_2) = \mathrm{Tr}(\Gamma_2\Gamma_1) \tag{2}$$

$$\mathrm{Tr}(\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = -4i\epsilon^{\mu\nu\rho\sigma} \tag{3}$$

- Inconsistent in $D \neq 4 \implies \gamma_5$-problem
- Give up at least one property
  - Breitenlohner-Maison/'t Hooft-Veltman (BMHV) scheme [HV'72, BM'77]
  - Alternative γ_5-schemes (Naive anticommuting? Reading point? ...)

Breitenlohner-Maison/'t Hooft-Veltman (BMHV) Scheme

Abandoning anticommutativity of $\gamma_5 \longrightarrow$ BMHV algebra

- Decomposing the D dimensional space

$$\mathbb{M} = \mathbb{M}_4 \oplus \mathbb{M}_{-2\epsilon}, \qquad \eta_{\mu\nu} = \overline{\eta}_{\mu\nu} + \hat{\eta}_{\mu\nu}, \qquad X^\mu = \overline{X}^\mu + \hat{X}^\mu$$

- BMHV algebra

$$\{\gamma^\mu, \gamma_5\} = \{\hat{\gamma}^\mu, \gamma_5\} = 2\,\hat{\gamma}^\mu\,\gamma_5, \qquad \{\overline{\gamma}^\mu, \gamma_5\} = 0, \qquad [\hat{\gamma}^\mu, \gamma_5] = 0$$

- Breaks gauge invariance in intermediate steps
- Broken symmetry has to be restored $\Longrightarrow$ more complicated renormalization

$$S_{\text{ct}} = S_{\text{sct}} + S_{\text{fct}} = S_{\text{sct,inv}} + S_{\text{sct,non-inv}} + S_{\text{fct}}$$

- Complicated
- But: unitary and proven to be consistent to all orders!

Chiral Gauge Theories in D Dimensions and the BMHV Scheme

Abelian chiral gauge theory — Lagrangian of the considered theory

$$\mathcal{L} = i\overline{\psi}_i\gamma^\mu\partial_\mu\psi_i - e\mathcal{Y}_{Rij}A_\mu\overline{\psi}_i\mathbb{P}_\mathrm{L}\gamma^\mu\mathbb{P}_\mathrm{R}\psi_j + \dots$$

Challenges in D dimensions

1. Kinetic term, i.e. γ^μ, must be D-dimensional

$$\overline{\psi}\partial\!\!\!/\psi = \overline{\psi_L}\overline{\partial\!\!\!/}\psi_L + \overline{\psi_R}\overline{\partial\!\!\!/}\psi_R + \overline{\psi_L}\widehat{\partial\!\!\!/}\psi_R + \overline{\psi_R}\widehat{\partial\!\!\!/}\psi_L$$

   $\Longrightarrow$ kinetic term necessarily mixes chiralities

2. Analytic continuation to D dimensions not unique
   - $\mathbb{P}_\mathrm{L}\gamma^\mu\mathbb{P}_\mathrm{R}$ admits many inequivalent but equally correct extensions
   - use the most symmetric option $\Longrightarrow$ most natural and symmetric choice

$$\overline{\psi}\mathbb{P}_\mathrm{L}\gamma^\mu\mathbb{P}_\mathrm{R}\psi = \overline{\psi}\mathbb{P}_\mathrm{L}\overline{\gamma}^\mu\mathbb{P}_\mathrm{R}\psi = \overline{\psi_R}\overline{\gamma}^\mu\psi_R$$

Always: Mismatch D versus 4 $\Longrightarrow$ breaks gauge invariance

Symmetries

Symmetries — Slavnov-Taylor Identity

- The Slavnov-Taylor identity reflects symmetries in the full quantum theory
- The classical Symmetry can, and may in general, be broken by the Regularization

$$\mathcal{S}(\Gamma_{\text{reg}}) \neq 0$$

- The Slavnov-Taylor identity needs to be obeyed after renormalization for consistency
  - unphysical states / negative norm
  - unitary and gauge independent physical S-matrix
- Require the validity of symmetries as part of the definition of the theory

$$\mathcal{S}(\Gamma_{\text{ren}}) \stackrel{!}{=} 0$$

- Regularization induced symmetry breakings need to be restored
  - Symmetries usually valid in DReg
  - However, no gauge-invariant regularization that preserves chiral symmetry $\Longrightarrow$ γ_5-problem

Ward Identities in abelian chiral Gauge Theories

Abelian gauge theories: Slavnov-Taylor identity $\mathcal{S}(\Gamma) = 0 \implies$ Ward identities

$$
\begin{aligned}
p_\nu \, \widetilde{\Gamma}^{\nu\mu}_{AA} &= 0, \\
p_\sigma \, \widetilde{\Gamma}^{\sigma\rho\nu\mu}_{AAAA} &= 0, \\
\widetilde{\Gamma}^{ji,\mu}_{\psi\overline{\psi}A}(q=0) &= -e\,\mathcal{Y}_{Rjk}\,\frac{\partial}{\partial p_\mu}\widetilde{\Gamma}^{ki}_{\psi\overline{\psi}}
\end{aligned}
$$

Distinguish two cases

1. Regularization preserves symmetries
   $\implies$ standard multiplicative renormalization
2. Regularization breaks symmetries
   $\implies$ include symmetry-restoring counterterms, satisfying $\mathcal{S}(\Gamma^{(n)}_{\text{subren}}) = -bS^n_{\text{ct}}$

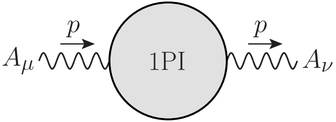
[OP,SS'95, HB et al.'23]

Symmetry Restoration

Symmetry Restoration — Explicit Evaluation

Explicit evaluation of all Green functions in $\mathcal{S}_D(\Gamma^{(n)}_{\text{subren}} + S^n_{\text{sct}})$ [HB et al. '21,'23]

$$i\,\widetilde{\Gamma}^{\nu\mu}_{AA}(p)\Big|^1 = $$



$$\Big|^1$$

$$= \frac{ie^2}{16\pi^2}\frac{\text{Tr}(\mathcal{Y}_R^2)}{3}\Bigg\{\frac{2}{\epsilon}\left[\left(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{\eta}^{\mu\nu}\right) - \frac{1}{2}\widehat{p}^2\overline{\eta}^{\mu\nu}\right]$$

$$+\left[\left(\frac{10}{3} - 2\ln(-p^2)\right)\left(\overline{p}^\mu\overline{p}^\nu - \overline{p}^2\overline{\eta}^{\mu\nu}\right) - \left(\overline{p}^2 + \widehat{p}^2\left(\frac{8}{3} - \ln(-p^2)\right)\right)\overline{\eta}^{\mu\nu}\right]\Bigg\}$$

Exhibit the breaking explicitly — probe the gauge boson Ward identity

$$[\mathcal{S}(\Gamma)]^1_{A^\mu c} = i\,p_\nu\widetilde{\Gamma}^{\nu\mu}_{AA}(p)\Big|^1 = \frac{ie^2}{16\pi^2}\frac{\text{Tr}(\mathcal{Y}_R^2)}{3}\left[-\frac{1}{\epsilon}\widehat{p}^2\overline{p}^\mu - \overline{p}^2\overline{p}^\mu\right] \neq 0$$

Symmetry Restoration — Quantum Action Principle

- The ultimate symmetry requirement is the Slavnov-Taylor identity

$$\underset{D\to 4}{\mathrm{LIM}}\left(\mathcal{S}_D(\Gamma_{\mathrm{DRen}})\right)=0$$

- Regularized Quantum Action Principle of Dimensional Regularization

DReg:[BM'77], DRed:[DS'05]
Rev:[OP,SS'95, HB et al.'23]

$$\mathcal{S}_D(\Gamma_{\mathrm{DRen}})=(\widehat{\Delta}+\Delta_{\mathrm{ct}})\cdot\Gamma_{\mathrm{DRen}}$$

- Possible symmetry breaking can be rewritten as a composite operator insertion

$$\widehat{\Delta}=\mathcal{S}_D(S_0),\qquad \widehat{\Delta}+\Delta_{\mathrm{ct}}=\mathcal{S}_D(S_0+S_{\mathrm{ct}})$$

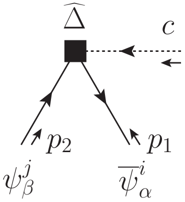
Symmetry Restoration — QAP: Practical Application

- Perturbative requirement from the Slavnov-Taylor identity

$$\underset{D\to 4}{\mathrm{LIM}}\left(\widehat{\Delta}\cdot\Gamma^{n}_{\mathrm{DRen}}+\sum_{k=1}^{n-1}\Delta^{k}_{\mathrm{ct}}\cdot\Gamma^{n-k}_{\mathrm{DRen}}+\Delta^{n}_{\mathrm{ct}}\right)=0$$

- Tree-level breaking: $\widehat{\Delta}$-operator reflects the breaking of chiral gauge invariance

$$\mathcal{S}_D(S_0)=\widehat{\Delta}=-\int d^Dx\, e\,\mathcal{Y}_{Rij}\, c\left\{\overline{\psi}_i\left(\overleftarrow{\widehat{\partial\!\!\!/}}\mathbb{P}_R+\overrightarrow{\widehat{\partial\!\!\!/}}\mathbb{P}_L\right)\psi_j\right\}$$



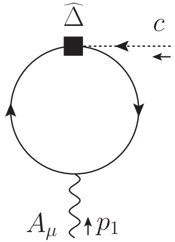
$$=-e\,\mathcal{Y}_{Rij}\left(\widehat{p\!\!\!/}_1\mathbb{P}_{\mathrm{R}}+\widehat{p\!\!\!/}_2\mathbb{P}_{\mathrm{L}}\right)_{\alpha\beta}$$

- Compute Green functions involving an insertion of the composite operator $\widehat{\Delta}+\Delta_{\mathrm{ct}}$

Symmetry Restoration — QAP: 1-Loop Example Revisited

Breaking of the gauge boson self energy at 1-loop [HB et al. '21,'23]

$$i\left(\widehat{\Delta}\cdot\widetilde{\Gamma}^{1}_{\mathrm{DRen}}\right)_{A_\mu c} =$$



$$= \frac{e^2}{16\pi^2}\frac{\mathrm{Tr}(\mathcal{Y}_R^2)}{3}\left[-\frac{1}{\epsilon}\widehat{p}^2\overline{p}^\mu - \overline{p}^2\overline{p}^\mu\right]$$

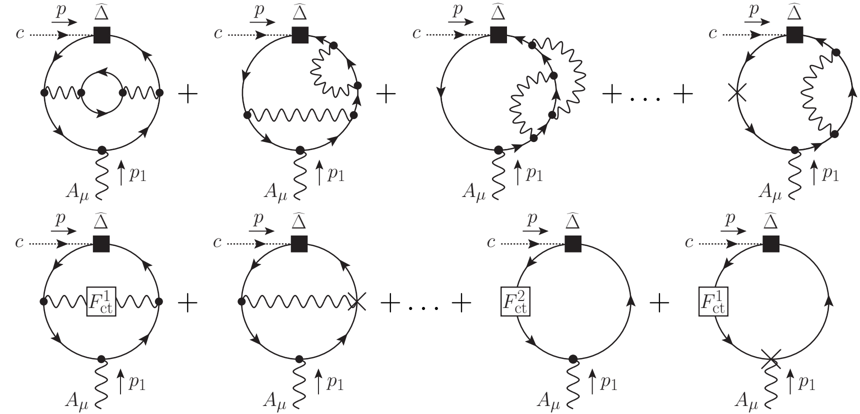
$$= -i\left[\mathcal{S}(\Gamma)\right]^1_{A^\mu c} = p_\nu \widetilde{\Gamma}^{\nu\mu}_{AA}(p)\Big|^1$$

1-loop breaking of the Slavnov-Taylor identity

$$\left(\widehat{\Delta}\cdot\widetilde{\Gamma}\right)^1 = -\frac{1}{16\pi^2}\int d^Dx\,\frac{e^2\,\mathrm{Tr}\left(\mathcal{Y}_R^2\right)}{3}\left[\frac{1}{\epsilon}\,c\,\overline{\partial}_\mu\,\widehat{\partial}^2\,\overline{A}^\mu + c\,\overline{\partial}_\mu\,\overline{\partial}^2\,\overline{A}^\mu\right] + \ldots$$

Symmetry Restoration — QAP: 3-Loop Application

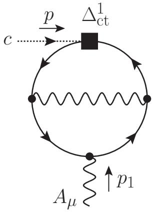
$$i\left(\widehat{\Delta}\cdot\widetilde{\Gamma}^{3}_{\mathrm{DRen}}\right)_{A_\mu c} =$$



$$+ \quad \dots$$

Symmetry Restoration — QAP: 3-Loop Application

$$i\left(\Delta^1_{\mathsf{ct}} \cdot \widetilde{\Gamma}^2_{\mathrm{DRen}}\right)_{A_\mu c} =$$



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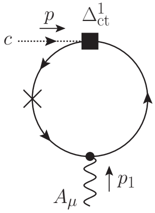
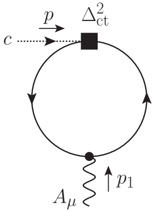


$+ \dots$

$$i\left(\Delta^2_{\mathsf{ct}} \cdot \widetilde{\Gamma}^1_{\mathrm{DRen}}\right)_{A_\mu c} =$$



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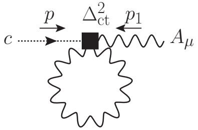

Quantum Action Principle vs. Explicit Evaluation

Simplification is threefold

+ Only UV-divergent part of Green functions contributes
+ Only power-counting divergent Green functions required
+ In general fewer diagrams with Δ insertion

Cons

- Requires special Δ-operator inserted Green functions

$\Longrightarrow$ Procedure with the Quantum Action Principle is much more efficient!

Computational Setup

Computational Setup

Two different computational frameworks

1. Mathematica based setup
   - Diagram generation with FeynArts [TH '00]
   - Special symbolic manipulation (Dirac algebra, ...) with FeynCalc [RM,VS et al. '91,'16,'20]
   - IBP-reduction with FIRE C++, interfaced with FeynHelpers [AS et al. '19]
   - Tested up to 3-Loop
2. FORM based setup [JV et al., '89]
   - Diagram generation with QGRAF [PN '91]
   - FORM code generation partially with Mathematica
   - Integral family and sector symmetries with Feynson and Reduze2 [VM '22; AM,CS '12]
   - IBP-reduction with FIRE C++
   - Tested up to 4-Loop (so far ...)

Results

Right-Handed Model

Gauge Boson Self Energy in an abelian chiral Gauge Theory

1-loop result [HB et al. '21,'23]

$$i\widetilde{\Gamma}^{\nu\mu}_{AA}(p)\Big|^{1}_{\text{div}} = \frac{i\,e^2}{16\pi^2}\frac{\text{Tr}\left(\mathcal{Y}_R^2\right)}{3}\left[\frac{2}{\epsilon}\left(\overline{p}^{\mu}\overline{p}^{\nu} - \overline{p}^2\overline{\eta}^{\mu\nu}\right) - \frac{1}{\epsilon}\widehat{p}^2\overline{\eta}^{\mu\nu}\right]$$

2-loop result

$$i\widetilde{\Gamma}^{\nu\mu}_{AA}(p)\Big|^{2}_{\text{div}} = \frac{i\,e^4}{(16\pi^2)^2}\frac{\text{Tr}\left(\mathcal{Y}_R^4\right)}{3}\left[\frac{2}{\epsilon}\left(\overline{p}^{\mu}\overline{p}^{\nu} - \overline{p}^2\overline{\eta}^{\mu\nu}\right) - \left(\frac{1}{2\epsilon^2} - \frac{17}{24\epsilon}\right)\widehat{p}^2\overline{\eta}^{\mu\nu}\right]$$

- Transversality of the gauge boson self energy is violated
- Violation is local
- Symmetry restoration necessary and possible

Gauge Boson Self Energy in an abelian chiral Gauge Theory

3-loop result [DS,MW'23]

$$
\begin{aligned}
i\widetilde{\Gamma}^{\nu\mu}_{AA}(p)\Big|^{3}_{\text{div}} = &- \frac{i}{(16\pi^2)^3}\, e^6 \left[\mathcal{B}^{3,\text{inv}}_{AA}\,\frac{1}{\epsilon^2} + \mathcal{A}^{3,\text{inv}}_{AA}\,\frac{1}{\epsilon}\right]\left(\overline{p}^{\mu}\overline{p}^{\nu} - \overline{p}^2\overline{\eta}^{\mu\nu}\right) \\
&- \frac{i}{(16\pi^2)^3}\, e^6 \left[\widehat{\mathcal{C}}^{3,\text{break}}_{AA}\,\frac{1}{\epsilon^3} + \widehat{\mathcal{B}}^{3,\text{break}}_{AA}\,\frac{1}{\epsilon^2} + \widehat{\mathcal{A}}^{3,\text{break}}_{AA}\,\frac{1}{\epsilon}\right]\widehat{p}^2\,\overline{\eta}^{\mu\nu} \\
&+ \frac{i}{(16\pi^2)^3}\, e^6\, \overline{\mathcal{A}}^{3,\text{break}}_{AA}\,\frac{1}{\epsilon}\,\overline{p}^2\,\overline{\eta}^{\mu\nu},
\end{aligned}
$$

with

$$
\mathcal{B}^{3,\text{inv}}_{AA} = \frac{4}{162}\Big(3\,\text{Tr}\big(\mathcal{Y}_R^6\big) - 5\,\text{Tr}\big(\mathcal{Y}_R^4\big)\text{Tr}\big(\mathcal{Y}_R^2\big)\Big),
$$

$$\dots$$

$$
\widehat{\mathcal{A}}^{3,\text{break}}_{AA} = \frac{1}{64800}\Big(\big(156672\,\zeta_3 - 49427\big)\text{Tr}\big(\mathcal{Y}_R^6\big) - 8374\,\text{Tr}\big(\mathcal{Y}_R^4\big)\text{Tr}\big(\mathcal{Y}_R^2\big)\Big),
$$

$$\dots$$

Violation of the Gauge Boson Transversality

3-loop result [DS,MW'23]

$$i\Big(\big[\widehat{\Delta}+\Delta_{\text{ct}}\big]\cdot\widetilde{\Gamma}\Big)^3_{A_\mu c} = i\big(\widehat{\Delta}\cdot\widetilde{\Gamma}^3\big)_{A_\mu c} + i\big(\Delta^1_{\text{ct}}\cdot\widetilde{\Gamma}^2\big)_{A_\mu c} + i\big(\Delta^2_{\text{ct}}\cdot\widetilde{\Gamma}^1\big)_{A_\mu c}$$
$$= -\frac{e^6}{(16\pi^2)^3}\left[\widehat{\mathcal{C}}^{3,\text{break}}_{AA}\frac{1}{\epsilon^3}+\widehat{\mathcal{B}}^{3,\text{break}}_{AA}\frac{1}{\epsilon^2}+\widehat{\mathcal{A}}^{3,\text{break}}_{AA}\frac{1}{\epsilon}\right]\widehat{p}^2\,\overline{p}^\mu$$
$$+\frac{e^6}{(16\pi^2)^3}\left[\overline{\mathcal{A}}^{3,\text{break}}_{AA}\frac{1}{\epsilon}+\mathcal{F}^{3,\text{break}}_{AA}\right]\overline{p}^2\,\overline{p}^\mu,$$

with the same Laurent-coefficients as before and

$$\mathcal{F}^{3,\text{break}}_{AA} = -\frac{1}{21600}\Big(\big(35242+8448\,\zeta_3\big)\text{Tr}\big(\mathcal{Y}^6_R\big)+1639\,\text{Tr}\big(\mathcal{Y}^4_R\big)\text{Tr}\big(\mathcal{Y}^2_R\big)\Big)$$

Singular Counterterm Action - Bilinear Gauge Boson Contributions

[DS,MW'23]

$$
\begin{aligned}
S_{\text{sct}} = &- \frac{e^2}{16\pi^2} \frac{\text{Tr}\left(\mathcal{Y}_R^2\right)}{3} \frac{1}{\epsilon} \int d^D x \left[2\left(-\frac{1}{4} \overline{F}^{\mu\nu} \overline{F}_{\mu\nu} \right) + \frac{1}{2} \overline{A}_\mu \widehat{\partial}^2 \overline{A}^\mu \right] + \ldots \\
&- \frac{e^4}{(16\pi^2)^2} \frac{\text{Tr}\left(\mathcal{Y}_R^4\right)}{3} \int d^D x \left[\frac{2}{\epsilon} \left(-\frac{1}{4} \overline{F}^{\mu\nu} \overline{F}_{\mu\nu} \right) + \left(\frac{1}{2\epsilon^2} - \frac{17}{24\epsilon} \right) \frac{1}{2} \overline{A}_\mu \widehat{\partial}^2 \overline{A}^\mu \right] + \ldots \\
&+ \frac{e^6}{(16\pi^2)^3} \left[\mathcal{B}_{AA}^{3,\text{inv}} \frac{1}{\epsilon^2} + \mathcal{A}_{AA}^{3,\text{inv}} \frac{1}{\epsilon} \right] \int d^D x \left(-\frac{1}{4} \overline{F}^{\mu\nu} \overline{F}_{\mu\nu} \right) \\
&- \frac{e^6}{(16\pi^2)^3} \left[\widehat{\mathcal{C}}_{AA}^{3,\text{break}} \frac{1}{\epsilon^3} + \widehat{\mathcal{B}}_{AA}^{3,\text{break}} \frac{1}{\epsilon^2} + \widehat{\mathcal{A}}_{AA}^{3,\text{break}} \frac{1}{\epsilon} \right] \int d^D x \, \frac{1}{2} \overline{A}_\mu \widehat{\partial}^2 \overline{A}^\mu \\
&+ \frac{e^6}{(16\pi^2)^3} \overline{\mathcal{A}}_{AA}^{3,\text{break}} \frac{1}{\epsilon} \int d^D x \, \frac{1}{2} \overline{A}_\mu \overline{\partial}^2 \overline{A}^\mu + \cdots \\
&+ \ldots
\end{aligned}
$$

Finite Counterterm Action

[DS,MW'23]

$$
\begin{aligned}
S_{\text{fct}} = &- \frac{e^2}{16\pi^2}\,\mathcal{F}^{1,\text{break}}_{AA} \int d^4x\,\frac{1}{2}\,\overline{A}_\mu\,\overline{\partial}^2\,\overline{A}^\mu + \frac{e^2}{16\pi^2}\,\mathcal{F}^{1,\,\text{break}}_{\psi\overline{\psi},\,ji} \int d^4x\,\overline{\psi}_j\,i\,\overline{\partial\!\!\!/}\,\mathbb{P}_\mathrm{R}\,\psi_i \\
&+ \frac{e^4}{16\pi^2}\,\mathcal{F}^{1,\,\text{break}}_{AAAA} \int d^4x\,\frac{1}{8}\,\overline{A}_\mu\overline{A}^\mu\overline{A}_\nu\overline{A}^\nu \\
&+ \frac{e^4}{(16\pi^2)^2}\,\mathcal{F}^{2,\text{break}}_{AA} \int d^4x\,\frac{1}{2}\,\overline{A}_\mu\,\overline{\partial}^2\,\overline{A}^\mu - \frac{e^4}{(16\pi^2)^2}\,\mathcal{F}^{2,\,\text{break}}_{\psi\overline{\psi},\,ji} \int d^4x\,\overline{\psi}_j\,i\,\overline{\partial\!\!\!/}\,\mathbb{P}_\mathrm{R}\,\psi_i \\
&- \frac{e^6}{(16\pi^2)^2}\,\mathcal{F}^{2,\,\text{break}}_{AAAA} \int d^4x\,\frac{1}{8}\,\overline{A}_\mu\overline{A}^\mu\overline{A}_\nu\overline{A}^\nu \\
&+ \frac{e^6}{(16\pi^2)^3}\,\mathcal{F}^{3,\text{break}}_{AA} \int d^4x\,\frac{1}{2}\,\overline{A}_\mu\,\overline{\partial}^2\,\overline{A}^\mu - \frac{e^6}{(16\pi^2)^3}\,\mathcal{F}^{3,\,\text{break}}_{\psi\overline{\psi},\,ji} \int d^4x\,\overline{\psi}_j\,i\,\overline{\partial\!\!\!/}\,\mathbb{P}_\mathrm{R}\,\psi_i \\
&- \frac{e^8}{(16\pi^2)^3}\,\mathcal{F}^{3,\,\text{break}}_{AAAA} \int d^4x\,\frac{1}{8}\,\overline{A}_\mu\overline{A}^\mu\overline{A}_\nu\overline{A}^\nu \\
&+ \ldots
\end{aligned}
$$

Left- and Right-Handed Interactions

Towards the Standard Model

Left- and Right-Handed Interactions in the BMHV Scheme

Lagrangian with 4-dimensional chiral interaction currents

$$\mathcal{L} = i\overline{\psi}_i\gamma^\mu\partial_\mu\psi_i - e\mathcal{Y}_{Rij}A_\mu\overline{\psi}_i\mathbb{P}_\mathrm{L}\gamma^\mu\mathbb{P}_\mathrm{R}\psi_j - e\mathcal{Y}_{Lij}A_\mu\overline{\psi}_i\mathbb{P}_\mathrm{R}\gamma^\mu\mathbb{P}_\mathrm{L}\psi_j + \ldots$$

Evanescent interaction currents

$$\mathcal{L} \supset -e\mathcal{Y}_{LRij}A_\mu\overline{\psi}_i\mathbb{P}_\mathrm{R}\gamma^\mu\mathbb{P}_\mathrm{R}\psi_j - e\mathcal{Y}_{RLij}A_\mu\overline{\psi}_i\mathbb{P}_\mathrm{L}\gamma^\mu\mathbb{P}_\mathrm{L}\psi_j$$

Additional tree-level breaking

$$\mathcal{S}_D(S_0) = \widehat{\Delta} = \widehat{\Delta}_{c\overline{\psi}\psi} + \widehat{\Delta}_{cA\overline{\psi}\psi}$$

- electric charge conservation (globally) $\Longrightarrow$ $[Q, \mathcal{Y}_k] = 0$
- Hermiticity and CPT-invariance $\Longrightarrow$ $\mathcal{Y}_R = \mathcal{Y}_R^\dagger,\ \mathcal{Y}_L = \mathcal{Y}_L^\dagger,\ \mathcal{Y}_{LR} = \mathcal{Y}_{RL}^\dagger$

Counterterm Action with generalised Couplings

1-loop result

[PE,PK,DS,MW'24] to be published

$$S^1_{\textsf{sct}} \supset -\frac{e^2}{16\pi^2}\frac{1}{\epsilon}\,\widehat{\mathcal{A}}^{1,\,\textsf{break}}_{\psi\overline{\psi},\mathrm{LR},\,ji}\int d^4x\left\{\overline{\psi}_j\,i\,\widehat{\partial\!\!\!/}\,\psi_i - e\,\mathcal{Y}_{LR\,kj}\,\overline{\psi}_k\,\widehat{A\!\!\!/}\,\psi_i\right\}$$

$$\begin{aligned}
S^1_{\textsf{fct}} = &\frac{e^2}{16\pi^2}\,\mathcal{F}^{1,\textsf{break}}_{AA}\int d^4x\,\frac{1}{2}\,\overline{A}_\mu\,\overline{\partial}^2\,\overline{A}^\mu + \frac{e^4}{16\pi^2}\,\mathcal{F}^{1,\,\textsf{break}}_{AAAA}\int d^4x\,\frac{1}{8}\,\overline{A}_\mu\overline{A}^\mu\overline{A}_\nu\overline{A}^\nu \\
&+\frac{e^2}{16\pi^2}\int d^4x\,\overline{\psi}_j\,i\,\overline{\partial\!\!\!/}\left[\mathcal{F}^{1,\,\textsf{break}}_{\psi\overline{\psi},\mathrm{R},\,ji}\,\mathbb{P}_\mathrm{R} + \mathcal{F}^{1,\,\textsf{break}}_{\psi\overline{\psi},\mathrm{L},\,ji}\,\mathbb{P}_\mathrm{L}\right]\psi_i \\
&+\frac{e^3}{16\pi^2}\,\mathcal{F}^{1,\,\textsf{break}}_{\psi A\overline{\psi},\,ji}\int d^4x\,\overline{\psi}_j\,\overline{A\!\!\!/}\,\left(\mathbb{P}_\mathrm{R} - \mathbb{P}_\mathrm{L}\right)\psi_i
\end{aligned}$$

Conclusions

Conclusions

1. Renormalization of a right-handed abelian chiral Gauge Theory
   - New singular counterterm structures arise at higher loop-level, e.g.:

$$4 \text{ dim. singular bilin. gauge boson ct.} \propto \frac{e^6}{(16\pi^2)^3}\frac{1}{\epsilon}\int d^Dx \frac{1}{2}\overline{A}_\mu \overline{\partial}^2 \overline{A}^\mu$$

$$\text{singular quartic gauge boson ct.} \propto \frac{e^8}{(16\pi^2)^3}\frac{1}{\epsilon}\int d^Dx \overline{A}_\mu \overline{A}^\mu \overline{A}_\nu \overline{A}^\nu$$

   - Finite symmetry-restoring counterterm action maintains structure
2. Chiral gauge Theory with generalised interaction currents
   - New evanescent breakings
   - New counterterm structures, not only in S_{sct}, but also in S_{fct}
3. Outlook
   - Higher loop levels: 4-loop and beyond?
   - Renormalization of non-abelian chiral gauge theories
   - Application to the Standard Model

References

References

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