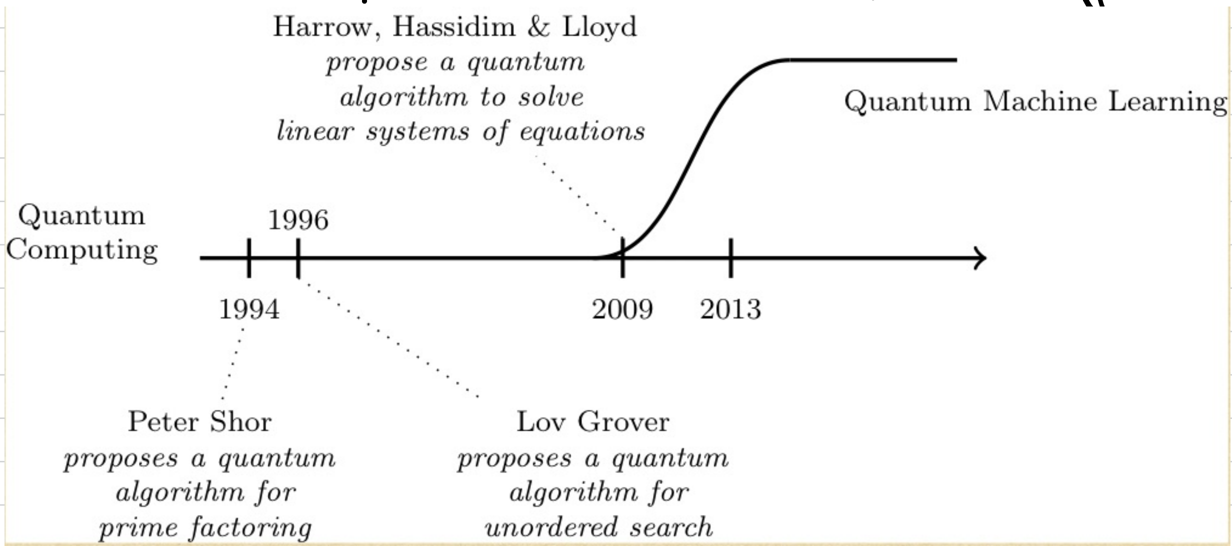
Recap of quantum computing

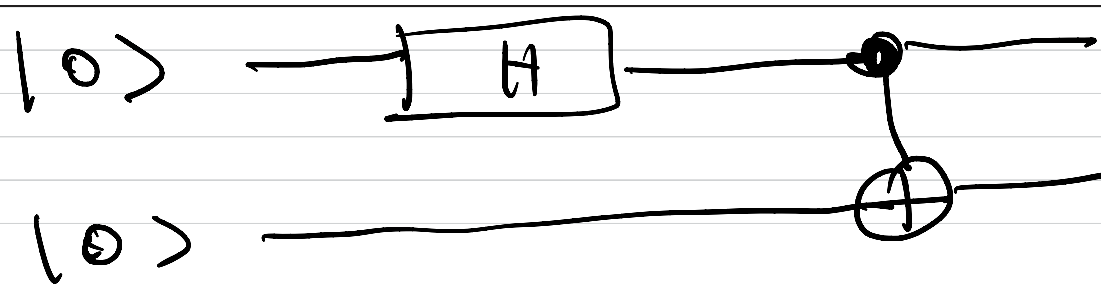


timeline of quantum computing
& quantum machine learning

Theorem: any quantum evolution can be approxi. by a sequence of only a handful of elementary manipulations called **Quantum Gates** which act on one or two qubits at a time

Consequence: quantum algorithms are mainly as quantum circuits of these elementary gates.

example entangelment Quantum circuit



A quantum computer: physical implementation of n-qubits with precise control on the evolution of the state

A quantum algorithm: a controlled manipulation of the quantum system with measurements to retrieve information

→ a quantum computer → A sampling device

quantum bits examples: superconducting qubits, photonic setup, ion traps, topological properties of quasi-particles

Bits & Qubits: Recap

Concept: classical to quantum

$$0 \longrightarrow |0\rangle$$

$$1 \longrightarrow |1\rangle$$

$|0\rangle$ & $|1\rangle$ form an othonormal basis in a 2d Hilbert space $\mathcal{H}$, aka computationals basis

superposition: General state of a qubit $|\psi\rangle$

$$|\psi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle, \quad \alpha_0, \alpha_1 \in \mathbb{C}$$

$$|\alpha_0|^2 + |\alpha_1|^2 = 1$$

Vetor notation a qubit is represented as $\underline{\alpha} = \begin{pmatrix} \alpha_0 \\ \alpha_1 \end{pmatrix}$

example! $|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ both in $\mathbb{C}^2$

Bloch Sphere

$$|\psi\rangle = e^{i\gamma}\left(\cos\frac{\theta}{2}\,|0\rangle + e^{i\theta}\sin\frac{\theta}{2}\,|1\rangle\right)$$

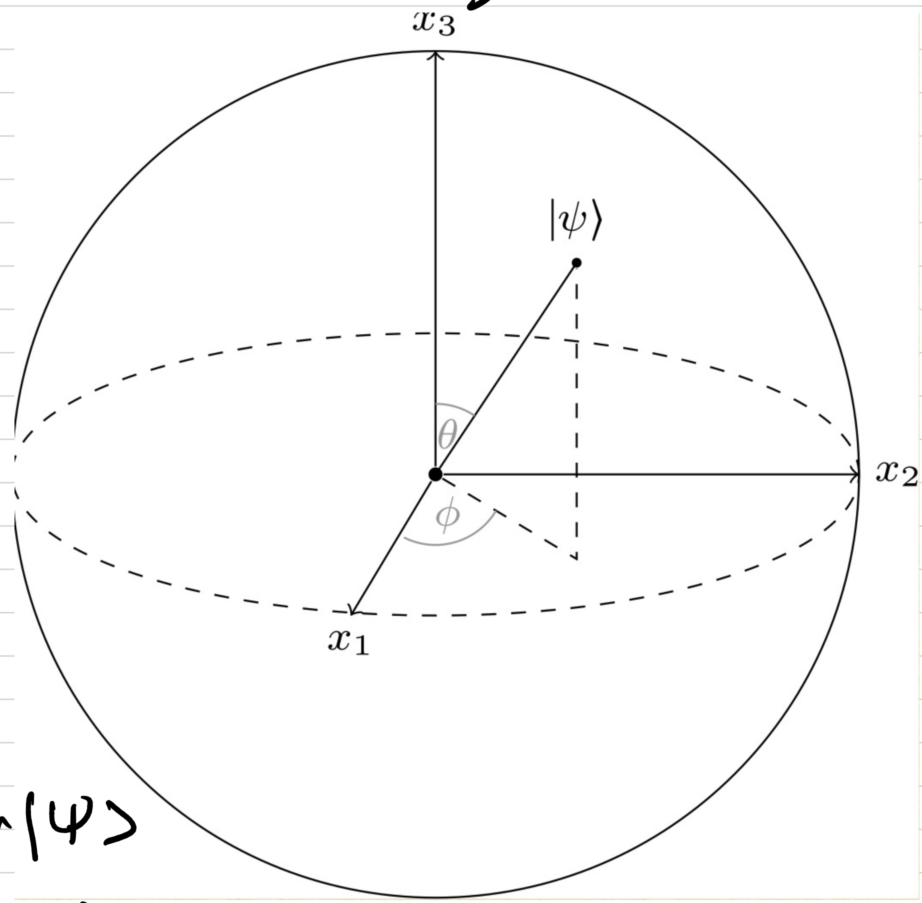
general phase
can be omitted

θ, ϕ & $\gamma \in \mathbb{R}$

spherical coordinates

$0 \leq \phi < 2\pi$

$0 \leq \theta < \pi$



the Hilbert space vector $|\psi\rangle$

can be visualized as $\mathbb{R}^3$ vector

$$\begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\phi \end{pmatrix}$$

given m on n unentangled qubits $|q_1\rangle \cdots |q_n\rangle$

$$|\psi\rangle = |q_1\rangle \otimes |q_2\rangle \otimes \cdots \otimes |q_n\rangle$$

shorthand

$$|ab\rangle = |a\rangle \otimes |b\rangle$$

$$|\psi\rangle = \sum_{i=0}^{2^{n-1}} \alpha_i |i\rangle$$

binary equivalent.

ex: $\alpha_4 = |100\rangle = |4\rangle$

$\alpha_7 = |111\rangle = |7\rangle$

Q. state: using the density matrix notation: ρ

pure state: $\rho_{pure} = |\psi\rangle\langle\psi| = \sum_{i,j=0}^{2^n-1} \alpha_i^* \alpha_j |i\rangle\langle j|$

mixed state: $\rho_{mixed} = \sum_{i,j=0} \alpha_{ij} |i\rangle\langle j|$ $\qquad \alpha_{ij} \in \mathbb{C}$

Quantum gates

Concepts: There are 2 basic operations that are fundamental to quantum computing:

1) quantum logic gates QLG.
2) computational basis measurements CBM.

QLG: realised as unitary transformations $\underline{\underline{U}}$;

$$\underline{\underline{U}}^{-1} = \underline{\underline{U}}^{+}$$

maintain lengths as:

$$\underline{\underline{U}}\,\underline{\alpha} = \underline{\alpha}' \; ; \; \sum |\alpha_k|^2 = \sum |\alpha'_k|^2$$

example: Not gate represented as $\underline{\underline{X}} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$$\underline{\underline{X}}\,|0\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} = |1\rangle \; ; \; \underline{\underline{X}}\,\underline{\underline{X}}^{+} = \underline{\underline{I}}$$

Quantum gates:

example: 2qubit gate ; the CNOT gate

$$\underline{\underline{U}}_{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$|00\rangle \to |00\rangle$; $|00\rangle = |0\rangle \otimes |0\rangle$

$|01\rangle \to |01\rangle$

$|10\rangle \to |11\rangle$

$|11\rangle \to |10\rangle$

other ex: Hadamard gate, Pauli rotation gates ....

Measuring Qubits in the computational basis

concept: A computational basis measurement is used to measure whether the individual qubits are in state $|0\rangle$ or $|1\rangle$

$\longrightarrow$ apply a circuit U just before meas.

concept: pre-meas. circuits are basis transformation of the quantum state

theory: A CBM of $|\psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle$ is represented by the projectors on the two possible eigenspaces P_0 & P_1 with $P_0 = |0\rangle\langle 0|$ & $P_1 = |1\rangle\langle 1|$

results: The probability of obtaining the measurement outcome 0 is:

$$p(0) = \mathrm{Tr}\left(P_0|\psi\rangle\langle\psi|\right) = \langle\psi|P_0|\psi\rangle = |\alpha_0|^2$$

$P(1) = |\alpha_1|^2$

Case 0 — if the outcome is 0, the qubit is now in the state

Q. M.

$$|\psi\rangle \leftarrow \frac{P_0|\psi\rangle}{\sqrt{\langle\psi|P_0|\psi\rangle}} = |0\rangle$$

→ the full observable corresponding to CBM is simply the Pauli Z- observable

$$\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$$

reading the eigenvalues $+1 \rightarrow$ observation $|0\rangle$

$-1 \rightarrow$ observation $|1\rangle$

seperate CBMs performed on multiple qubits is similar to drawing a sample of a binary string of length n where n is the number of qubits from a distribution defined by the quantum state.

$\langle \sigma_z \rangle \in [-1, 1]$

in practice run an algorithm s-times to sample $S \subset \{-1, 1\}$ $\rightarrow$ average of bits

The nb of sample S required for an error ε is equivalent to estimating the probability p when sampling from a Bernoulli distrib.

confi. interv.: $[p-\varepsilon, p+\varepsilon]$, Wald inter. large S

$$\varepsilon = z\sqrt{\frac{\hat{p}(1-\hat{p})}{S}}$$

$\hat{p}$: share of the sample being in state $|1\rangle$

z: the z-value

much detailed analysis will follow (ex: Wilson score)

$$\hat{p} \approx 0 \rightarrow \varepsilon \leq \frac{z}{2\sqrt{S}}$$

1) Deutsch- Josza Algorithm
2) Quantum encoding (Very important)
3) Survey of quantum algorithms