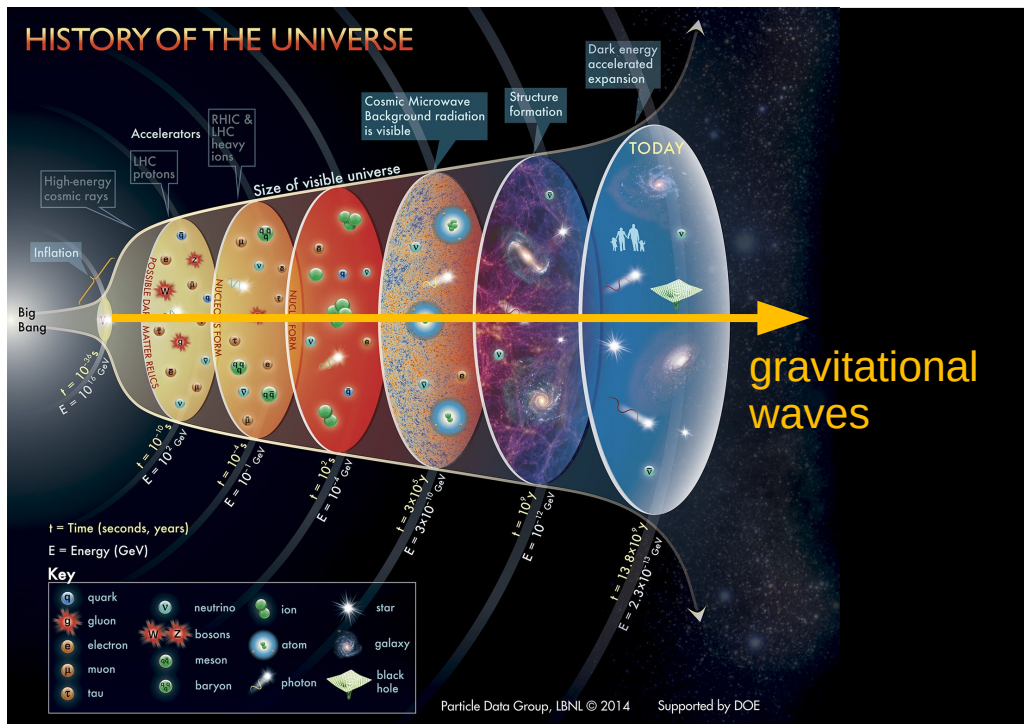


# Gravitational Waves

# as a probe of the early Universe

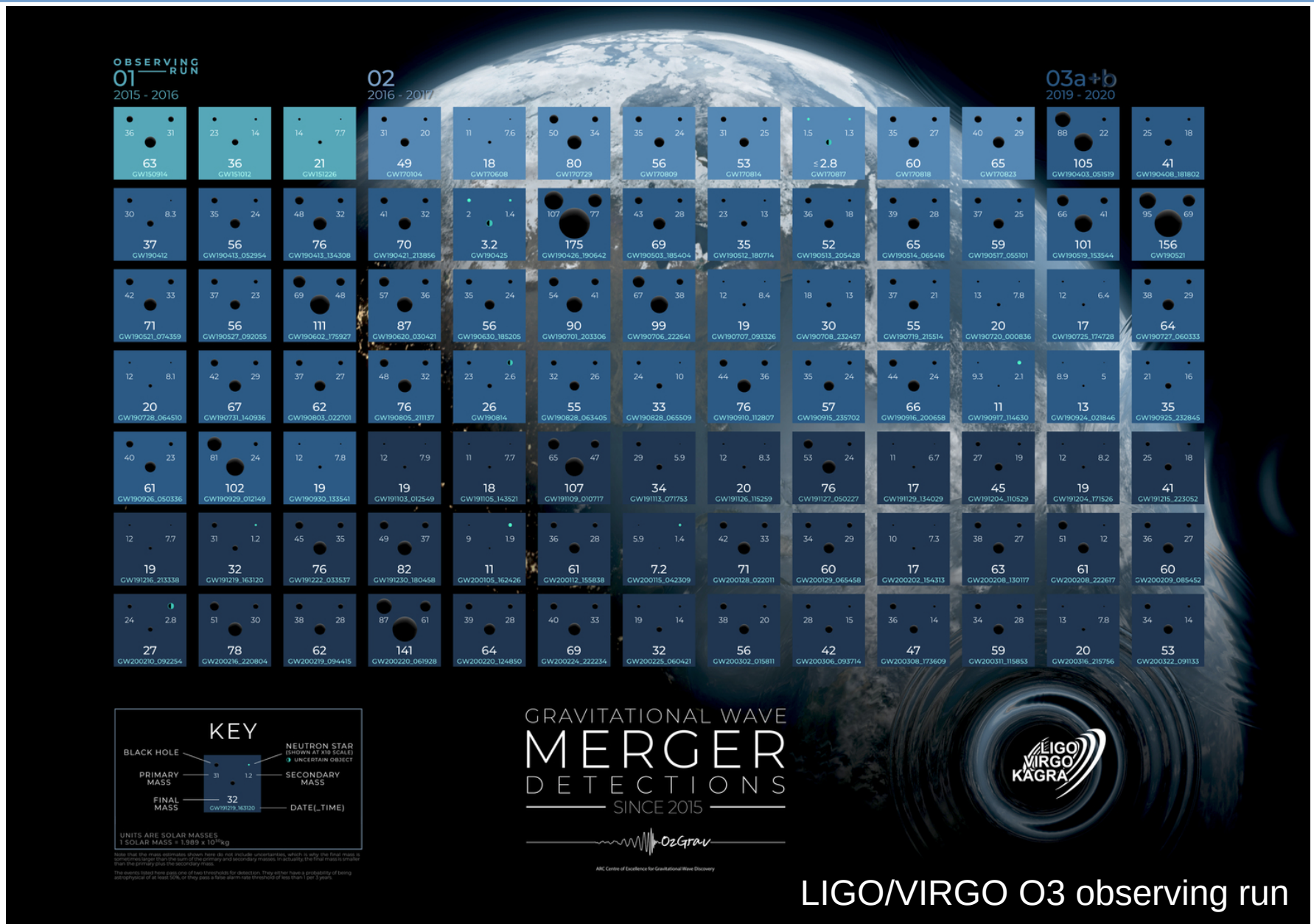


Valerie Domcke  
CERN

*Colloquium at DESY, Hamburg*  
May 16 2023

based on work with  
Wilfried Buchmüller, Camilo Garcia-Cely,  
Sung Mook Lee, Nick Rodd, Kai Schmitz  
and members of the  
LISA Cosmology Working group,  
and UHF-GW initiative.

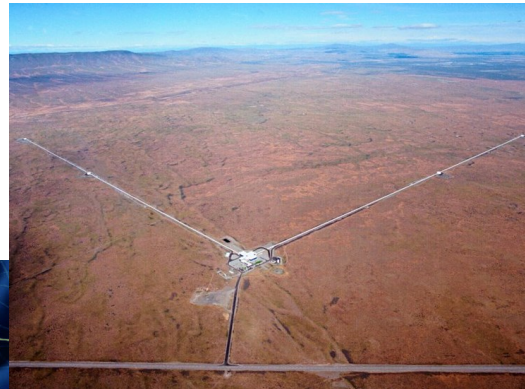
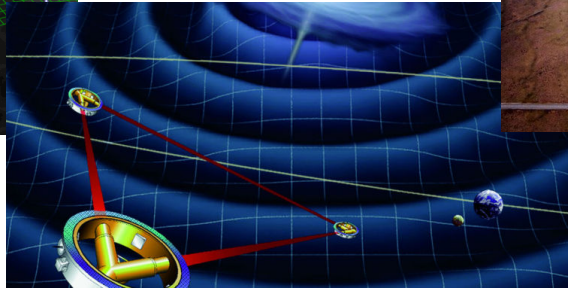
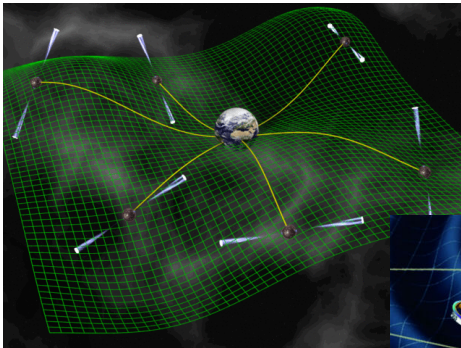
# GW observations today



# GW observations today

pulsar timing arrays

interferometers



LIGO

LISA

?

nHz

mHz

kHz

frequency

QCD PT

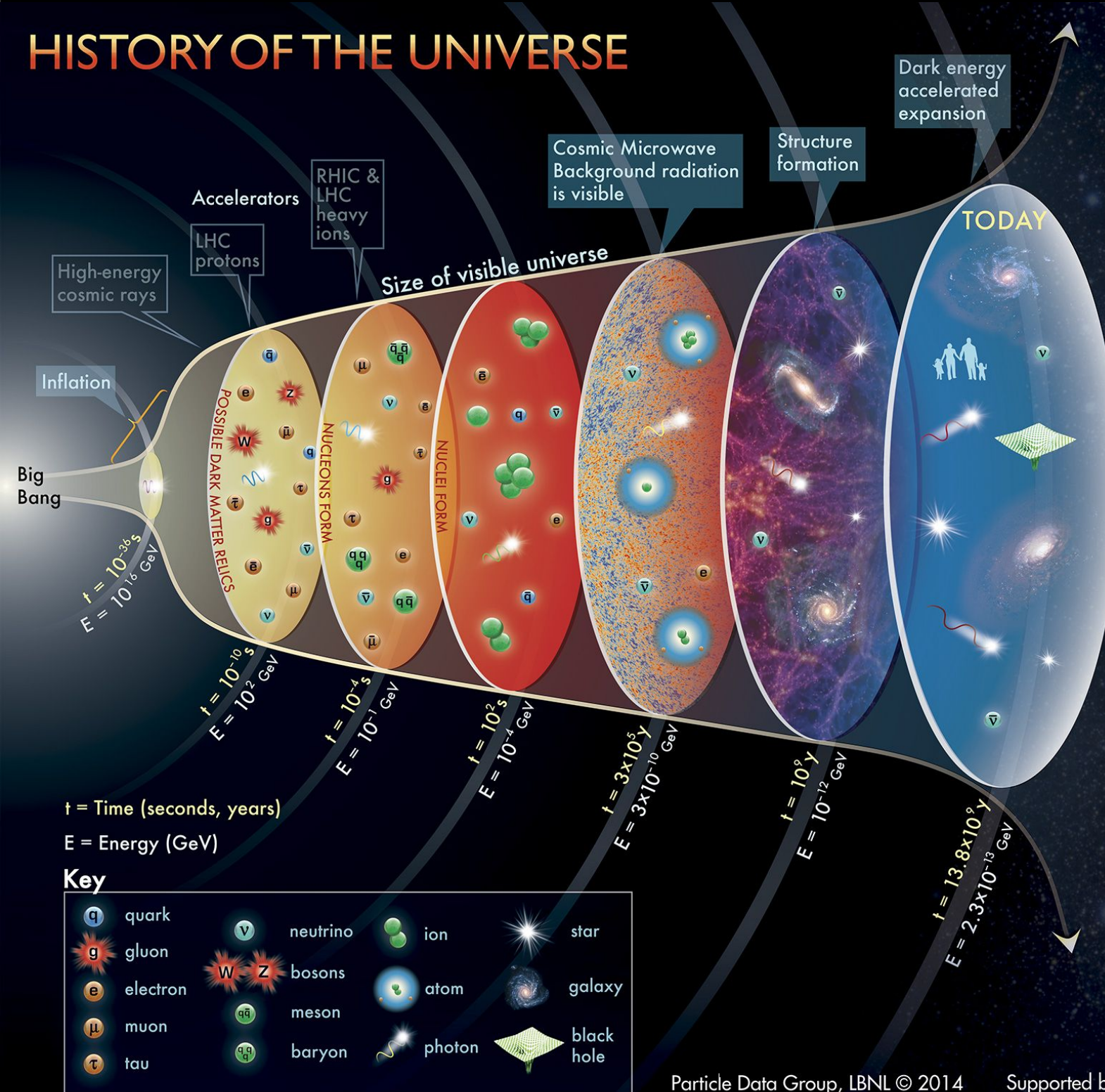
EW PT

GUT PT

mass (merging compact objects)  
time (cosmological events)

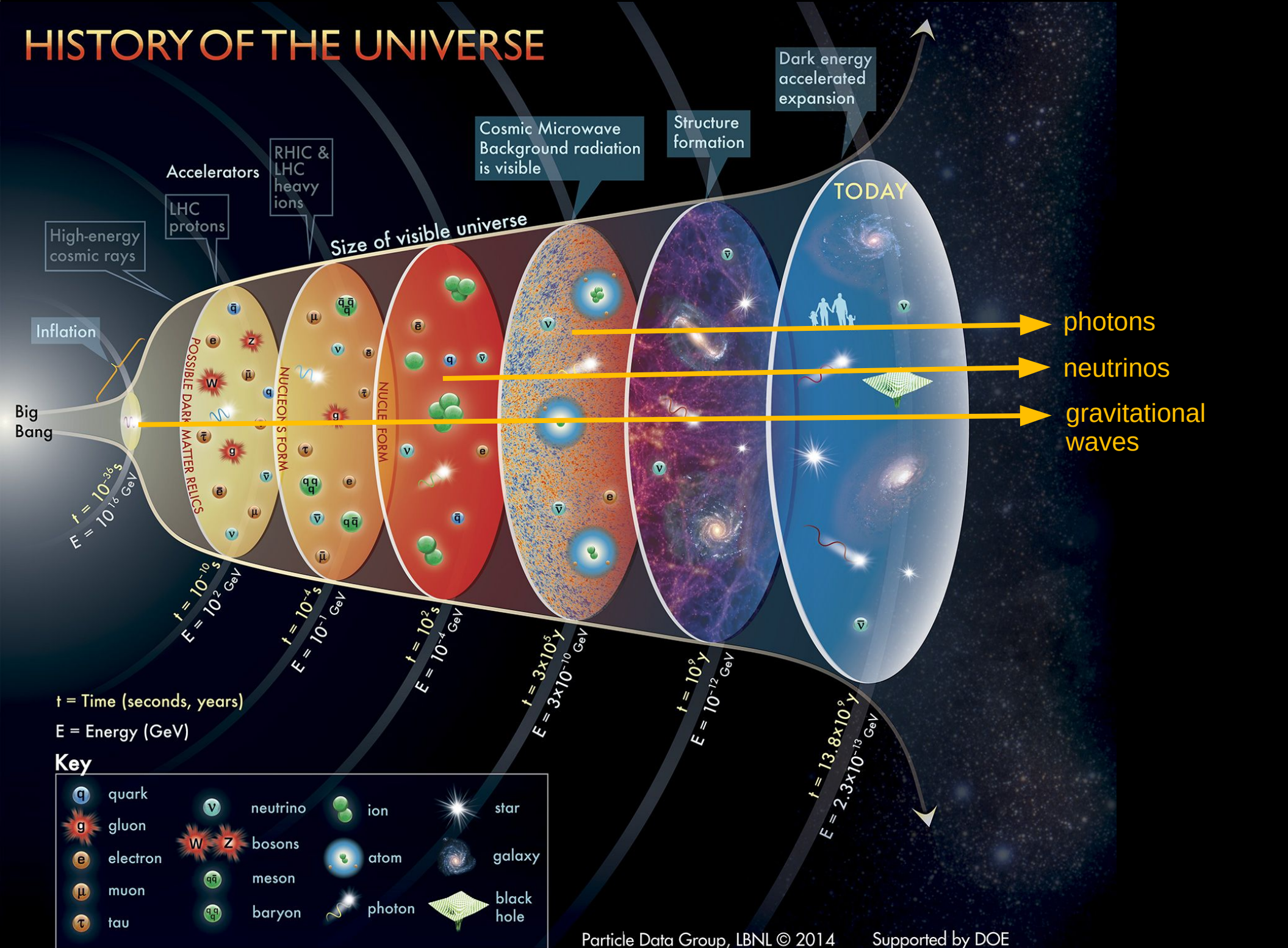


# HISTORY OF THE UNIVERSE



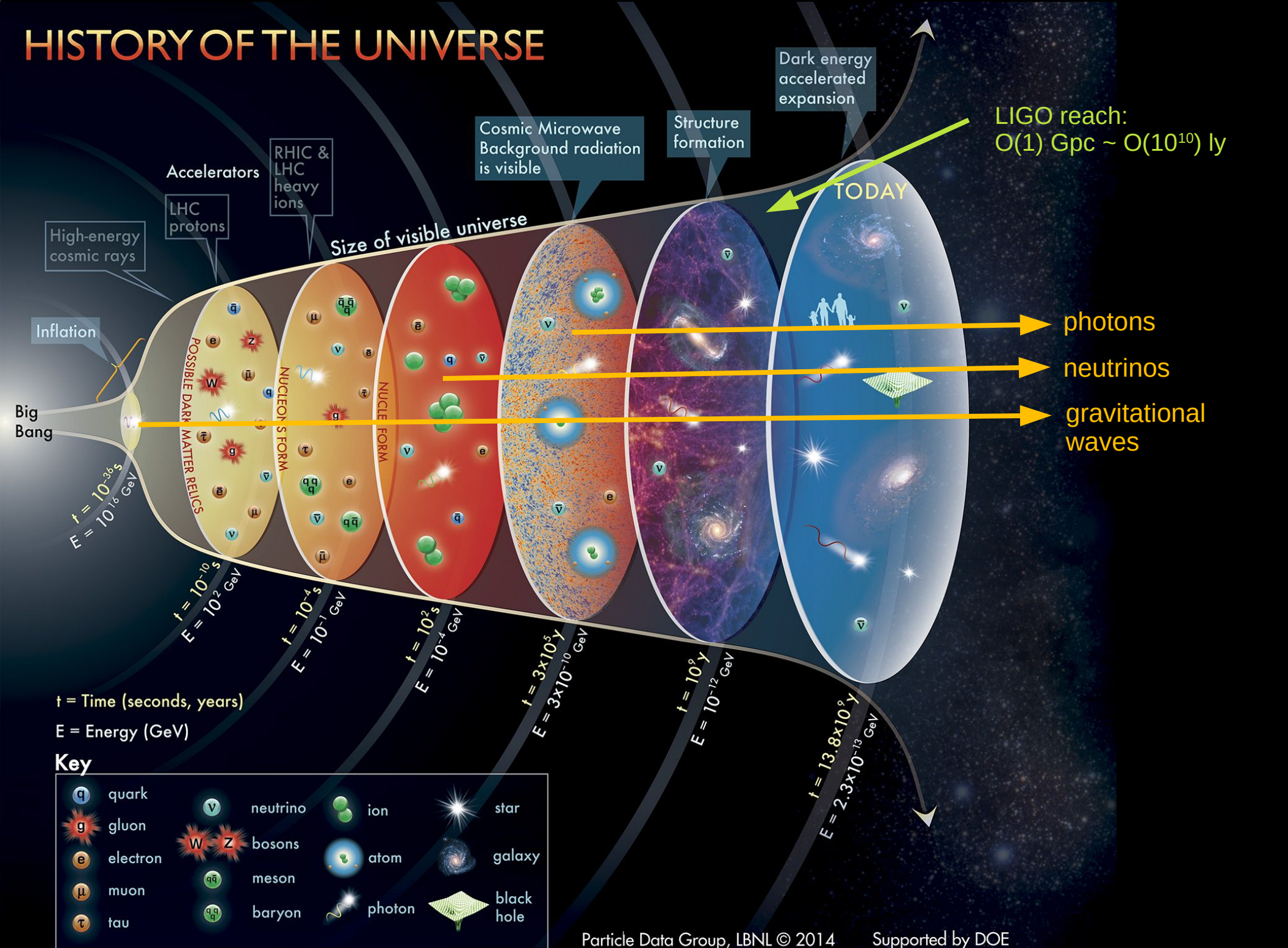


# HISTORY OF THE UNIVERSE



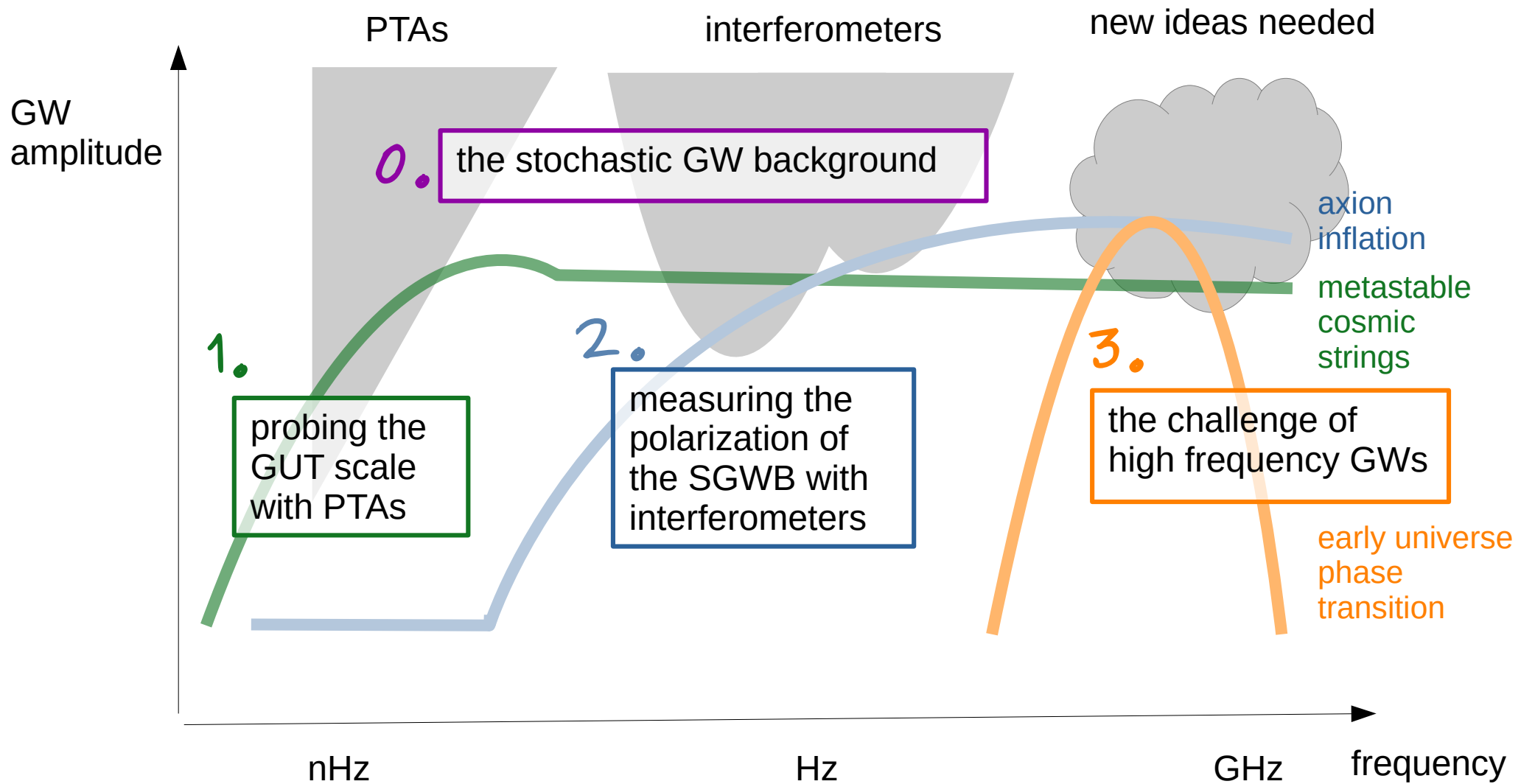


# HISTORY OF THE UNIVERSE

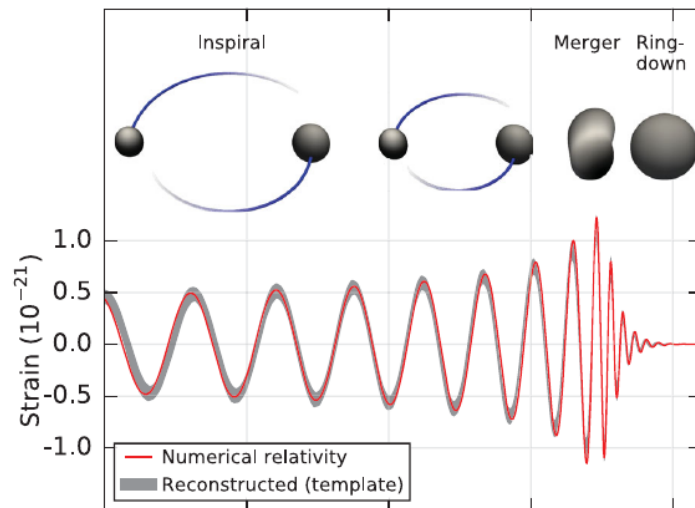




# Outline



# transient and stochastic signals



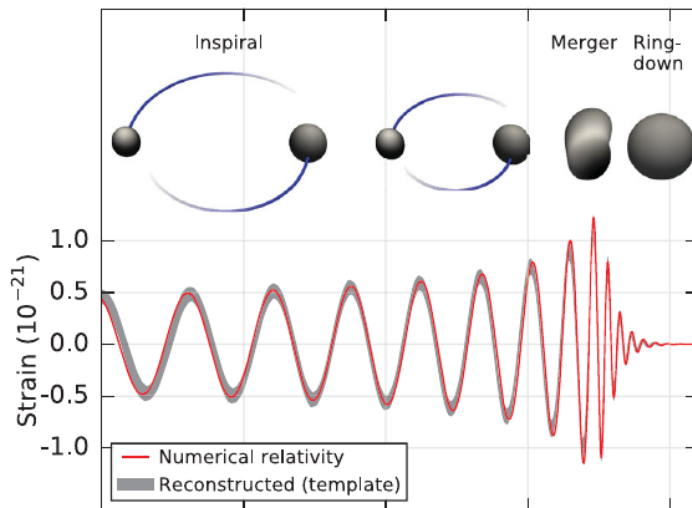
LIGO Livingston, USA



2015: first direct observation of GWs, collision of two black holes a billion years ago



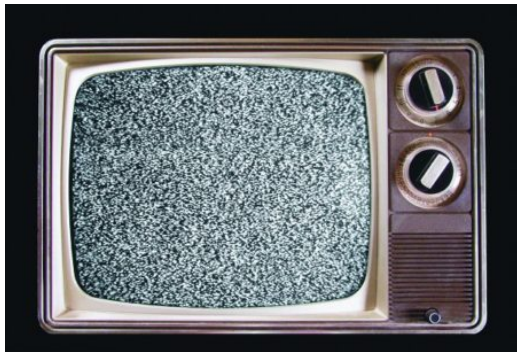
# transient and stochastic signals



LIGO Livingston, USA



2015: first direct observation of GWs, collision of two black holes a billion years ago



next challenge:  
stochastic gravitational  
wave background

analogous to CMB



Penzias, Wilson '64

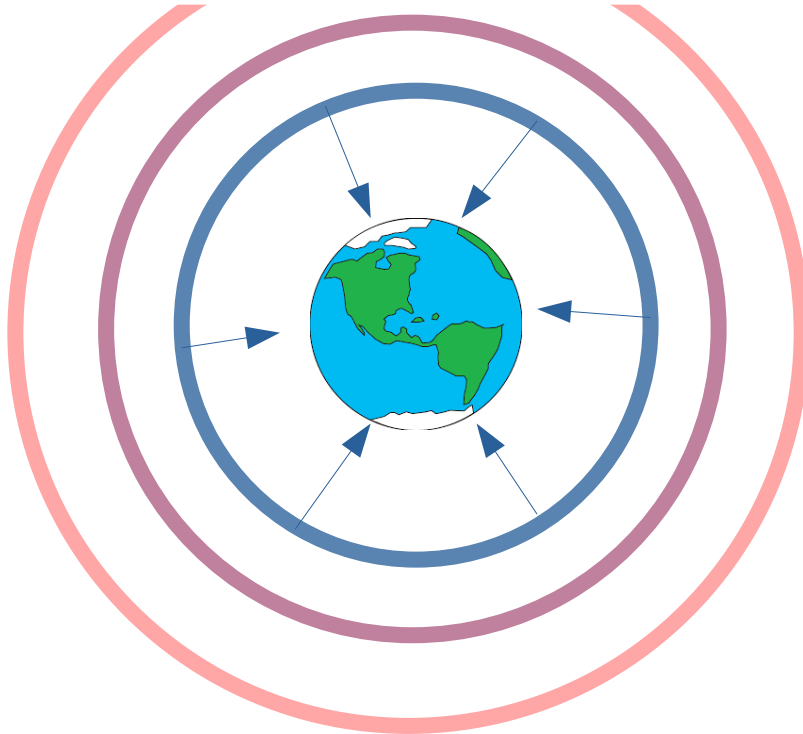
astrophysical  
and  
cosmological  
contributions

possible hint  
by PTAs  
(pulsar timing  
arrays)

# prelude: stochastic gravitational wave background

eg review by Caprini, Figueroa '18

stochastic gravitational wave background (SGWB):



astrophysical sources:

unresolved mergers  
of compact objects (BH, NS, ..)

cosmological sources:

SM: inflation, thermal fluctuations  
→ very small

BSM: inflation, (p)reheating,  
phase transitions, ...

$$f \sim \text{mHz} (0.01/\epsilon_*) (T_*/100 \text{ GeV})$$

$$\epsilon_* \lesssim 1$$

primary observable:

$$\Omega_{GW} = \frac{1}{\rho_c} \frac{\partial \rho_{GW}(f, \tau)}{\partial \ln f}$$

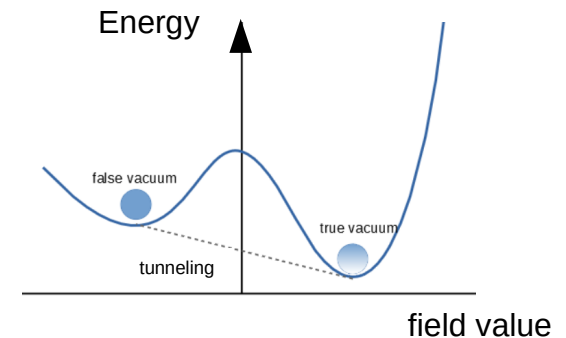


# example : phase transition

Water boiling: first order phase transition

Standard Model: electroweak symmetry breaking through Higgs acquiring a vacuum expectation value

.. and beyond: extended symmetry groups (eg GUTs) spontaneously broken in cooling Universe

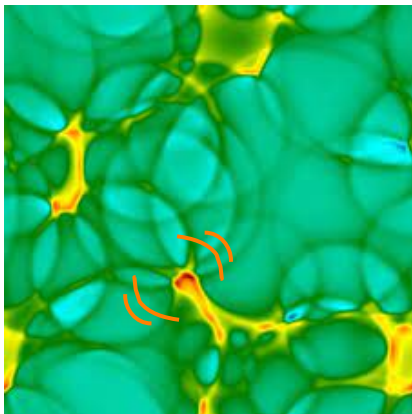
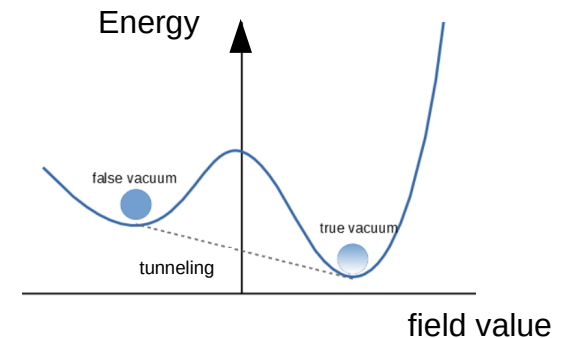


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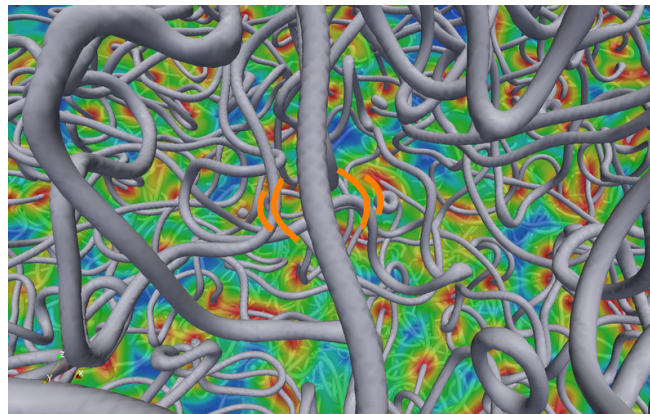
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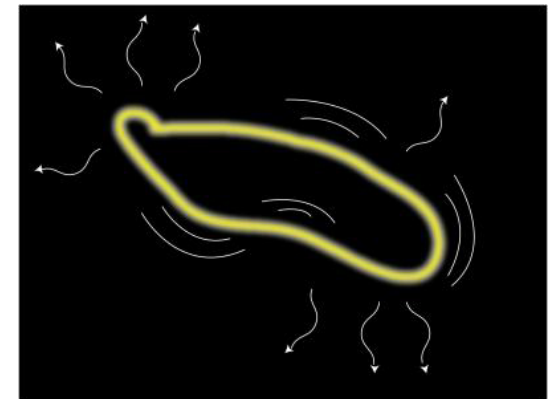
.. and beyond: extended symmetry groups (eg GUTs) spontaneously broken in cooling Universe



1st order PT sources GWs

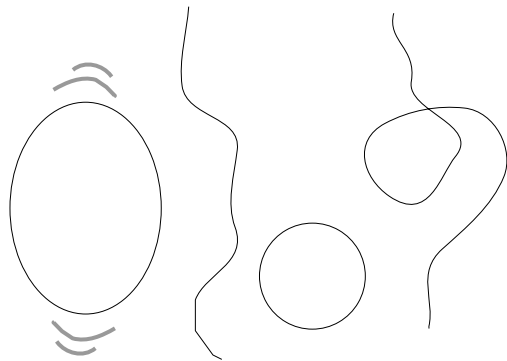


topological defects formed during PT radiate GWs

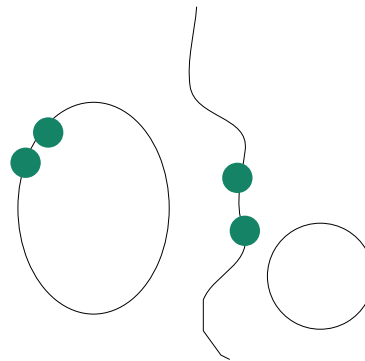




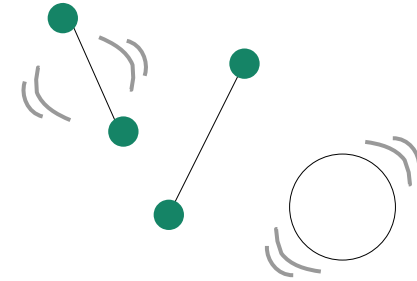
# example : metastable cosmic strings



scaling regime  
(long strings & loops)



$$t_s = 1/\Gamma_d^{1/2}$$



segments & loops

[see also Leblond, Shlaer, Simons '09]

cosmic strings formed  
in phase transition, eg

$$G_{SM} \times U(1)_{B-L} \rightarrow G_{SM}$$

spontaneous creation of monopoles  
due to GUT embedding

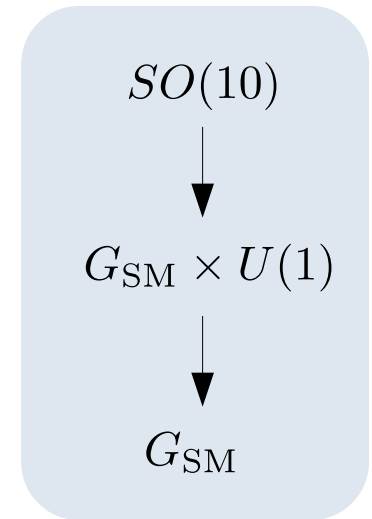
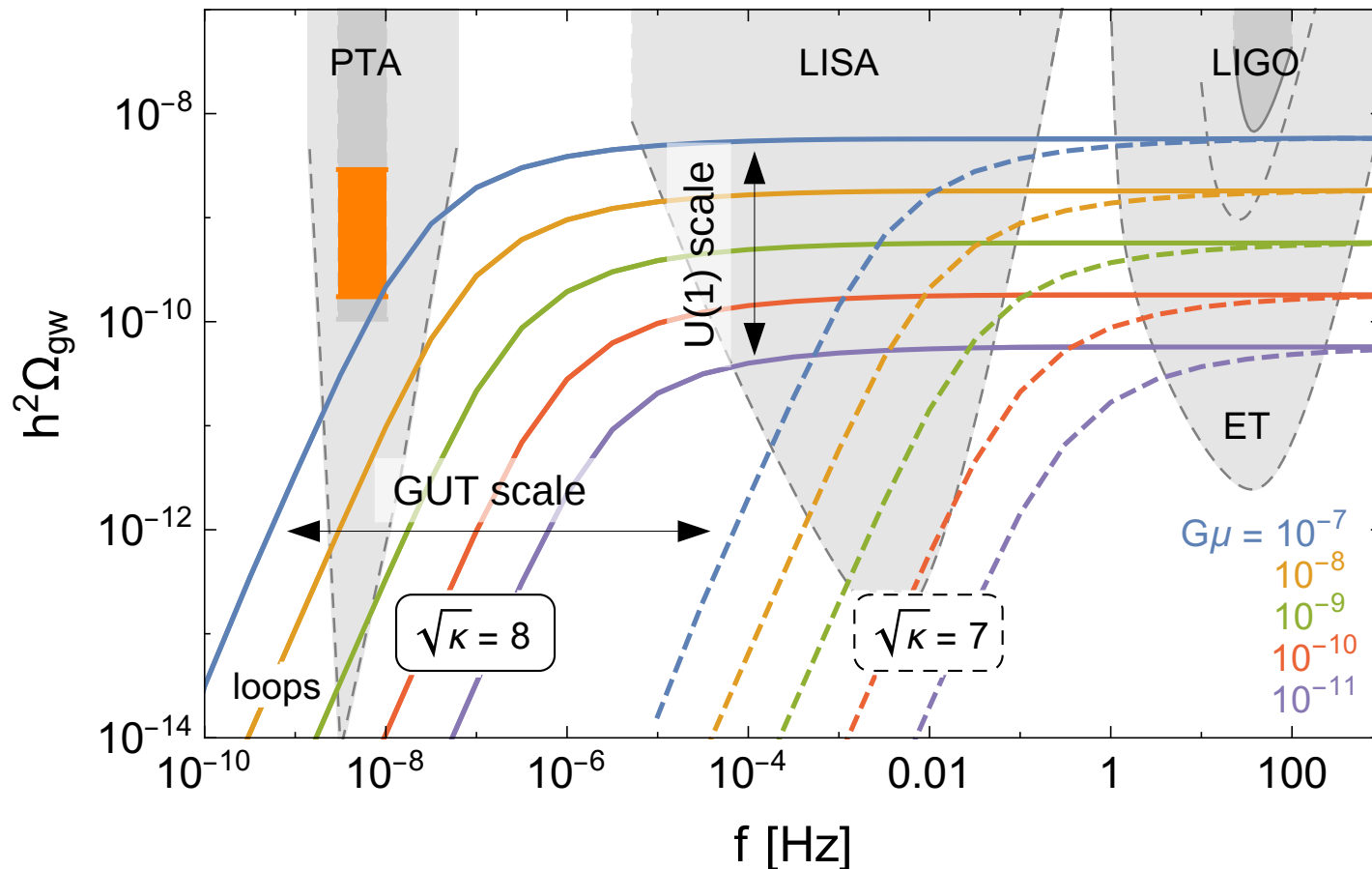
$$G_{SM} \times U(1)_{B-L} \subset SO(10)$$

$$\Gamma_d \sim \mu \exp(-\pi\kappa^2), \quad \kappa^2 = m^2/\mu$$

$$\begin{aligned} \mu &\sim v_{B-L}^2 && \text{string tension} \\ m &\sim v_{GUT} && \text{monopole mass} \end{aligned}$$

# example: metastable cosmic strings

Buchmüller, VD, Schmitz '21



$$\sqrt{\kappa} \sim v_{SO(10)}/v_{U(1)}$$

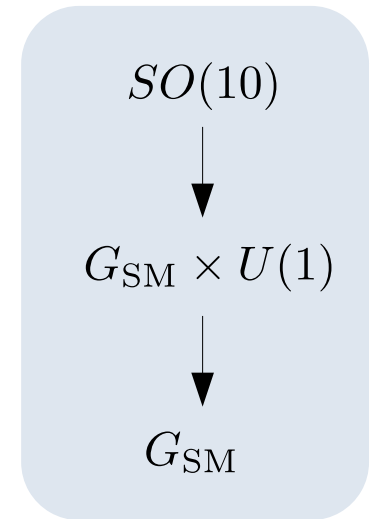
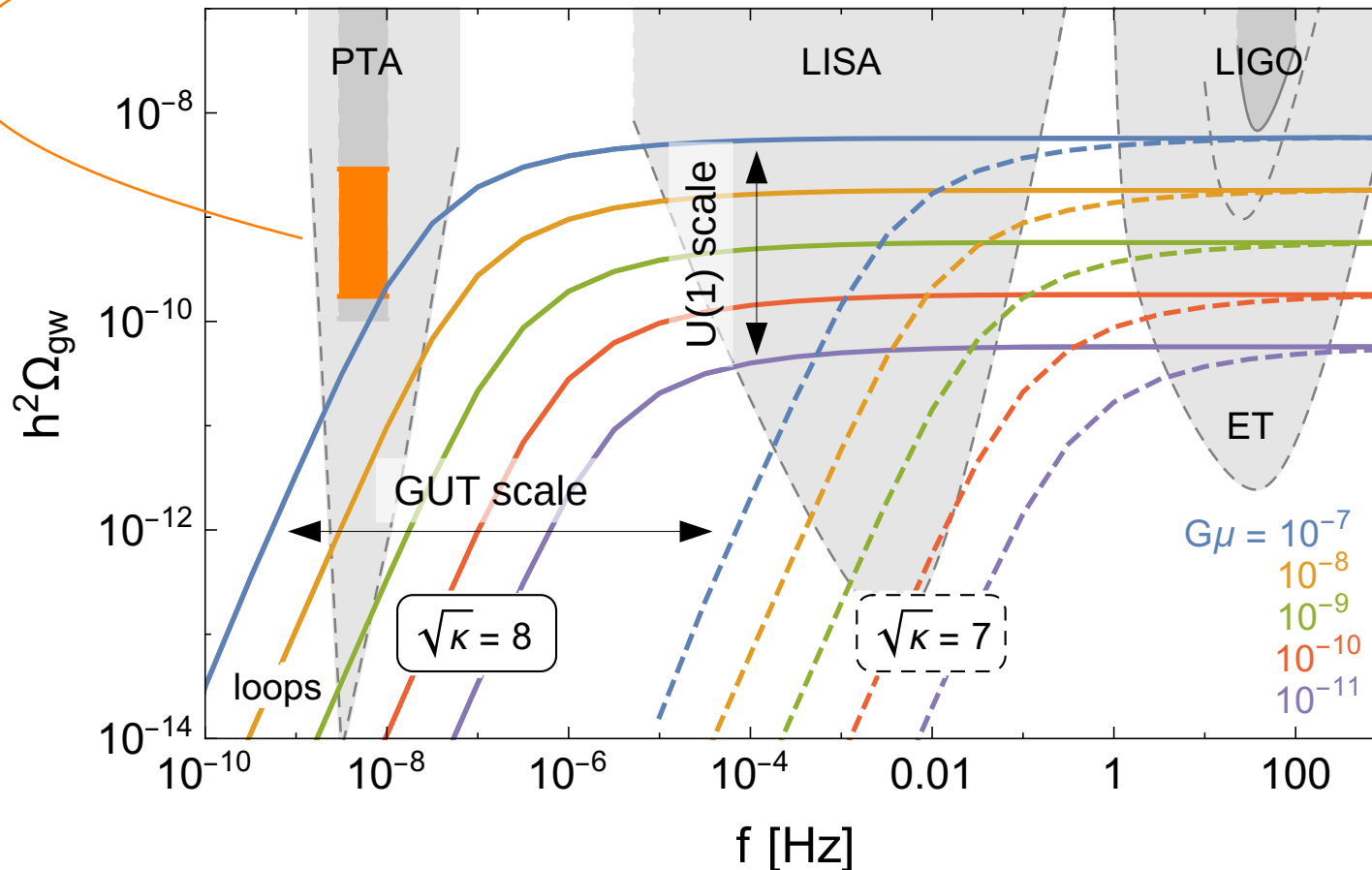
$$G\mu \sim (v_{U(1)}/M_P)^2$$

GUT-scale U(1) phase transition can be tested with GWs

# example: metastable cosmic strings

Buchmüller, VD, Schmitz '21

first tentative hint of SGWB at pulsar timing arrays

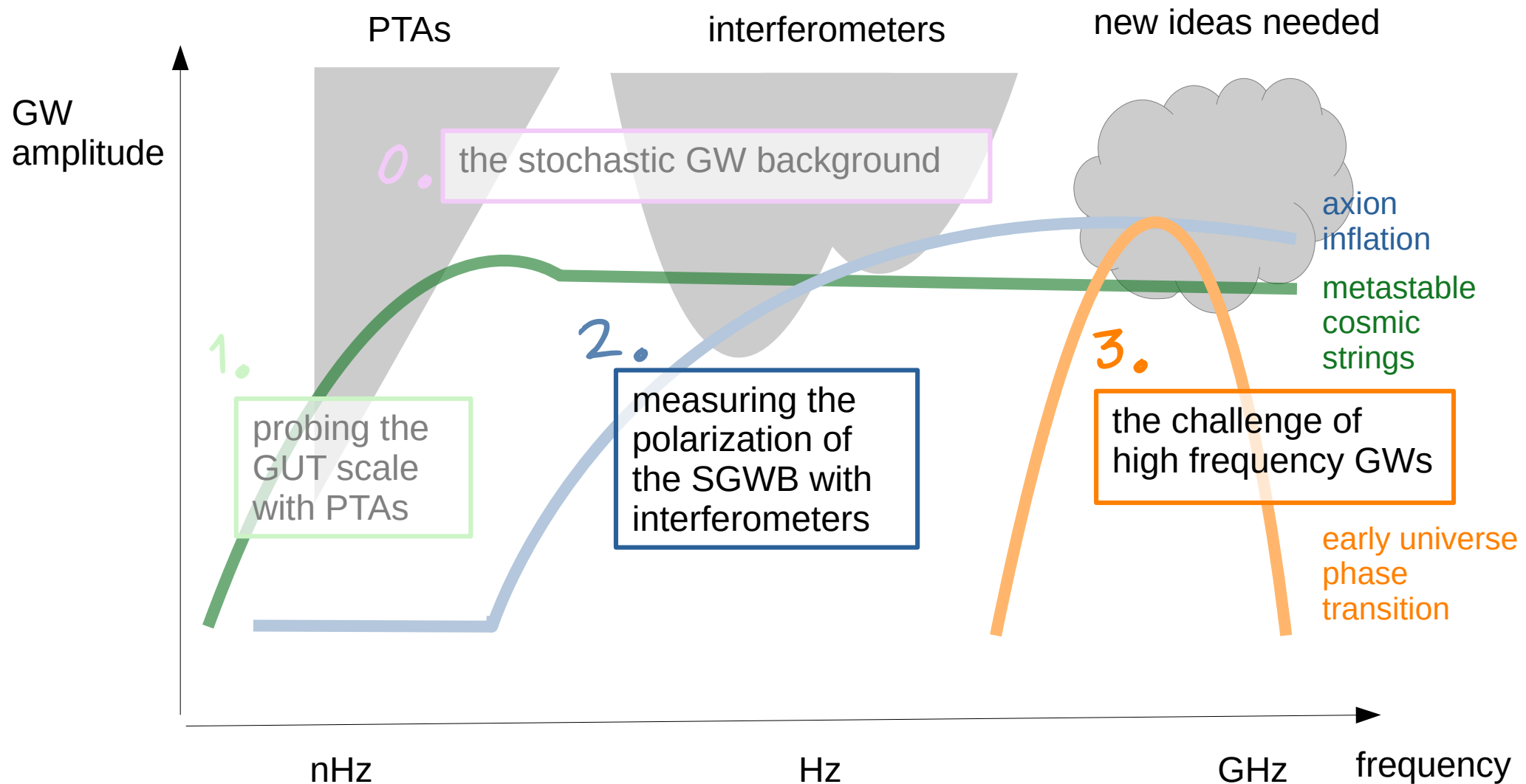


$$\sqrt{\kappa} \sim v_{SO(10)}/v_{U(1)}$$

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# Outline

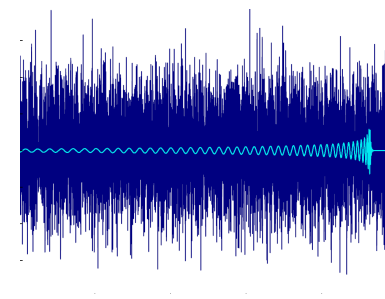




# decoding the SGWB

signal vs background discrimination is very challenging!

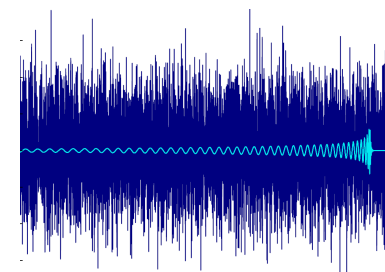
- signal cannot be shielded, noise models have uncertainties
- expected signal shape is model and parameter dependent
- cosmological and astrophysical contributions superimposed



# decoding the SGWB

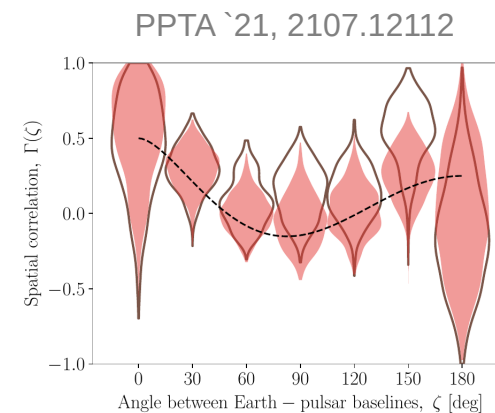
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possible avenues

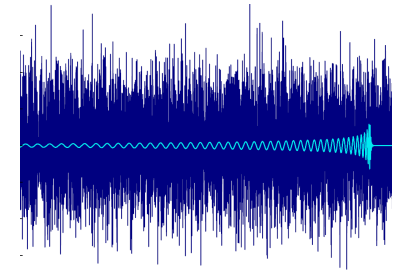
- signal vs noise channels / cross-correlation
- spectral shape
- anisotropies and polarization
- ...



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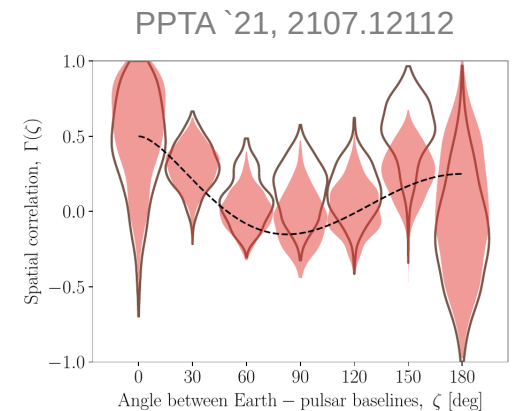
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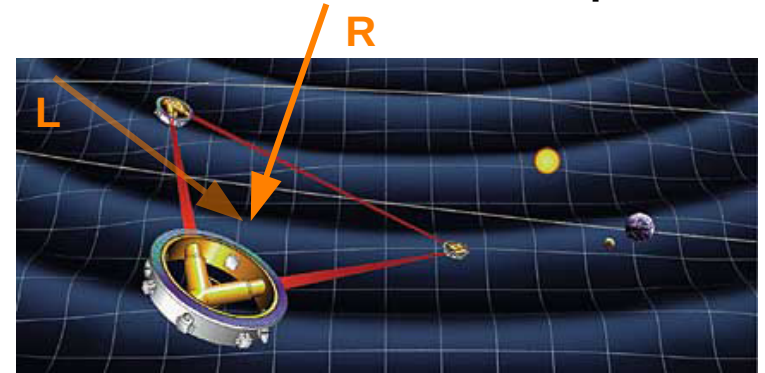
# decoding the SGWB : polarization

some CP-violating models predict a **chiral** stochastic gravitational wave (GW) spectrum

⚡ a **planar** detector cannot distinguish left- and right-handed GWs from an **isotropic** source



LIGO Livingston - US



LISA (launch 2030s)



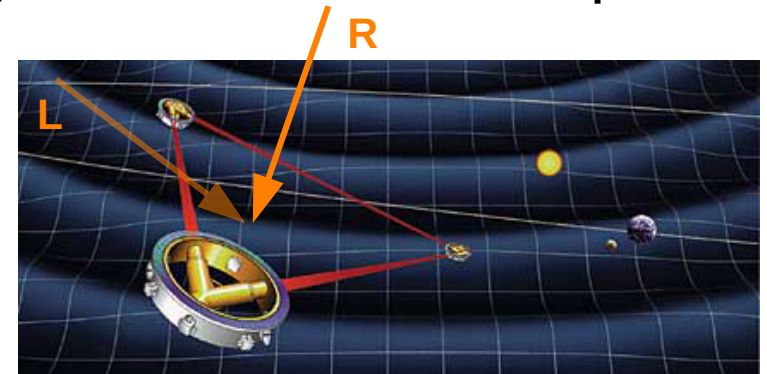
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LIGO Livingston - US



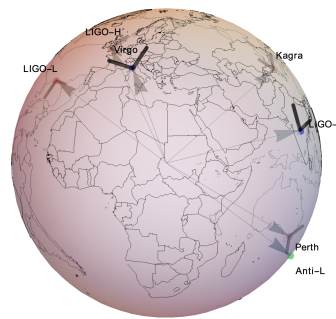
LISA (launch 2030s)

VD, Garcia-Bellido, Peloso, Pieroni, Ricciardone, Sorbo, Tasinato '20 [LISA Cosmology WG]

ground-based detectors:  
network breaks

Seto, Taruya '07; Crowder et al '12

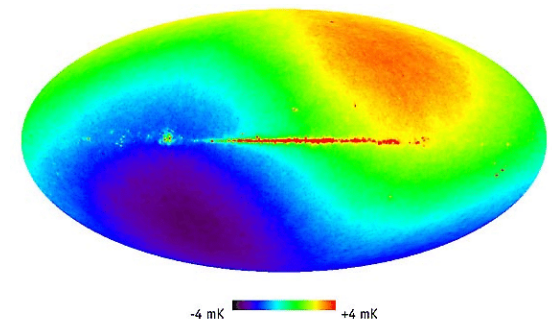
→ sensitive to maximally  
chiral scale-invariant  
spectrum if  $\text{SNR} > 10^3$



LISA (or single ET):  
kinematic cosmic dipole breaks isotropy

Seto '06

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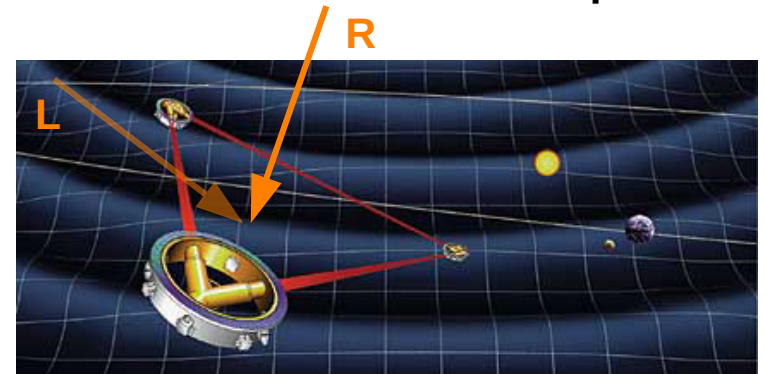
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LIGO Livingston - US



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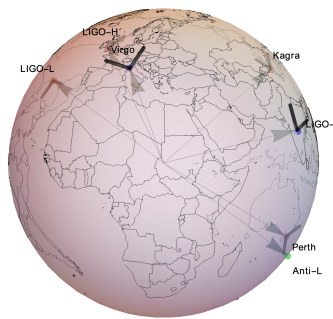
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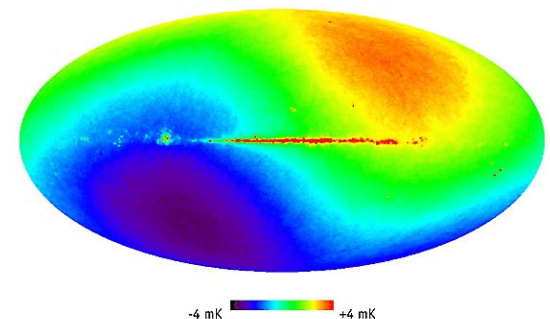
possible for  $\text{SNR} > 10^3$



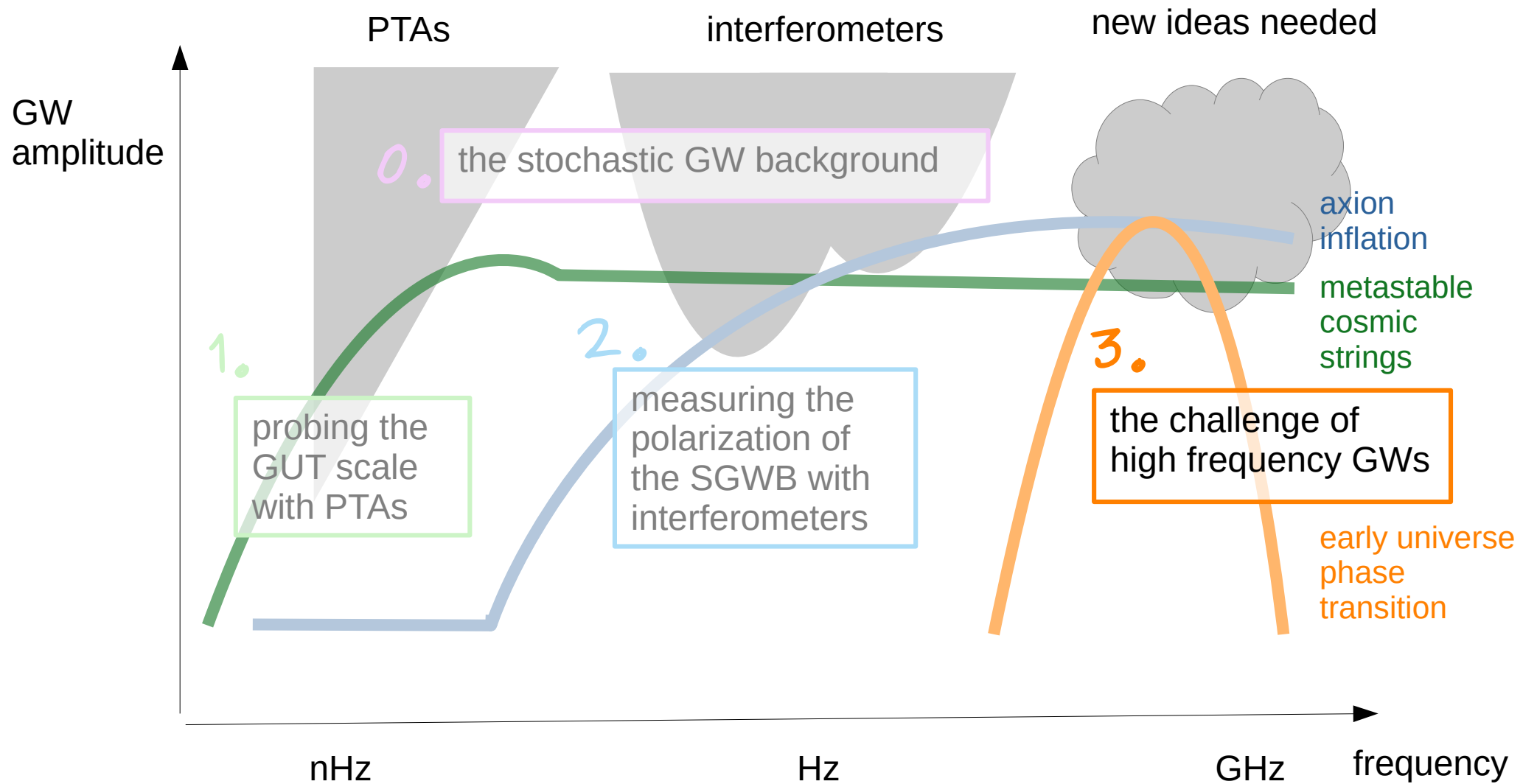
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Seto '06

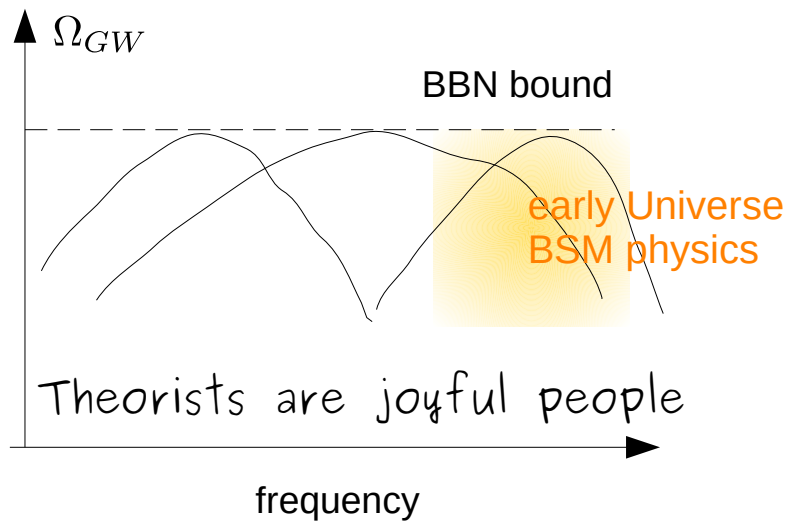
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# Outline



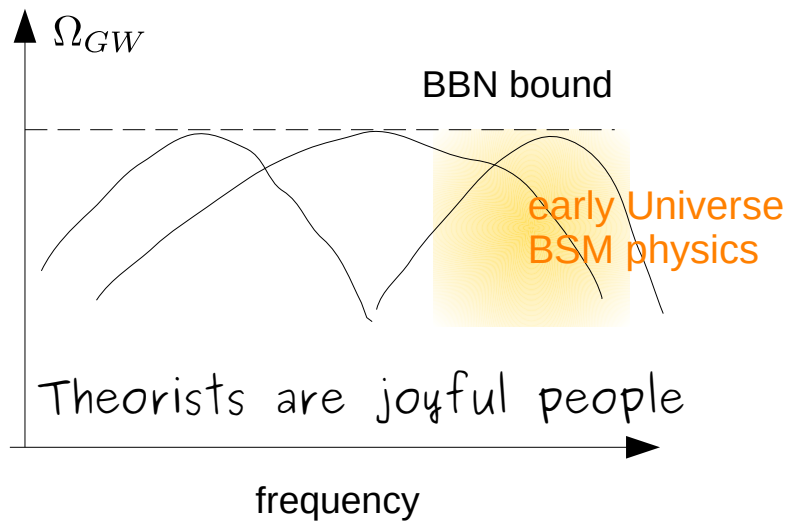
# challenges in UHF GW detection



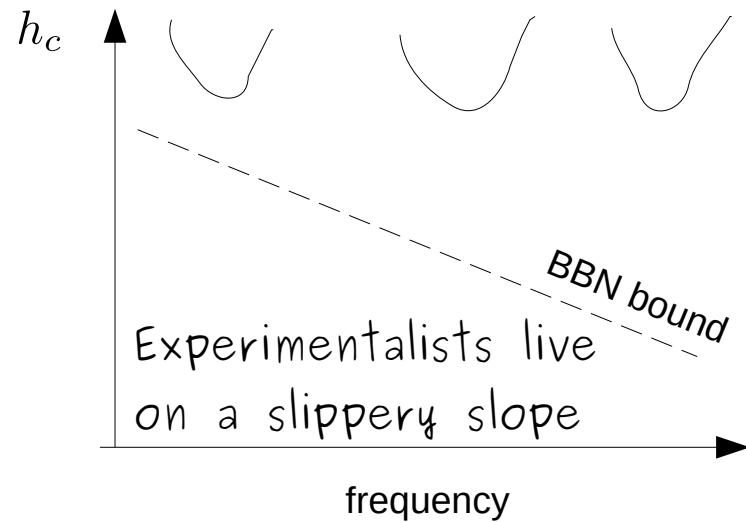
CMB/BBN bound constrains energy



# challenges in UHF GW detection



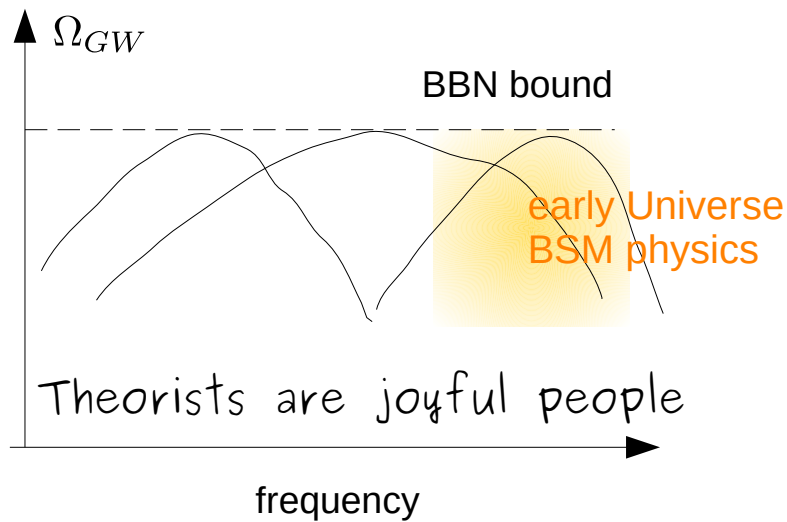
CMB/BBN bound constrains energy



experiments measure displacement

$$\Omega_{GW} \propto f^2 h_c^2$$

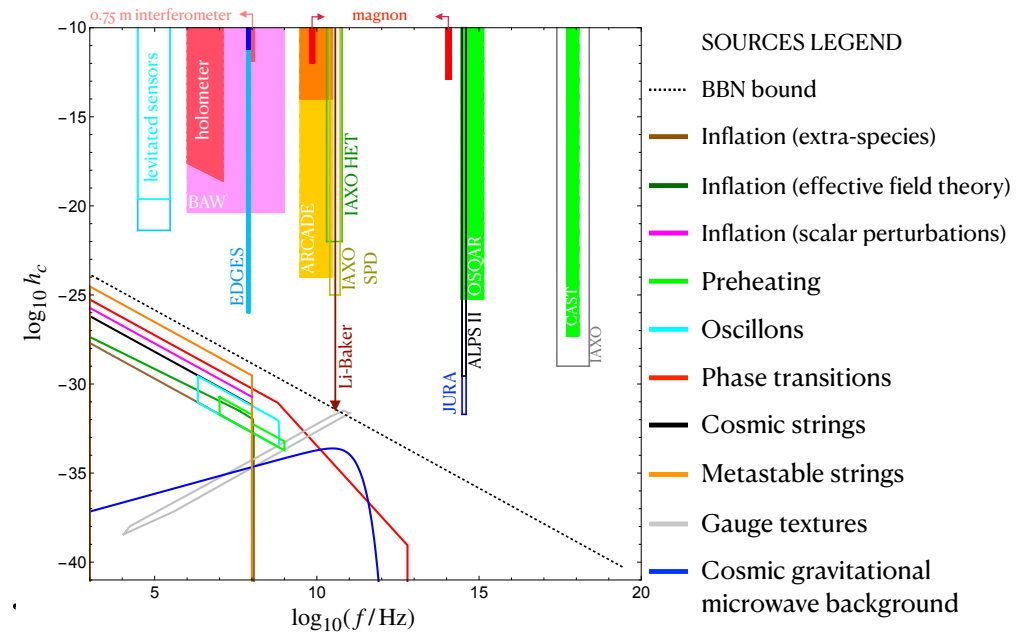
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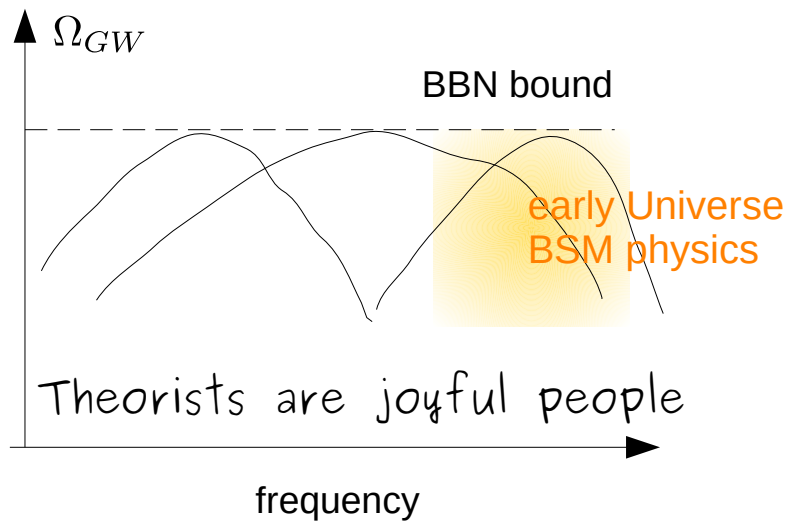
$$\Omega_{GW} \propto$$

UHG GW initiative Living Review:  
<https://arxiv.org/abs/2011.12414>



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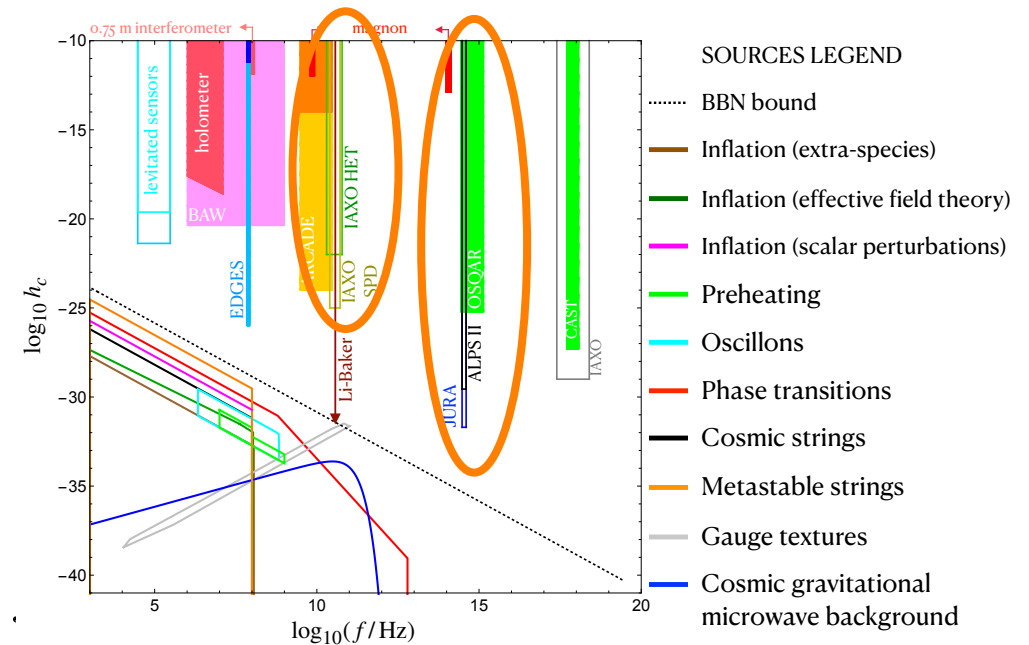
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CMB/BBN bound constrains energy

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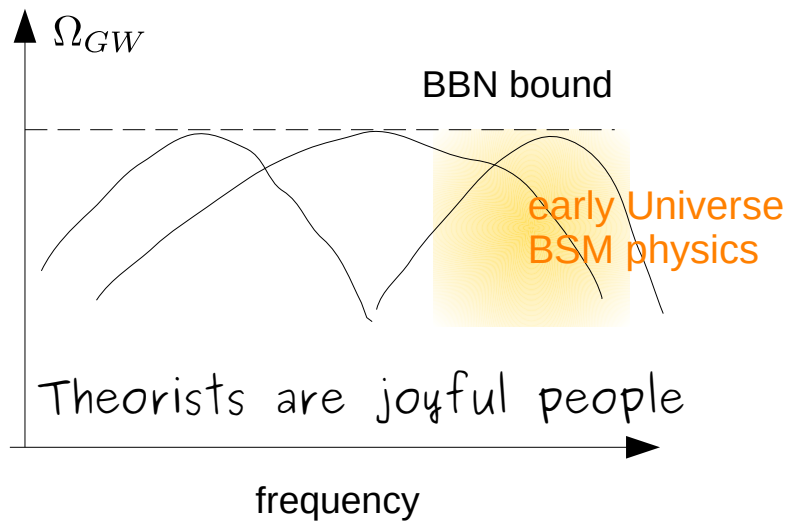
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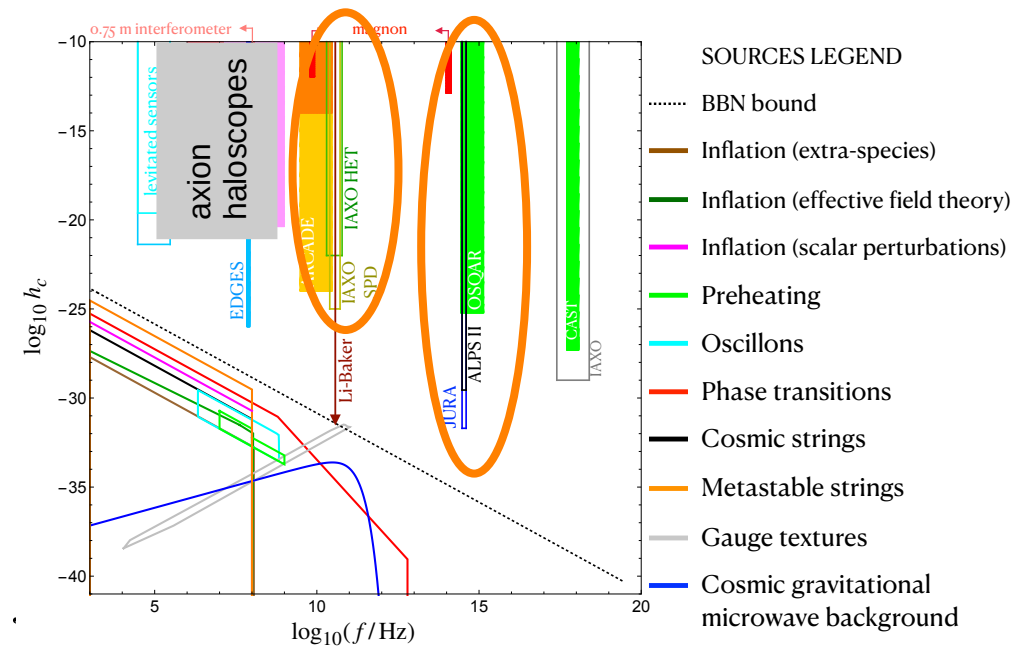


# challenges in UHF GW detection



## CMB/BBN bound constrains energy

UHG GW initiative Living Review:  
<https://arxiv.org/abs/2011.12414>



experiments measure displacement

# GW electrodynamics

Classical electrodynamics + linearized GR,  $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$  :

$$\partial_\nu F^{\mu\nu} = j_{\text{eff}}^\mu = (-\nabla \cdot \mathbf{P}, \nabla \times \mathbf{M} + \partial_t \mathbf{P})$$

effective current  
effective polarization vector  
effective magnetization vector

with

$$\begin{aligned} P_i &= -h_{ij}E_j + \frac{1}{2}hE_i + h_{00}E_i - \epsilon_{ijk}h_{0j}B_k, \\ M_i &= -h_{ij}B_j - \frac{1}{2}hB_i + h_{jj}B_i + \epsilon_{ijk}h_{0j}E_k, \end{aligned}$$

induced at linear order in h  
in presence of external E,B field

VD, Garcia-Cely, Rodd `22

Direct analogy with axion electrodynamics

$$\mathcal{L} \supset g_{a\gamma\gamma}a \mathbf{E} \cdot \mathbf{B} \quad \rightarrow \quad \mathbf{P} = g_{a\gamma\gamma}a \mathbf{B}, \quad \mathbf{M} = g_{a\gamma\gamma}a \mathbf{E}$$

McAllister et al `18  
Tobar, McAllister, Goryachev `19  
Quellet, Bogorad `19

effective source terms in Maxwell's equation due to GW

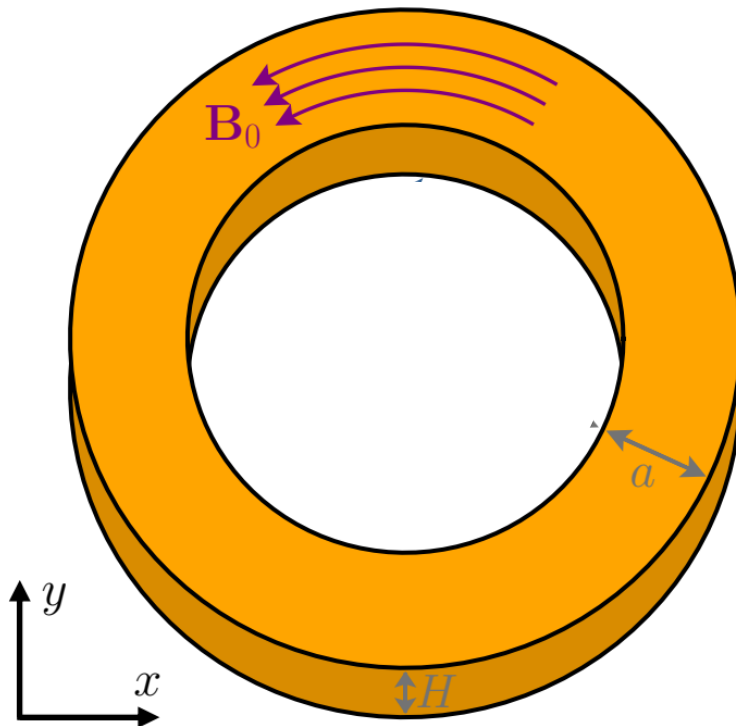
# GW signal in axion haloscopes

eg ABRACADABRA, SHAFT, DM Radio:

VD, Garcia-Cely, Rodd '22

VD, Garcia-Cely, Lee, Rodd (in progress)

static magnetic field



# GW signal in axion haloscopes

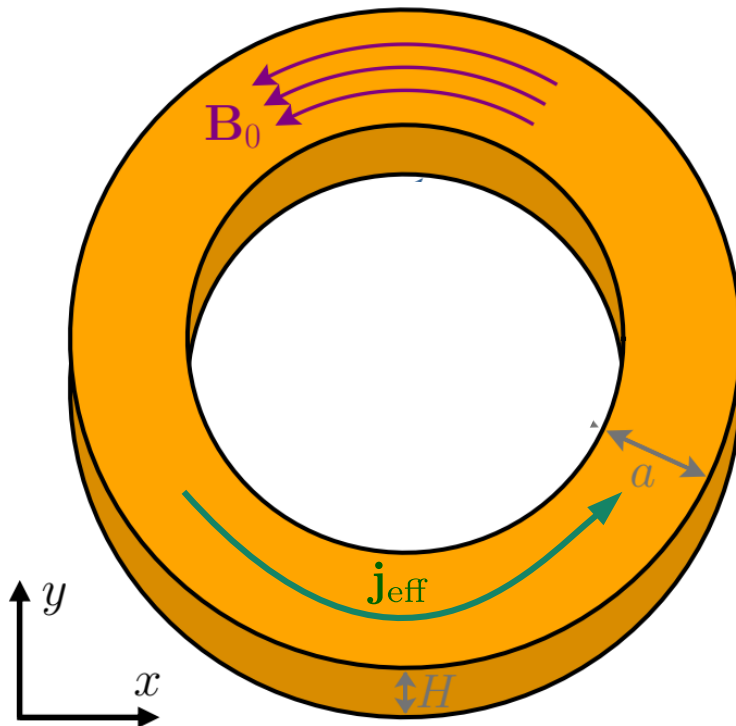
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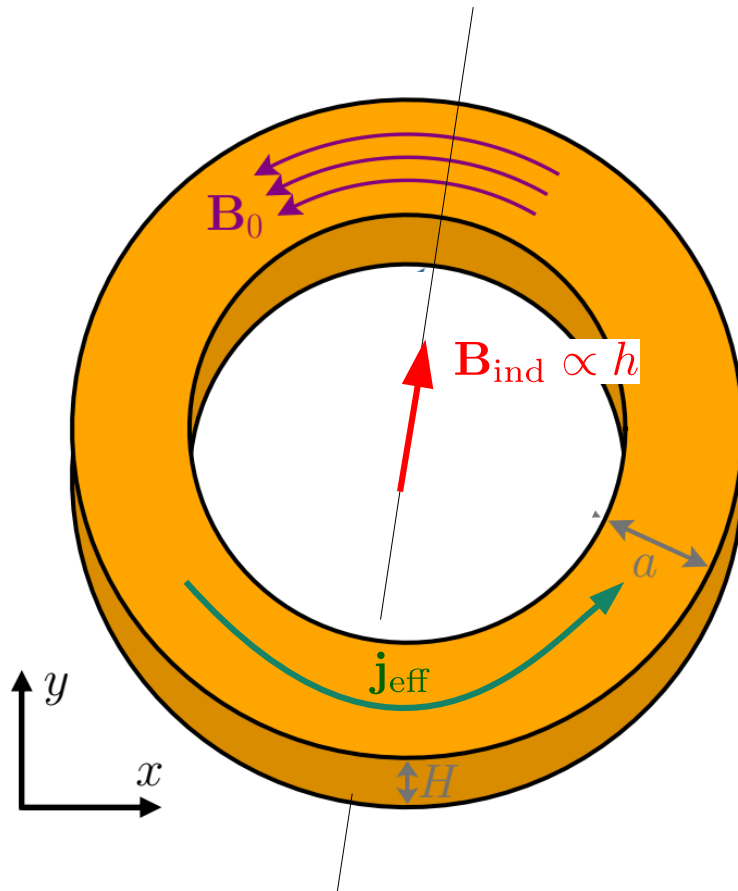


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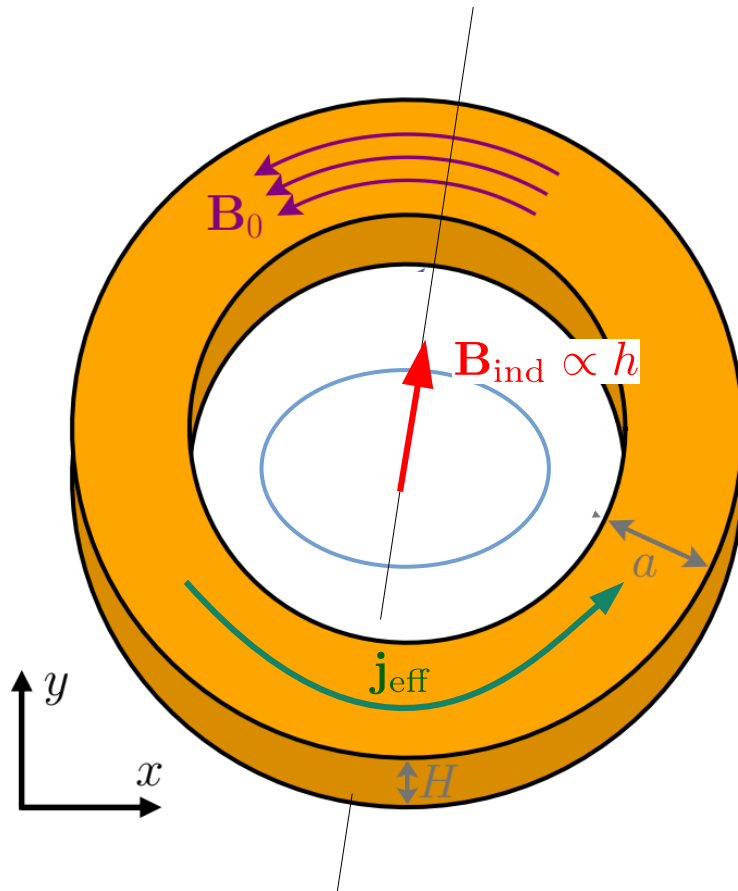
induced oscillating magnetic field

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eg ABRACADABRA, SHAFT, DM Radio:

VD, Garcia-Cely, Rodd '22

VD, Garcia-Cely, Lee, Rodd (in progress)



static magnetic field

effective current

induced oscillating magnetic field

measure magnetic flux ( $\sim h$ )  
through pickup loop

at leading order in  $(\omega R)$  :

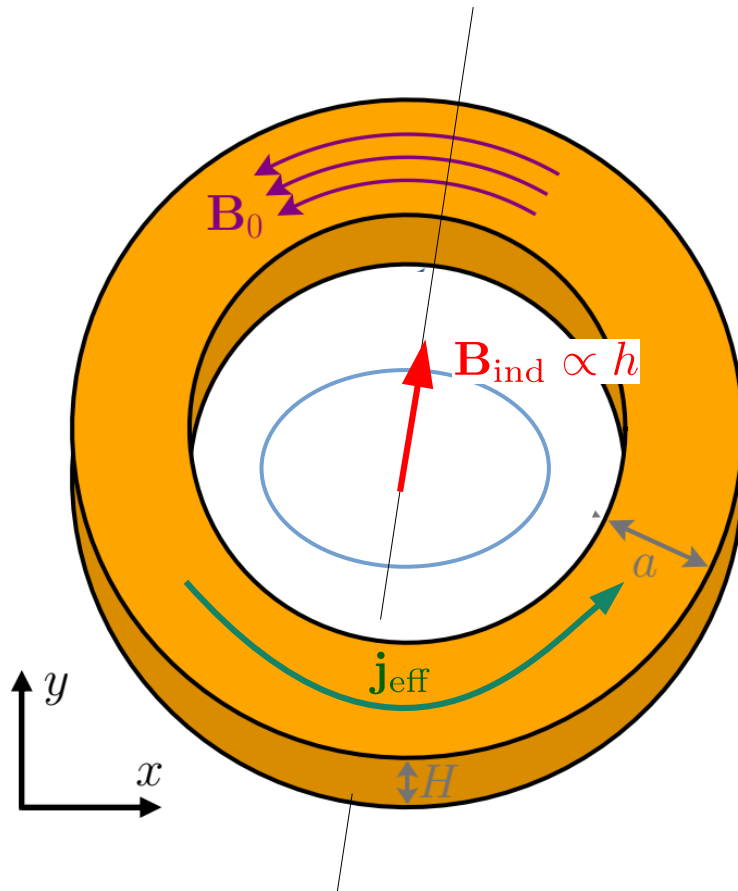
$$\Phi_{\text{gw}} = \frac{i e^{-i\omega t}}{16\sqrt{2}} h^\times \omega^3 B_0 \pi r^2 R a (a + 2R) s_{\theta_h}^2$$

# GW signal in axion haloscopes

eg ABRACADABRA, SHAFT, DM Radio:

VD, Garcia-Cely, Rodd '22

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static magnetic field

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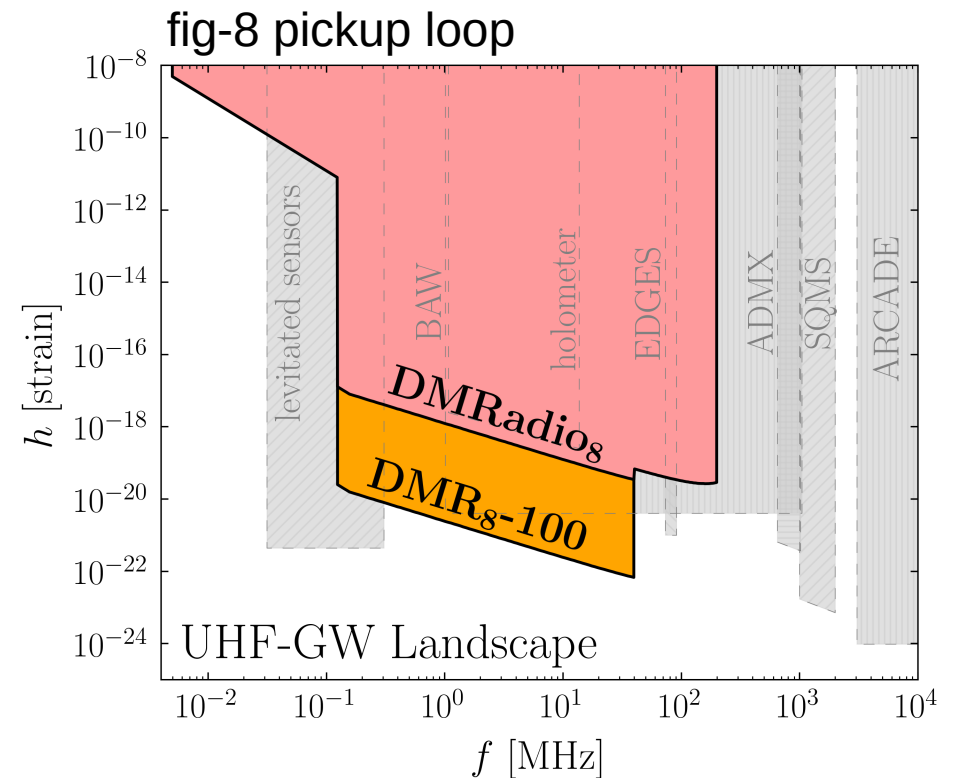
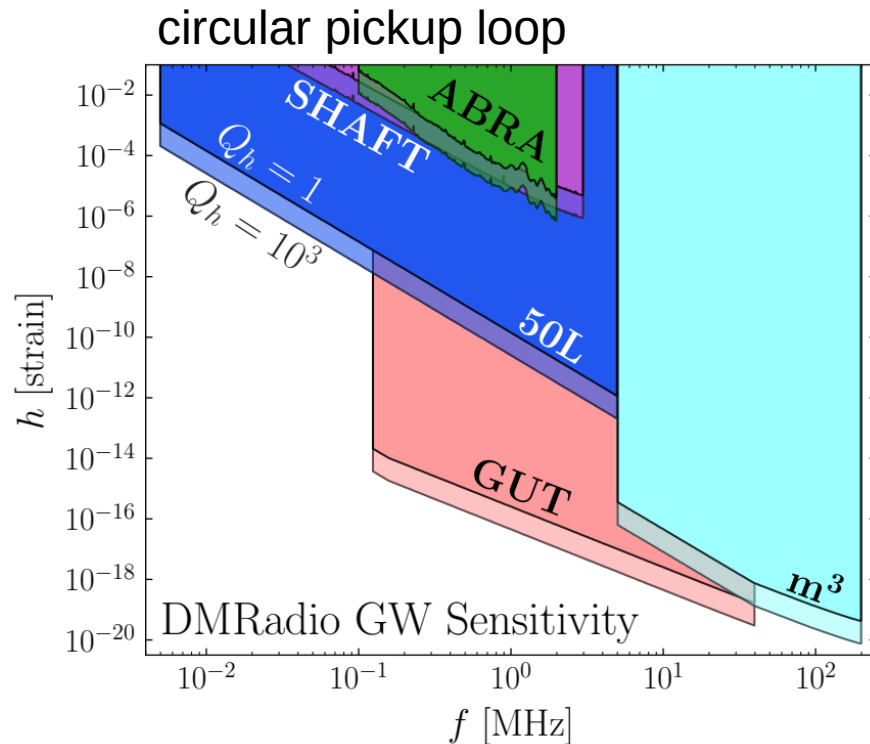
match to axion induced flux to recast  
axion-photon coupling bounds as GW bounds

$$\Phi_a = e^{-i\omega t} g_{a\gamma\gamma} \sqrt{2} B_0 \pi r^2 R \ln(1 + a/R)$$

- set bounds recasting existing axion searches
- parametric improvement w modified pick-up loop

# axion haloscopes: bounds and prospects

VD, Garcia-Cely, Rodd '22



bounds from recasting ABRA [2102.06722] and SHAFT limits [2003.03348]

prospects for DM Radio proposals [Snowmass Letters of Interest CF2]

still far away from BBN bound, but clear synergies with axion searches

see also Ejlli et al '19, Berlin et al '21, '23



# Conclusions and Outlook

## **The stochastic gravitational wave background**

- astrophysical and cosmological contributions expected
- possibly first hint at pulsar timing arrays – stay tuned!
- further characterization (spectrum, anisotropies, polarization..) will be crucial for BSM interpretations

## **The search for high-frequency gravitational waves**

- GW signals  $\gg$  kHz would be a smoking gun of BSM physics
- GW electrodynamics has clear similarities with axion electrodynamics: Important synergies between axion searches and UHF GW searches
- New bounds and prospects for low-mass axion haloscopes as GW detectors

**Thank you!**

backup slides

# BBN bound

radiation energy after electron decoupling:

$$\rho_{rad} = \frac{\pi^2}{30} \left( 2 + \frac{7}{4} \left( \frac{4}{11} \right)^{4/3} (3.046 + \Delta N_{eff}) \right) T^4$$

photons
neutrinos
BSM

at BBN or CMB decoupling:

$$\rho_{GW}(T) < \Delta \rho_{rad}(T) \quad \Rightarrow \quad \left( \frac{\rho_{GW}}{\rho_{\gamma}} \right)_{T_{BBN, CMB}} \leq \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} \Delta N_{eff} \simeq 0.05$$

➔ at BBN, CMB decoupling ~ 5 % GW energy density allowed

today:

$$\frac{\rho_{GW}^0}{\rho_c^0} = \Omega_{\gamma}^0 \left( \frac{g_s^0}{g_s(T)} \right)^{4/3} \frac{\rho_{GW}(T)}{\rho_{\gamma}(T)} \leq 10^{-5} \Delta N_{eff} \simeq 10^{-6}$$

note: constraint  
on *total* GW energy

➔ today, energy fraction < 10<sup>-6</sup> (for GWs present at BBN / CMB decoupling)

# metastable cosmic strings

consider  $SO(10) \rightarrow G_{SM} \times U(1)_{B-L} \rightarrow G_{SM}$

Vilenkin '82; Leblond, Shlaer, Siemens '09;  
Monin, Voloshin '08/09; Dror et al '19

$$\Pi_1(G_{SM} \times U(1)/G_{SM}) = \Pi_1(U(1)) \neq 1 \quad \rightarrow$$

cosmic strings

$$\Pi_1(SO(10)/G_{SM}) = 1 \quad \rightarrow$$

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resolution: no topologically stable cosmic strings

$$SO(10) \rightarrow G_{SM} \times U(1)_{B-L}$$

generates monopoles

$$G_{SM} \times U(1)_{B-L} \rightarrow G_{SM}$$

generates cosmic strings,

metastable  
string &  
monopole  
network



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resolution: no topologically stable cosmic strings

$$SO(10) \rightarrow G_{SM} \times U(1)_{B-L}$$

generates monopoles

cosmic inflation

dilutes monopoles

$$G_{SM} \times U(1)_{B-L} \rightarrow G_{SM}$$

generates cosmic strings,

decay via nucleation of monopoles

metastable  
string &  
monopole  
network

$$\Gamma_d \sim \mu \exp(-\pi \kappa^2), \quad \kappa^2 = m^2/\mu$$

$$\begin{aligned} \mu &\sim v_{B-L}^2 && \text{string tension} \\ m &\sim v_{GUT} && \text{monopole mass} \end{aligned}$$

# gravitational wave signal - SGWB

see eg. Auclair, Blanco-Pillado, Figueroa et al `19

gravitational wave emission from integration over loop distribution function:

$$\Omega_{\text{GW}}(f) = \frac{8\pi f (G\mu)^2}{3H_0^2} \sum_{q=1}^{\infty} C_q(f) P_q$$

$$C_q(f) = \frac{2q}{f^2} \int_0^{z_{\text{max}}} dz \frac{n(\ell(z), t(z))}{H(z)(1+z)^6}$$

GW power spectrum of a single loop

$$P_q = \Gamma / (\zeta(4/3) q^{4/3})$$

# of loops emitting GWs  
observed at frequency  $f$  today

# of loops with length  $\ell$  at time  $t$

with  $\ell = 2q / ((1+z)f)$

cosmological history

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$$n(\ell, z) = n(\ell, z)_{\kappa \rightarrow \infty} \times e^{-\Gamma_d [\ell(t-t_s) + 1/2 \Gamma G\mu (t-t_s)^2]} \times \Theta(\alpha t_s - \ell(t_s))$$

finite CS life time

number density  
for stable strings

decay due to monopole  
production and GW  
emission

loop production only  
in scaling regime

$$n_r(\ell, t) = 0.18 t^{-3/2} (\ell + 50 G\mu t)^{-5/2}$$

Blanco-Pillado, Olum, Shlaer '14

Buchmüller, VD, Schmitz '21

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cosmological history

analogous for contribution from segments

$$n(\ell, z) = n(\ell, z)_{\kappa \rightarrow \infty} \times e^{-\Gamma_d [\ell(t-t_s) + 1/2 \Gamma G\mu (t-t_s)^2]} \times \Theta(\alpha t_s - \ell(t_s))$$

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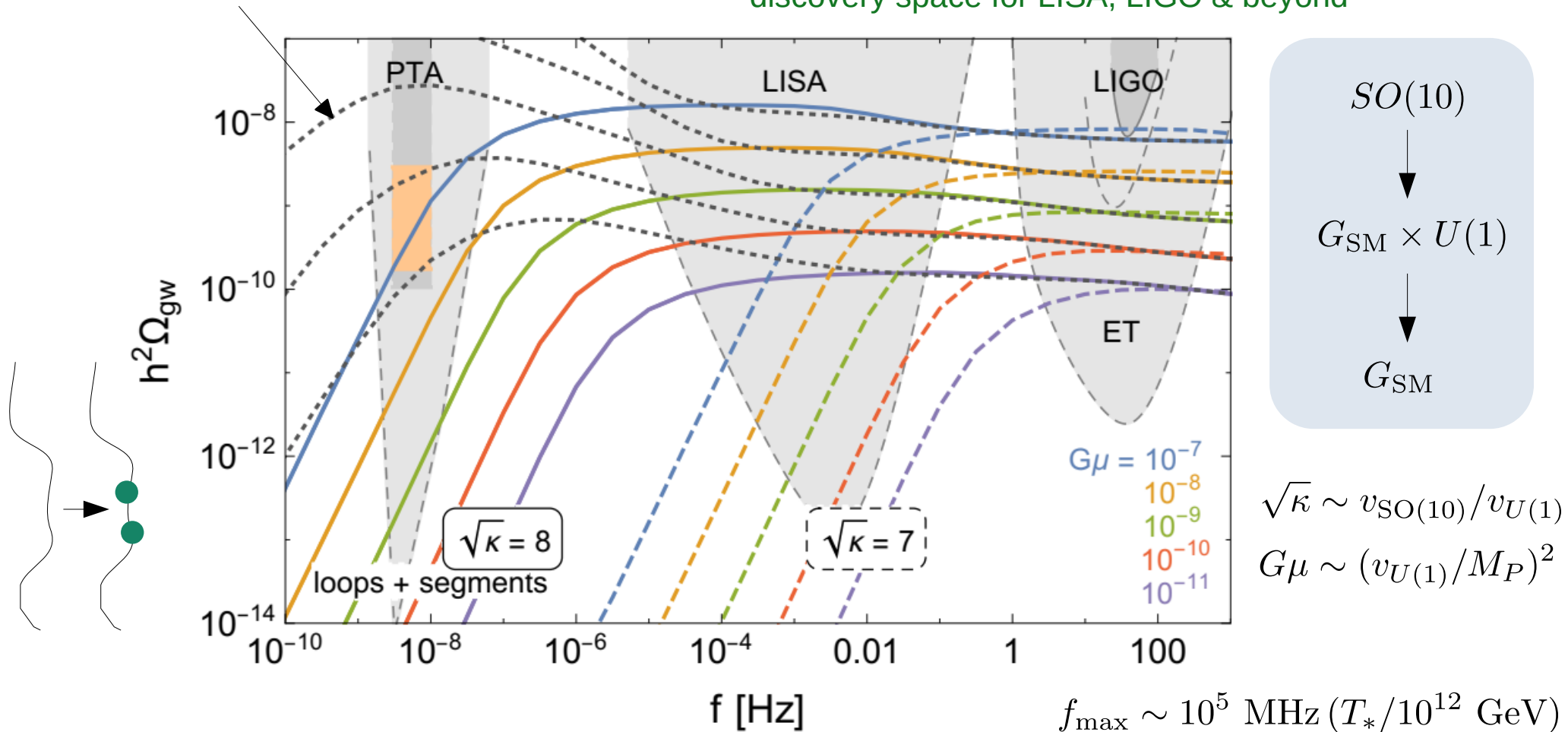
Buchmüller, VD, Schmitz '21

# example: metastable cosmic strings

topologically stable cosmic strings  
(highly constrained by PTA)

Buchmüller, VD, Schmitz '21

metastable cosmic strings:  
discovery space for LISA, LIGO & beyond



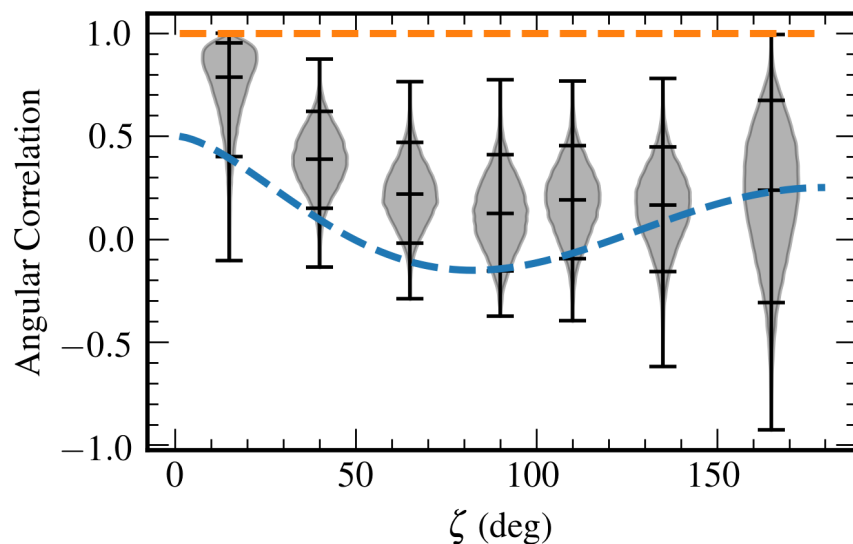
GUT-scale U(1) phase transition can be tested with GWs



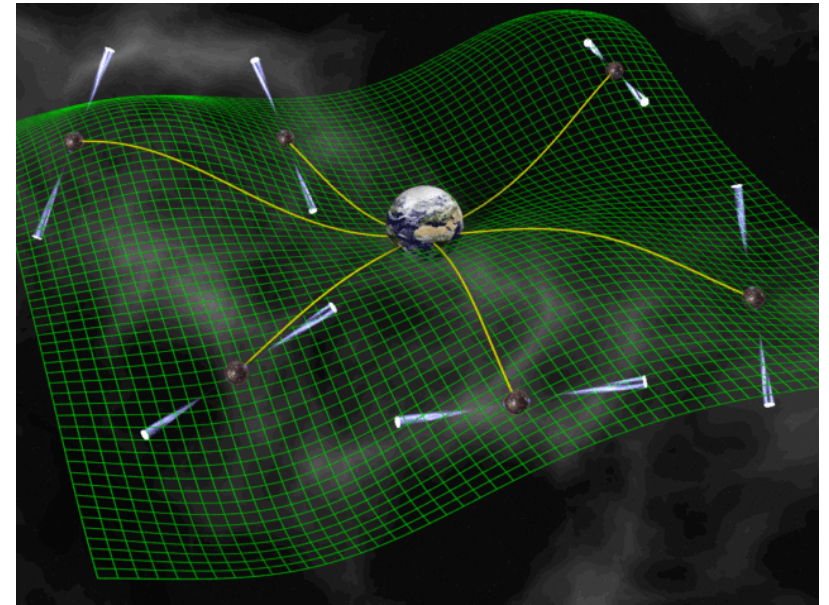
# NANOGrav: A first glimpse of the SGWB?

Pulsar timing array NANOGrav, Sept 2020:

“Our analysis finds strong evidence of a stochastic process, modeled as a power-law, with common amplitude and spectral slope across pulsars.”

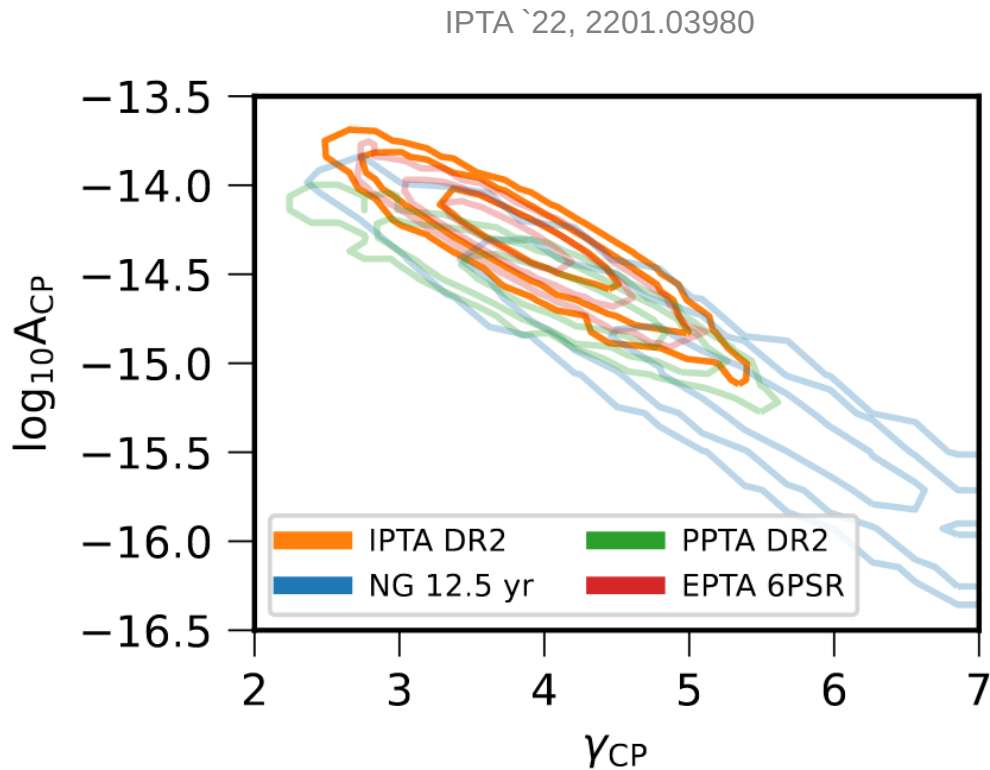


NANOGrav collaboration `20

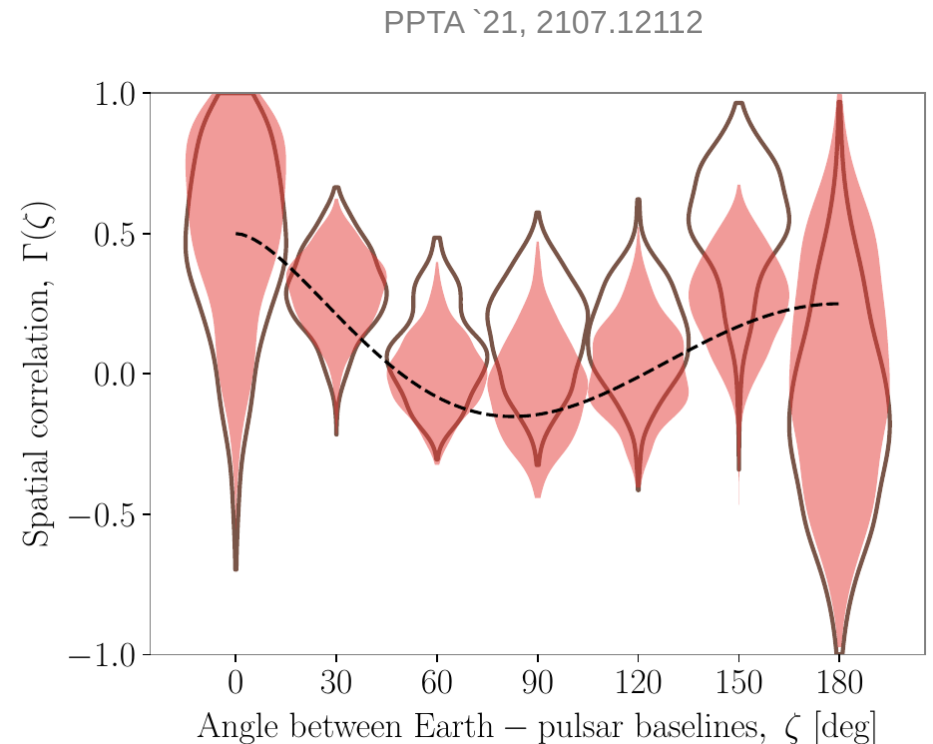


„However, we find no statistically significant evidence that this process has quadrupolar spatial correlations, which we would consider necessary to claim a GWB detection consistent with General Relativity.“

# PPTA, EPTA and IPTA results



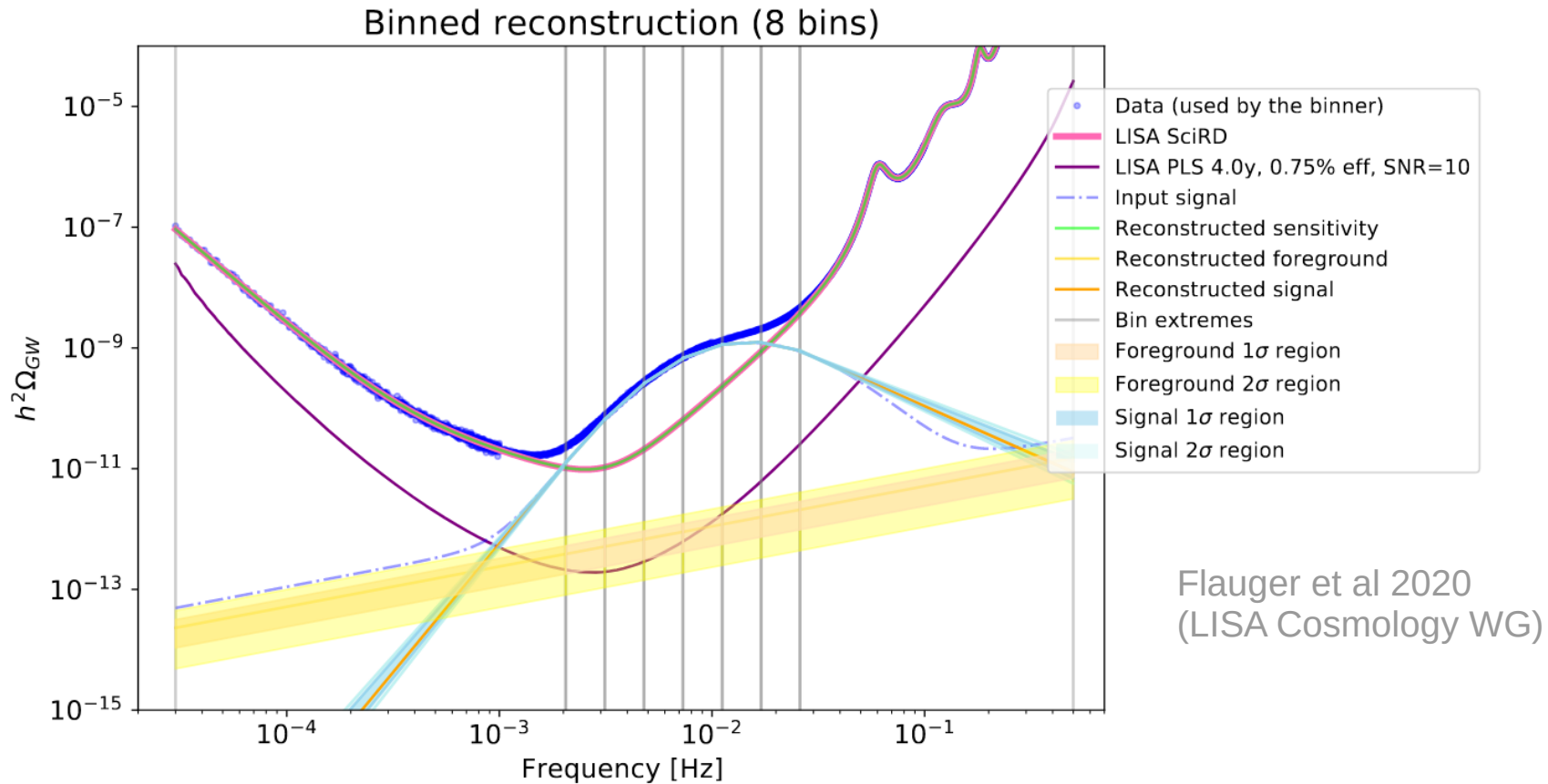
amplitude and spectral tilt  
competitive with NANOGrav



no significant detection of  
quadrupolar spatial correlation

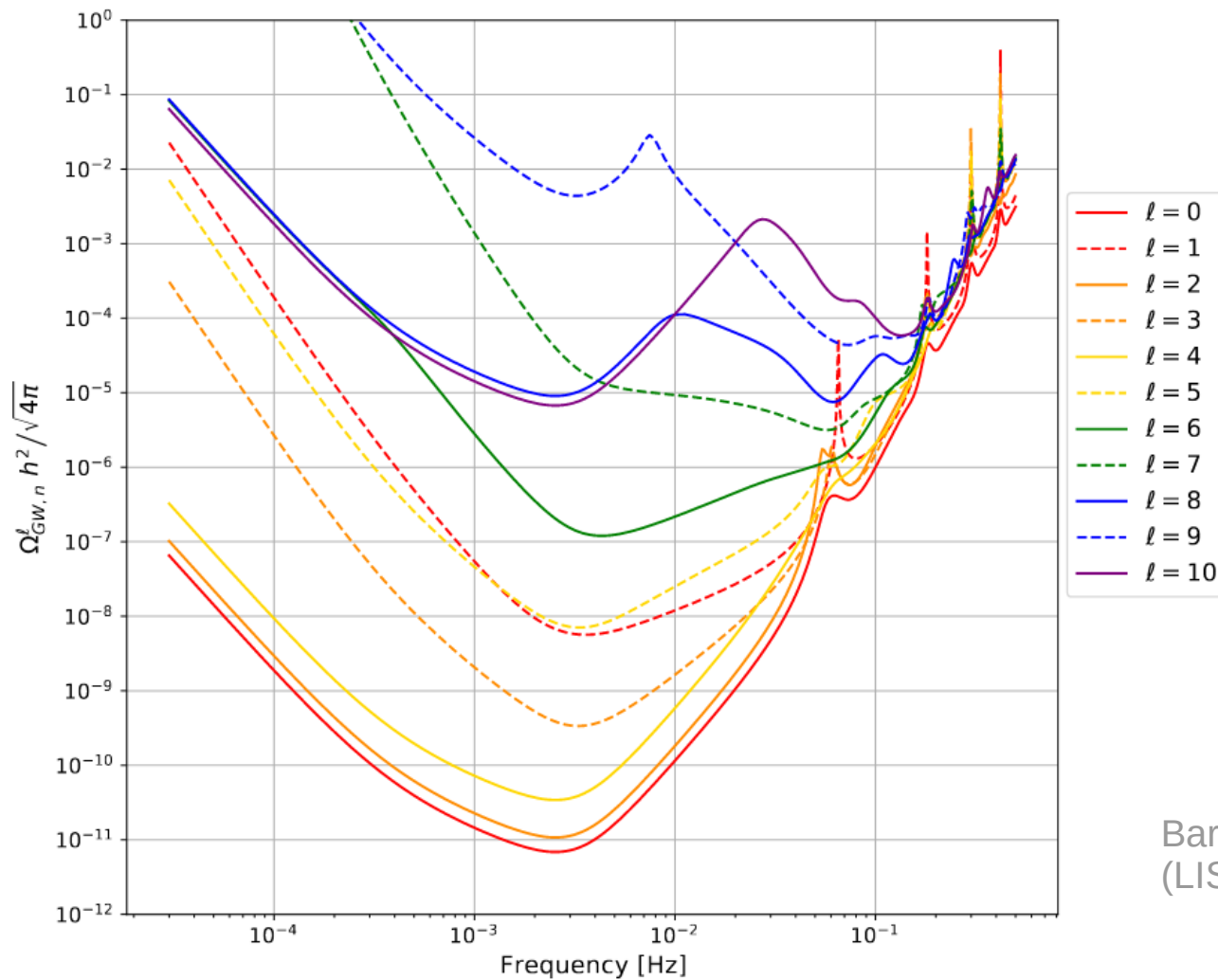
Maybe. Stay tuned for more data!

# decoding the SGWB: spectrum

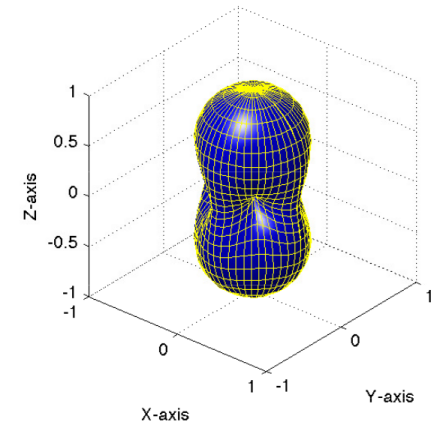


Challenge: simultaneous reconstruction of noise, 'foreground' and 'signal'

# decoding the SGWB : anisotropies



LISA sensitivity to different multipoles

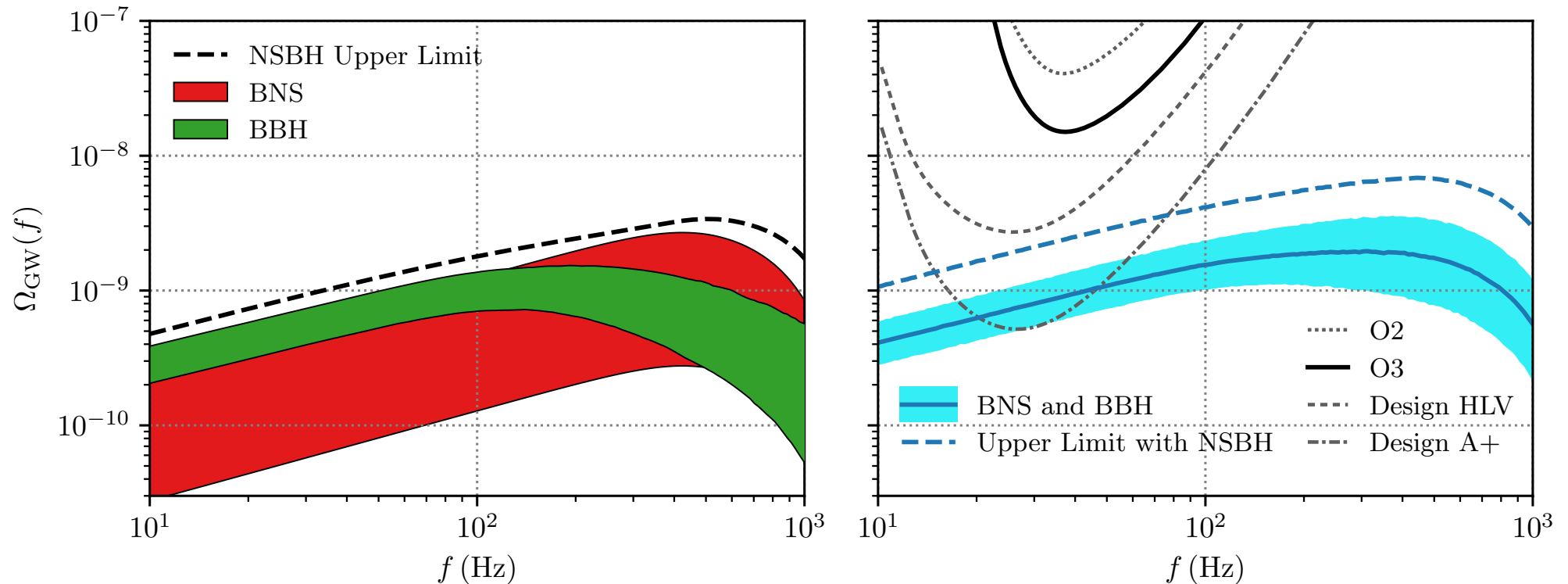


limited angular resolution due to broad antenna pattern of GW interferometers

Bartolo et al 2022  
(LISA Cosmology WG)

# LIGO stochastic backgrounds

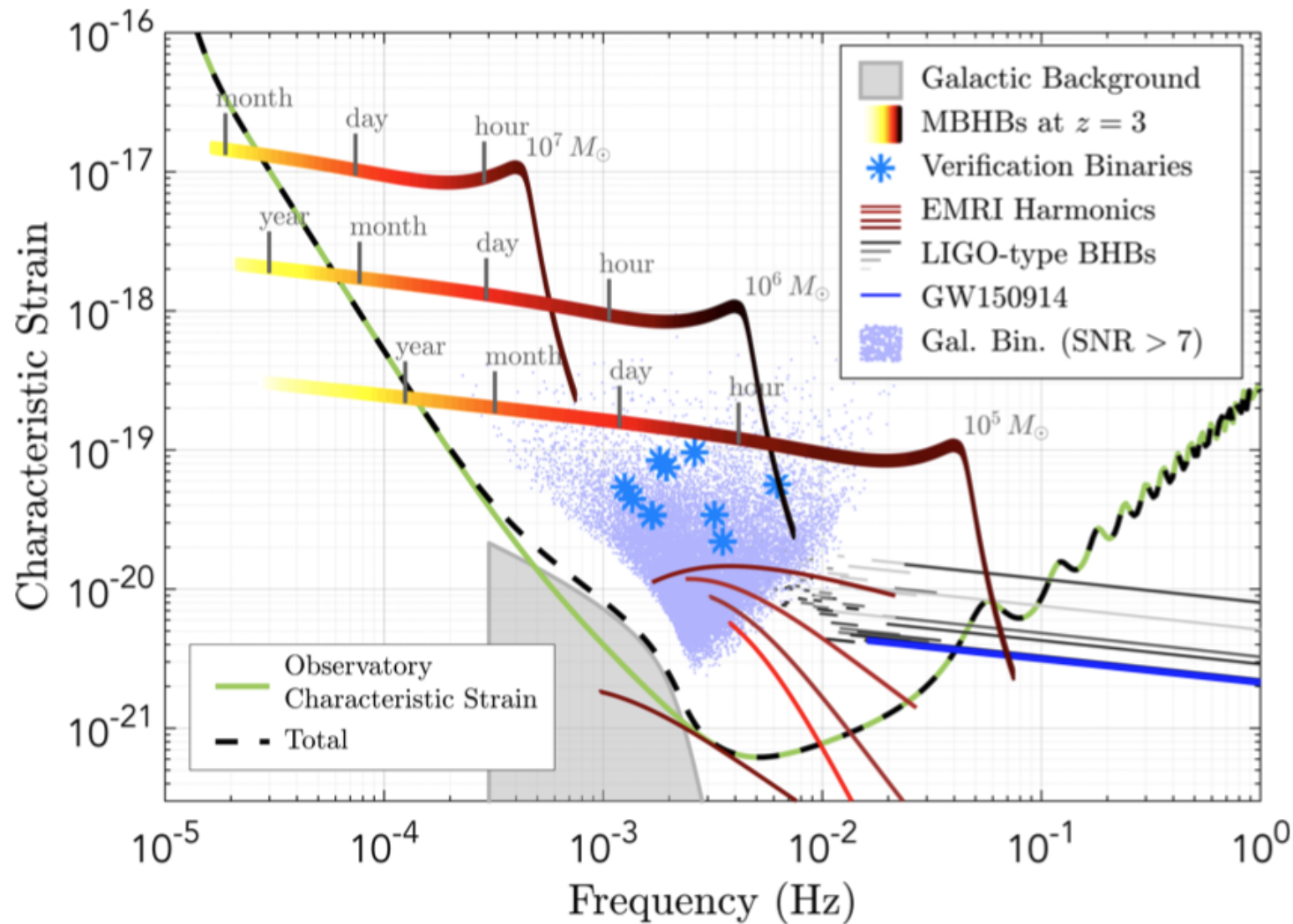
LIGO VIRGO O3 run, 2021



- possibly within reach of advanced LIGO / VIRGO / KAGRA
- not an intrinsically stochastic background: ET / Cosmic Explorer can resolve all BBHs in the Universe



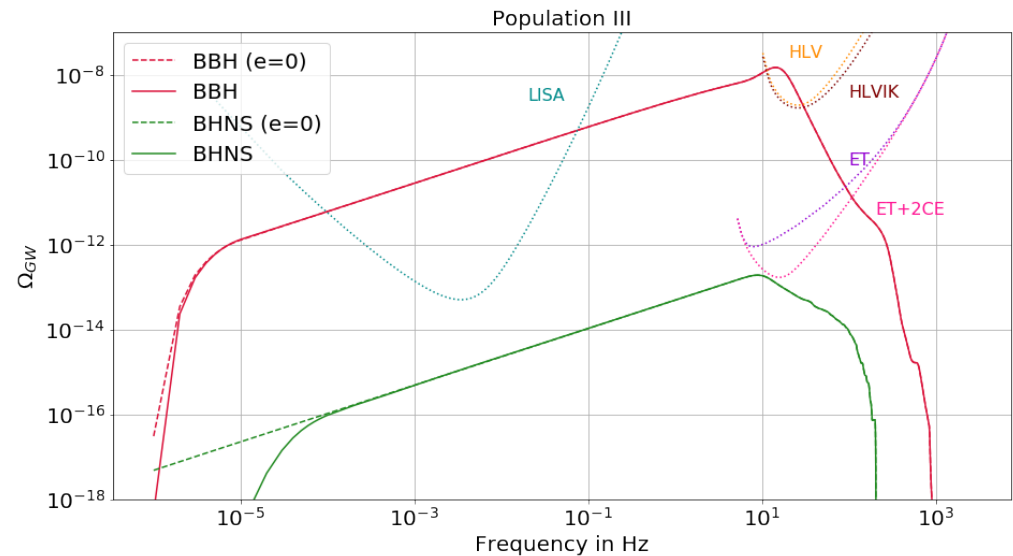
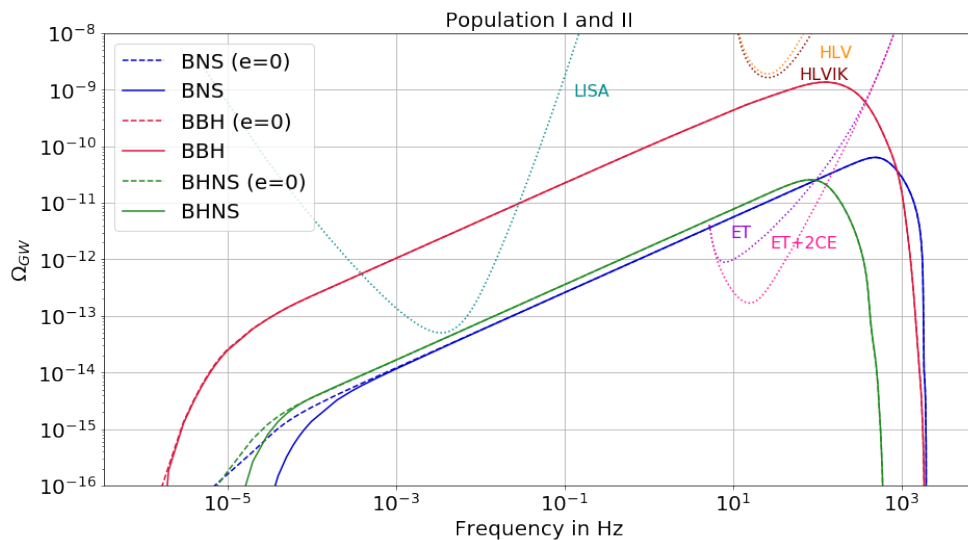
# LISA and backgrounds



LISA mission proposal '17

# LISA stochastic backgrounds

Périgois, Belczynski, Bulik, Regimbau '21



- merger rate will be well measured by LIGO/VIRGO/KAGRA
- thousands of resolved BHBs, millions of unresolved BHBs [Sesana '16]
- overlapping signals → confusion noise. Isotropic? Gaussian? ...

# [ a note on frames ]

GR is invariant under coordinate transformations, but linearized GR is not

## Transverse traceless (TT) gauge

$$h_{ij}^{TT} = (h^+ e_{ij}^+(\phi_h, \theta_h) + h^\times e_{ij}^\times(\phi_h, \theta_h)) e^{i(\mathbf{k} \cdot \mathbf{r} - \omega t)}$$

- coordinates fixed by freely falling test masses
- GW takes very simple form  $h_{0\mu} = 0, h_i^i = 0, \partial_j h^{ij} = 0$
- rigid body seems to ‘oscillate’ in presence of GW

## Proper detector frame

- coordinates fixed by laboratory frame
- GW takes a more involved form
- description of experimental setup and observables is straightforward

$$\begin{aligned} h_{00} &= \omega^2 F(\mathbf{k} \cdot \mathbf{r}) \mathbf{b} \cdot \mathbf{r}, & b_j &\equiv r_i h_{ij}^{TT}|_{\mathbf{r}=0}, \\ h_{0i} &= \frac{1}{2} \omega^2 [F(\mathbf{k} \cdot \mathbf{r}) - i F'(\mathbf{k} \cdot \mathbf{r})] (\hat{\mathbf{k}} \cdot \mathbf{r} b_i - \mathbf{b} \cdot \mathbf{r} \hat{k}_i), \\ h_{ij} &= -i \omega^2 F'(\mathbf{k} \cdot \mathbf{r}) (|\mathbf{r}|^2 h_{ij}^{TT}|_{\mathbf{r}=0} + \mathbf{b} \cdot \mathbf{r} \delta_{ij} - b_i r_j - b_j r_i), \end{aligned}$$

VD, Garcia-Cely, Rodd `22  
s.a. Berlin et al `21

we will consider a plane wave plane wave in the proper detector frame