QCD Phase Diagram and LHC Analysis

Introduction

I **Phase diagram at** $\mu = 0$

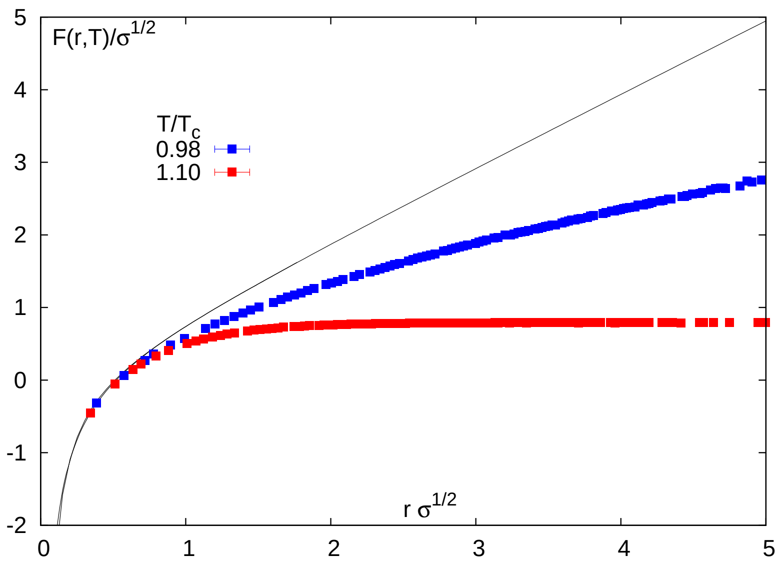
II **Phase diagram at** $\mu \neq 0$

at high temperature and/or density

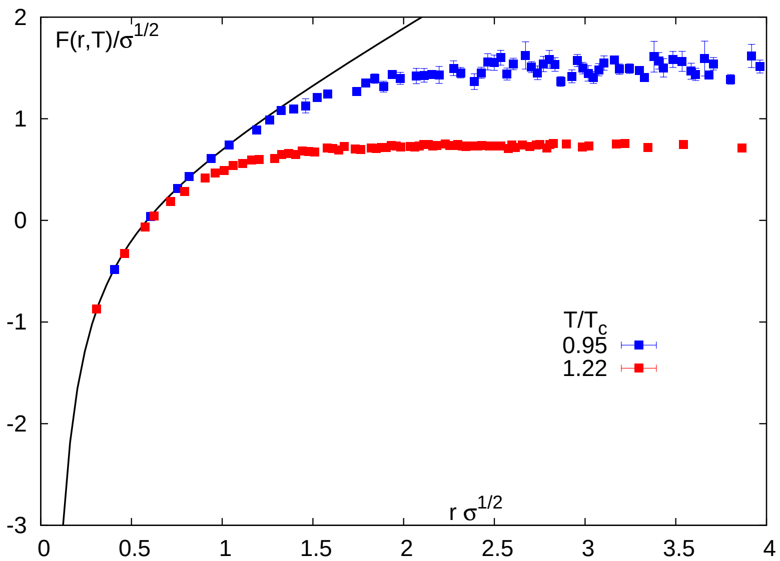
QCD undergoes a transition from the hadronic phase to the quark-gluon plasma

confinement - deconfinement

quenched:

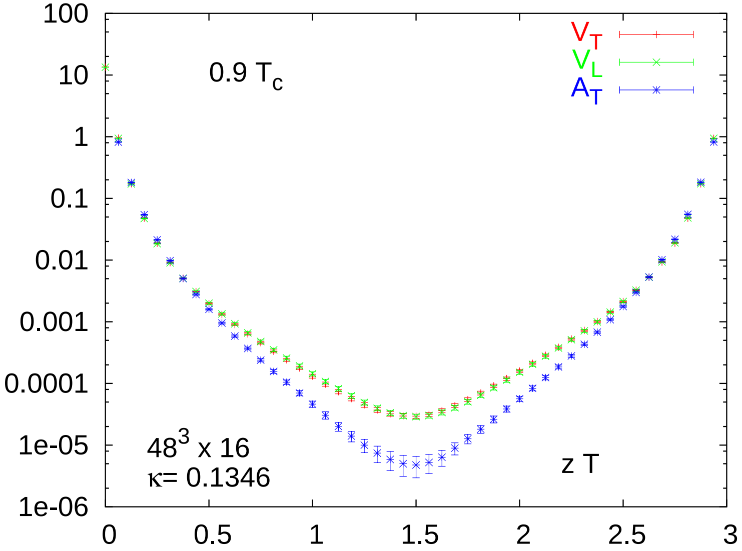


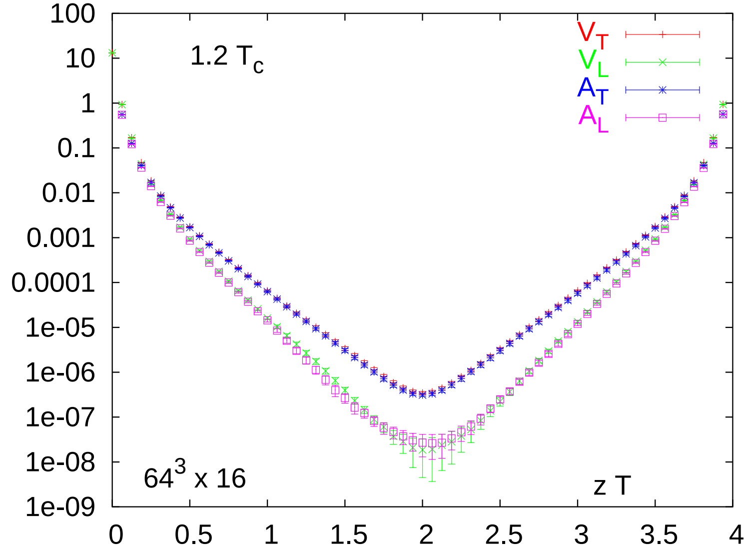
full QCD:



↑ string breaking

Chiral symmetry restoration $SU_V(N_F) \to SU_L(N_F) \times SU_R(N_F)$





$T < T_c$: $V_T = V_L \neq A_T = A_L$ $T > T_c$: $V_T = A_T \neq V_L = A_L$

- at $T > T_c$, chiral symmetry restoration: $V = A$
- at $T \neq 0$, for spatial correlations: rotational $SO(3) \to SO(2) \times Z(2)$

$\Rightarrow$ $V_T \neq V_L$, $A_T \neq A_L$ possible

Quantum Statistics in equilibrium :

partition function $Z = \mathrm{tr}\left\{e^{-\hat{H}/T}\right\}$

→ **Feynman path integral**

$$Z(T,V) = \int \mathcal{D}\phi(\vec{x},\tau)\exp\left\{-\int_0^{1/T} d\tau \int_0^V d^3\vec{x}\,\mathcal{L}_E[\phi(\vec{x},\tau)]\right\}$$

- integral over all configurations $\phi(\vec{x},\tau)$
- weighted by Boltzmann factor $\exp(-S_E)$
- euclidean "time" $\tau = it$
- (anti-) periodic boundary conditions in τ

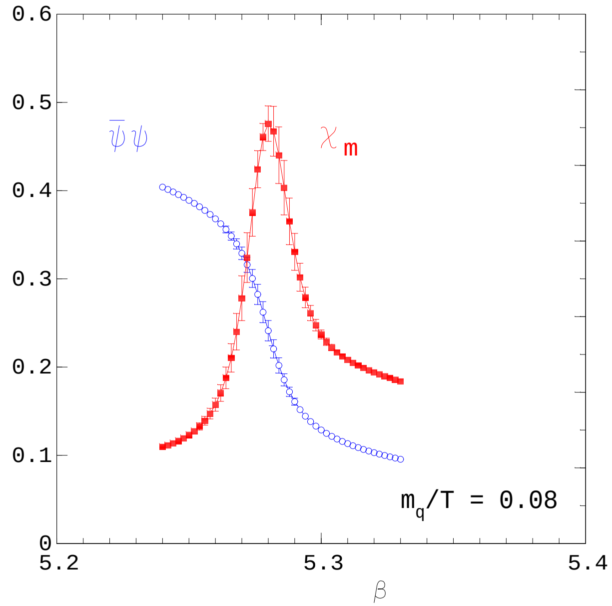
apply standard thermodynamic relations, e.g.

energy density $\epsilon = \frac{T^2}{V}\frac{\partial \ln Z}{\partial T}\Big|_V$

specific heat $c_V = \frac{1}{VT^2}\frac{\partial^2 \ln Z}{\partial(1/T)^2}\Big|_V$

in general

$$\langle \mathcal{O} \rangle = \frac{1}{Z}\mathrm{tr}\left\{\hat{\mathcal{O}}e^{-\hat{H}/T}\right\} = \frac{1}{Z}\int \mathcal{D}\phi\, \mathcal{O}[\phi]e^{-S_E[\phi]}$$

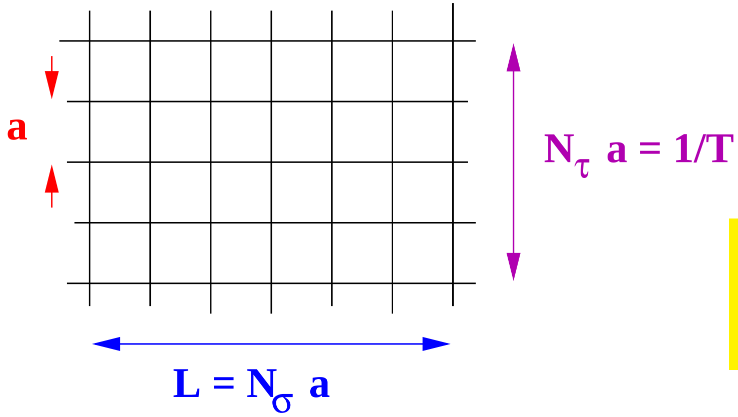


also : starting point of perturbation theory i.e. expansion in coupling strength g

numerical treatment of QCD $\Rightarrow$ discretize (Euclidean) space-time

$\Rightarrow$ **lattice**

$N_\sigma^3 \times N_\tau$



$$Z(T,V) = \int \prod_{i=1}^{N_\tau N_\sigma^3} d\phi(x_i) \exp\left\{-S[\phi(x_i)]\right\}$$

finite yet high-dimensional path integral

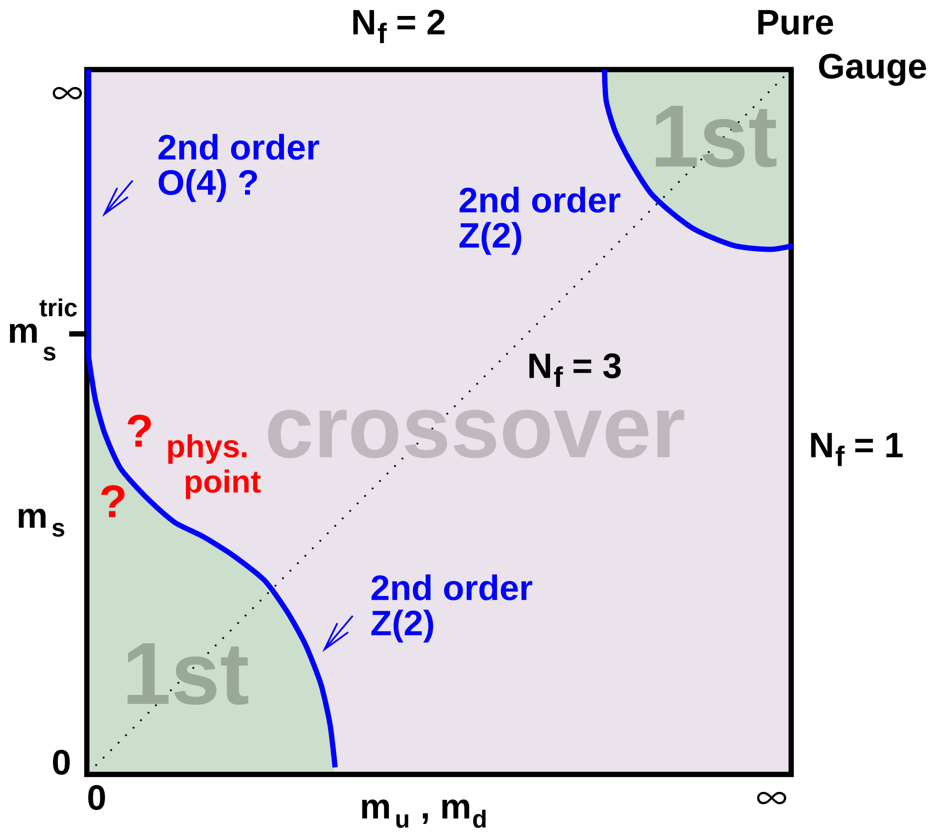
$\longrightarrow$ **Monte Carlo**

- thermodynamic limit, IR - cut-off effects $\quad LT = \frac{N_\sigma}{N_\tau} \longrightarrow \infty \quad$ (finite size scaling)
- continuum limit, UV - cut-off effects $\quad aT = \frac{1}{N_\tau} \longrightarrow 0 \quad$ improved actions
- chiral limit $\quad m \longrightarrow m_{\text{phys}} \simeq 0$

  numerical effort $\sim (1/m)^p$

Phase diagram at vanishing baryon density $(\mu = 0)$

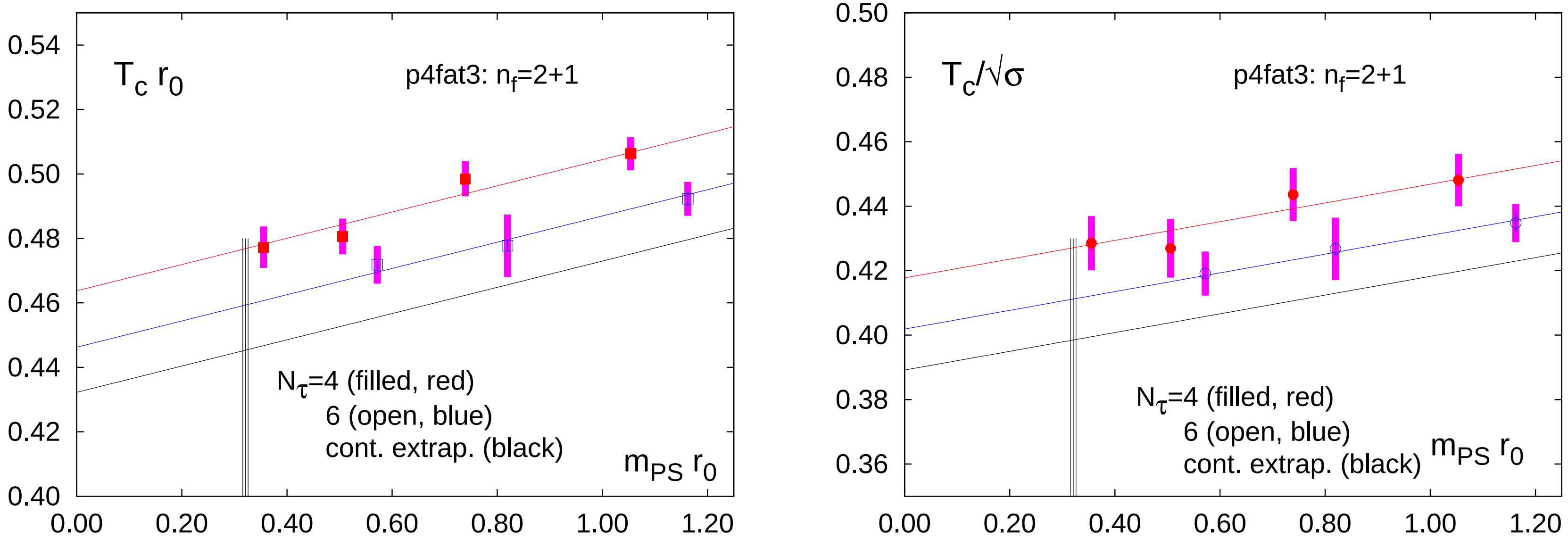
as expected in the $m_{u,d} - m_s$ plane



borne out of spin models in the same (?) universality class

[Pisarski, Wilczek; Rajagopal, Wilczek; Gocksch et al.]

critical temperature $N_F = 2+1$, physical K mass



combined continuum/chiral extrapolation ($d = 1.08$ for $O(4)$, $d = 2$ for first order)

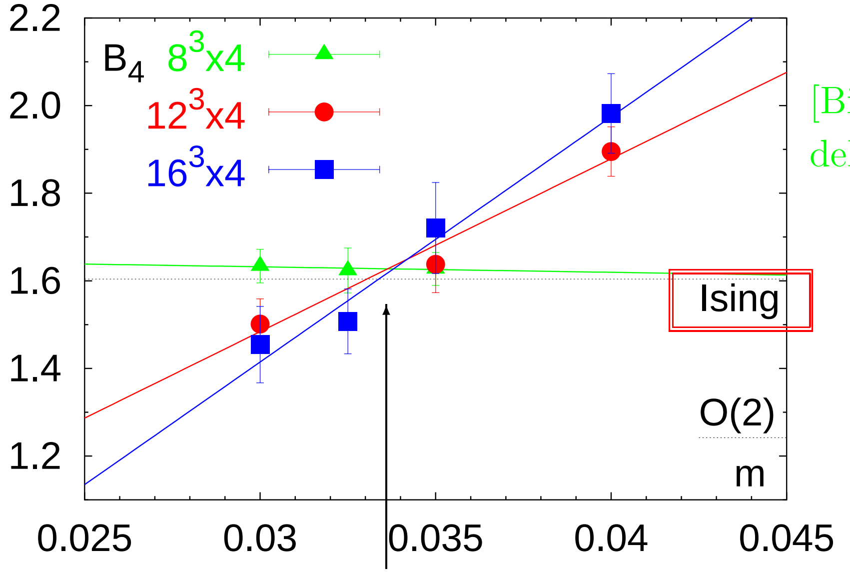
$$(T_c r_0)_{m_l, N_\tau} = T_c r_0 + A(m_{PS} r_0)^d + B/N_\tau^2$$

chiral limit $T_c r_0 = 0.444(6)^{+12}_{-2}$ $T_c/\sqrt{\sigma} = 0.399(5)^{+10}_{-1}$

phys. point $T_c r_0 = 0.456(7)^{+3}_{-1}$ $T_c/\sqrt{\sigma} = 0.408(7)^{+3}_{-1}$

with new $T = 0$ MILC (lattice) results for $r_0 = 0.469(7)$fm obtain: $T_c = 192(5)(4)$MeV

$N_F = 3$



[Bielefeld; deForcrand, Philipsen]

Binder cumulant B_4

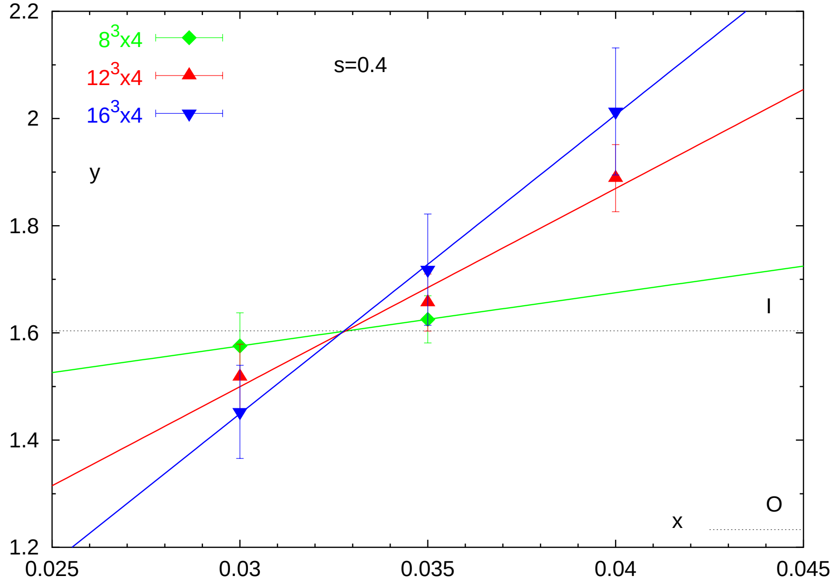
$$B = \frac{\langle \delta M^4 \rangle}{\langle \delta M^2 \rangle^2}$$

- intersection for various V yields critical value of m
- value of B_4 is universal
- corrections from V finite and 'order parameter not matched correctly'

$m_c \simeq 4\, m_u^{phys} \longrightarrow m_{PS} \simeq 290$ MeV std

$\longrightarrow$ 70 MeV improved

indicating large lattice artefacts



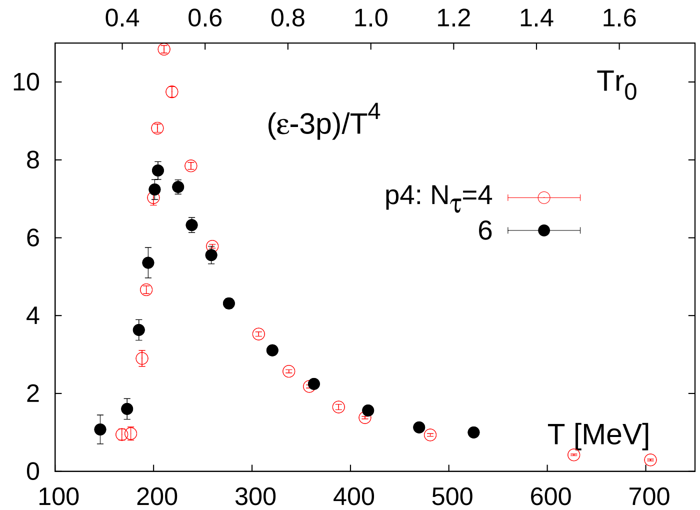
magnetization-like order parameter $\mathcal{M}$

not identical with chiral condensate $\langle \overline{q}q \rangle$

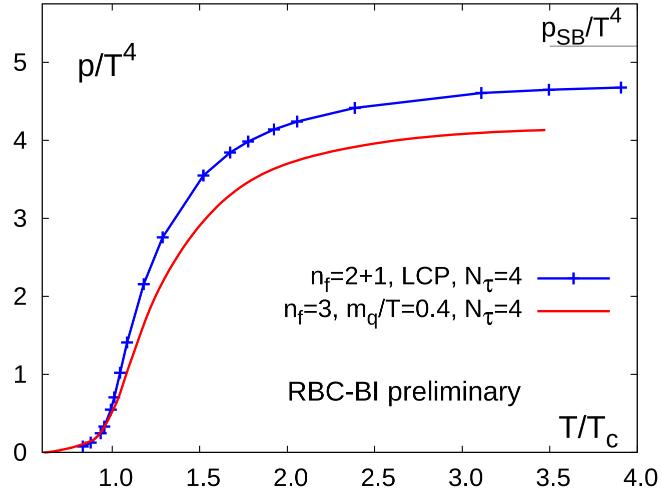
(chiral symmetry broken by $m_q \neq 0$ anyway)

$$\mathcal{M} = \langle \overline{q}q \rangle \; + \; s\, S$$

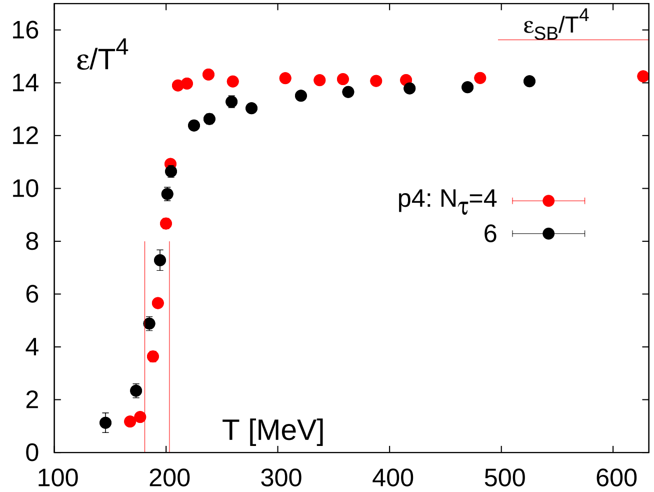
$\Theta^{\mu}_{\mu}(T)/T^4$



pressure $p(T)/T^4 = \int^T dT' \;\; \Theta^{\mu}_{\mu}(T')/T^5 + c$



energy density $\varepsilon(T)$



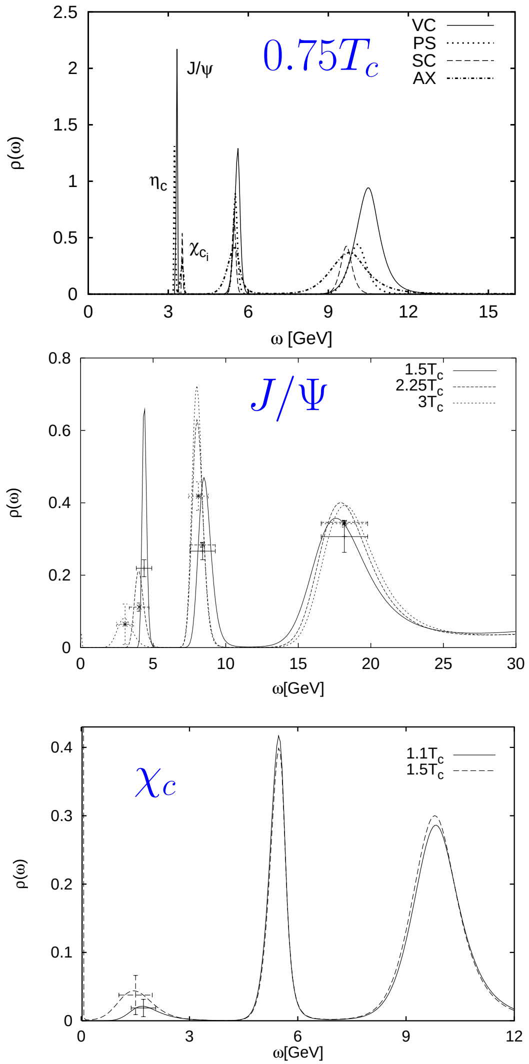
at $\gtrsim 2T_c$ almost no discretization effects

at $\gtrsim 2T_c$ 10% deviations from ideal gas

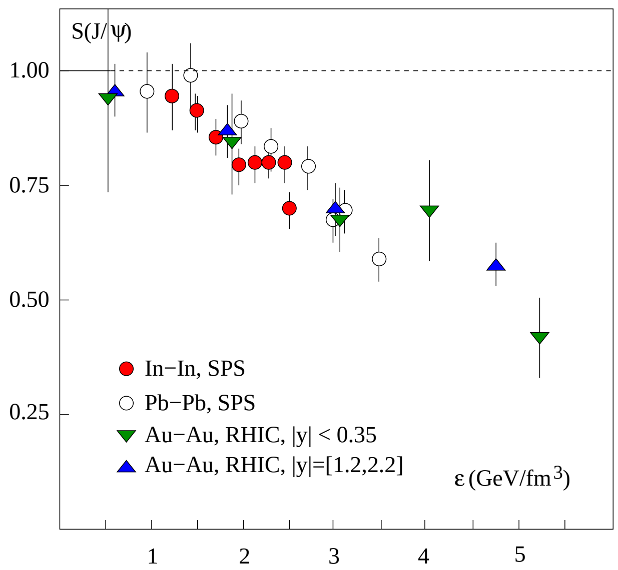
LHC: $\varepsilon \lesssim 1\,\mathrm{TeV/fm}^3 \;\hat{=}\; T \simeq 900$ MeV

LHC $\rightarrow$ properties of the Quark-Gluon Plasma

e.g. J/Ψ suppression



- individual "melting" temperatures $> T_c$
- suppression patterns



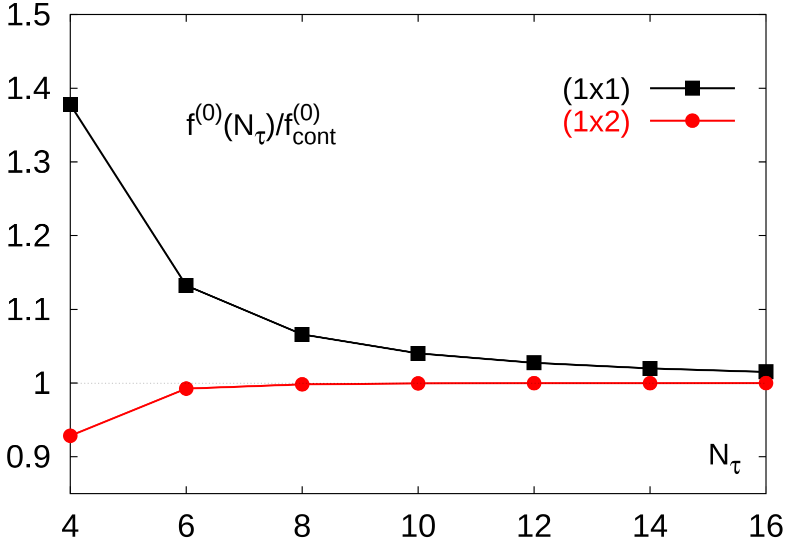
LHC:
- large abundance of c-quarks
- feed down from b-quarks

the difficulty

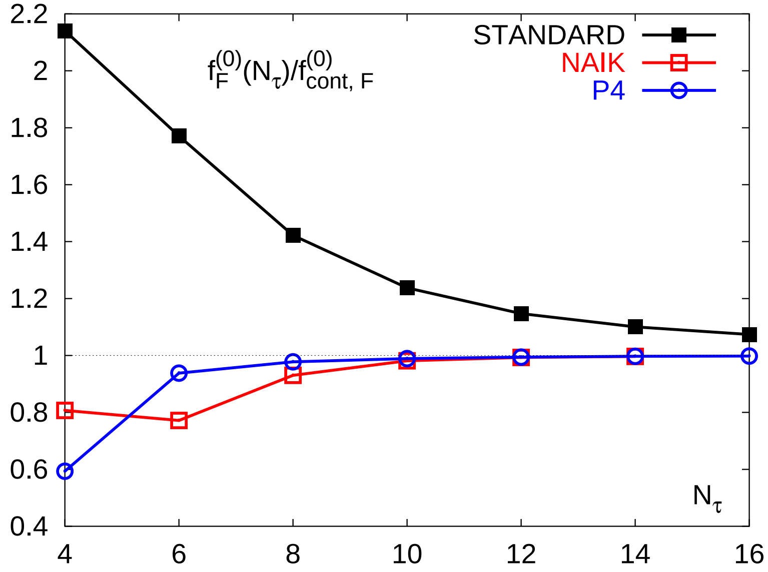
$$\frac{p}{T^4} = N_\tau^4 \times \{\text{signal} \pm \text{statistical error}\}$$

ideal gas limit at finite $aT = 1/N_\tau$:

gluon part



fermion part



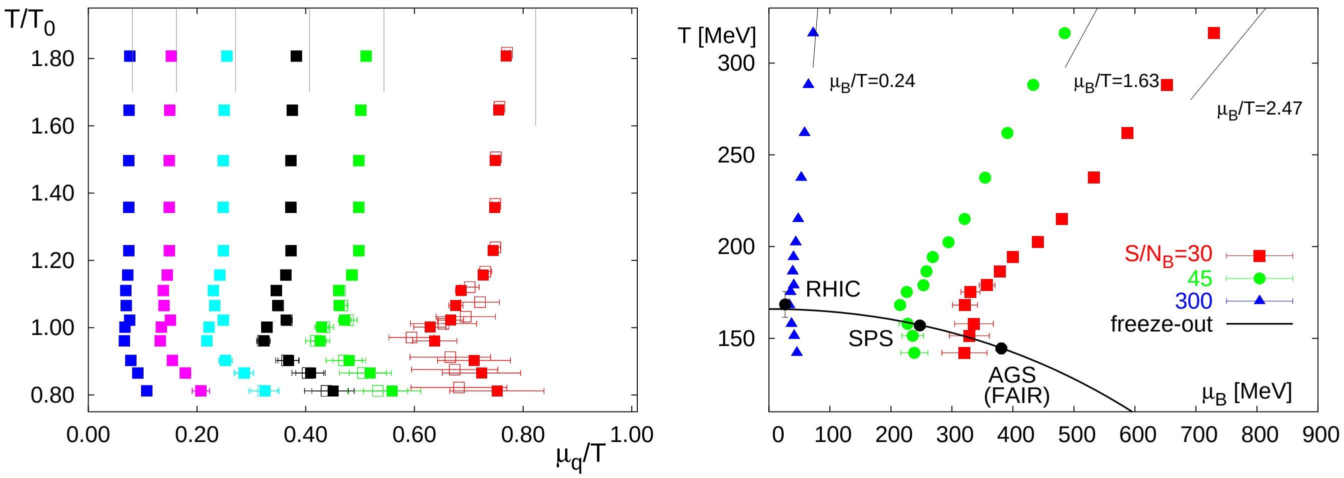
$\Rightarrow$ improved actions mandatory

improving thermodynamics: Naik action, p4 action

improving flavor symmetry: fat links, stout links

Phase diagram at non-vanishing baryon density $(\mu_B = 3\mu_q \neq 0)$

it is generally believed that the fireball expansion follows a line of fixed S/N_B



in the ideal gas limit
$$\frac{S}{N_B} = 3\frac{\frac{37\pi^2}{45} + \left(\frac{\mu_q}{T}\right)^2}{\frac{\mu_q}{T} + \frac{1}{\pi^2}\left(\frac{\mu_q}{T}\right)^3} \qquad \Rightarrow \frac{\mu_q}{T} = \text{const (vertical lines)}$$

isentropic expansion lines for

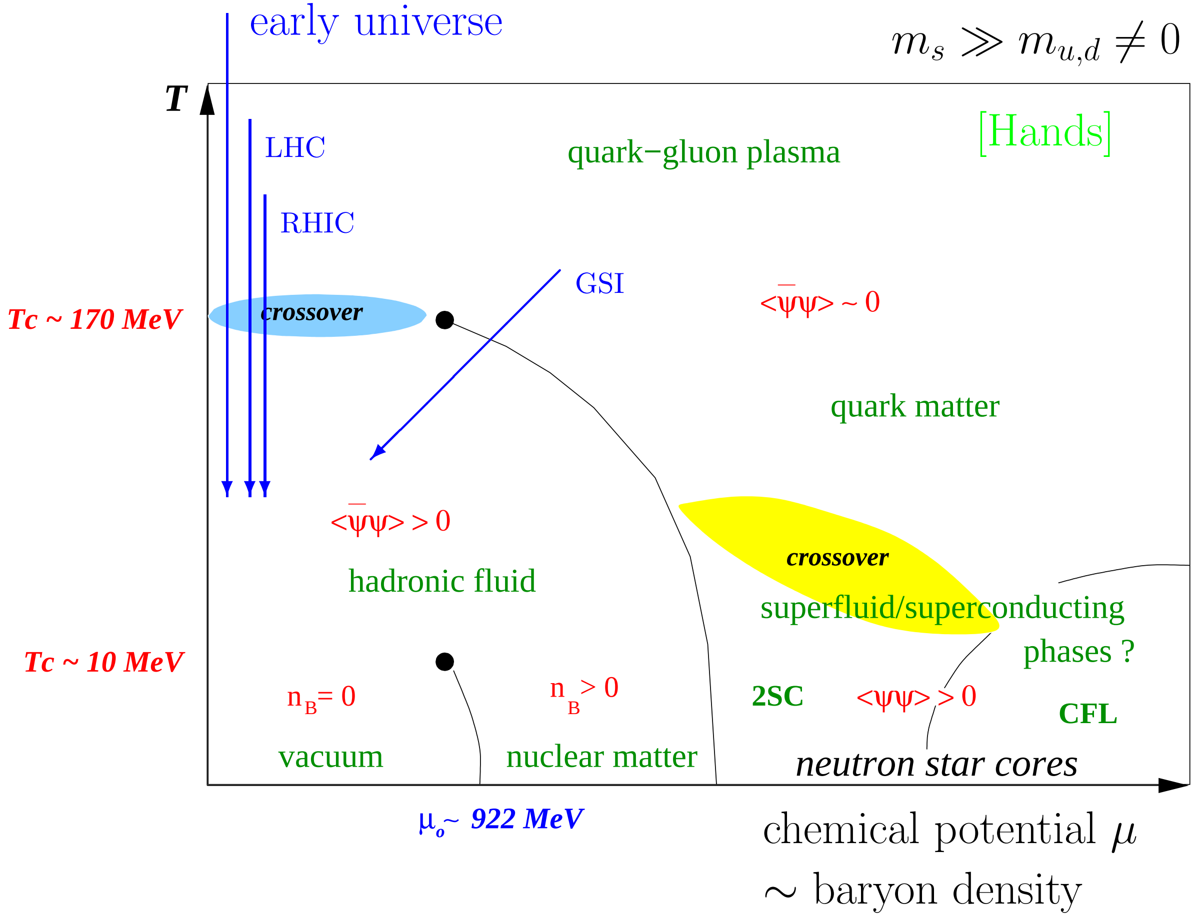
FAIR: $S/N_B \simeq 30$

SPS: $S/N_B \simeq 45$

RHIC: $S/N_B \simeq 300$

LHC: < 300

expected properties :



in detail dependent on masses of light flavors

$m_{u,d} \ll m_s \quad N_F = 2$

$m_{u,d} < m_s \quad N_F = 2 + 1$

$m_{u,d} \simeq m_s \quad N_F = 3$

[see e.g. Rajagopal, Wilczek]

the problem :

$$Z_{GC}(T,V,\mu) = \int \mathcal{D}U_\mu \, \mathcal{D}q \, \mathcal{D}\overline{q} \; \exp\{-S_G(U) + \overline{q}M(\mu)q\}$$

integrate over quark fields

$$Z_{GC}(T,V,\mu) = \int \mathcal{D}U_\mu \; \det M(\mu) \; \exp\{-S_G(U)\}$$

- for $\mu \neq 0$: $\det M(\mu)$ complex $\Rightarrow$ can not be used as statistical weight in Monte Carlo
- reformulate: $\det M(\mu) = |\det M(\mu)|\, e^{i\Theta}$ and use phase Θ as (part of the) observable:

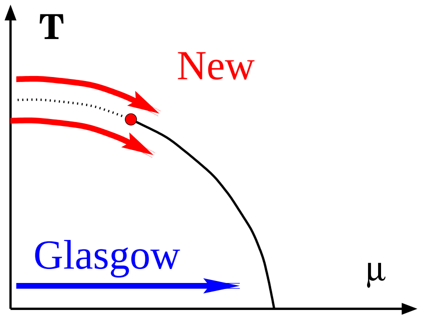
$$\langle\, \mathcal{O} \,\rangle_{\det M} = \langle\, \mathcal{O}\; e^{i\Theta} \,\rangle_{|\det M|} \;/\; \langle\, e^{i\Theta} \,\rangle_{|\det M|}$$

- but : $\langle\, e^{i\Theta} \,\rangle_{|\det M|} \sim e^{-V}$ 'sign problem'

* 'Reweighting'

[Glasgow; Fodor,Katz]

simulate at parameters $p_0 = (g, m, \mu)_0$ and reweight to $p = (g, m, \mu)$



$$\mathcal{D}U e^{-S_G(p)} \det M(p) = \underbrace{\mathcal{D}U e^{-S_G(p_0)} \det M(p_0)}_{\text{simulation}} \star \underbrace{e^{-[S_G(p)-S_G(p_0)]} \frac{\det M(p)}{\det M(p_0)}}_{\text{correction-factor}}$$

- limited by overlap

* 'Taylor-expansion'

[Bielefeld-Swansea; Gavai,Gupta]

$$\langle \mathcal{O} \rangle \left(\frac{\mu}{T}\right) = \langle \mathcal{O} \rangle_{\mu=0} + \langle \tilde{\mathcal{O}}_2 \rangle_{\mu=0} \star \left(\frac{\mu}{T}\right)^2 + \langle \tilde{\mathcal{O}}_4 \rangle_{\mu=0} \star \left(\frac{\mu}{T}\right)^4 + \ldots \qquad \text{with} \qquad \tilde{\mathcal{O}}_k = \frac{1}{k!} \frac{\partial^k \mathcal{O} \det M}{\partial \mu^k}$$

- limited by convergence radius

* 'imaginary μ'

[Forcrand,Philipsen; D'Elia,Lombardo]

- $\mu = i\mu_I \Rightarrow \det M$ real and positive
- analytic continuation to real μ

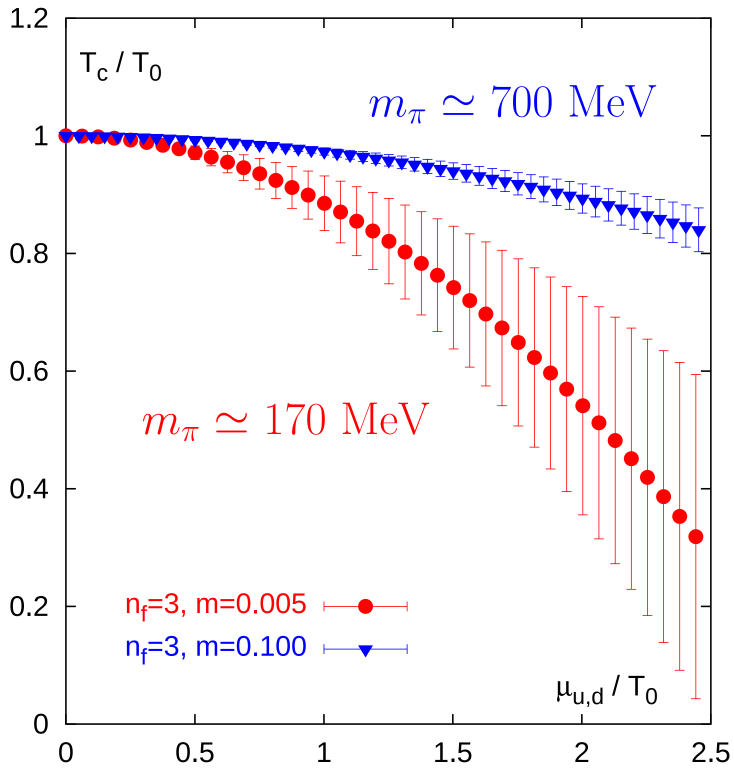
- limited by $Z(\mu_I/T) = Z(\mu_I/T + 2\pi/3)$

∗ **'canonical'** [Forcrand,Kratochvila]

$$Z_C(B) = \frac{1}{2\pi} \int d\left(\frac{\mu_I}{T}\right) \exp\left\{-i3B\frac{\mu_I}{T}\right\} Z_{GC}(\mu = \mu_I)$$

- sample at fixed μ_I
- Fourier transform each determinant $\longrightarrow$ work $\sim N_\sigma^9 \times N_\tau$
- combine with reweighting in μ_I
- back to $Z_{GC}(\mu)$ by $\Sigma_B \exp\left\{+3B\frac{\mu}{T}\right\} Z_C(B)$

- applicable at small values for μ in the phenomenologically relevant range for RHIC, LHC
- first, exploratory results in qualitative agreement, further systematic investigations required
- in particular at smaller quark masses :



$N_F = 3$, improved action

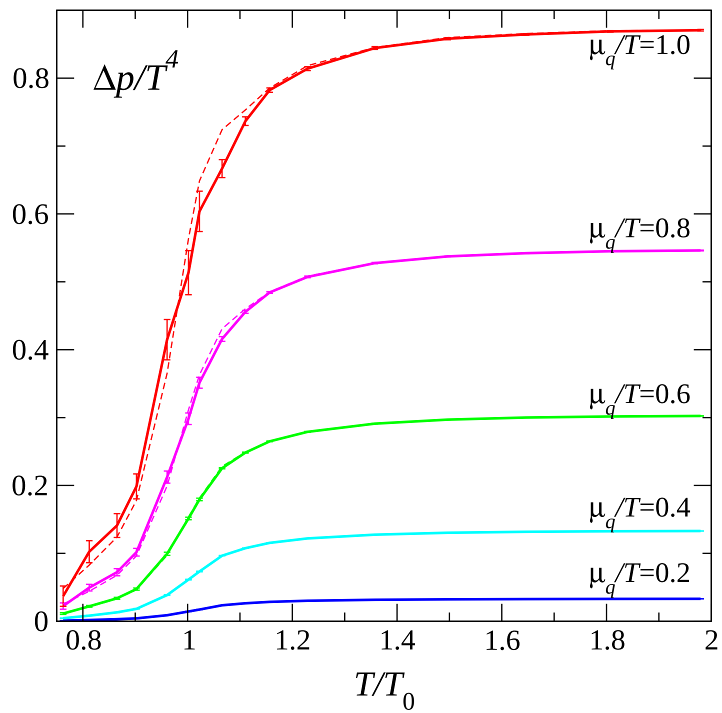
$$\frac{T_c(\mu)}{T_c(0)} = 1 - 0.025(6) \left(\frac{\mu}{T_c(0)} \right)^2$$

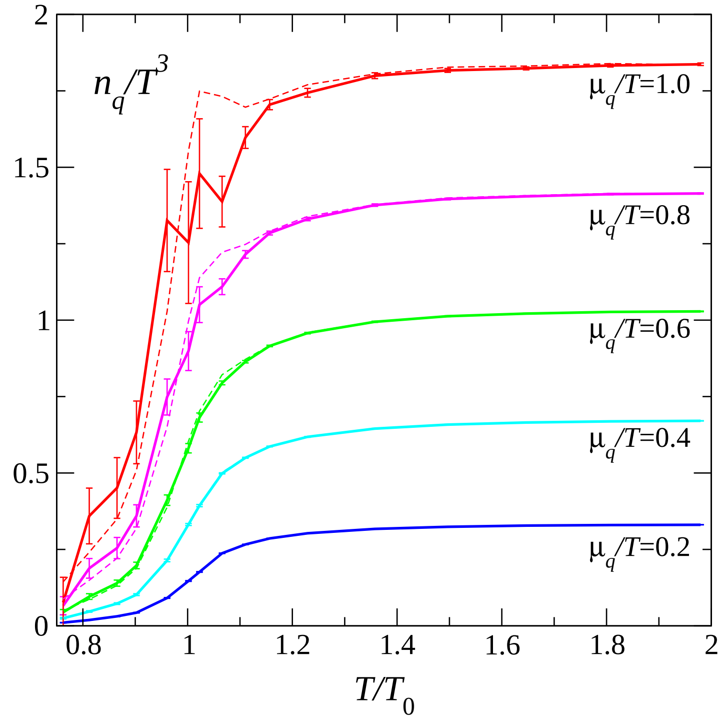
$$\frac{T_c(\mu)}{T_c(0)} = 1 - 0.114(46) \left(\frac{\mu}{T_c(0)} \right)^2$$

(perturbative β-function $d\beta_c / d \ln a$)

- considerable quark mass dependence

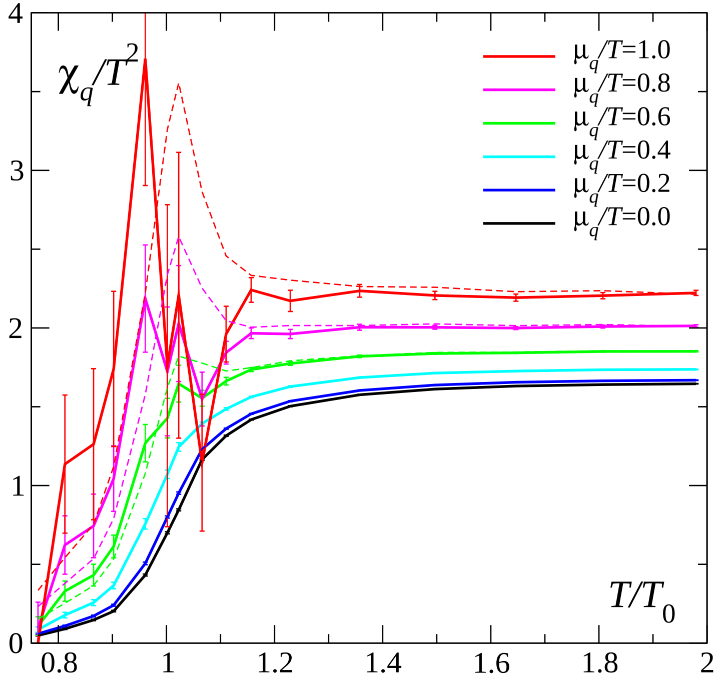
pressure **quark number density**

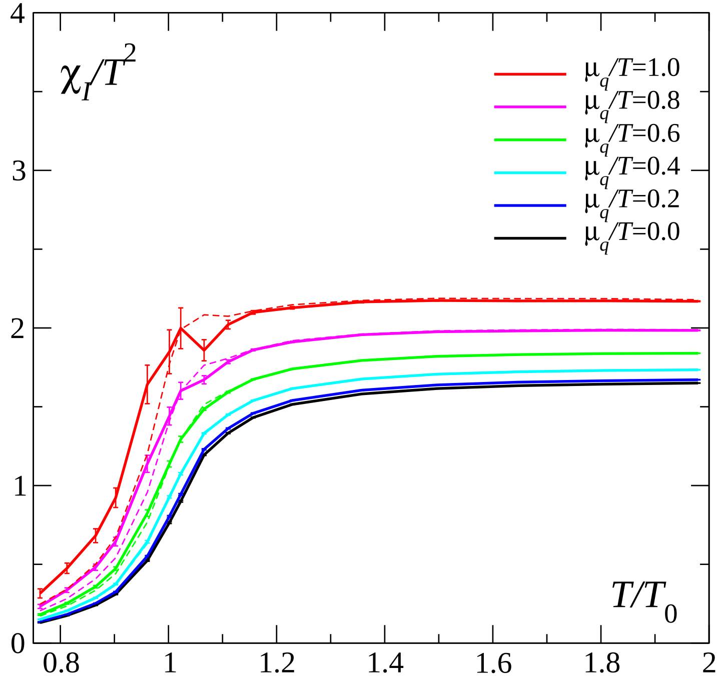




- comparison: up to $\mathcal{O}(\mu^4)$ (dashed) with up to $\mathcal{O}(\mu^6)$ (full) suggests rapid convergence
- contribution to total p is small: $p(\mu = 0)/T^4 \simeq \mathcal{O}(4)$

quark number susceptibility **isovector susceptibility**



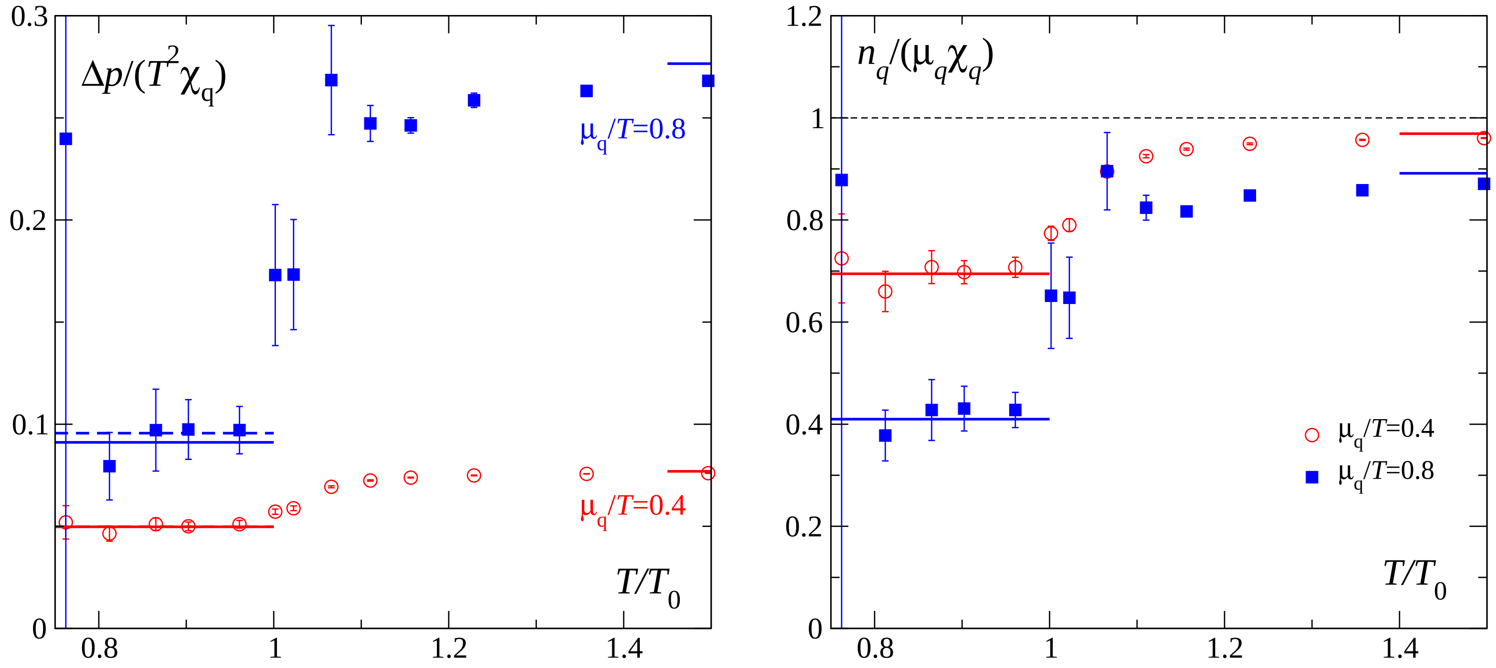


$\chi_q \sim \langle (n_u + n_d)^2 \rangle$ $\chi_I \sim \langle (n_u - n_d)^2 \rangle$

- peak in χ_q developing with increasing μ
- no peak in χ_I

$\Rightarrow$ around T_0 strong correlations between u and d fluctuations

- χ_q rises rapidly with increasing μ_q, but rise to a large extent due to rise in pressure



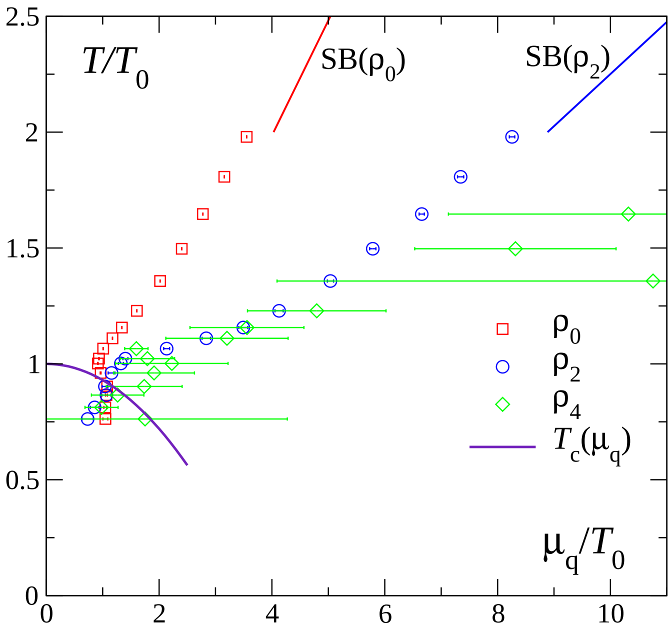
- $$\frac{\partial p}{\partial n_q} = \frac{\partial p/\partial \mu_q}{\partial n_q/\partial \mu_q} = \frac{n_q}{\chi_q} = \frac{1}{\kappa_T\, n_q} \to 0$$ at 2[nd] order phase transition (isothermal compressibility $\kappa_T \to \infty$)

- no indication of criticality

- but, for $T \leq 0.96 T_c$, consistency with hadron resonance gas model (at $\mu_I = 0$)

$$\frac{n_q^{HRG}}{\mu_q \chi_q^{HRG}} = \frac{T}{3\mu_q} \tanh\left(\frac{3\mu_q}{T}\right)$$

convergence radius



nearest (complex) singularity determines convergence radius

$$\rho = \lim_{k\to\infty} \rho_k = \lim_{k\to\infty} \sqrt{\left|\frac{c_k}{c_{k+2}}\right|}$$

- SB limit: $\rho_k = \infty$ for $k \geq 4$
- for T big: approaching SB limit

- at $T_c(\mu)$: $\rho_k \simeq 1$

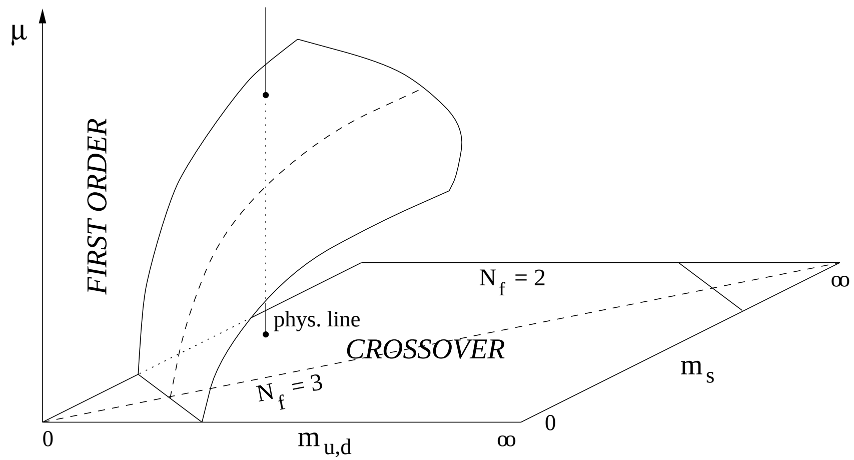
- $c_k > 0 \Rightarrow$ convergence radius indicates critical point

Results:

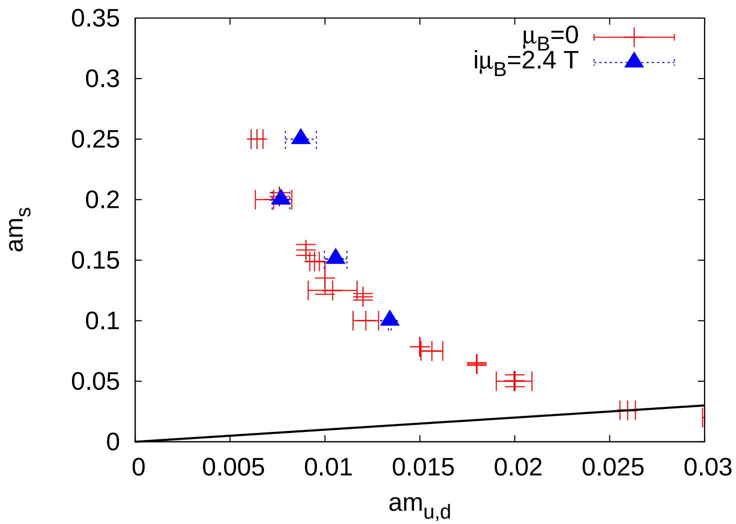
- Gavai, Gupta: $\mu_{B,E} \simeq 180\text{MeV}$ (Taylor expansion)
- Fodor, Katz: $\mu_{B,E} \simeq 360\text{MeV}$ (Lee-Yang zeroes)
- Bi-Swansea: LGT consistent with HRG, HRG analytic (Taylor expansion)

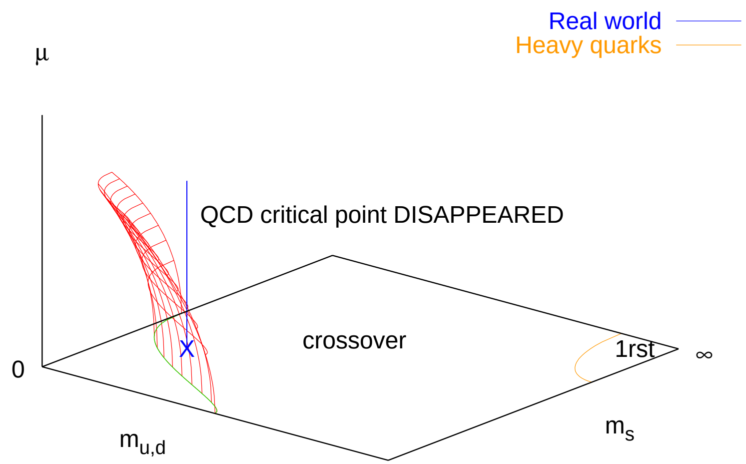
existence of a critical endpoint ?

standard scenario :



at $\mu = i\mu_I$:





critical region has the tendency to grow with μ_I $\Rightarrow$ shrink with real μ [deForcrand, Philipsen]

under investigation on finer lattices/ with improved actions