$\mathcal T$ rilinear $\mathcal Higgs\ \mathcal C$ oupling and $\mathcal Higgs\ \mathcal P$ otential

Milada Margarete Mühlleitner (CERN/LAPTH)

SYMPOSIUM Collider Physics at the Tera-Scale

May 29, 2007



Outline

- \ast \mathcal{M} otivation
- $* \quad \mathcal{I}nternational \ \mathcal{L}inear \ \mathcal{C}ollider$
 - Self-Couplings of the SM Higgs Boson
 - Self-Couplings of the MSSM Higgs Bosons
- $* \hspace{0.1in} \mathcal{H} iggs \hspace{0.1in} \mathcal{S}elf\text{-} \mathcal{C}ouplings \hspace{0.1in} at \hspace{0.1in} the \hspace{0.1in} \mathcal{L}arge \hspace{0.1in} \mathcal{H}adron \hspace{0.1in} \mathcal{C}ollider$
- * Conclusions

Motivation

${\cal H}iggs ~{\cal M}echanism$

\mathcal{P} roduction of \mathcal{M} asses via \mathcal{S} pontaneous \mathcal{S} ymmetry \mathcal{B} reaking (\mathcal{SSB})

- \mathcal{P} articles acquire \mathcal{M} asses through \mathcal{I} nteraction with a \mathcal{S} calar Field.
- Self-Interaction of the Scalar Field \rightsquigarrow Non-Zero Field Strength in the Ground State $\rightsquigarrow SSB$.
- $\mathcal{V}eV v = 246 \text{ GeV} \neq 0 \mathcal{I}nduced$ by the typical $\mathcal{F}orm$ of the $\mathcal{H}iggs$ Potential.
- \mathcal{W} eak iso-doublet \mathcal{S} calar Field via $\mathcal{SSB} \rightsquigarrow \mathcal{H}$ iggs Particle.

Motivation

Higgs Mechanism How establish experimentally?



Motivation

Higgs Mechanism How establish experimentally?



The SM Higgs Sector

The Higgs potential: [v = 246 GeV]

$$V(\Phi) = \lambda [\Phi^{\dagger} \Phi - \frac{v^2}{2}]^2 \qquad \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v+H \end{pmatrix} \rightarrow$$

 $V(H) = \frac{1}{2!}\lambda_{HH}H^{2} + \frac{1}{3!}\lambda_{HHH}H^{3} + \frac{1}{4!}\lambda_{HHHH}H^{4}$



${\cal H}$ iggs boson mass	$\lambda_{HH} = M_H^2 = 2\lambda v^2$	
${\mathcal T}$ rilinear coupling	$\lambda_{HHH} = 3 \frac{M_H^2}{v}$	· · · · · · · · · · · · · · · · · · ·
$\mathcal Q$ uartic coupling	$\lambda_{HHHH} = 3 \frac{M_H^2}{v^2}$	· · · · · · · · · · · · · · · · · · ·

${\cal M}$ easurement of the Higgs Self-Couplings	${\mathcal E}$ stablish the Scalar
and	$ ho$ ${\cal S}$ ector of the Higgs Mechanism
${\cal R}$ econstruction of the Higgs Potential $$	\mathcal{E} xperimentally

Determination of the Higgs Self-Couplings

Determination of the Higgs self-couplings at future colliders:

- λ_{HHH} via Higgs pair production
- λ_{HHHH} via triple Higgs production

Higgs-strahlung, WW/ZZ fusion, gg fusion

Boudiema.Chopin

Also, but not treated here

* At Photon Collider	Ginzburg et al. Ilyin et al.;Telnov et al.	in $\gamma\gamma$ fusion	La-Zhen,Yao-Yang;Zhu et al. Gounaris,Layssac,Porfyriadis,Renard; Belusevic,Jikia;
* At ILC: $e^+e^- \rightarrow b\bar{b}\bar{b}$	$HH, tar{t}HH$		Gutiérrez-Rodriguez et al.

Determination of the Higgs Self-Couplings

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Main HH production processes at the ILC



WW Fusion in e^+e^- Collisions



Dominant Production Amplitude: $W_L W_L \rightarrow HH$

$$\hat{\mathcal{M}}_{LL} = \frac{G_F \hat{s}}{\sqrt{2}} \left\{ \frac{\lambda_{HHH}}{(\hat{s} - M_H^2)/M_Z^2} (1 + \beta_W^2) + \frac{1}{\beta_W \beta_H} \left[\frac{1 - \beta_W^4 + (\beta_W - \beta_H x)^2}{x - x_W} + (x \to -x) \right] + (1 + \beta_W^2) \right\}$$
$$x = \cos \theta_W, \ x_W = (1 - 2M_H^2/\hat{s})/(\beta_W \beta_H), \ \beta_x = (1 - 4M_x^2/\hat{s})^{1/2}$$

Process $e^+e^- ightarrow W^+W^- ightarrow ar{ u}_e u_e HH$

Rough estimate: Equivalent particle approximation: W bosons partons in e^{\pm} , on-shell Kane et al.

$$\sigma = \int_{4M_H^2/s_{ee}}^1 d\tau \left(\frac{d\mathcal{L}}{d\tau}\right)_{WW/ee} \hat{\sigma}_{WW \to HH} (\hat{s} = \tau s_{ee})$$

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WW Fusion in e^+e^- Collisions



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Process $e^+e^-
ightarrow W^+W^-
ightarrow ar{
u}_e
u_e HH$ High-enery limit:

$$\sigma_{\infty}(e^+e^- \to \bar{\nu}_e \nu_e HH) \approx \left(\frac{\alpha}{4\pi \sin^2 \theta_W}\right)^2 \left[\frac{1}{2}\ln^2\left(\frac{4M_H^2}{s_{ee}}\right) + 2\ln\left(\frac{4M_H^2}{s_{ee}}\right) + 3\right] \frac{M_W^2 G_F^2}{2\pi}$$

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SM WW Double Higgs Fusion



 \circ polarized e^{\pm} beams

Why Polarisation?

Polarisation of e^+, e^- beams:

 W^- bosons only couple to left-handed $e^- \rightsquigarrow$





SM WW Double Higgs Fusion

CompHEP: Boos et al. 10 F SM: $e^+ e^- \rightarrow \nu \nu HH$ σ^{pol} [fb] 1.6 TeV 1 1 TeV 0.1 500 GeV 0.01 100 120 140 160 180 $M_{H}[GeV]$

o polarized e[±] beams → $\sigma^{pol} = 4\sigma^{unpol}$ o cross sections increase with higher energy

Double Higgs-strahlung in e^+e^- Collisions

Contributing diagrams:



Double differential cross section

$$\frac{d\sigma(e^+e^- \to ZHH)}{dx_1 dx_2} = \frac{G_F^3 M_Z^6(v_e^2 + a_e^2)}{384\sqrt{2}\pi^3} \frac{1}{s(1-\mu_Z)^2} \left\{ \frac{M_Z^2[(y_1+y_2)^2 + 8\mu_Z]}{4s[-1+x_1+x_2+\mu_Z-\mu_H]^2} \lambda_{HHH}^2 + \dots \right\}$$

scaled energies: $x_i = E_i / \sqrt{s}$ reduced masses: $\mu_i = M_i^2 / s$

SM Double Higgs-strahlung at $\sqrt{s} = 500$ GeV, 800 GeV



Djouadi, Kilian, MM, Zerwas

• σ small; polarisation of $e^{\pm} \rightsquigarrow \sigma^{\text{pol}} = 2\sigma^{\text{unpol}}$ • σ shows scaling behaviour • $\sqrt{s} = 500$ GeV good choice for $M_H = 120$ GeV: σ large, sensitivity to λ_{HHH} large

Expected Accuracies



Higher Higgs masses

$$\begin{split} HH\nu\bar{\nu}~@~\sqrt{s} &= 3~\text{TeV},~\int\mathcal{L} = 5~\text{ab}^{-1} \qquad M_H = 120...180~\text{GeV} \qquad \boxed{\delta\lambda/\lambda = 8\%} \quad \text{Battaglia, Boos, Yack and Battaglia, Boos, Yack and Solve and$$

The Quartic Coupling λ_{HHHH}

Processes involving λ_{HHHH}

triple Higgs-strahlung $e^+e^- \rightarrow ZHHH$

WW triple Higgs fusion $e^+e^- \rightarrow \nu_e \bar{\nu}_e HHH$

Size of the cross section $\sigma_{HHH} \approx 10^{-3} \sigma_{HH}$

 λ_{HHHH} is suppressed with respect to λ_{HHH}

One more particle in the final state

CompHep/Boos et al.

		$\sigma(e^+e^- \rightarrow ZHHH)$ [ab]
$\sqrt{s}=1~{ m TeV}$	$M_H = 110 {\rm GeV}$	0.44
$\sqrt{s} = 1.6 \text{ TeV}$	$M_H = 110 \mathrm{GeV}$	0.30

The quartic coupling is not measurable for the time being.

[idem LHC/ $gg \rightarrow HHH$:Plehn,Rauch]

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The MSSM Higgs Sector

MSSM Higgs sector – SUSY & anomaly free theory \Rightarrow 2 complex Higgs doublets \rightarrow 5 Higgs states

M_h	\lesssim	$140~{ m GeV}$
$M_{A,H,H^{\pm}}$	\sim	$\mathcal{O}(v)1~{ m TeV}$

Ellis et al;Okada et al;Haber,Hempfling; Hoang et al;Carena et al;Heinemeyer et al; Zhang et al;Brignole et al;...

6 CP-invariant neutral trilinear Higgs couplings

$$\begin{split} \lambda_{hhh} &= 3\cos 2\alpha \sin(\beta + \alpha) &+ 3\frac{\epsilon}{M_Z^2} \frac{\cos \alpha}{\sin \beta} \cos^2 \alpha \\ \lambda_{Hhh} &= 2\sin 2\alpha \sin(\beta + \alpha) - \cos 2\alpha \cos(\beta + \alpha) &+ 3\frac{\epsilon}{M_Z^2} \frac{\sin \alpha}{\sin \beta} \cos^2 \alpha \\ \lambda_{HHh} &= -2\sin 2\alpha \cos(\beta + \alpha) - \cos 2\alpha \sin(\beta + \alpha) &+ 3\frac{\epsilon}{M_Z^2} \frac{\cos \alpha}{\sin \beta} \sin^2 \alpha \\ \lambda_{HHH} &= 3\cos 2\alpha \cos(\beta + \alpha) &+ 3\frac{\epsilon}{M_Z^2} \frac{\sin \alpha}{\sin \beta} \sin^2 \alpha \\ \lambda_{hAA} &= \cos 2\beta \sin(\beta + \alpha) &+ \frac{\epsilon}{M_Z^2} \frac{\cos \alpha}{\sin \beta} \cos^2 \beta \\ \lambda_{HAA} &= -\cos 2\beta \cos(\beta + \alpha) &+ \frac{\epsilon}{M_Z^2} \frac{\sin \alpha}{\sin \beta} \cos^2 \beta \end{split}$$

Radiative Corrections one-loop leading m_t^4 approximation parametrized by

$$\epsilon = \frac{3G_f m_t^4}{\sqrt{2}\pi^2 \sin^2 \beta} \ln \left(1 + \frac{M_S^2}{m_t^2}\right) \qquad \tan \beta = \frac{v_2}{v_1}$$

The Trilinear Couplings $\lambda_{hhh}, \lambda_{Hhh}$

Subsequent analysis:

Carena, Espinosa, Quiros, Wagner Djouadi, Kalinowski, Spira

- * Dominant one-loop and two-loop corrections to MSSM Higgs masses and couplings included.
- * Corrections involve mixing paramters A and μ .



- $\circ \quad {\rm Mixing:} \ A=\mu=1 \ {\rm TeV}.$
- $\circ \quad \lambda_{hhh}, \lambda_{Hhh}$ can become zero.
- Couplings not strongly affected by mixing effects when evaluated for the physical Higgs masses.

The MSSM Higgs Self-Couplings



h

h

sensitive to $\lambda_{hhh}, \lambda_{Hhh}$

 e^{-}

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h, H

h

The Sensitivity Areas

Processes sensitive	λ	double Higgs-strahlung Zhh ZHh ZHH ZAA			triple Higgs-production Ahh AHh AHH AAA					
10 $\lambda_{3\Phi}$	hhh	×				×				System columbia
	Hhh	×	×			×	×			for all λ 's
	HHh		×	×			×	×		up to discrete
	HHH			×			J	×		ambiguities
	hAA				×	×	×		×	
	HAA				×		×	×	×	

Sensitivity Areas

Djouadi, Kilian, MM, Zerwas

$$\begin{aligned} (i) & \sigma[\lambda] > 0.01 \text{ fb [and relaxed to } 0.1 \text{ fb}] \\ (ii) & \text{eff}\{\lambda \to 0\} > 2 \text{ st.dev. for } \int \mathcal{L} = 2 \text{ ab}^{-1} \end{aligned}$$



Regions of no sensitivity: $\lambda \sin(\beta - \alpha), \lambda \cos(\beta - \alpha)$ small



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Regions of no sensitivity: $\lambda \sin(\beta - \alpha), \lambda \cos(\beta - \alpha)$ small

large M_A : sensitivity criteria not fulfilled due to

- phase space effects
- \circ suppression of the H, A propagators for large masses



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The Trilinear Higgs Self-Coupling at the LHC

Determination of λ_{HHH} at the LHC

Djouadi, Kilian, MM, Zerwas

In processes analogous to those at e^+e^- colliders and in gluon gluon fusion

double Higgs-strahlung:	q ar q	\rightarrow	W/Z + HH	Barger, Han, Phillips
WW/ZZ fusion:	qq	\rightarrow	qq + HH	Dicus, Kallianpur, Willenbrock Abbasabadi, Repko, Dicus, Vega Dobrovolskaya, Novikov Eboli, Marques, Novaes, Natale
gluon gluon fusion:	gg	\rightarrow	HH	Glover,van der Bij Plehn,Spira,Zerwas Dawson,Dittmaier,Spira

Gluon Gluon Fusion - dominant process



Comments:

- 2-loop QCD corrections known for $m_t \to \infty$: $K \sim 1.9$
- Single Higgs production $\rightsquigarrow K \rightarrow K \pm 10\%$ for large M_H

Dawson, Dittmaier, Spira Spira, Djouadi, Graudenz, Zerwas

SM Double Higgs Production at the LHC

Djouadi, Kilian, MM, Zerwas



Expected Accuracies on λ_{HHH} at the LHC

Smallness of the signal + large QCD backgrounds ~-> challenging!

$$\begin{split} \underline{M_H} &< 140 \text{ GeV}: \underline{gg} \rightarrow \underline{HH} \rightarrow b\bar{b}\gamma\gamma: \\ &\circ \text{SLHC } [\int \mathcal{L} = 6 \text{ ab}^{-1}]: \\ M_H &= 120 \text{ GeV} \qquad \text{rule out } \lambda_{HHH} = 0 \qquad \text{at } 90\% \text{ CL} \\ &\circ \text{VLHC } [\sqrt{s} = 200 \text{ TeV}]: \\ M_H &= 120 \text{ GeV}: \qquad \delta\lambda_{HHH}/\lambda_{HHH} = 20 - 40\% \qquad \text{at } 1 \sigma \\ \\ \underline{M_H} &= 120 \text{ GeV}: \qquad \delta\lambda_{HHH}/\lambda_{HHH} = 20 - 40\% \qquad \text{at } 1 \sigma \\ \\ \underline{M_H} &> 140 \text{ GeV}: \underline{gg} \rightarrow \underline{HH} \rightarrow \underline{W^+W^-W^+W^-:} \\ &\circ \text{LHC } [\int \mathcal{L} = 300 \text{ fb}^{-1}]: \\ 150 \leq M_H \leq 200 \text{ GeV}: \quad \text{rule out } \lambda_{HHH} = 0 \qquad \text{at } 95\% \text{ CL} \\ &\circ \text{ SLHC } [\int \mathcal{L} = 3 \text{ ab}^{-1}]: \\ 150 < M_H < 200 \text{ GeV} \qquad \delta\lambda_{HHH}/\lambda_{HHH} = 10 - 30\% \qquad \text{at } 1 \sigma \end{split}$$

MSSM Higgs Self-Couplings at the LHC

Determination of $\lambda_{3\Phi}$

- * gluon gluon fusion
- * double Higgs-strahlung
- * VV double Higgs fusion
- * triple Higgs production

Example $gg \rightarrow hh$



Comments:

- Involves the Higgs self-couplings λ_{hhh} , λ_{Hhh} .
- 2-loop QCD corrections (large m_t limit) included.

Dawson, Dittmaier, Spira

Dawson,Dittmaier,Spira; Plehn,Spira,Zerwas Belyaev,Drees,Eboli,Mizkoshi,Novaes; Djouadi,Kilian,MM,Zerwas

MSSM hh Production in gg Fusion at the LHC



◦ Enhancement: resonant H production & H → hh.
 ◦ ↑ variation: 0.5λ_{Hhh}...1.5λ_{Hhh}.
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Standard Model λ_{HHH} at the ILC

- * Double Higgs-strahlung & WW double Higgs fusion are sensitive to λ_{HHH} .
- * For $M_H = 120 \text{ GeV } \sqrt{s} = 500 \text{ GeV}$ is a good choice: sensitivity to λ_{HHH} is large.
- * Accuracy at ILC: $\delta \lambda_{HHH} / \lambda_{HHH} \lesssim 20$ %.

 $\lambda_{HHH}^{SM} = 3 \frac{M_H^2}{v} \rightsquigarrow \text{Measurement of } \lambda_{HHH} \text{ allows for a first consistency check.}$

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MSSM trilinear Higgs self-couplings at the ILC

* Zhh, ZHh, Ahh sensitive to $\lambda_{hhh}, \lambda_{Hhh}$ in parts of the parameter space $\tan \beta - M_A$.

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Higgs self-couplings at the LHC

- \ast Larger cross sections face large QCD background.
- * May rule out $\lambda_{HHH}^{SM} = 0.$ * λ_{Hhh}^{MSSM} might be accessible in resonance region.

