Sleptons at ILC

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1. Overview

2. Sleptons as a precision laboratory

3. Sleptons as a challenge for theorists

Overview

SUSY analysis program:

1. Establish supersymmetry breaking pattern

   $\hookrightarrow \begin{cases} \text{Determine slepton masses and mixings} \\ \text{Extrapolate to high scales} \end{cases}$ $\longrightarrow$ talk of W. Porod

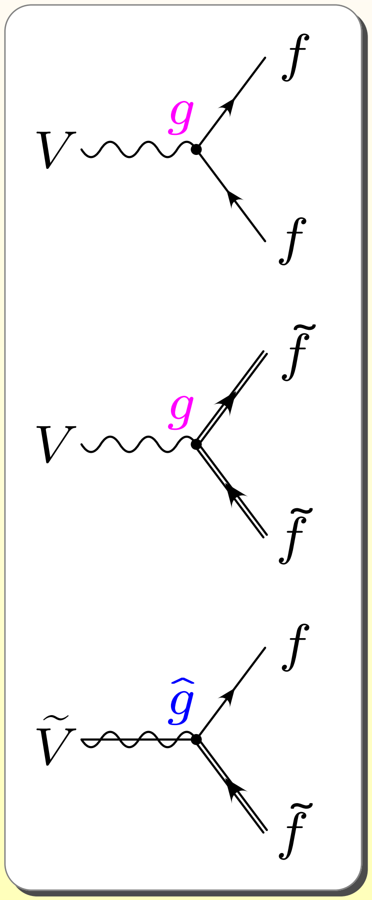
2. Establish fundamental supersymmetry relation

   Gauge coupling g = Yukawa coupling $\hat{g}$

   $\longrightarrow$ required to resolve hierarchy problem

   $\longrightarrow$ compare precise cross-section measurements with theoretical predictions

3. Identify particle spins



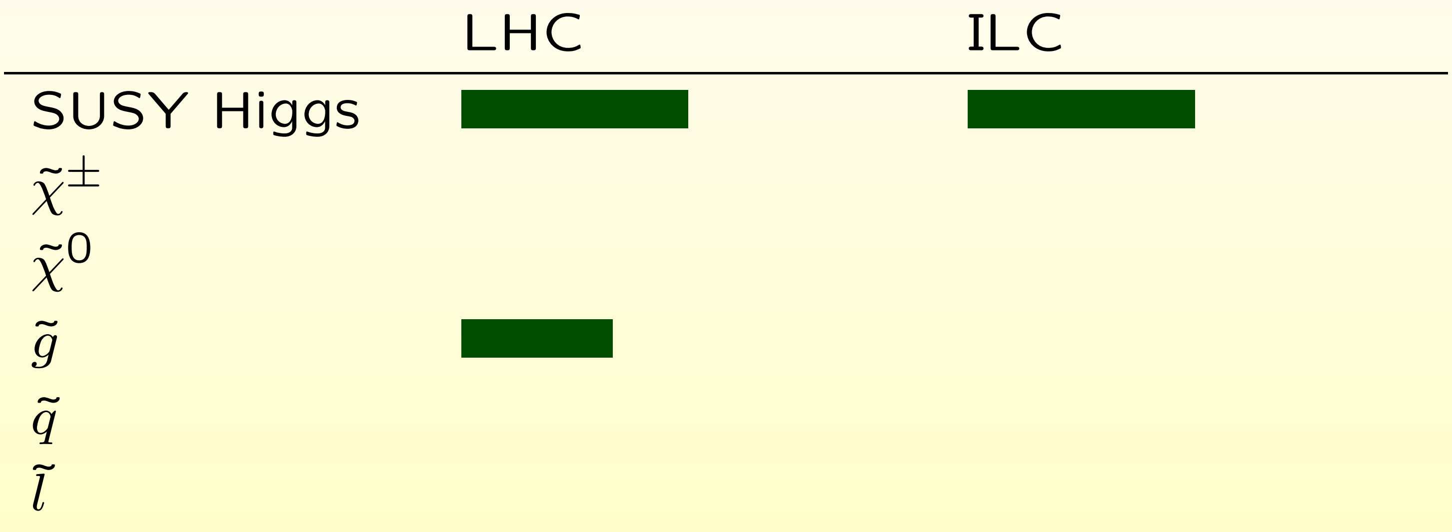
SUSY Phenomenology (Zerwasonomy)

An inaccurate timeline: **1993**

	LHC	ILC
SUSY Higgs	■	■
$\tilde{\chi}^{\pm}$		
$\tilde{\chi}^{0}$		
$\tilde{g}$		
$\tilde{q}$		
$\tilde{l}$		

SUSY Phenomenology (Zerwasonomy)

An inaccurate timeline: **1995**



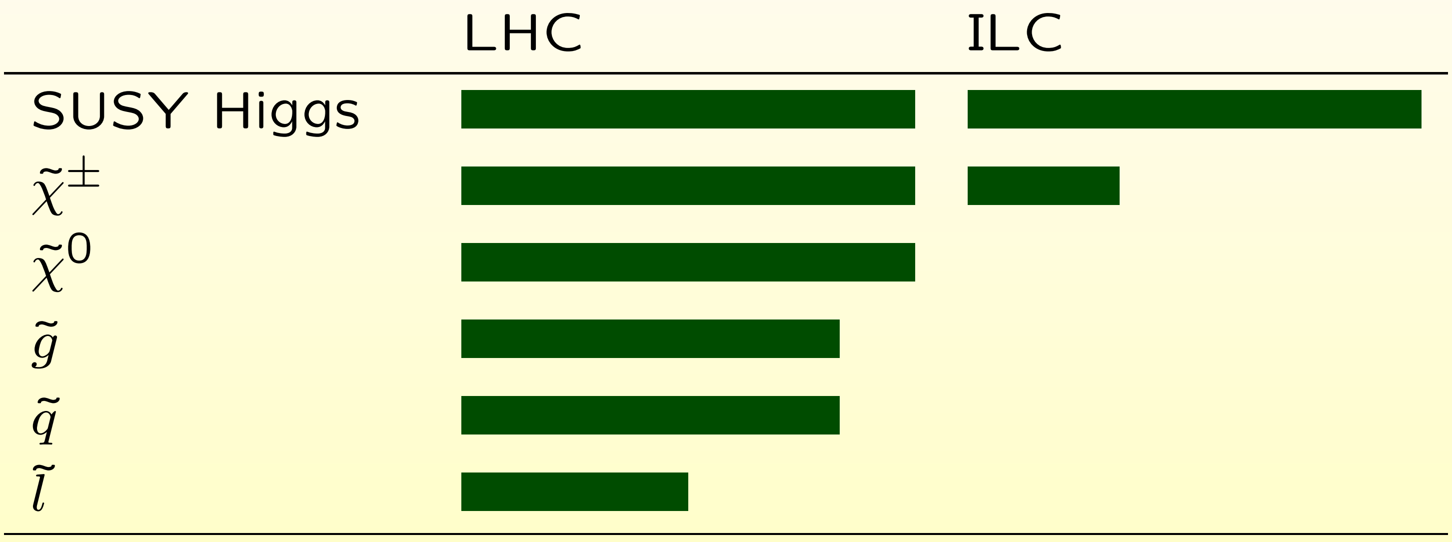
SUSY Phenomenology (Zerwasonomy)

An inaccurate timeline: **1997**

	LHC	ILC
SUSY Higgs		
$\tilde{\chi}^{\pm}$		
$\tilde{\chi}^{0}$		
$\tilde{g}$		
$\tilde{q}$		
$\tilde{l}$		

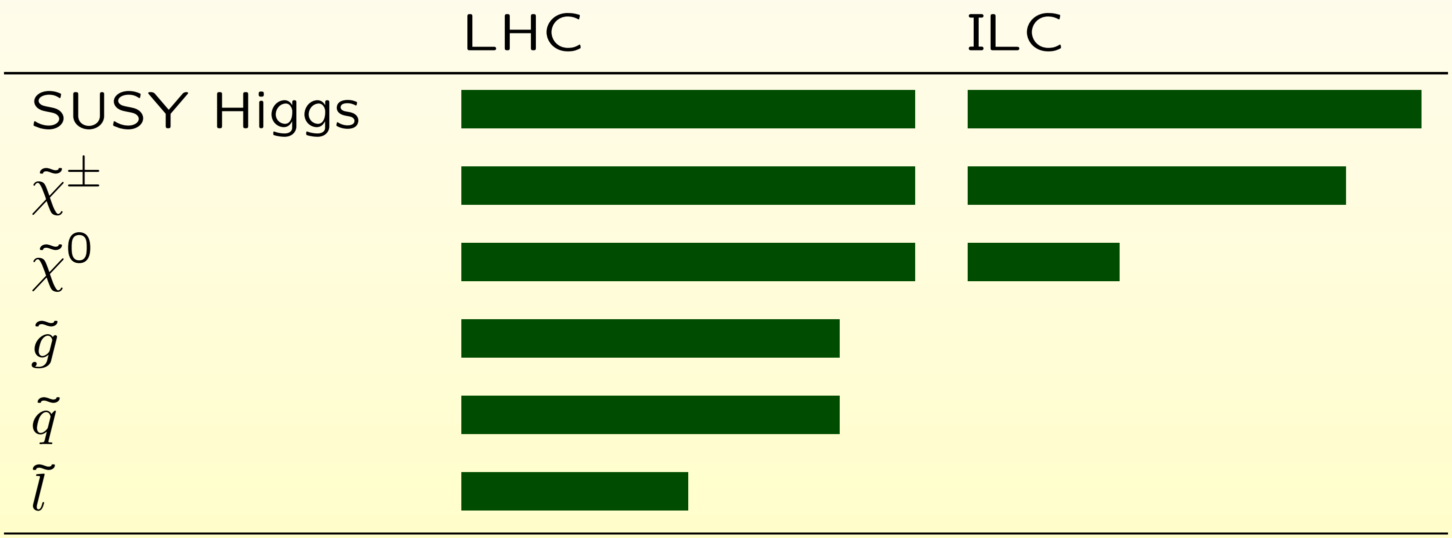
SUSY Phenomenology (Zerwasonomy)

An inaccurate timeline: **1999**



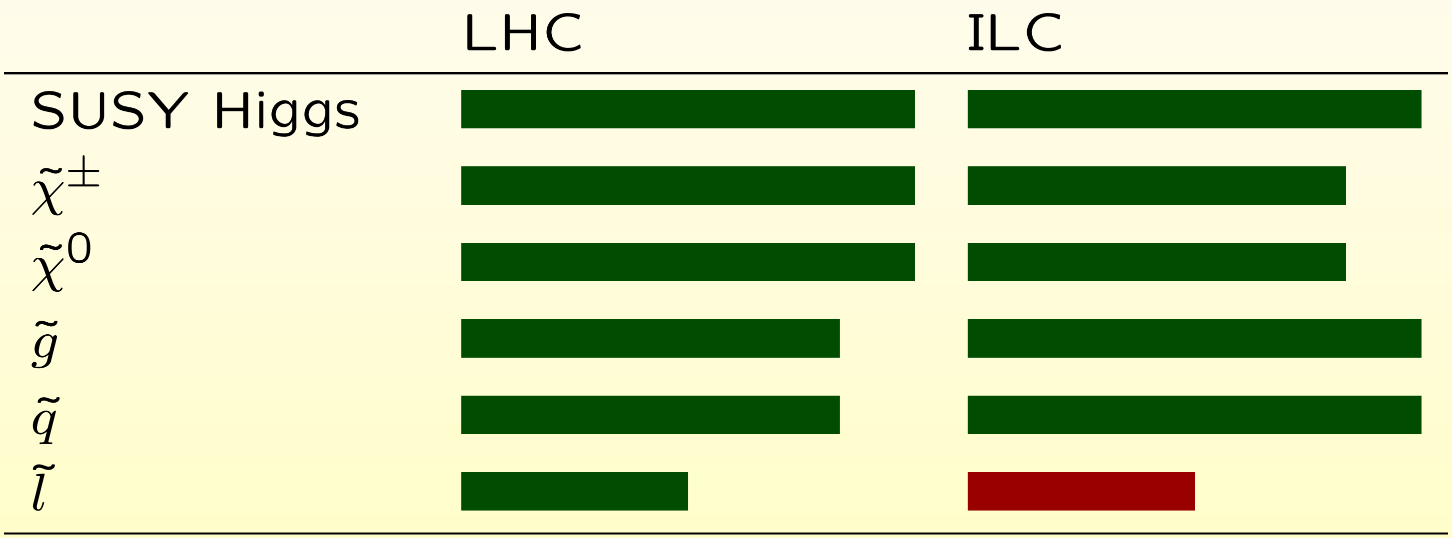
SUSY Phenomenology (Zerwasonomy)

An inaccurate timeline: **2001**



SUSY Phenomenology (Zerwasonomy)

An inaccurate timeline: **2003**



SUSY Phenomenology (Zerwasonomy)

An inaccurate timeline: **2007**

	LHC	ILC
SUSY Higgs	green	green
$\tilde{\chi}^{\pm}$	green	green
$\tilde{\chi}^{0}$	green	green
$\tilde{g}$	green	green
$\tilde{q}$	green	green
$\tilde{l}$	green	red

Sleptons as a precision laboratory

Are sleptons just one more particle to study?

Sleptons can play a special role
in exploring SUSY experimentally

- Simple
- Clean
- Relevant

Simple: Masses and decay characteristics

SPS1a scenario

Sleptons among lightest sparticles
In many SUSY breaking scenarios

$\Rightarrow$ very clean signature:

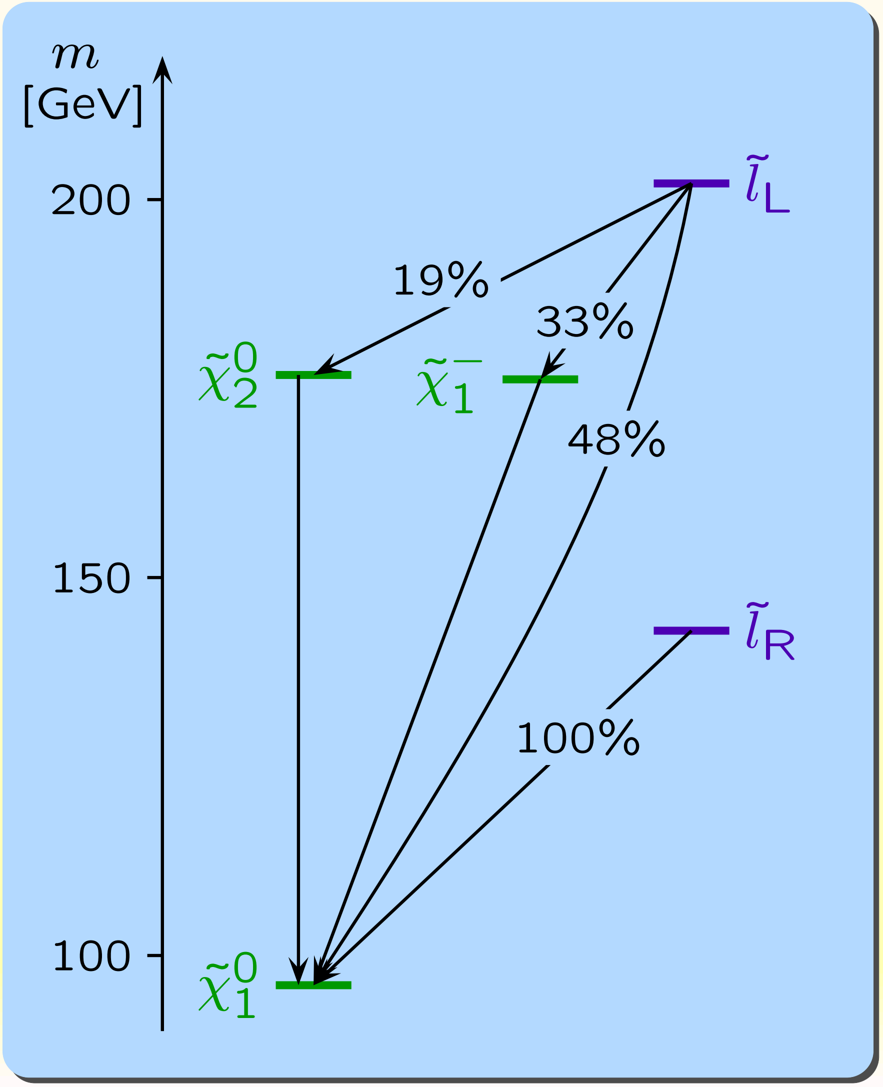
few leptons $+ \not{E}$

1st/2nd generation sleptons expected (almost) not to mix

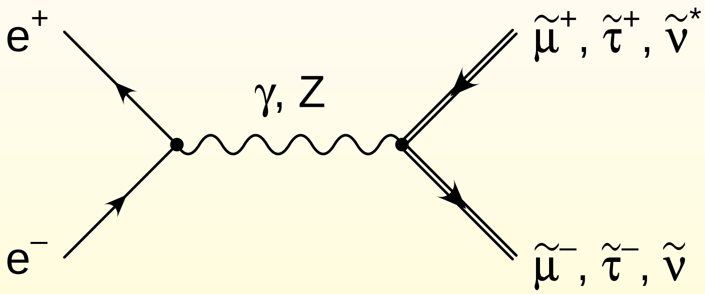
Different decay modes for $\tilde{l}_R, \tilde{l}_L$

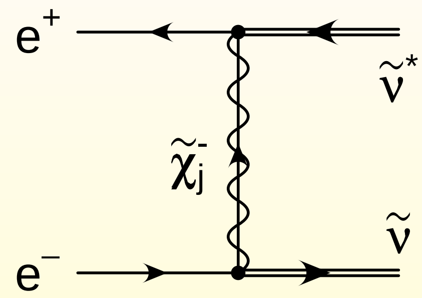
$$\tilde{l}_R \to l^- \, \tilde{\chi}_1^0$$

$$\tilde{l}_L \to l^- \, \tilde{\chi}_2^0$$
$$\hookrightarrow \tau^+ \tau^- \, \tilde{\chi}_1^0$$
$$\to \nu_l \, \tilde{\chi}_1^-$$
$$\hookrightarrow \tau^- \nu_\tau \, \tilde{\chi}_1^0$$



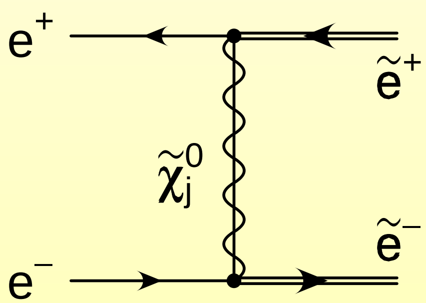
Slepton production

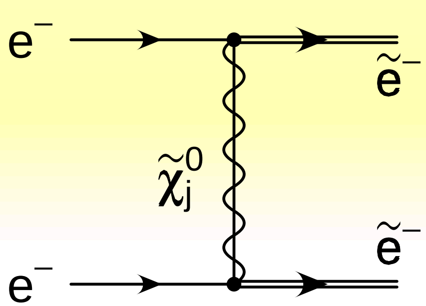




In general P-wave excitation $\propto \beta^3$

Additional t-channel neutralino exchange for selectrons:





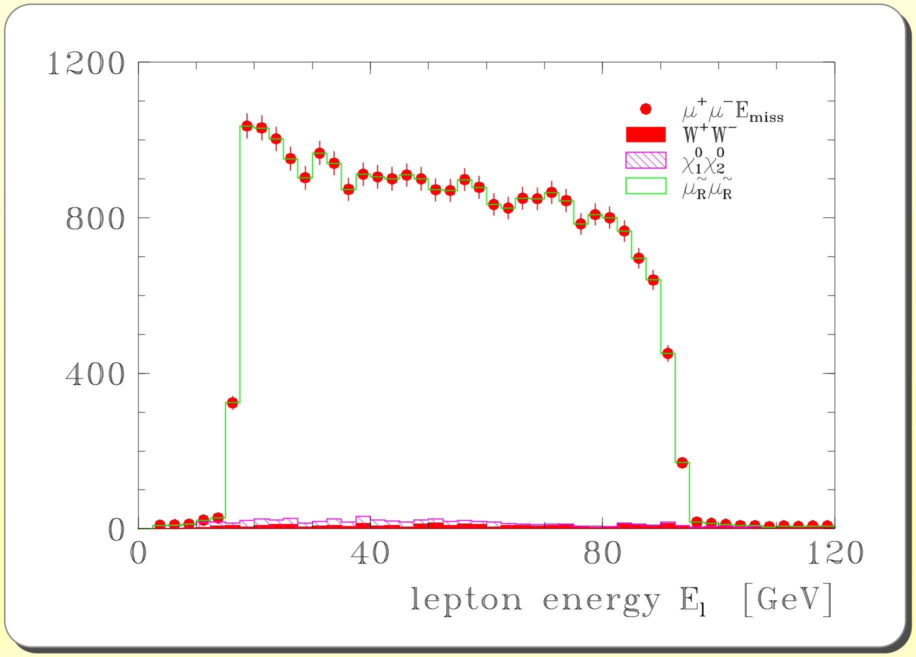
$e^+e^- \to \tilde{e}^+_R \tilde{e}^-_R$	$[\gamma, Z, \tilde{\chi}^0]$	$\propto \beta^3$	(P-wave)
$e^+e^- \to \tilde{e}^+_R \tilde{e}^-_L$	$[\tilde{\chi}^0]$	$\propto \beta$	(S-wave)
$e^+e^- \to \tilde{e}^+_L \tilde{e}^-_L$	$[\gamma, Z, \tilde{\chi}^0]$	$\propto \beta^3$	(P-wave)
$e^-e^- \to \tilde{e}^-_R \tilde{e}^-_R$	$[\tilde{\chi}^0]$	$\propto \beta$	(S-wave)
$e^-e^- \to \tilde{e}^-_R \tilde{e}^-_L$	$[\tilde{\chi}^0]$	$\propto \beta^3$	(P-wave)
$e^-e^- \to \tilde{e}^-_L \tilde{e}^-_L$	$[\tilde{\chi}^0]$	$\propto \beta$	(S-wave)

Clean: mass measurements

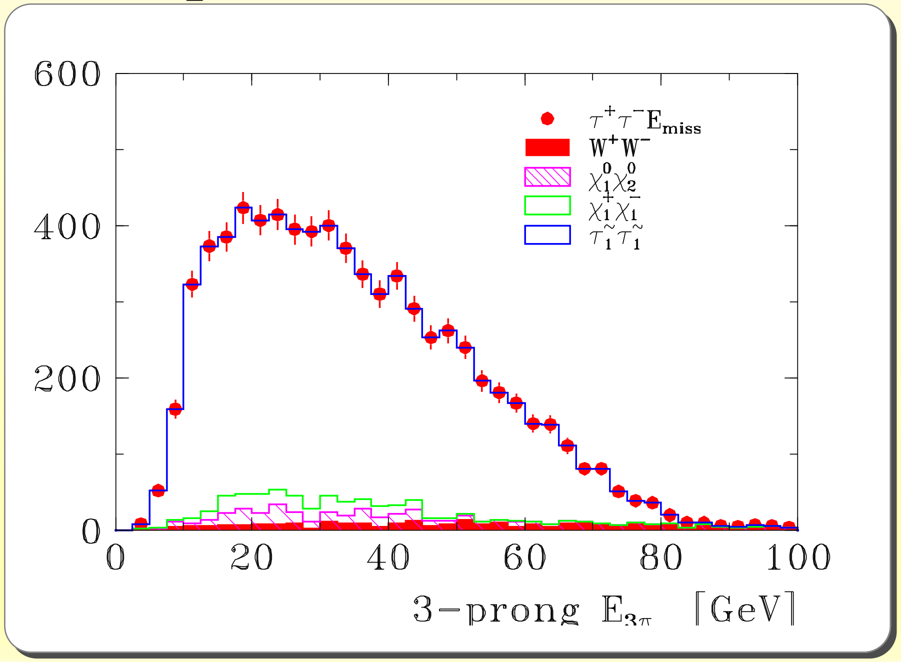
- From edges in decay energy distributions

Examples: Martyn '03

$\tilde{\mu}_R \to \mu \tilde{\chi}_1^0$



$\tilde{\tau}_1 \to \tilde{\chi}_1^0 \tau \to 3\pi + \not{E}$



Threshold scans

Freitas, v.Manteuffel, Martyn, Zerwas '00–04

in general P-waves $\propto \beta^3$

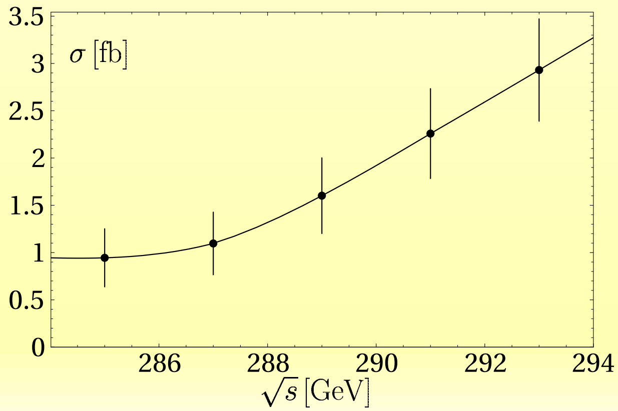
exceptions: (S-waves)
$$\left.\begin{array}{l} e^+_L e^-_L \to \tilde{e}^+_R \tilde{e}^-_L \\ e^-_R e^-_R \to \tilde{e}^-_R \tilde{e}^-_R, \ldots \end{array}\right\} \propto \beta$$

typically

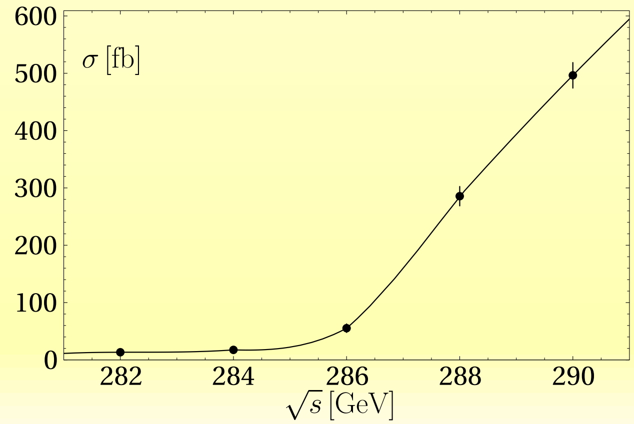
$5 \times 10\ \text{fb}^{-1}$ in e^+e^-

$5 \times \ \ 1\ \text{fb}^{-1}$ in e^-e^-

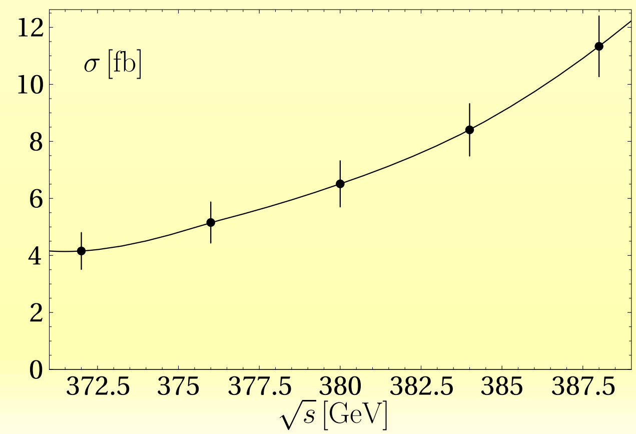
$$e^+e^- \to \tilde{\mu}^+_R \tilde{\mu}^-_R \to \mu^+ \mu^- + \not{E}$$
$\propto \beta^3$



$$e^-e^- \to \tilde{e}^-_R \tilde{e}^-_R \to e^- e^- + \not{E}$$
$\propto \beta$



$$e^+e^- \to \tilde{\nu}^*_e \tilde{\nu}_e \to e^+ \tilde{\chi}^0_1\ \nu_e \tilde{\chi}^-_1 \to e^+ \tau^- + \not{E}$$
$\propto \beta^3$



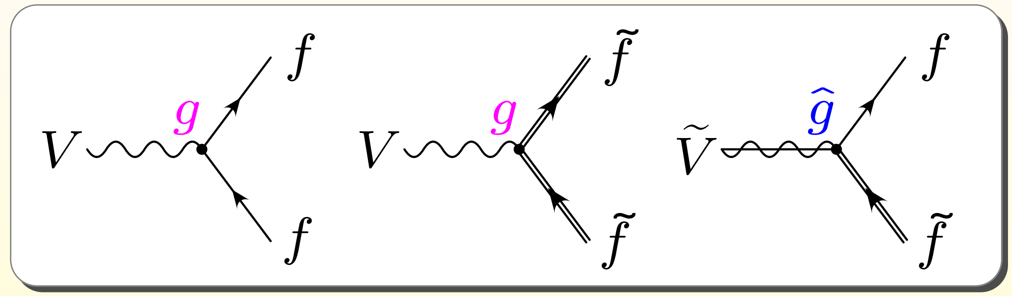
Results for SPS1a:

	m [GeV]	Δm [GeV] spectra	thr. scans	combine	Γ [GeV]
$\tilde{\chi}_1^0$	96.1	0.10	–	0.065$^{(a)}$	–
$\tilde{e}_R$	143.0	0.08	0.05	0.05	0.21 ± 0.05
$\tilde{e}_L$	202.1	0.8	0.2	0.2	0.25 ± 0.02
$\tilde{\nu}_e$	186.0	1.2	1.1	1.1	$0.16^{+0.7}_{-0.5}$
$\tilde{\mu}_R$	143.0	0.2	0.2	0.085$^{(b)}$	0.2 ± 0.2
$\tilde{\mu}_L$	202.1	–	0.5$^{(c)}$		?
$\tilde{\tau}_1$	133.2	0.3	?		?
$\tilde{\tau}_2$	133.2	?	1.1$^{(d)}$		?

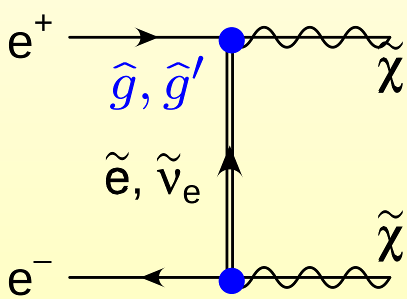
$^{(a)}$ from $\tilde{e}_\mathrm{R}$ spectrum using selectron mass determined at threshold
$^{(b)}$ from $\tilde{\mu}_\mathrm{R}$ spectrum using $\tilde{\chi}_1^0$ mass as input
$^{(c,d)}$ estimate for threshold scan [P. Grannis]

Relevant: Test of SUSY coupling relations

Electroweak gauge & Yukawa couplings can be probed in



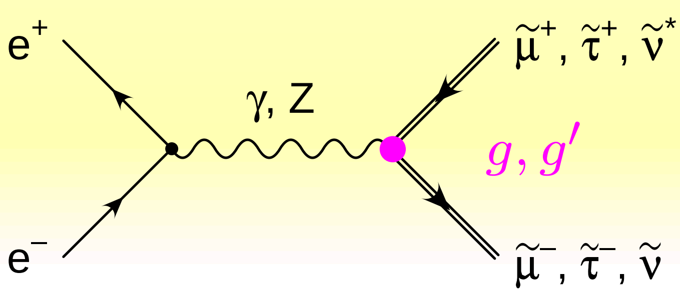
- Neutralino production

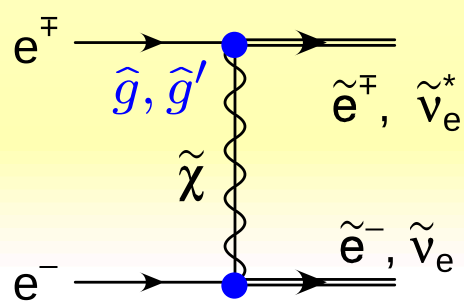


Choi, Kalinowski, Moortgat-Pick, Zerwas '01

- Slepton production

Freitas, v.Manteuffel, Zerwas '03





g' U(1) coupl.

g SU(2) coupl.

Determination of Yukawa couplings

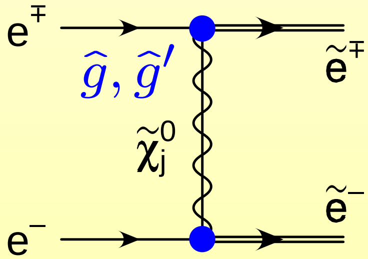
From selectron cross-sections

$$e^+e^- \to \tilde{e}^+_R \tilde{e}^-_R \to e^+e^- \tilde{\chi}^0_1 \tilde{\chi}^0_1$$

$$e^+e^- \to \tilde{e}^+_R \tilde{e}^-_L \to e^+e^- \tilde{\chi}^0_1 \tilde{\chi}^0_2$$

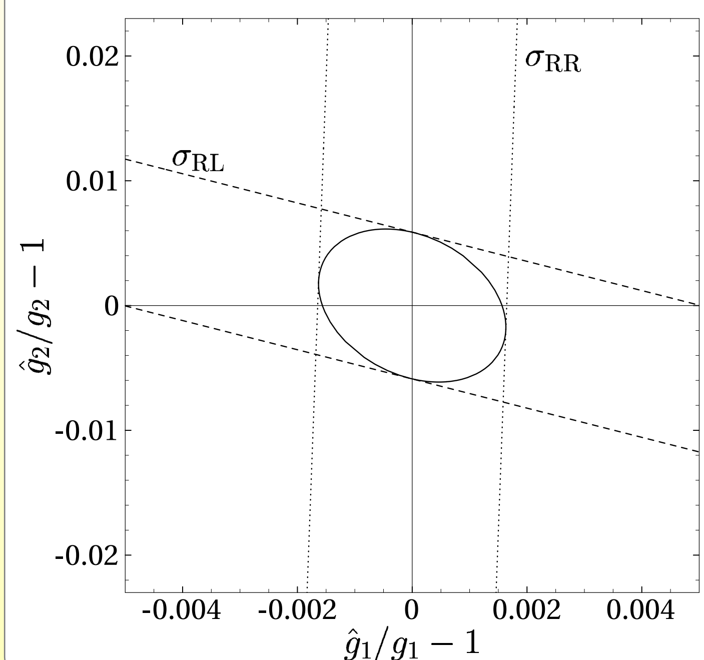
$$\hookrightarrow \tau^+\tau^- \tilde{\chi}^0_1$$

Use polarized beams to disentangle U(1) and SU(2) couplings



$$\frac{\delta \hat{g}'}{\hat{g}'} \approx 0.2\% \qquad \frac{\delta \hat{g}}{\hat{g}} \approx 0.7\%$$

$\sqrt{s} = 500$ GeV, $\int L = 500$ fb^{-1}



Relevant: Determination of spin

- Threshold behaviour:

  Fermion pairs: $\propto \beta$

  Scalar pairs: $\propto \beta^3$

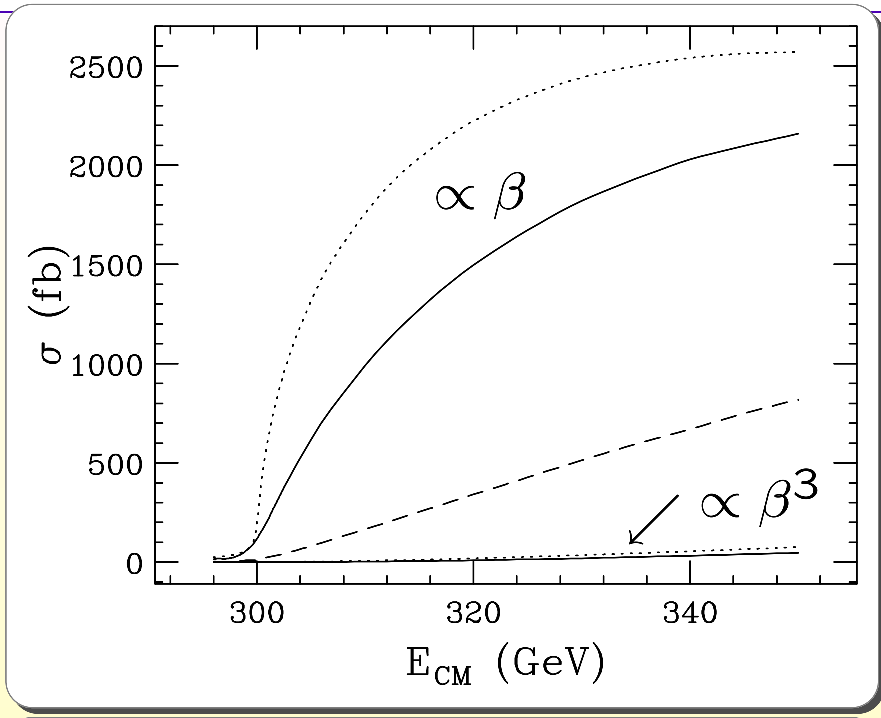
- Angular distribution

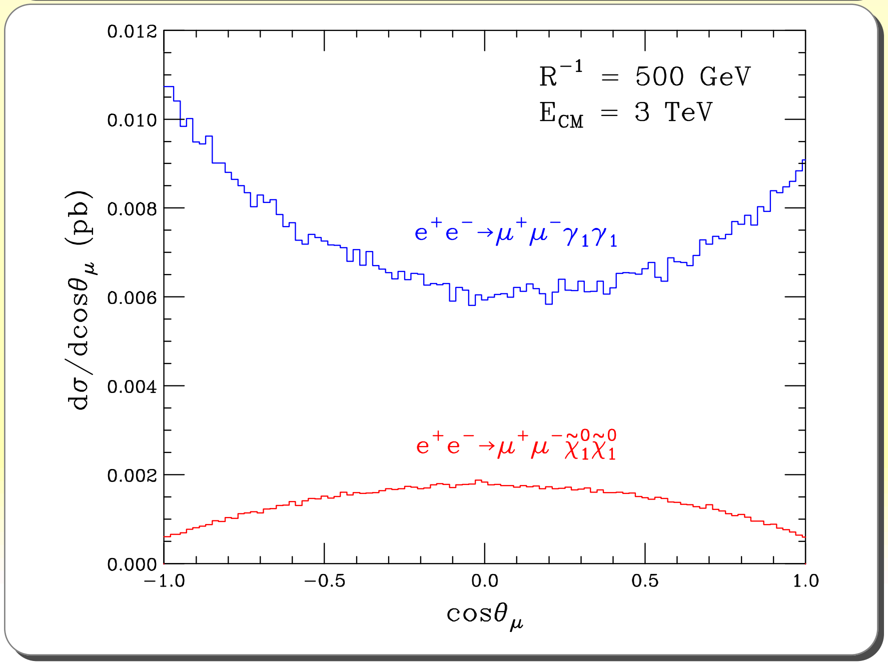
  Fermion pairs:

  $$\frac{d\sigma}{d\cos\theta} \sim 1 + \cos^2\theta$$

  Scalar pairs (Sfermions):

  $$\frac{d\sigma}{d\cos\theta} \sim 1 - \cos^2\theta$$





3rd generation: Determination of mixings

Determination of $\tilde{\tau}$ masses as before

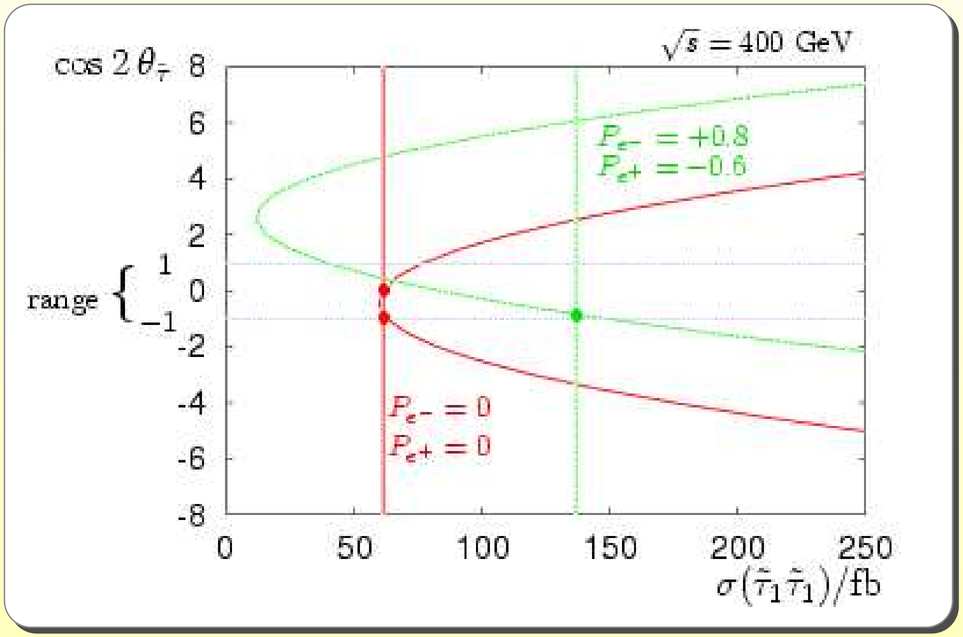
$m_{\tilde{\tau}_2}$ at SPS1a not yet clear

Mixing angle $\theta_{\tilde{\tau}}$ from $\sigma(\tilde{\tau}_1\tilde{\tau}_1)$ with polarized $e^{\pm}$ beams

$\rightarrow$ $\tilde{\tau}_1$, $\tilde{\tau}_2$ couple differently to Z

$\Rightarrow$ $\cos 2\theta_{\tilde{\tau}} = -0.84 \pm 0.04$

Martyn '03



Ultimate goal:

Extract A_τ using

$$A_\tau = \frac{m^2_{\tilde{\tau}_2} - m^2_{\tilde{\tau}_1}}{m_\tau} \sin 2\theta_{\tilde{\tau}} + \mu \tan\beta$$

from χ sector

intern: τ polarization

extern: χ or Higgs sector

Boos et al. '03

$\rightarrow$ difficult due to large cancellations

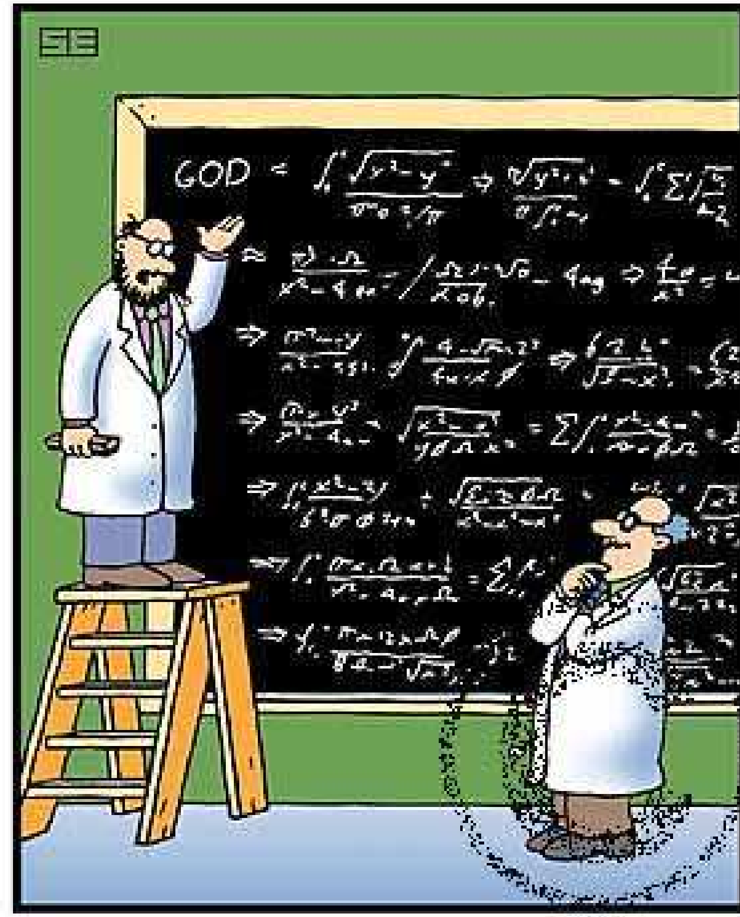
Sleptons as a challenge for theorists

Goal:
Measurements of slepton properties with high accuracy

Requires precise theoretical predictions:

- Near threshold ($\sqrt{s} \approx 2m_{\tilde{l}}$)
  $\longrightarrow$ Mass, width, spin

- In the continuum ($\sqrt{s} \gg 2m_{\tilde{l}}$)
  $\longrightarrow$ Couplings, mixings, spin



"I think you made your mistake right at the beginning!"

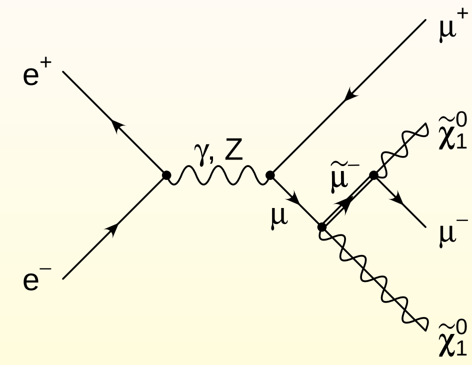
Threshold analysis

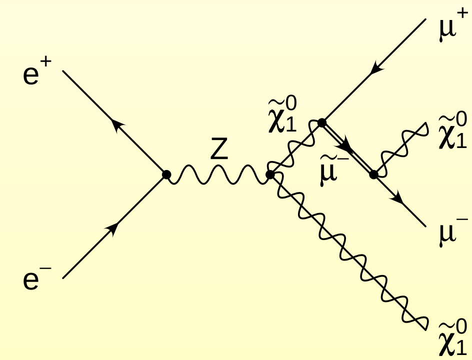
Non-zero width, gauge invariance

Gauge invariance can be violated by

- Production of off-shell Smuons

  → **Additional diagrams** with same final state



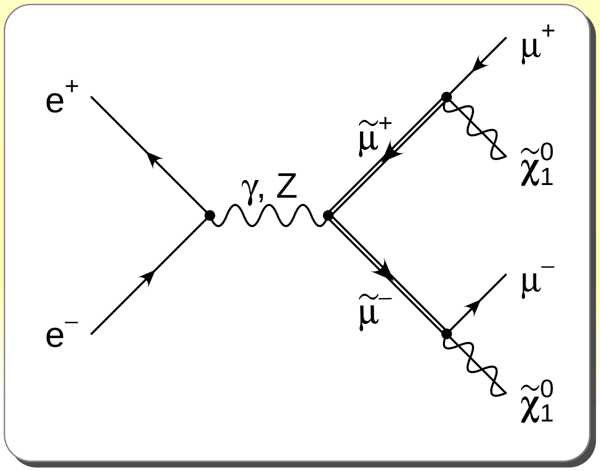


- Inclusion of finite widths (sub-class of higher order corrections)

  → **Complex mass:**

$$m_{\tilde{\mu}}^2 \to m_{\tilde{\mu}}^2 - i m_{\tilde{\mu}} \Gamma_{\tilde{\mu}}$$

  preserves all Ward identities

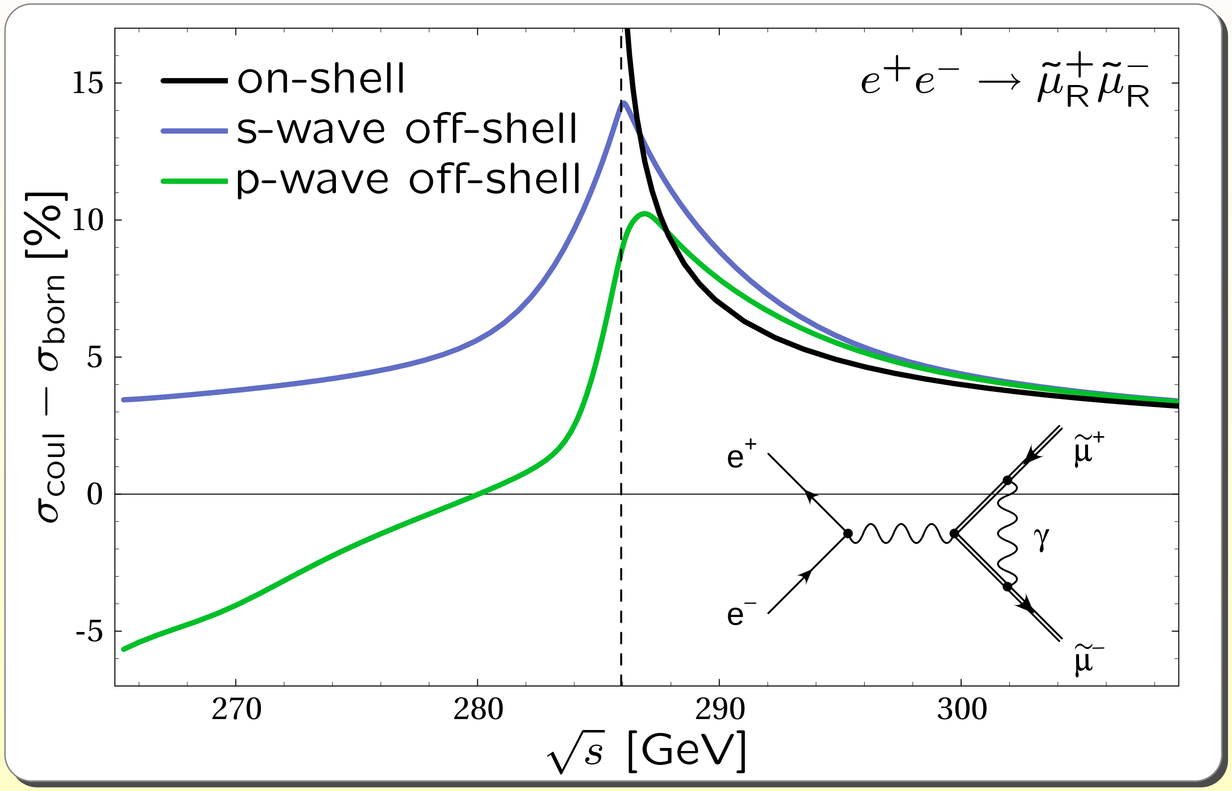


Coulomb correction

Slowly moving $\tilde{l}$'s near threshold

→ large corrections from γ exchange for

$$\beta = \sqrt{1 - \frac{4m^2}{s}} \to 0$$



Off-shellness of the $\tilde{l}$'s:

Effective screening of Coulomb singularity

$$\sigma_{coul} = \sigma_{born} \frac{\alpha\pi}{2\beta}\left[1 - \frac{2}{\pi}\arctan\frac{|\beta_M|^2 - \beta^2}{2\,\beta\,\Im m\,\beta_M}\right]\Re e\,\mathcal{C}_l \qquad \mathcal{C}_l = \left[\frac{\beta^2 + {\beta_M}^2}{2\beta^2}\right]^l$$

$$\beta_M = \frac{1}{s}\sqrt{(s - M_+^2 - M_-^2)^2 - 4M_+^2M_-^2}, \qquad M_\pm^2 = m_\pm^2 - im_\pm\Gamma_\pm$$

Slepton production in the continuum

Goal:

- Precise determination of supersymmetric couplings
- Mixing in third generation

$\rightarrow$ requires calculation of radiative corrections

$\mathcal{O}(\alpha)$ corrections completed for all relevant processes:

- Sfermion decay $\tilde{f} \to f\,\tilde{\chi}_i^0$, $\tilde{f} \to f'\,\tilde{\chi}_j^\pm$ — Guasch, Hollik, Solà '01
- Slepton production of first/second generation
  $e^+e^- \to \tilde{e}^+\tilde{e}^-, \tilde{\mu}^+\tilde{\mu}^-, \tilde{\nu}\tilde{\nu}^*$
  $e^-e^- \to \tilde{e}^-\tilde{e}^-$ — Freitas, v.Manteuffel, Zerwas '02,04
- Third generation slepton production
  $e^+e^- \to \tilde{\tau}^+\tilde{\tau}^-$ — Arhrib, Hollik '03; Kovařík, Weber, Eberl, Majerotto '04

Renormalization

SPA conventions: J.A. Aguilar-Saavedra et al. '05

- On-shell (pole mass) renormalization for masses
- SUSY Lagrange parameters in $\overline{\text{DR}}$ with $\tilde{\mu} = 1$ TeV
- Mixing angles and matrices in $\overline{\text{DR}}$ with $\tilde{\mu} = 1$ TeV

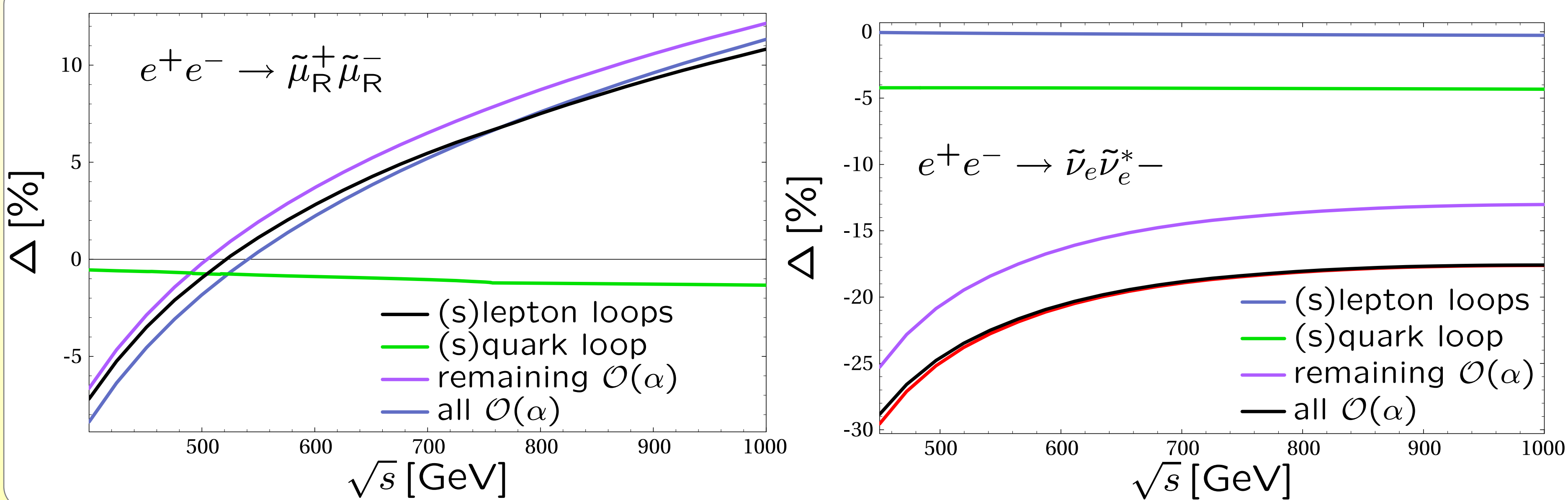
Allows also extrapolation to high-scales $\rightarrow$ talk by W. Porod

$\rightarrow$ for sleptons:

- Slepton masses fixed on-shell
- Neutralino/chargino system fixed through on-shell masses
- Slepton mixing angle and $\tan\beta$ fixed in $\overline{\text{DR}}$

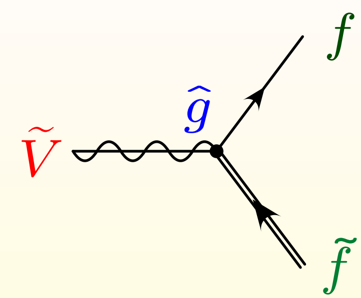
Typical examples

- Slepton production:

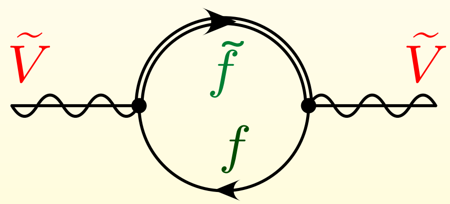


$$\Delta = \frac{\sigma_\alpha - \sigma_{\text{Born}}}{\sigma_{\text{Born}}}$$

Non-decoupling sfermion corrections

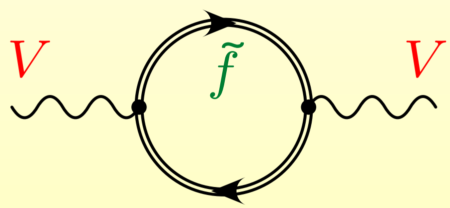


$f/\tilde{f}$ loop corrections to gaugino Yukawa vertex:



$$= C_1 \log \frac{m_{\tilde{f}}}{m_{\text{weak}}}$$

$\tilde{f}$ loop corrections to gauge renormalization:



$$= C_2 \log \frac{m_{\tilde{f}}}{m_{\text{weak}}}$$

$C_1 \neq C_2$

$\rightarrow$ non-decoupling contributions

$$\propto \log \frac{m_{\tilde{f}}}{m_{\text{weak}}}$$

Equivalence of effective gauge and Yukawa couplings

$$g_{\text{gauge}} = \widehat{g}_{\text{Yuk}}$$

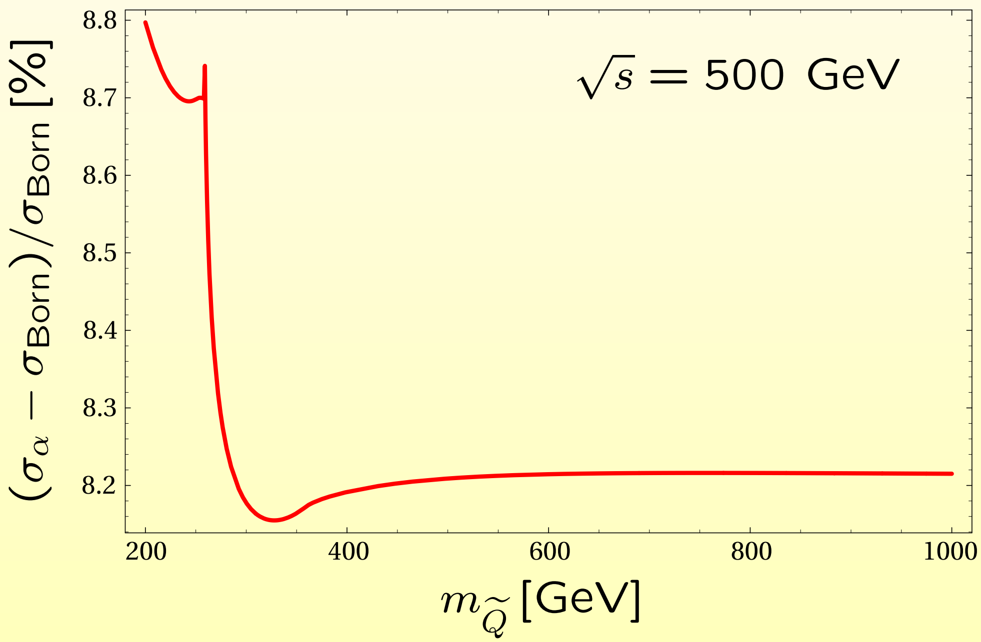
modified at higher orders:

$$g_{\text{gauge,eff}} \neq \widehat{g}_{\text{Yuk,eff}}$$

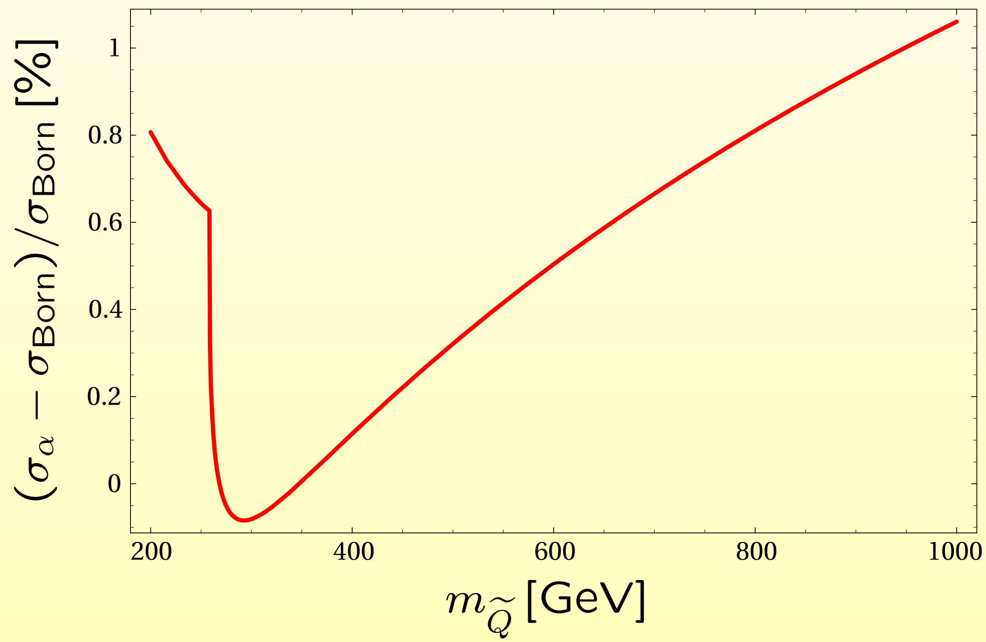
Effect of non-decoupling corrections:

Keep $m_{\tilde{L}}$ fixed, but vary $m_{\widetilde{Q}}$

$\tilde{\mu}_R\tilde{\mu}_R$ production



$\tilde{e}_R\tilde{e}_R$ production

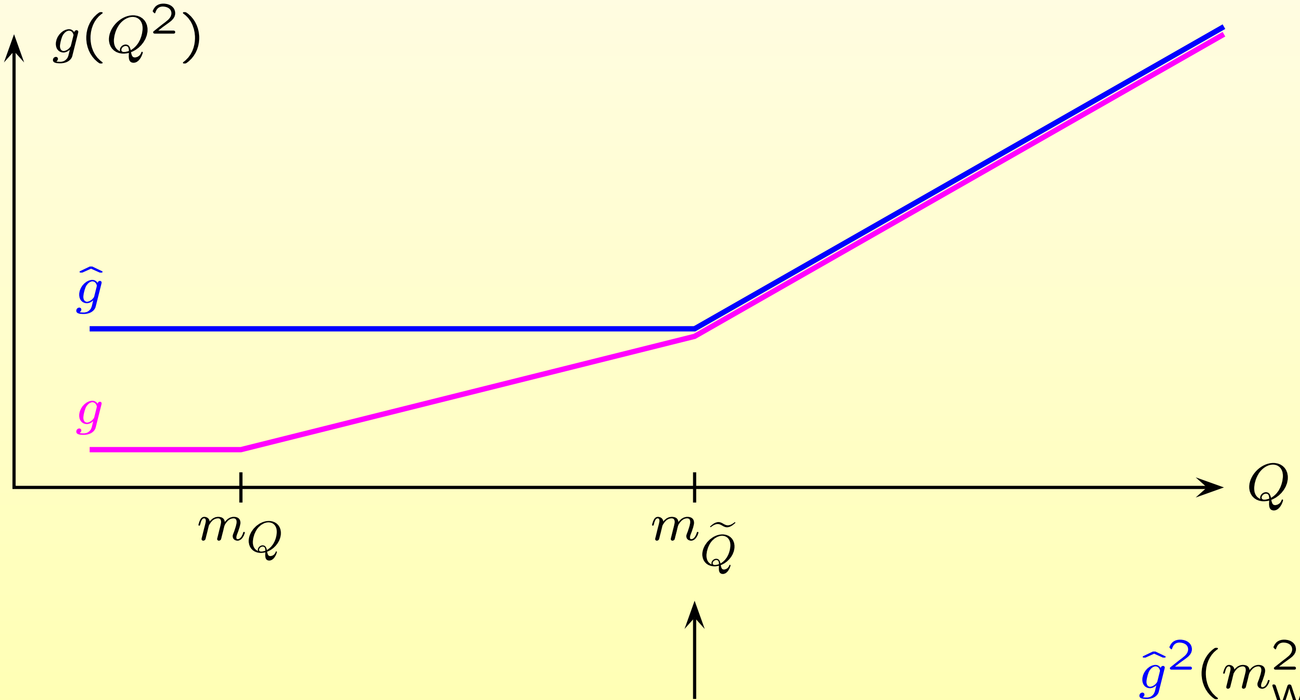


$\rightarrow$ corrections are of $\mathcal{O}(\%)$

Non-decoupling corrections in renormalization group evolution

Nojiri, Fujii, Tsukamoto '96
Cheng, Feng, Polonsky '97

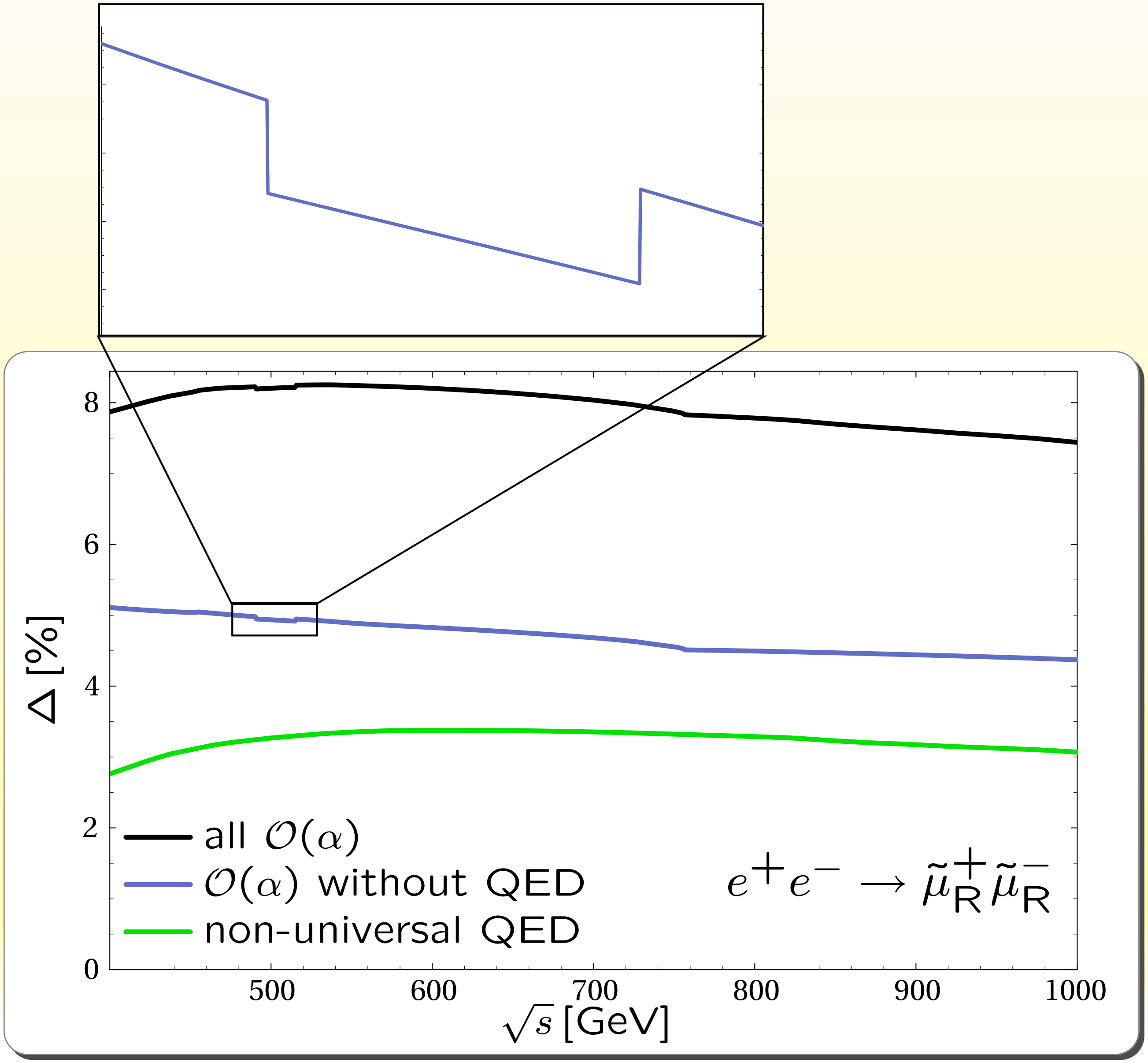
Consider only $q/\tilde{q}$ loops



$\tilde{q}$ decouple from g
q and $\tilde{q}$ decouple from $\hat{g}$

$$\rightarrow \quad \frac{\hat{g}^2(m^2_{\text{weak}})}{g^2(m^2_{\text{weak}})} - 1 \propto \log \frac{m_{\widetilde{Q}}}{m_{\text{weak}}}$$

Anomalous thresholds



Discontinuity in $\sigma(s)$

Singularity known as *anomalous threshold*

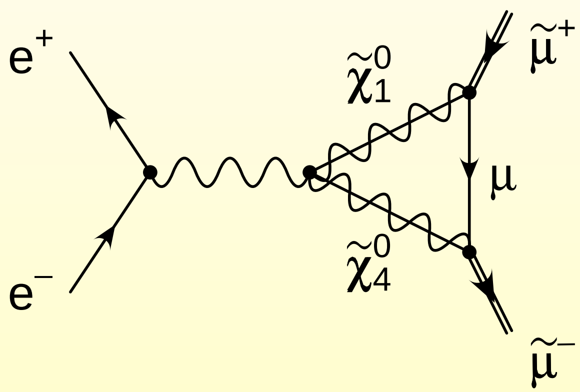
$\rightarrow$ Deuteron form factor

Karplus,Sommerfield, Wichmann '58

Landau '59

Anomalous thresholds in vertex graphs

Different in nature to normal two-particle thresholds



Normal threshold when e.g. $\tilde{\chi}^0_1$ and $\tilde{\chi}^0_4$ can be produced on-shell

Anomalous threshold:
All three loop particles $\tilde{\chi}^0_1$, $\tilde{\chi}^0_4$ and μ get on-shell

only possible for $m_{\tilde{\chi}^0_1} < m_{\tilde{\mu}} < m_{\tilde{\chi}^0_4}$

at the kinematical point $$s = \frac{m^2_{\tilde{\mu}}(m^2_{\tilde{\chi}^0_1} - m^2_{\tilde{\mu}})^2}{(m^2_{\tilde{\mu}} - m^2_{\tilde{\chi}^0_1})(m^2_{\tilde{\chi}^0_4} - m^2_{\tilde{\mu}})}$$

Conclusions

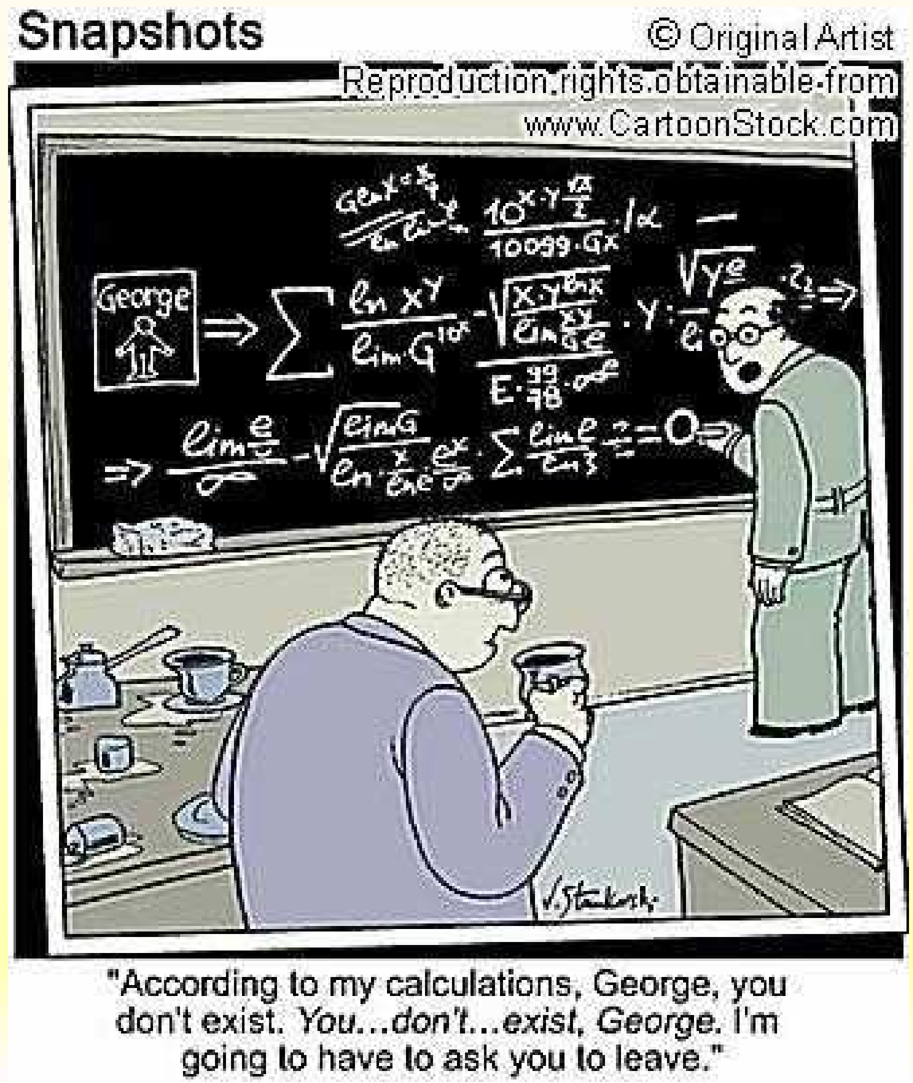
If sleptons exist at the reach of ILC......

- We cannot miss them
- We can measure them at the per-cent to per-mille level
- We can understand them theoretically at per-cent level
  $\longrightarrow$ more work might be needed
- We can test fundamental concepts of SUSY:
  relation between gauge and Yukawa couplings: $g = \hat{g}$
- We can precisely determine masses and mixings:
  $\longrightarrow$ base of reconstructing high scale theory of SUSY breaking

Conclusions

If sleptons don't exist at the reach of ILC......

A theorist always finds
a way out....



Neutralino/chargino renormalization

$$X = \begin{pmatrix} M_2 & \sqrt{2}M_W s_\beta \\ \sqrt{2}M_W c_\beta & \mu \end{pmatrix} \qquad Y = \begin{pmatrix} M_1 & 0 & -M_Z s_W c_\beta & M_Z s_W s_\beta \\ 0 & M_2 & M_Z c_W c_\beta & -M_Z c_W s_\beta \\ -M_Z s_W c_\beta & M_Z c_W c_\beta & 0 & -\mu \\ M_Z s_W s_\beta & -M_Z c_W s_\beta & -\mu & 0 \end{pmatrix}$$

$\rightarrow$ 3 parameters (M_1, M_2, μ) and 6 physical particles ($\tilde{\chi}^{\pm}_{1,2}$, $\tilde{\chi}^0_{1...4}$)

1. On-shell conditions for all particles
2. Determine counterterms for M_1, M_2, μ from conditions for e.g. $\tilde{\chi}^{\pm}_{1,2}$, $\tilde{\chi}^0_1$
3. Calculate other mass counterterms ($\tilde{\chi}^0_{2,3,4}$)
   $\rightarrow$ shift in $m_{\tilde{\chi}^0_{2,3,4}}$ predicted

Two technically different but equivalent prescriptions on market:

Eberl, Majerotto, Kincel, Yamada '01

Pierce Papadopoulos '94

Fritzsche,Hollik '02