Track reconstruction of charged particles using a 4D quantum algorithm

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HELMHOLTZ

Track reconstruction

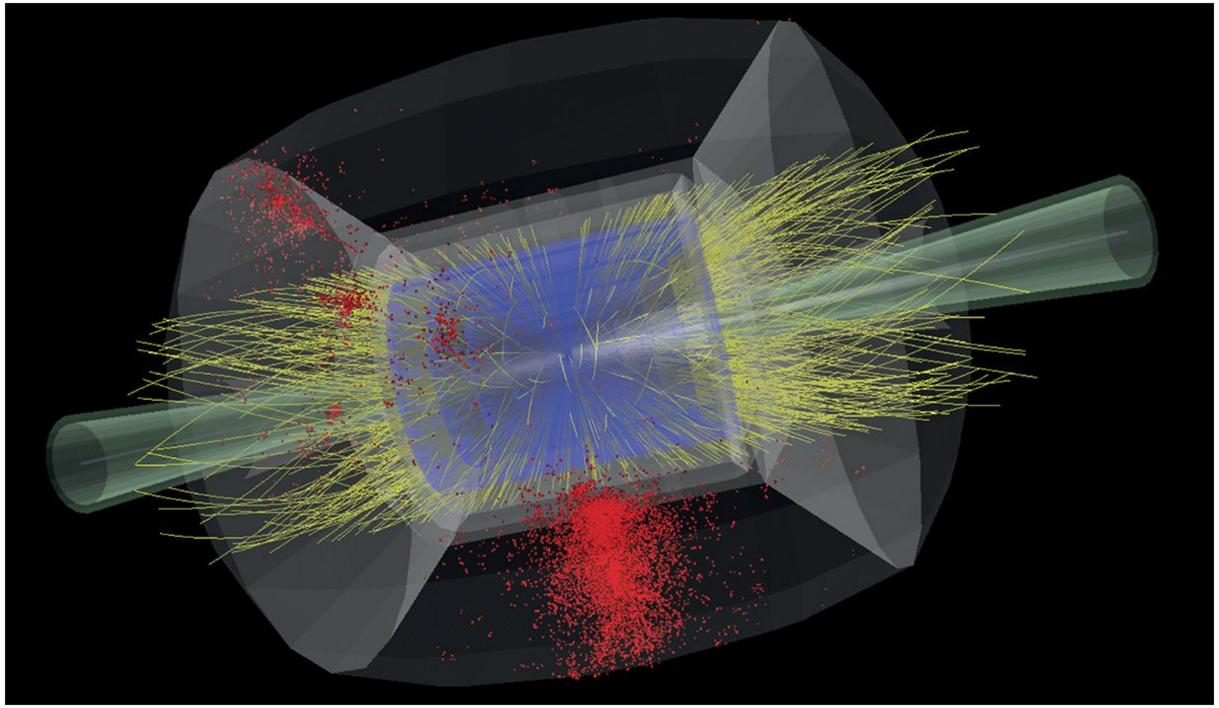
- Tracking can be challenging

  → good detectors and algorithms needed

- Future collider facilities plan to use timing capability

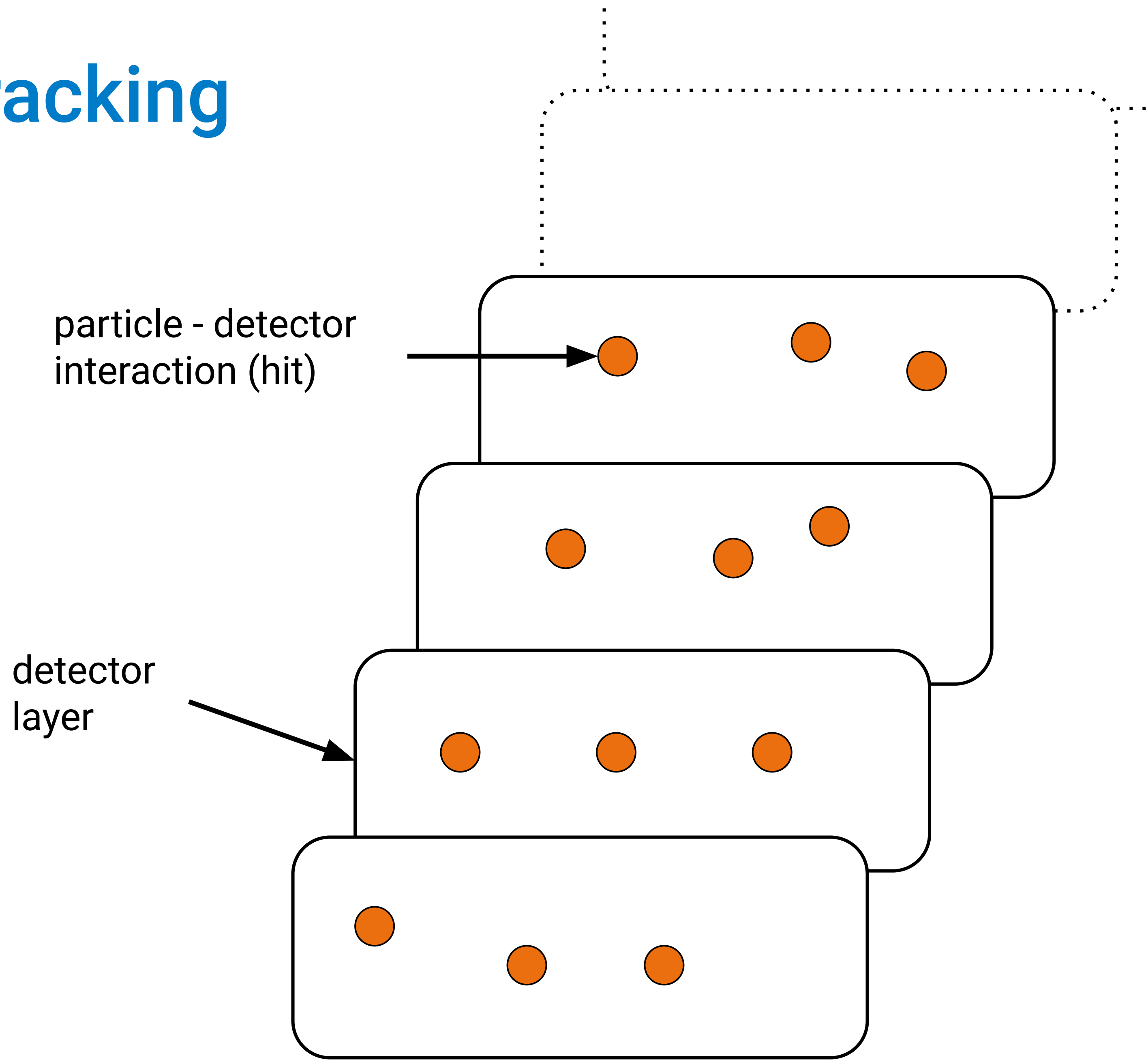
  → 4D algorithms

- This talk:
  - 4D quantum algorithm for pattern recognition of charged particle tracks
  - Results for a Muon Collider detector as an example



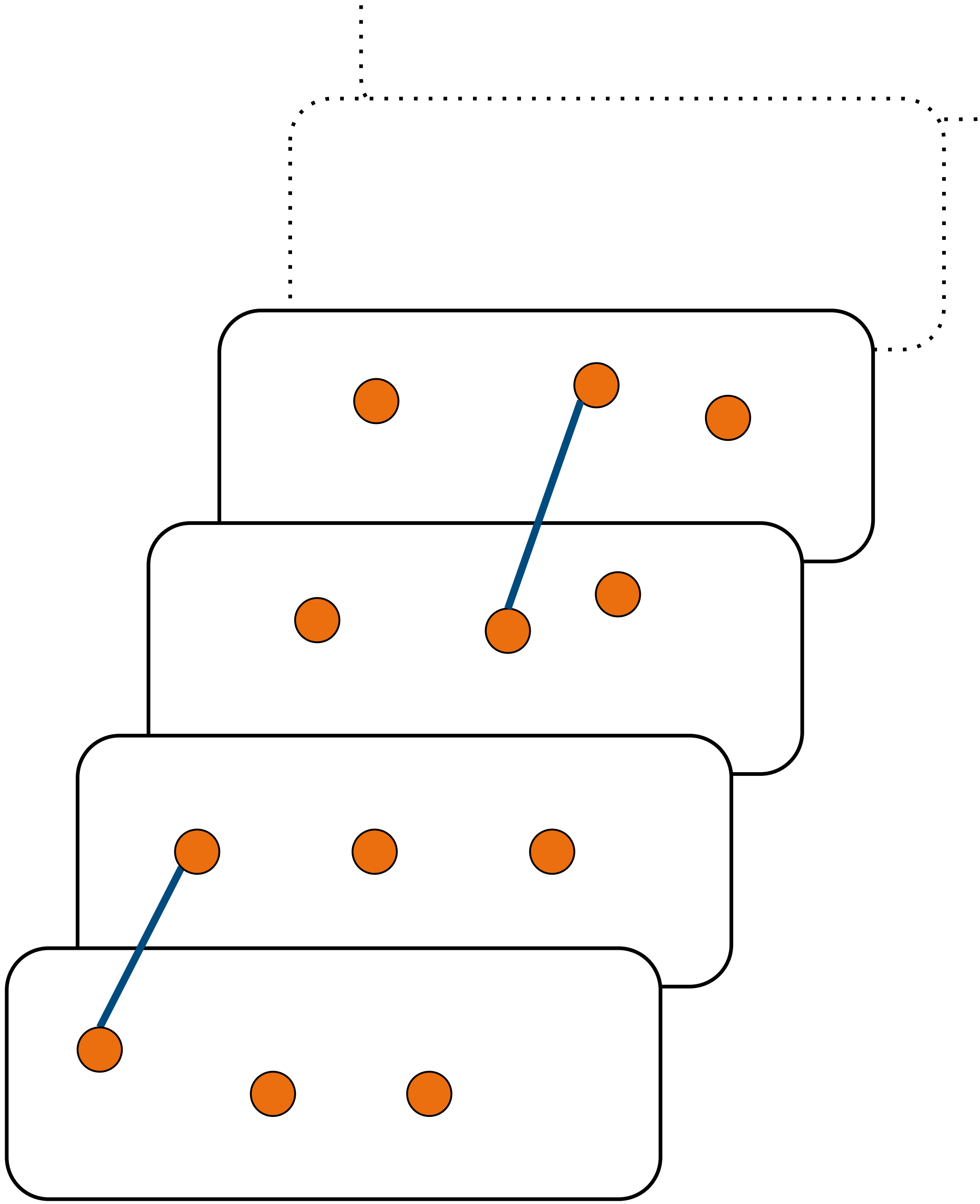
Source: https://home.cern/science/accelerators/muon-collider

Pattern recognition for tracking



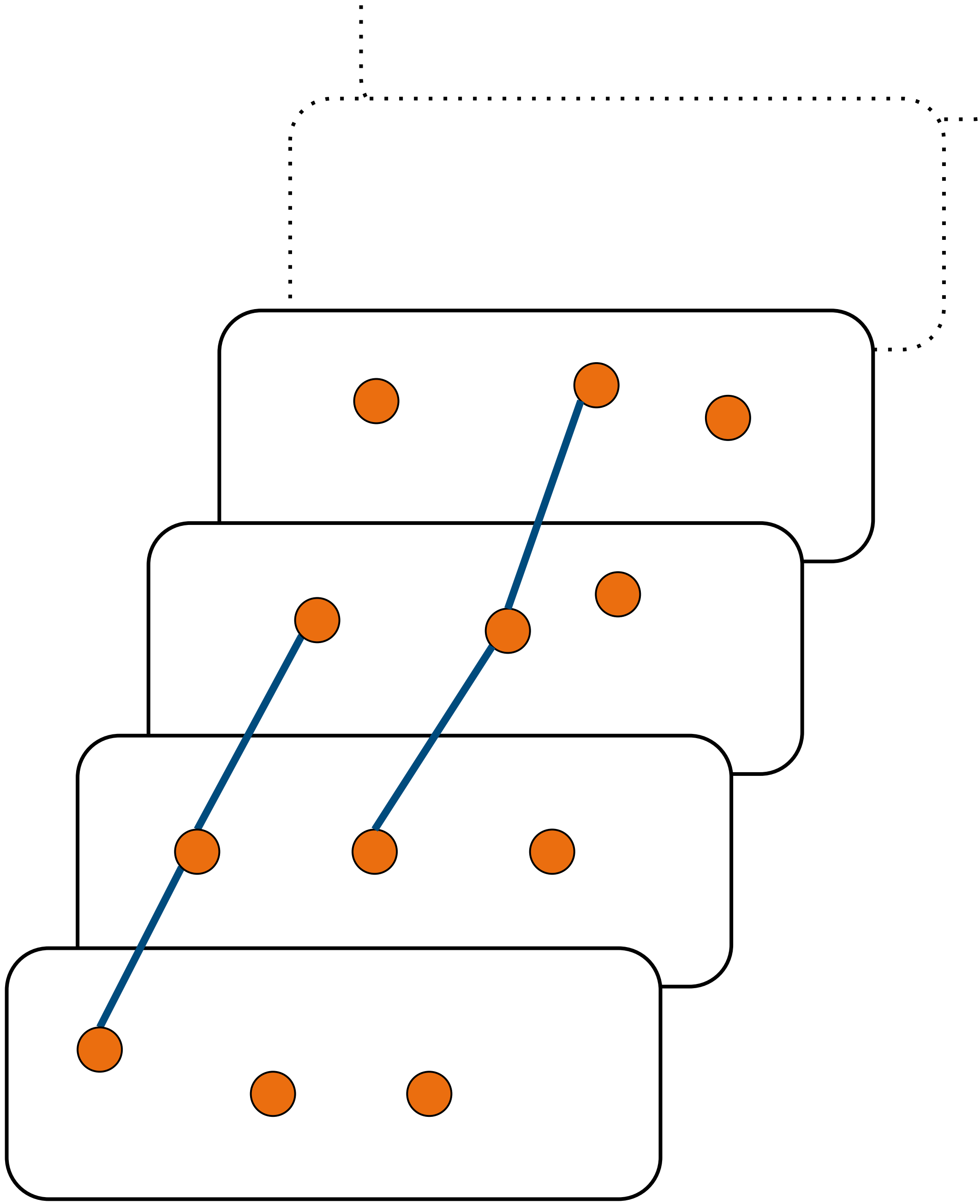
Pattern recognition for tracking

- **Doublets**



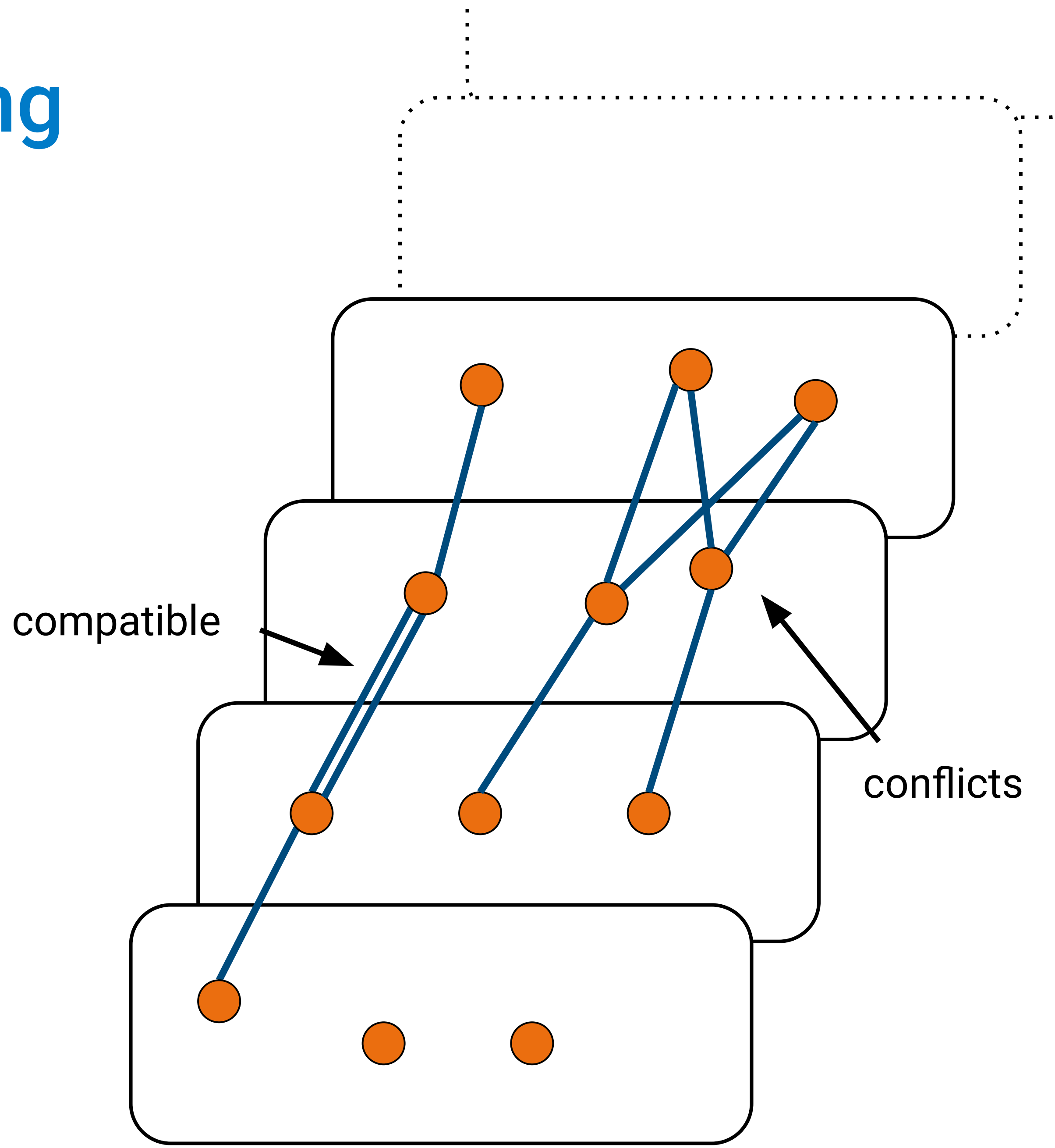
Pattern recognition for tracking

- **Doublets**
- **Triplets**



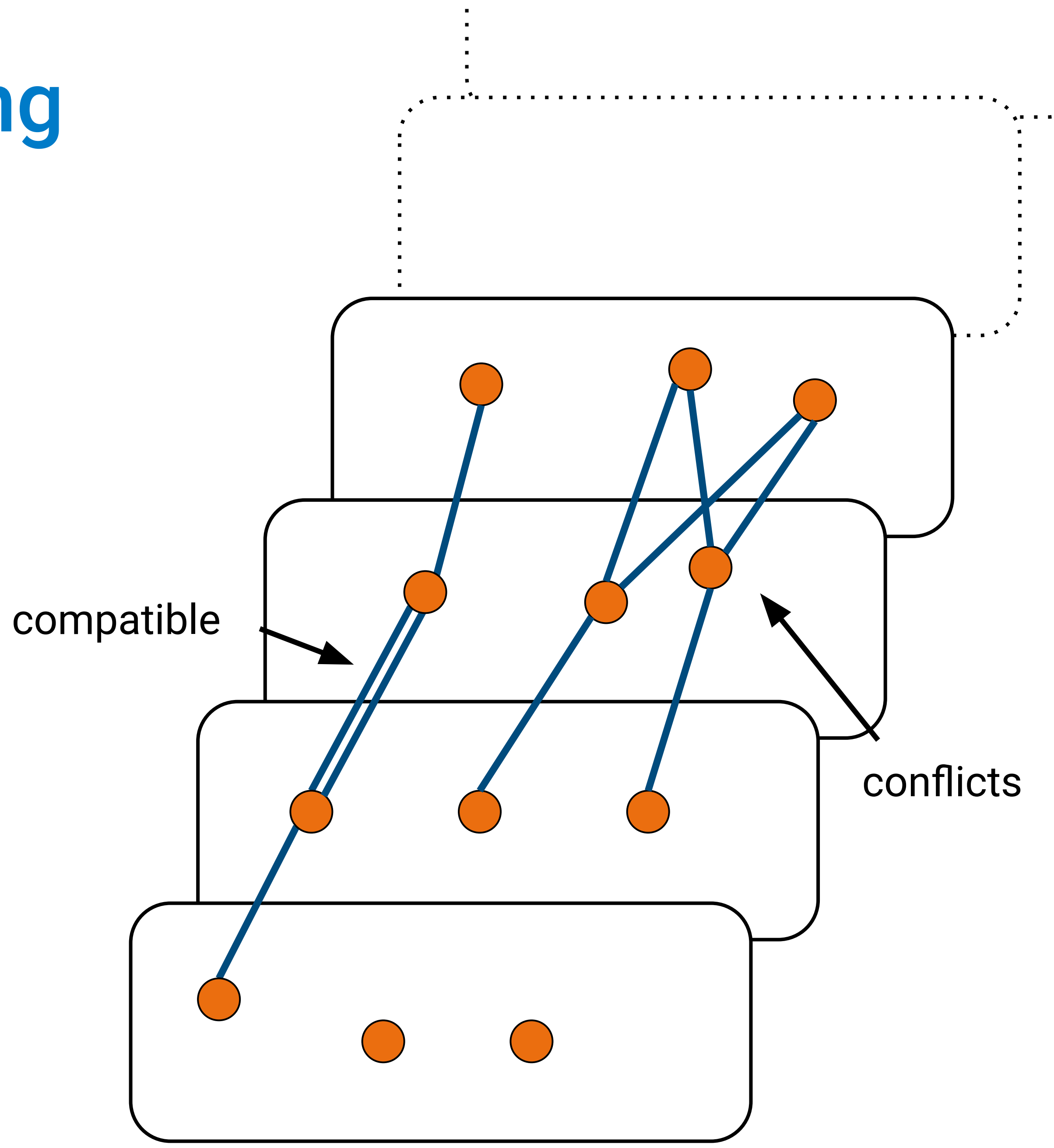
Pattern recognition for tracking

- **Doublets**
- **Triplets** → exploit triplet relations!



Pattern recognition for tracking

- **Doublets**
- **Triplets** → exploit triplet relations!



Goal:

Identify triplets stemming from a single particle and combine them to tracks

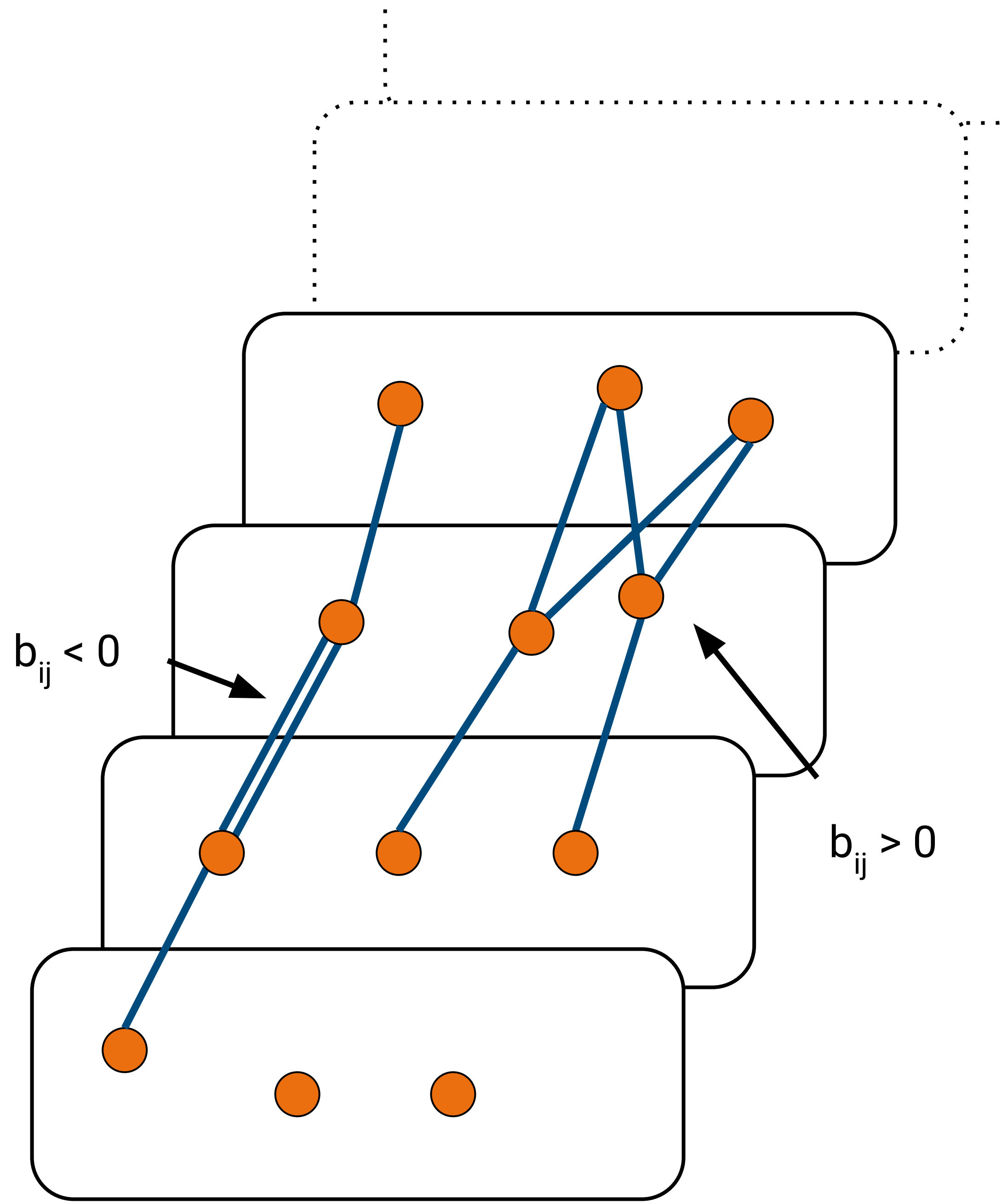
QUBO

Quadratic Unconstrained Binary Optimisation

$$\hat{H} = \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j + \sum_{i=1}^{N} a_i T_i \qquad \text{(QUBO)}$$

- $T_i \in \{0, 1\}$
- b_{ij} : interaction
- a_i : quality

values based on **spatial** and/or **temporal** information



QUBO

Quadratic Unconstrained Binary Optimisation

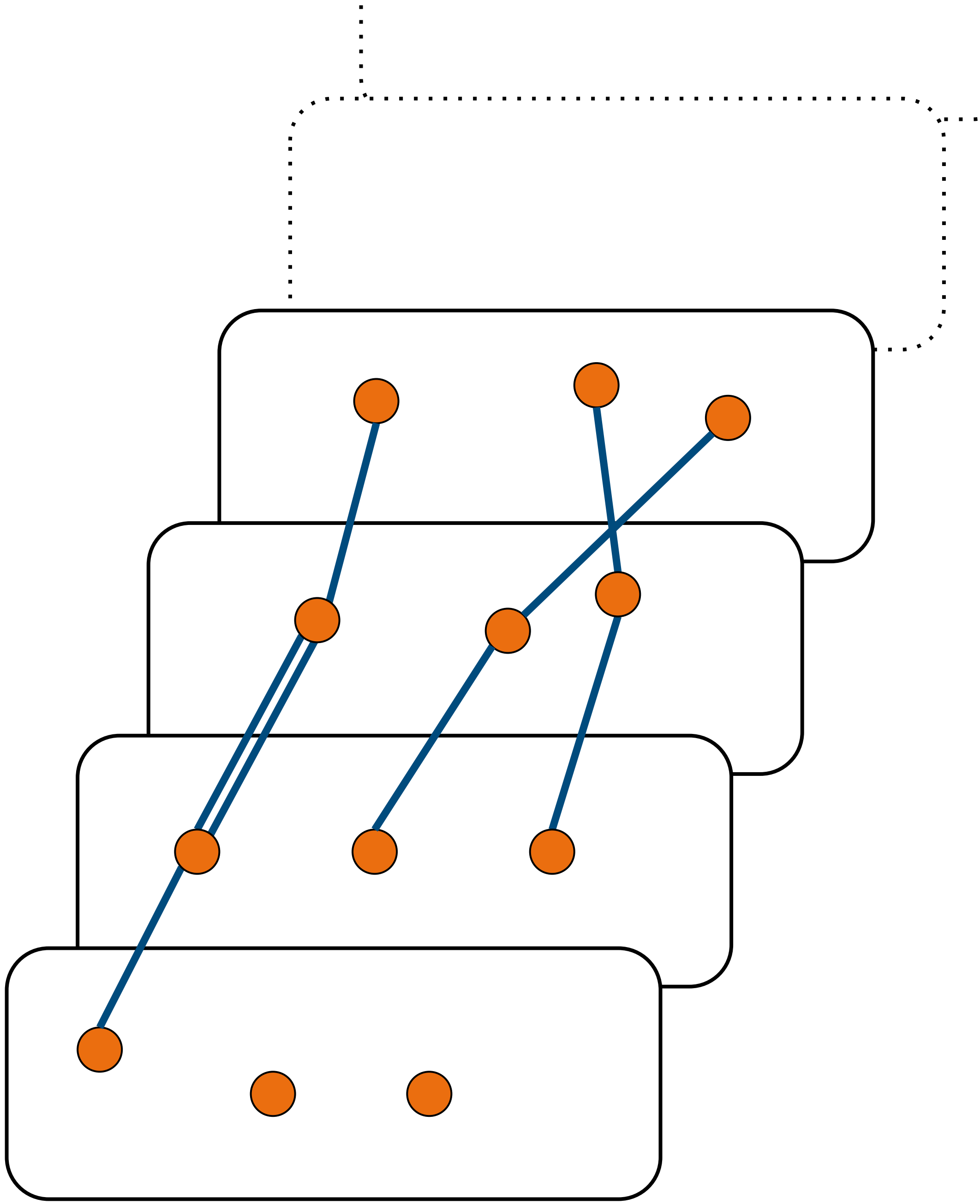
$$\hat{H} = \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j + \sum_{i=1}^{N} a_i T_i \qquad \text{(QUBO)}$$

Minimise Hamiltonian cost function:

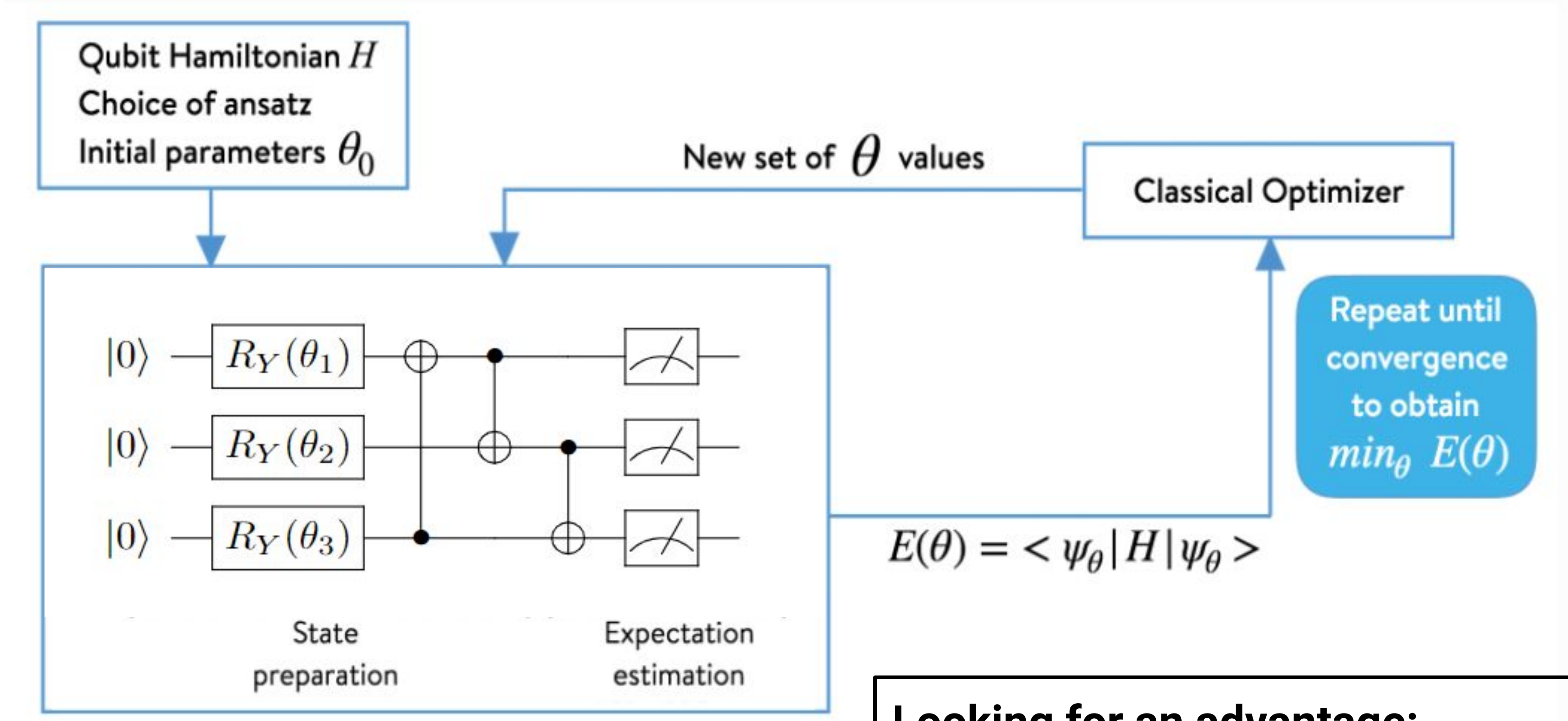
- Ground state → best set of triplets
- $\mathbf{v}_{\mathbf{binary}}$: [$T_1$, T_2, T_3, ... , T_N] → [0, 1, 1, ... , 0] as result

Computation:

- Matrix diagonalisation (analytic solution)
- Hybrid quantum-classical algorithm (VQE)



VQE - Variational Quantum Eigensolver



Source: edited from http://openqemist.1qbit.com/

$$H = \sum_{a=1}^{\mathcal{O}(Poly(N))} w_a P_a$$

$$P_a \in \{I, X, Y, Z\}^{\otimes N}$$

Looking for an advantage:

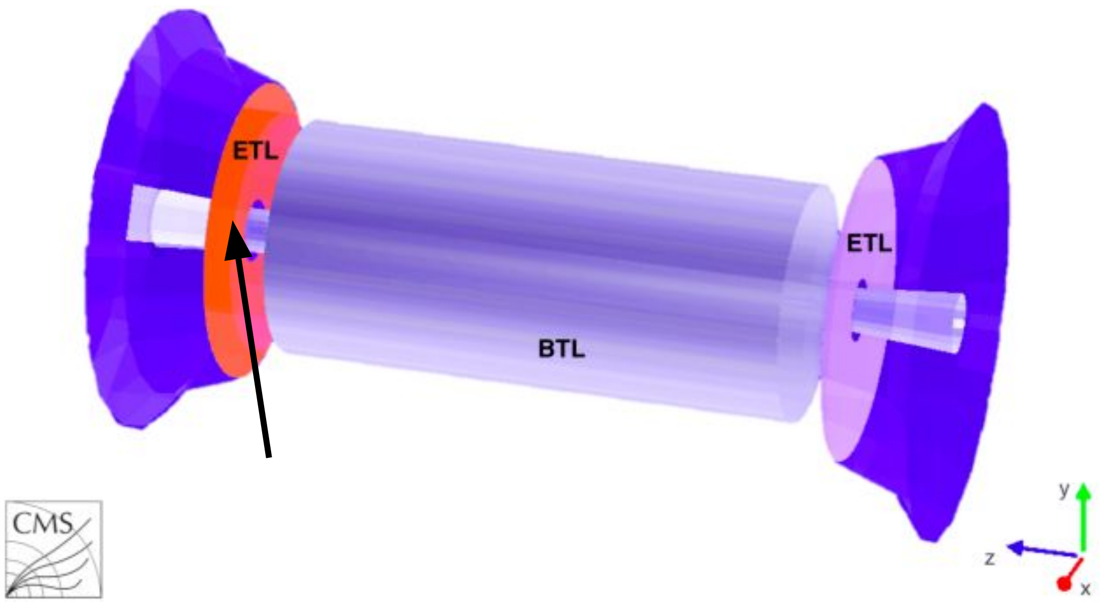
dim(H) = 2^n x 2^n, then <ψ|H|ψ> needs

- O(2^{2n}) operations classically
- Possibly only O(Poly(n)) operations with a quantum computer

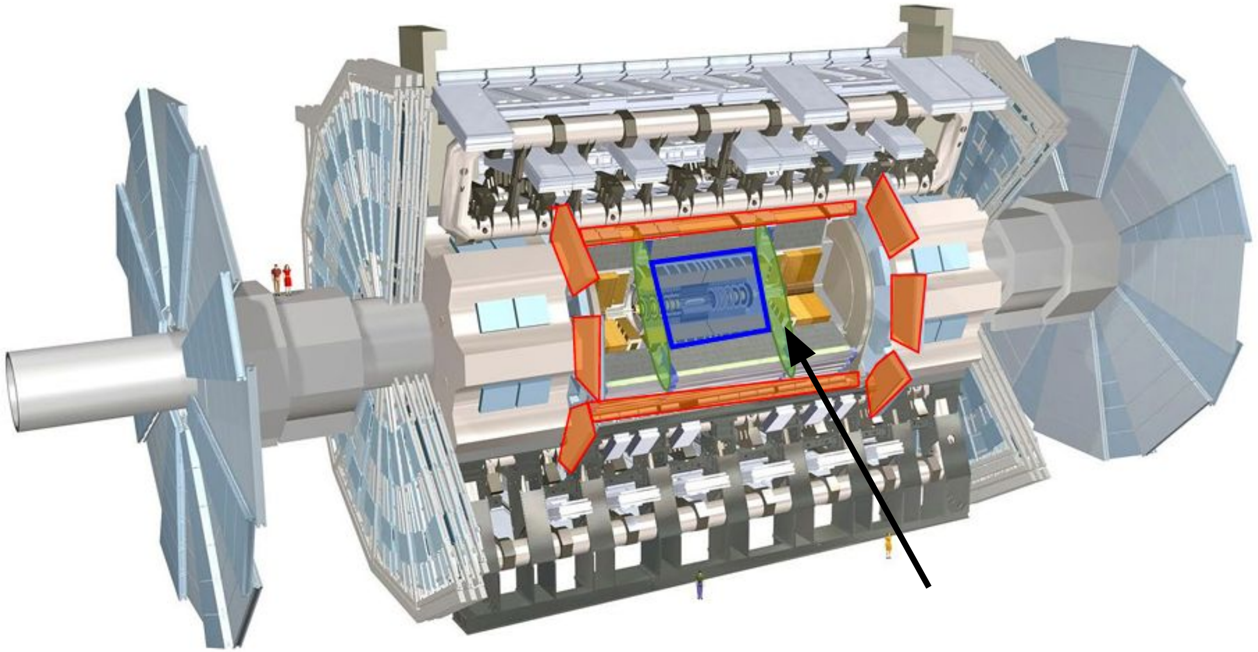
Timing information used for particle tracking

- Purpose: Reduce ambiguity and complexity of reconstructing trajectories
- Planned Phase-2 upgrade ATLAS(HGTD)/CMS(MIP):

  Timing layers in forward direction to reduce pileup background



Source: CERN-LHCC-2019-003

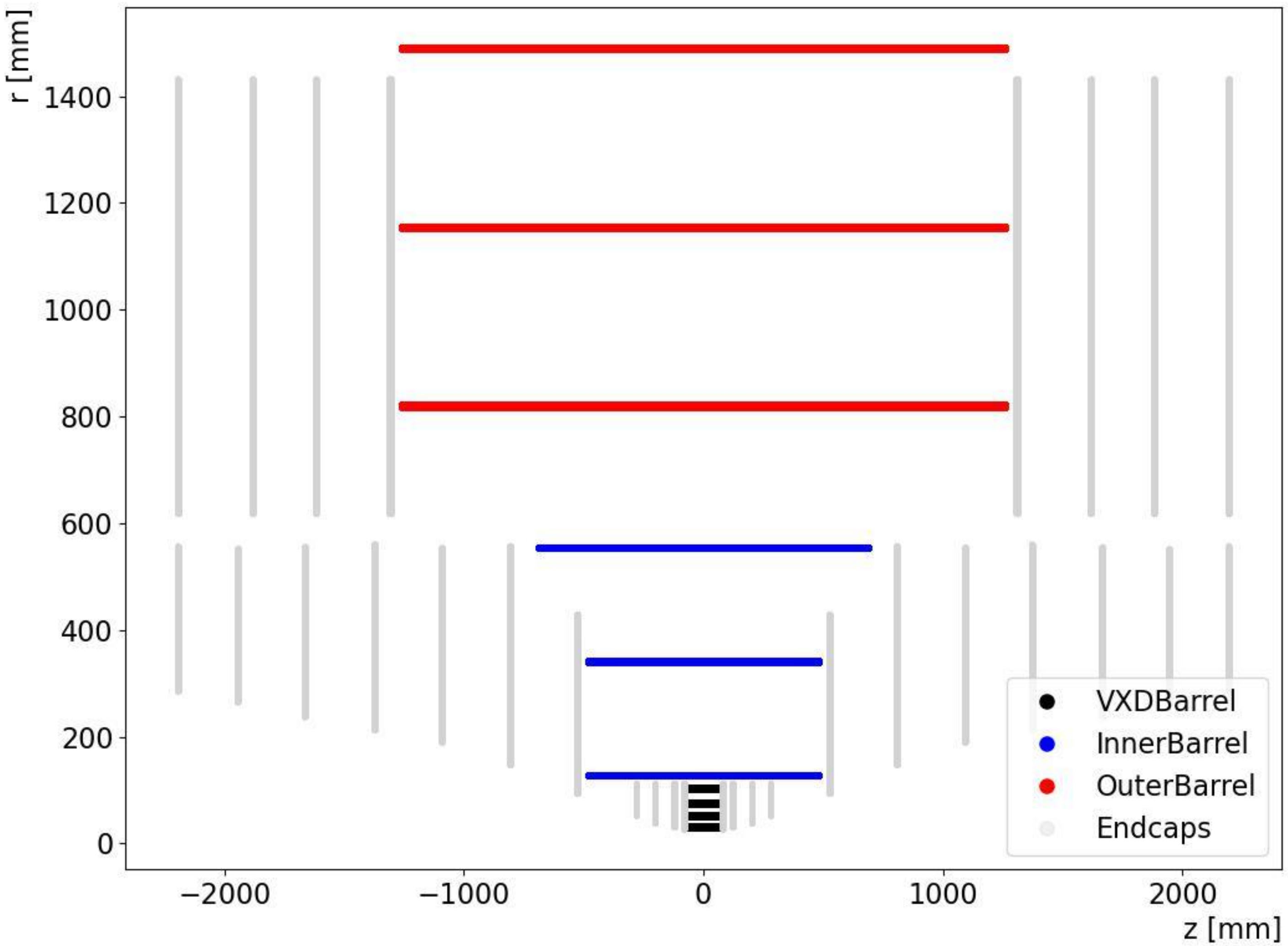


Source: CERN-LHCC-2020-007

Muon Collider Tracking Detector

- Spatial resolution:
  - VXD: 25x25 μm^2
  - Inner/Outer: 50x100 μm^2

- Temporal resolution:
  - VXD: σ_t = 30ps,
  - Inner/Outer :σ_t = 60ps



Detector geometry described as in arxiv:2303.08533

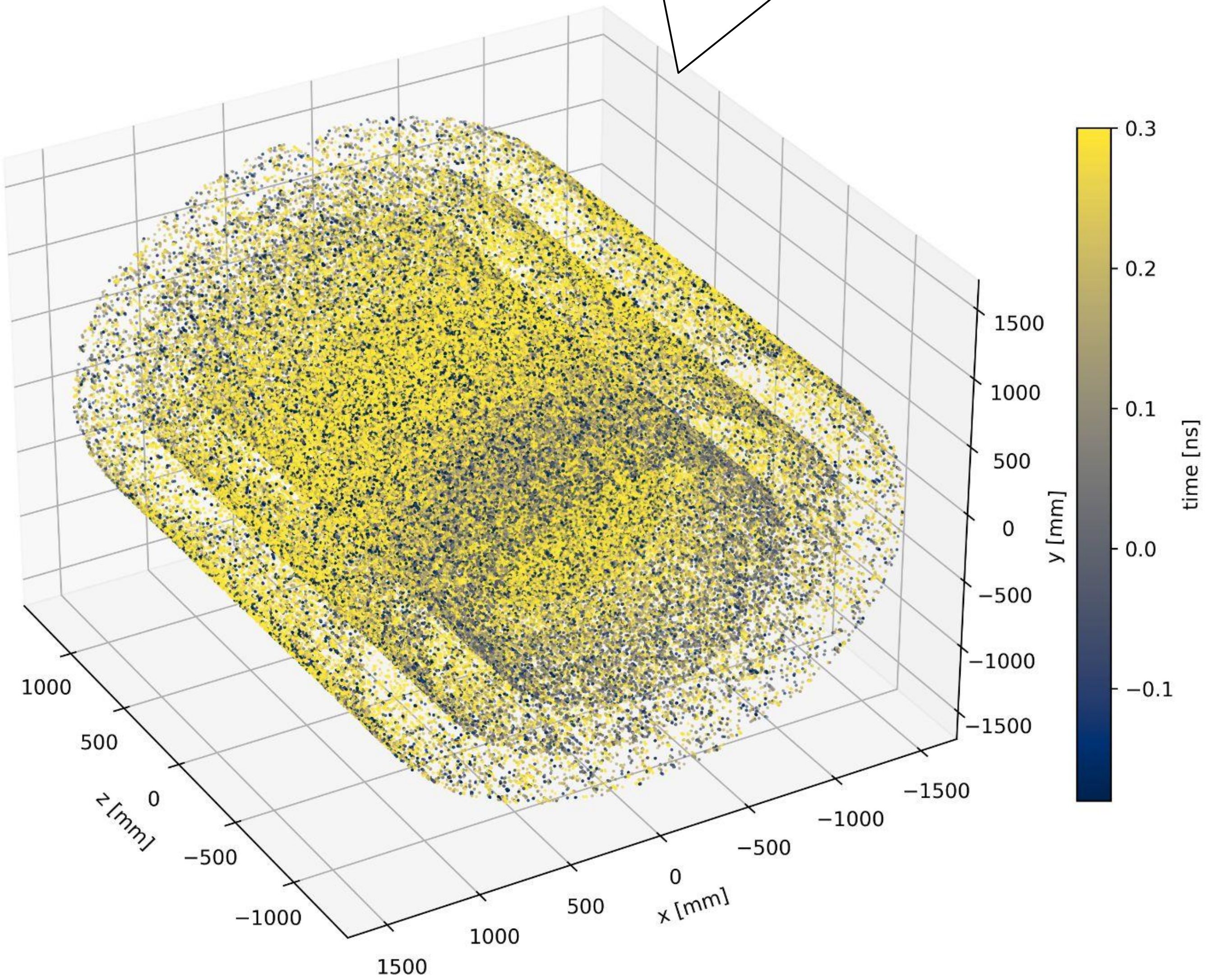
Tracking at Muon Collider

- Large **B**eam-**I**nduced-**B**ackground (BIB)

- Time information as a crucial component to suppress BIB particles

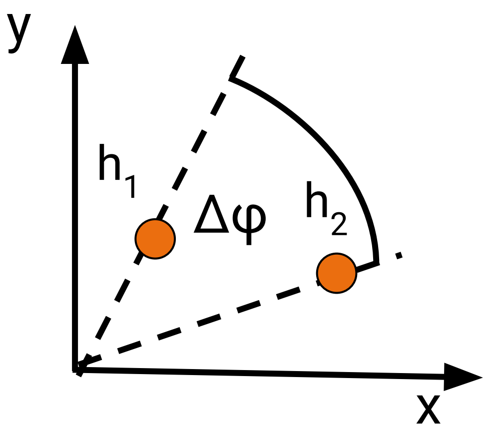
4D tracking with a quantum algorithm:
QUBO parameters determined by spatial + temporal information of detector hits

5TeV BIB overlay simulated with FLUKA

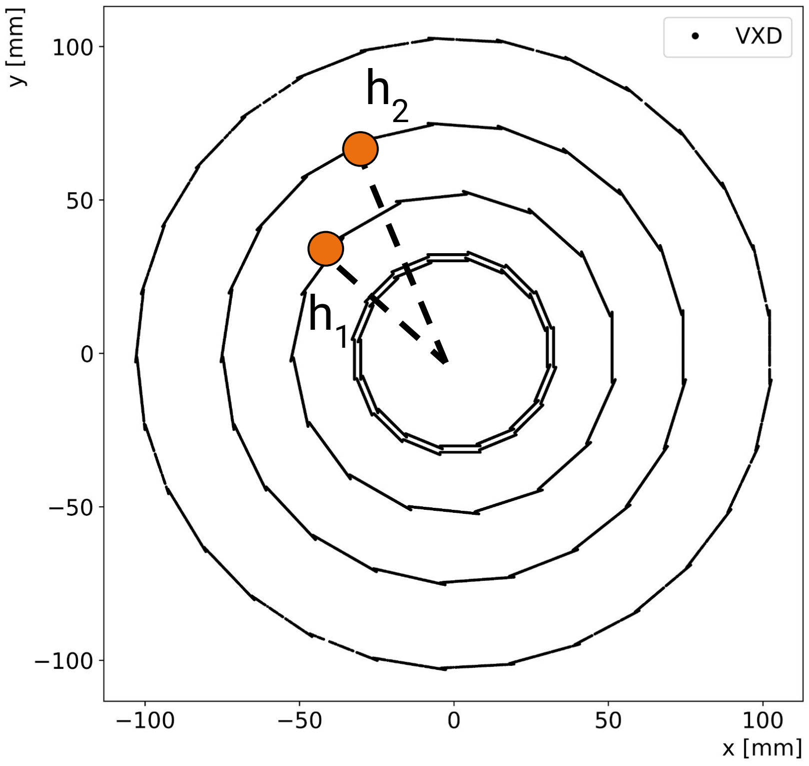


Pre-selection

- **Doublets:**
  - **Δφ:** max allowed curvature → min p_T

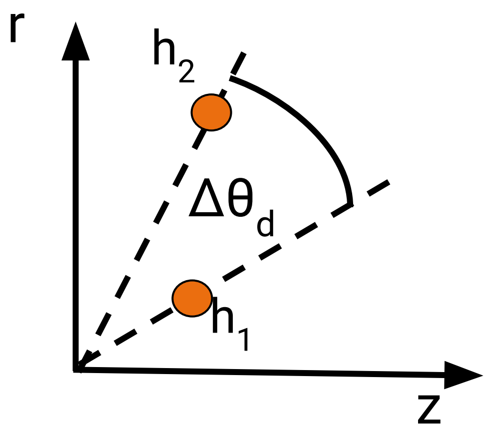


"curvature"

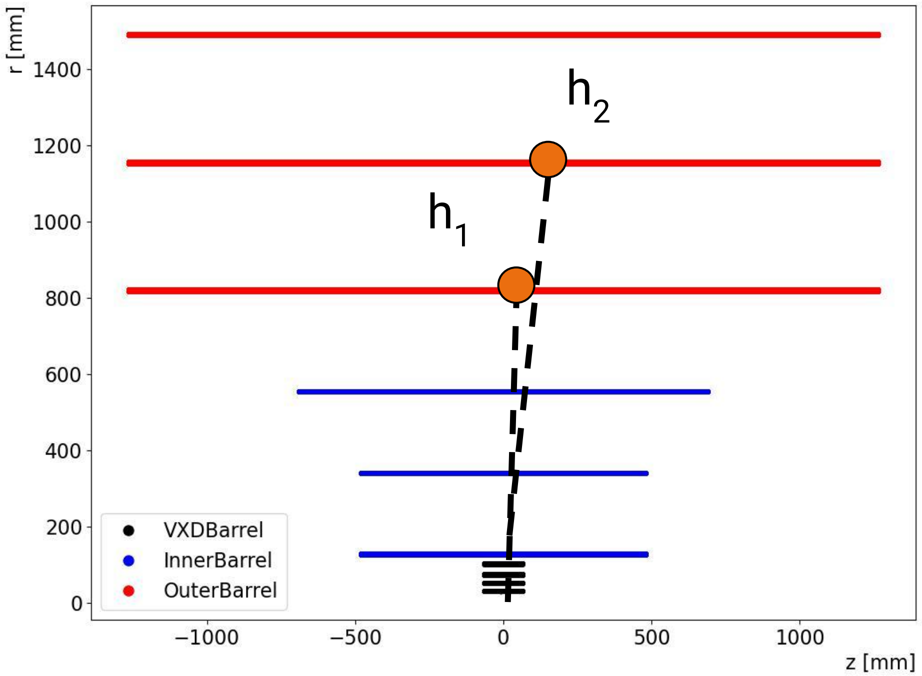


Pre-selection

- **Doublets:**
  - **Δφ:** max allowed curvature → min p_T
  - **$\Delta\theta_d$:** alignment in r-z plane → doublets compatible with primary vertex

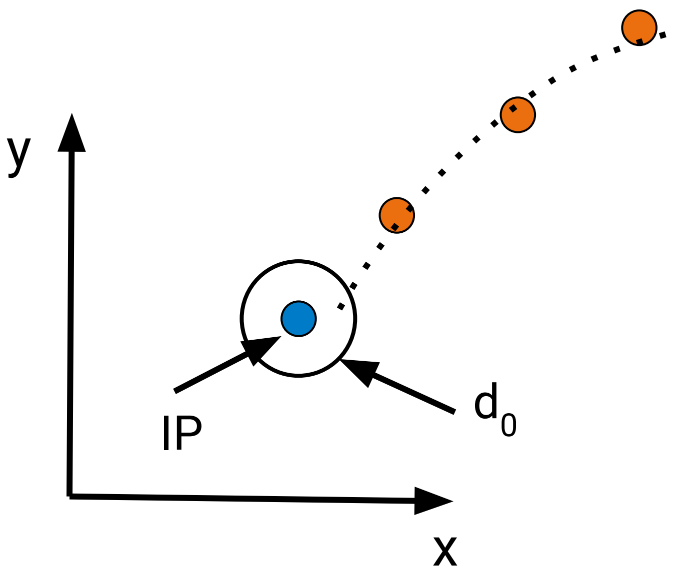


"primary vertex compatibility"



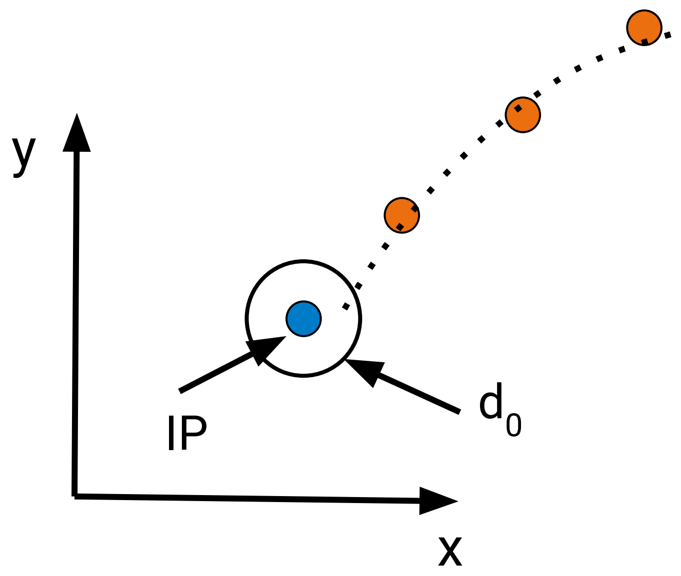
Pre-selection

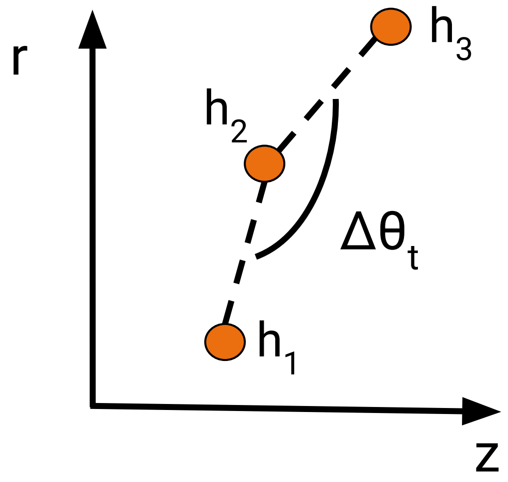
- **Triplets:**
  - **d_0** : transverse impact parameter → curvature compatible with primary vertex



Pre-selection

- **Triplets:**
  - $\mathbf{d_0}$: transverse impact parameter → curvature compatible with primary vertex
  - $\mathbf{\Delta\theta_t}$: alignment of triplet hits → scattering





4D QUBO

$$\hat{H} = \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j$$

a_i set to 0

4D QUBO

$$\hat{H} = \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j$$

Interaction b_{ij}:

- δ(curvature)

4D QUBO

$$\hat{H} = \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j$$

Interaction b_{ij}:

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- Alignment of hits in the r-z plane (scattering)

4D QUBO

$$\hat{H} = \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j$$

Interaction b_{ij}:

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- Alignment of hits in the r-z plane (scattering)
- Time compatibility of hits

4D QUBO

$$\hat{H} = \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j$$

Interaction b_{ij}:

- δ(curvature)
- Alignment of hits in the r-z plane (scattering)
- Time compatibility of hits

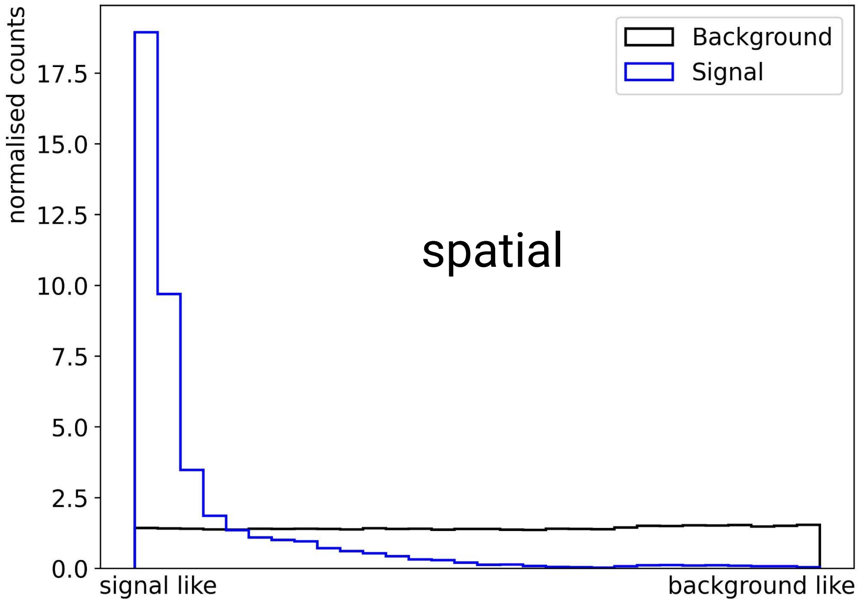
} weighted sum → $b_{ij} \in [-1, 0]$

→ **4D modeling of triplet interactions**

Triplet interactions b_{ij}

$$\hat{H} = \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j$$

- Optimise b_{ij} to distinguish interactions from signal and background triplets



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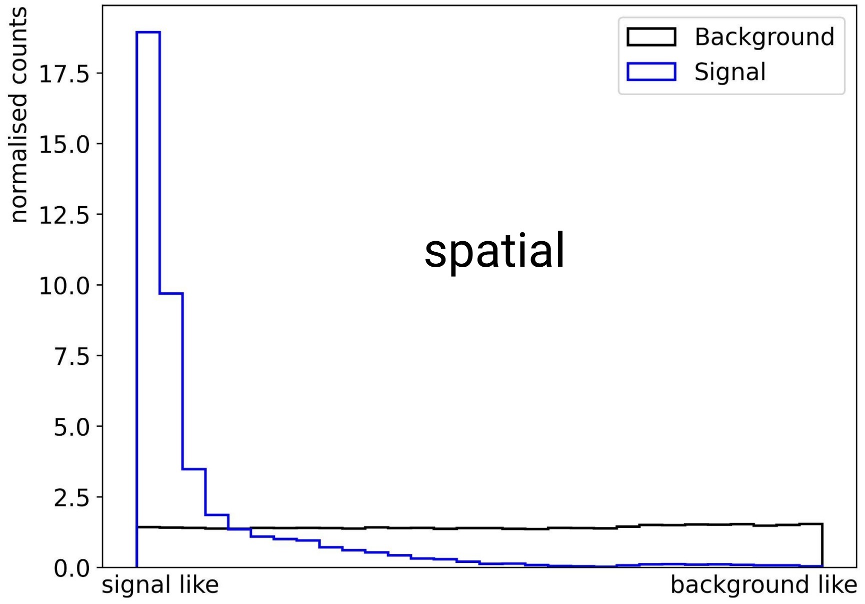
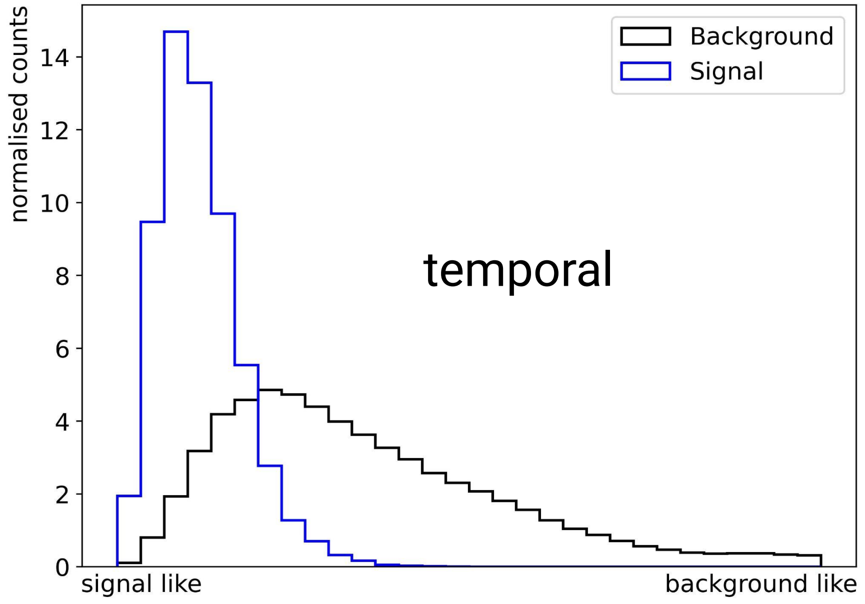


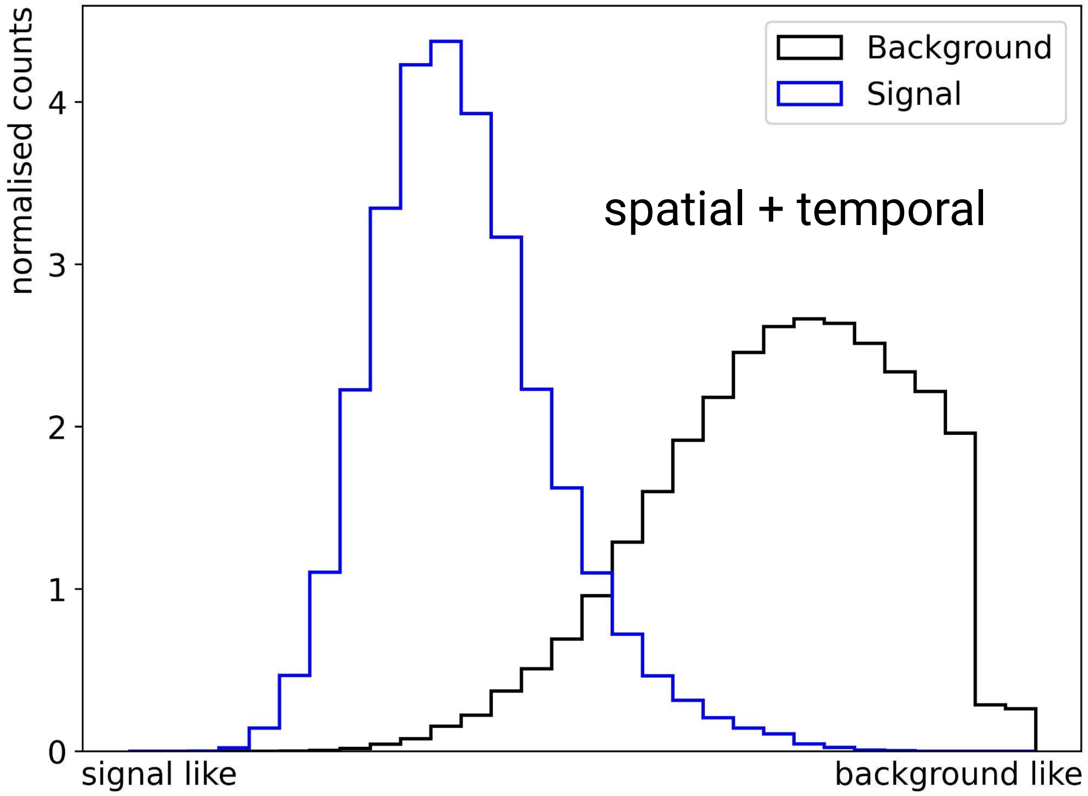


Triplet interactions b_{ij}

$$\hat{H} = \sum_{i}^{N} \sum_{j<i} b_{ij} T_i T_j$$

- Optimise b_{ij} to distinguish between interactions from signal and background triplets

- Combining position and time information improves the classification by b_{ij}

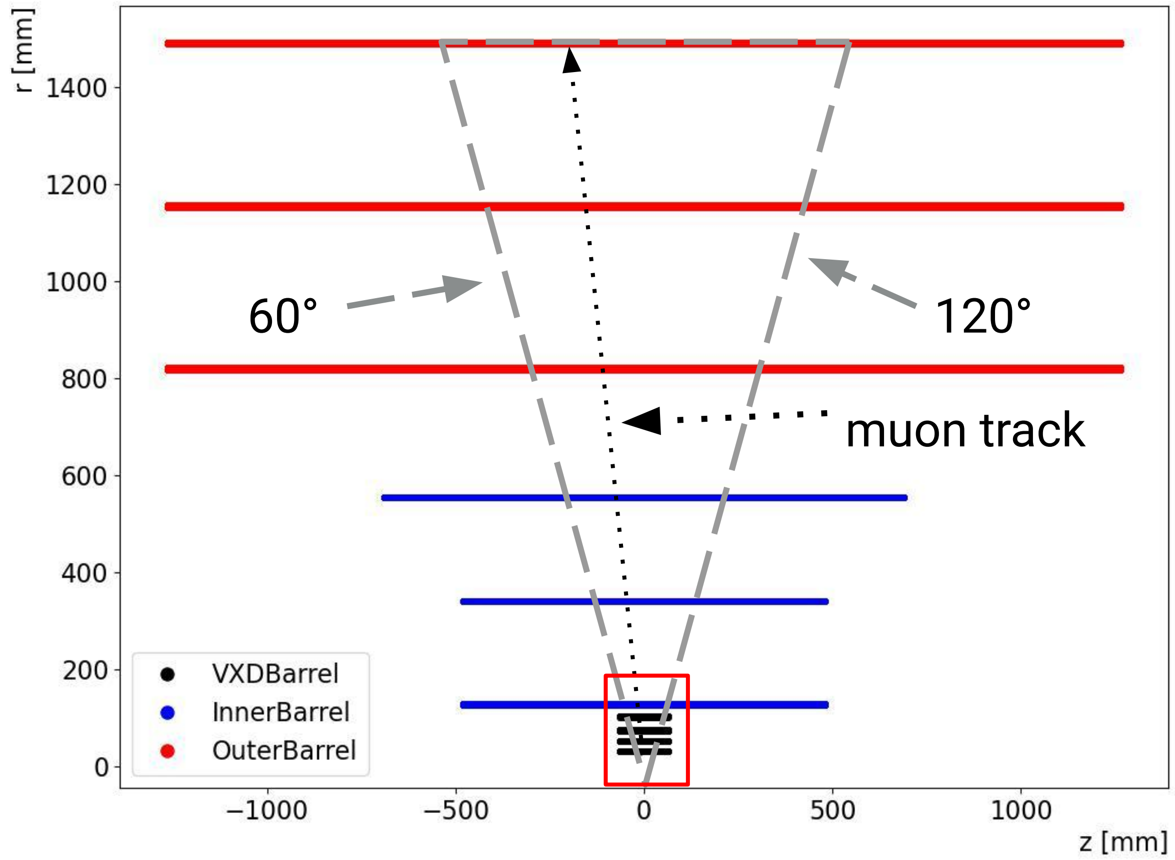


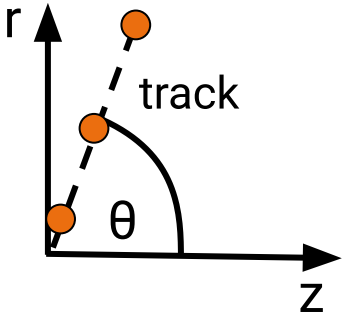
Reconstruct a single muon track within a large BIB

Setup

- Single muon events overlaid with BIB
- 0.5 GeV < p_T < 5.0 GeV
- 60° < θ < 120°
- Primary vertex (0, 0, 0), σ_z = 1.5 mm
- Pattern recognition limited to :
    - 1 double layer + 3 single layer (VXD)
    - 1 single layer (inner barrel)

Goal: Reconstruct a single muon track within a large BIB





Results

Data:

- O(30k) triplets after pre-selection

Optimisation

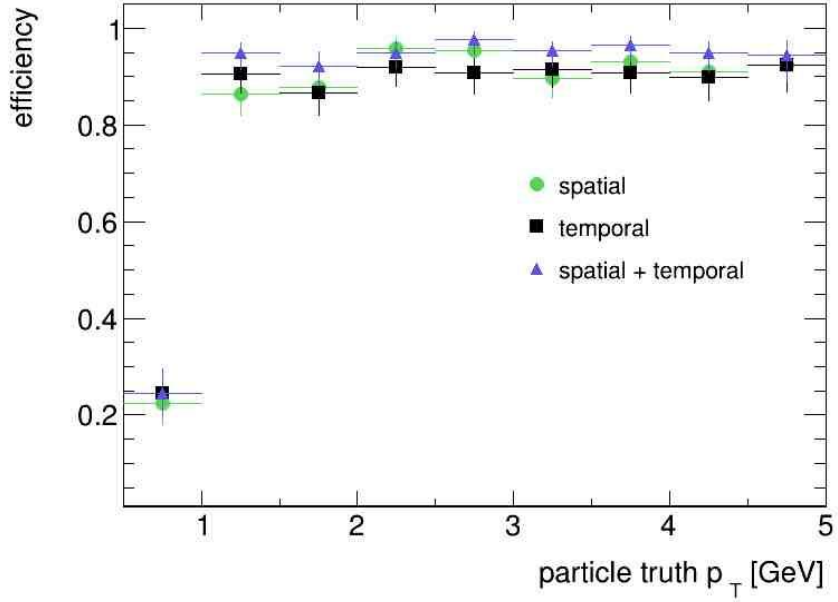
- Matrix diagonalisation, sub-QUBO size = 7

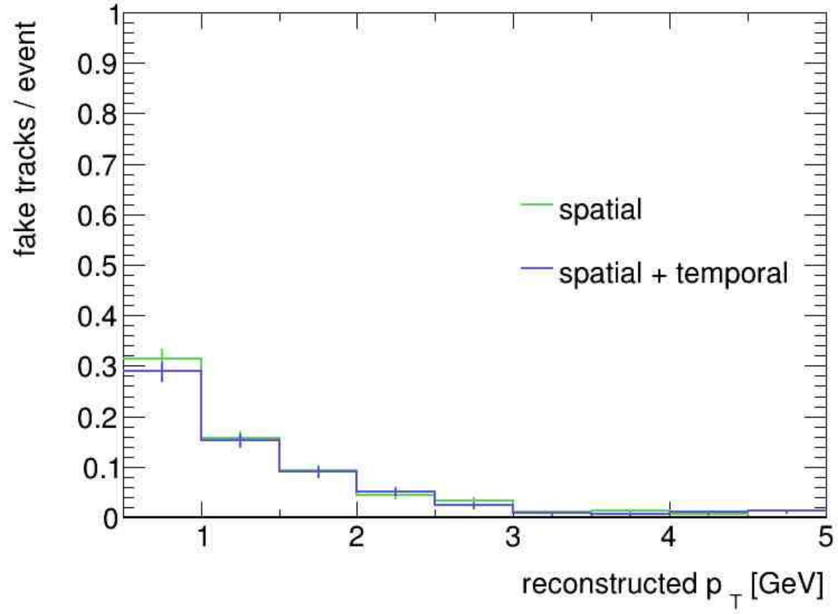
Criteria

- 6 hits required, all from same signal particle

Fake rate

- < 1 fake track per event on average





Summary

- A 4D quantum tracking approach using a QUBO formulation was presented
  → QUBO parameters as a combination of spatial and temporal information of detector hits

- Example test application shown for a future high-energy muon collider

- Combining time and position information improves the characterisation of QUBO parameters b_{ij}
  → increased track reconstruction efficiency

Outlook

- Further optimisation of the parameters b_{ij} is needed

  → find optimal b_{ij} = f(spatial, temporal)

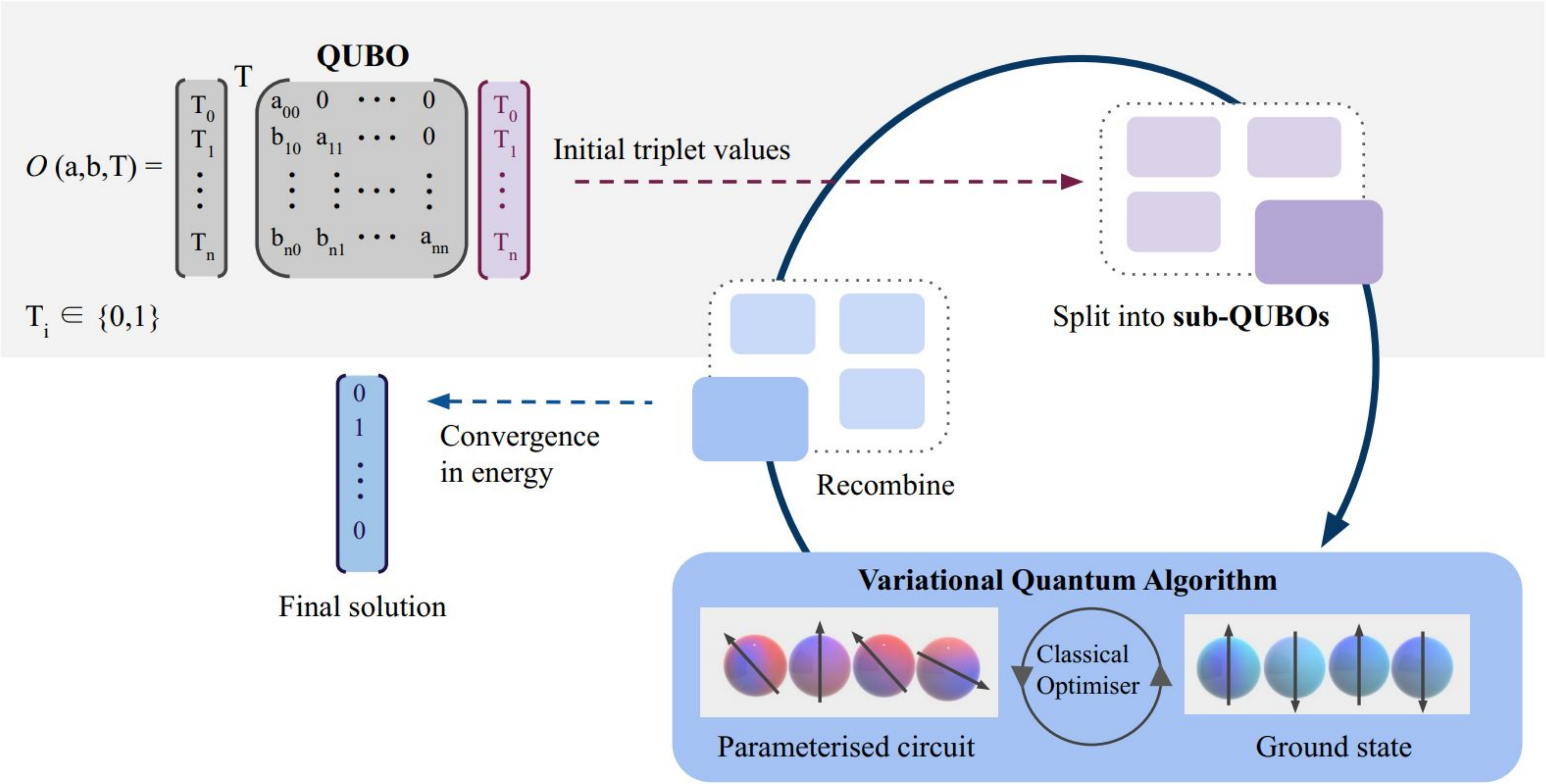
- Two options to proceed with the study:
  - Extend search to other tracker regions, maybe even the full tracker

    → computationally costly
  - Use found tracks as seed for CKF

    → proper track finding and fitting throughout the detector

Thank You!

Sub-QUBOs



Source: arxiv:2304.01690

Appendix: QUBO parameter settings

Trivial

- -1 if connection possible

Spatial

- $f(q/p_T) : \delta(\text{curvature}) \rightarrow \mathbf{\{0, 1\}}$
- $\max(\Delta\theta_t) \rightarrow \mathbf{[0, 1]}$

Temporal

- $\max(\Delta t) \rightarrow \mathbf{[0, 1]}$

Appendix: Preselection values

VXD:	$\Delta\varphi = 0.025$ [rad]	$\Delta\theta = 0.04$ [rad]	d0 = 1 [mm]
ITracker:	$\Delta\varphi = 0.25$ [rad]	$\Delta\theta = 0.02$ [rad]	d0 = 2 [mm]
OTracker:	$\Delta\varphi = 0.25$ [rad]	$\Delta\theta = 0.02$ [rad]	d0 = 5 [mm]

Appendix: χ^2 of spatial + temporal