Moduli dependent species entropy and species thermodynamics

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- EFTs are one of the main tools to study physics
- They are not fundamental, rather have certain validity regime
- They are characterized at least by two scales: Λ_{IR} and Λ_{UV}

This talk: I will study Λ_{UV} in EFTs with gravity.

What is Λ_{UV}? Can we calculate it from first principles?

Introduction and motivation

The UV cutoff in gravity

Gravity is strongly coupled at $M_P \sim 10^{18}\, GeV$.

Ratio of the 1-graviton to the 0-graviton amplitude $\sim (E/M_P)^2$
For $E > M_P$ gravity perturbation theory breaks down.

However, gravity could be strongly coupled already at $\Lambda_{UV} < M_P$.

For example, string theory suggests $\Lambda_{UV} = M_s < M_P$.

The species scale

String EFTs generically contain a high number N_{sp} of (light) fields/species, e.g. moduli, KK modes, winding, oscillators, ...

[Veneziano '01; Dvali '07; Dvali, Redi '07; Dvali, Lüst '09] proposed that gravity becomes strongly coupled at

$$\Lambda_{sp} = \frac{M_P}{N_{sp}^{\frac{1}{d-2}}} < M_P$$

called **species scale**. It is an upper bound on the UV cutoff

$$\Lambda_{UV} \lesssim \Lambda_{sp} < M_P$$

Note: $N_{sp} \gg 1$ for this effect to be non-trivial.
Is there an underlying "statistical" interpretation?

Arguments for the species scale (1/3)

Perturbative (QFT) [Dvali, Redi '09]

In the worst case, the perturbative expansion breaks down when 1-loop is comparable to tree-level.

The 1-loop graviton propagator coupled to N_{sp} (massless) scalars is proportional to the resummed vacuum polarization $G(p^2)$ such that

$$1/G(p^2) \simeq p^2 \left(1 - N_{sp} \left(\frac{p}{M_P} \right)^{d-2} \log \left(-\frac{p^2}{\mu^2} \right) \right)$$

Tree-level is comparable to 1-loop when

$$p = \frac{M_P}{N_{sp}^{\frac{1}{d-2}}} \equiv \Lambda_{sp}$$

Arguments for the species scale (2/3)

Higher-derivatives corrections to GR

The Einstein-Hilbert term is first in a series $\mathcal{O}(R^n)$.
The higher-derivative expansion

$$\mathcal{L} \sim M_P^{d-2} \left(R + \frac{N}{\Lambda^{d-2}} R^2 + \dots \right)$$

breaks down when

$$\Lambda = \frac{M_P}{N^{\frac{1}{d-2}}} \equiv \Lambda_{sp}$$

From this argument, one can also understand that N_{sp} is related to (gravitational) threshold corrections.
[Dixon, Kaplunovsky, Louis '91; Antoniadis, Narain, Taylor '91], recent work [Klaewer, Lee, Weigand, Wiesner '20]

Arguments for the species scale (3/3)

Non-perturbative (BH)

[Brustein, Dvali, Veneziano '09; Brustein, Medved '10]

The size of the smallest possible black hole reliably described by the EFT sets the species scale.

Consider a black hole of Planckian size, leading to

$$\mathcal{S}_{BH} \sim (M_P R_{BH})^{d-2} \sim (M_P L_P)^{d-2} \sim 1$$

However, in a theory with N_{sp} particles, the black hole should be able to emit or absorb all of them. Thus, we should rather find

$$\mathcal{S}_{BH} \sim N_{sp}$$

This happens if the smallest possible black hole is of size

$$R_{BH} = \Lambda_{sp}^{-1} > L_P.$$

The species scale $\Lambda_{sp} = M_P / N_{sp}^{\frac{1}{d-2}}$ is a relevant scale in EFTs of quantum gravity. It provides an upper bound on the UV cutoff.

In string theory, N_{sp} should be a function of the moduli

$$N_{sp} = N_{sp}(\phi),$$

for there are no free parameters.
To find such a function in general is challenging.

For the specific setup of type II compactifications on CY_3, [Vafa, Van de Heisteeg, Wiesner, Wu '22] proposed that such function is related to the **genus-one free energy** of the topological string

$$N_{sp} \simeq F_1$$

Moduli-dependent species scale and BH entropy

[NC, Lüst, Staudt '22]

Moduli-dependent species scale

[Vafa, Van de Heisteeg, Wiesner, Wu '22]

Analogy between 4d N=2 SUGRA with higher-derivatives

$$\mathcal{S}_{SUGRA} \supset \int F_1 R \wedge * R + \dots$$

and spontaneously broken 4d CFT

$$\mathcal{S}_{CFT} \supset \int a\, R \wedge * R + \dots$$

suggests correspondence $\quad a \longleftrightarrow F_1.$

Then, it is somehow natural to postulate

$$N_{sp} \simeq a \simeq F_1,$$

since the central charge counts degrees of freedom.

Comments and questions

- F_1 given by the topological string is an index. Hence, there could be unwanted cancellations when calculating N_{sp}. Furthermore, it is defined up to an additive constant, since zero modes are regulated away.

- Notice that to identify F_1 with the genus-one free energy of the topological string one needs the results of [Antoniadis, Gava, Narain, Taylor '93], who showed that the topological string is related to the superstring via the graviphoton background.

- Can we arrive at $N_{sp} \simeq F_1$ from black hole arguments? [NC, Lüst, Staudt '22]

Black holes and attractors

We work within 4d N=2 SUGRA, with a prepotential $F = F(X)$. BH solutions are further specified by a central charge

$$Z = q_\Lambda X^\Lambda - p^\Lambda \partial_\Lambda F$$

with $\Lambda = 0, 1, \ldots, n_V$. X^Λ are scalars in vector multiplets, $\partial_\Lambda F$ their symplectic duals and q_Λ, p^Λ electric, magnetic charges.

The attractor mechanism [Ferrara, Kallosh, Strominger '95; Ferrara, Kallosh '96] states that at the horizon

$$p^\Lambda = -2\mathrm{Im} X^\Lambda, \qquad q_\Lambda = -2\mathrm{Im}\partial_\Lambda F$$

and the entropy is

$$\mathcal{S}_{BH} = \frac{Area}{4} = \pi Z \bar{Z}$$

Probing the moduli space with BHs

[Bonnefoy, Ciambelli, Lüst, Lüst, '19] proposed a general method to study moduli spaces via BH solutions of the same compactification.

- At the horizon, moduli are functions of charges, $\phi = \phi(q, p)$. Consider e.g. the volume modulus

$$\mathcal{V} = \mathcal{V}(q, p)$$

  Then, we can express $\mathcal{S}_{BH} = \mathcal{S}_{BH}(q, p)$ as

$$\mathcal{S}_{BH} = \mathcal{S}_{BH}(q, \mathcal{V}), \qquad \text{or} \qquad \mathcal{S}_{BH} = \mathcal{S}_{BH}(\mathcal{V}, p)$$

- Large/small $\mathcal{S}_{BH}$ induced by large/small $\mathcal{V}$ (or any other modulus) and viceversa. Direct connection to **swampland distance conjecture**.

Application: Find smallest possible BH by tuning charges. Then, read moduli dependence of the species scale from the entropy.

Extremal BHs in heterotic on $K3 \times T^2$

Consider extremal dilatonic BHs in heterotic string on $K3 \times T^2$.

The entropy is given by [Ferrara, Kallosh '96]

$$\mathcal{S}_{BH} = \pi Z \bar{Z} = \pi p q$$

The attractor equations fix $g_s^{-2} = \frac{q}{p}$ and thus

$$\mathcal{S}_{BH} = \pi p^2 g_s^{-2}$$

The minimal entropy is reached for $p = 1$, giving

$$\mathcal{S}_{BH,min} \simeq g_s^{-2} \simeq N_{sp}.$$

It reproduces [Dvali, Lüst '10] for species = string states.

Extremal BHs in IIA on CY_3 (1/2)

A slightly more involved example is given by IIA on a CY_3.

At large volume, 4d N=2 SUGRA is described by

$$F(X) = \frac{1}{6} C_{ijk} \frac{X^i X^j X^k}{X^0}$$

where

$$z^i = \frac{X^i}{X^0}, \qquad i = 1, \ldots, h^{11} \equiv n_V$$

are complexified Kähler moduli (actual Kähler moduli: $t^i = \mathrm{Im} z^i$).

BHs can be supported by charges $q, p^i > 0$, giving an entropy

$$\mathcal{S}_{BH} = \pi Z \bar{Z} = 2\pi \sqrt{\frac{q}{6} C_{ijk} p^i p^j p^k}$$

Extremal BHs in IIA on CY_3 (2/2)

The attractor equations fix the CY volume modulus

$$\mathcal{V}_6 = \frac{1}{6} C_{ijk} t^i t^j t^k = \sqrt{\frac{q^3}{\frac{1}{6} C_{ijk} p^i p^j p^k}}$$

and thus

$$\mathcal{S}_{BH} = 2\pi \mathcal{V}_6^{\frac{1}{3}} \left(\frac{1}{6} C_{ijk} p^i p^j p^k \right)^{\frac{2}{3}}$$

The minimal entropy is reached for $\frac{1}{6} C_{ijk} p^i p^j p^k = 1$, giving

$$\mathcal{S}_{BH,min} \simeq \mathcal{V}_6^{\frac{1}{3}} \simeq N_{sp}$$

Species = KK modes of decompactification of a 2-cycle [Lee, Lerche, Weigand '19; Fierro Cota, Mininno, Weigand, Wiesner '22].
Smallest possible cycle on a simply connected CY_3.

Extremal BHs with R^2 corrections

Certain higher derivative corrections to supergravity are known. For example, we have

$$S_{corr} = \frac{1}{96\pi} \int \underbrace{c_{2i} t^i}_{F_1} R \wedge * R, \qquad c_{2i} = \int c_2(CY_3) \wedge \omega_i$$

from R^4 term in 11D [Antoniadis, Ferrara, Minasian, Narain '97].
It is one-loop and non-renormalized [Green, Gutperle '97].

There exist BPS black hole solutions with this correction. These are MSW black holes [Maldacena, Strominger, Witten '97].

The interaction S_{corr} can be supersymmetrized on a graviphoton background A [Cardoso, de Wit, Mohaupt '98]

$$F = \frac{1}{6} C_{ijk} \frac{X^i X^j X^k}{X^0} + c_{2i} \frac{X^i}{X^0} A \equiv F_0 + F_1 A$$

The modified entropy

In practice, one deals with 4d N=2 SUGRA with (X^Λ, A), $\Lambda = 0, 1, \ldots, n_V$. The attractor mechanism works the same [Behrndt, Cardoso, de Wit, Kallosh, Lüst, Mohaupt '96]

$$p^\Lambda = -2\,\mathrm{Im} X^\Lambda, \qquad q_\Lambda = -2\,\mathrm{Im}\partial_\Lambda F(X, A), \qquad A = -64.$$

Entropy is obtained with Wald formula [Wald '91]

$$\mathcal{S}_{BH} = 2\pi \int_{S^2} \frac{\delta \mathcal{L}}{\delta R_{\mu\nu\rho\sigma}} \epsilon_{\mu\nu}\epsilon_{\rho\sigma}$$

giving [Cardoso, de Wit, Mohaupt '98]

$$\mathcal{S}_{BH} = \pi \left[Z\bar{Z} + 4\mathrm{Im}\left(A\,\partial_A F(X, A)\right)\right]$$

Model-independent and valid for any $F(X, A)$.

The species scale and F_1

For black holes in IIA on CY_3, we find

$$\mathcal{S}_{BH} = \sqrt{\frac{q}{6}\left(C_{ijk}p^i p^j p^k + c_{2i}p^i\right)}$$

$$F_0 \qquad\qquad F_1$$

Entropy receives additive correction due to R^2 term.
Minimal entropy is now for $\frac{1}{6}C_{ijk}p^i p^j p^k = 0$ but $c_{2i}p^i \neq 0$, giving

$$\mathcal{S}_{BH,min} \simeq \sqrt{q c_{2i} p^i}$$

From the solution of the attractor equations, one finds

$$\sqrt{q c_{2i} p^i} \sim c_{2i} t^i \sim F_1$$

and thus we recover [Vafa, Van de Heisteeg, Wiesner, Wu '22]

$$N_{sp} \sim S_{BH,min} \sim F_1$$

Comments

- Our black hole argument employs SUGRA and can only detect large volume limit of F_1

  $$F_1 \sim c_{2i} t^i .$$

  To go beyond, one can use dualities [NC, Lüst to appear].

- A priori, the species scale we calculated in SUGRA is **unrelated** to the topological string and thus to [Vafa, Van de Heisteeg, Wiesner, Wu '22].
  Enforcing that the two scales are instead one and the same amounts to impose

  $$e^{\mathcal{F}_{BH}} = \mathcal{Z}_{BH} \equiv |\mathcal{Z}_{top}|^2 = e^{\mathcal{F}_{top} + \bar{\mathcal{F}}_{top}}$$

  This is the conjecture of [Ooguri, Strominger, Vafa '04], here recovered from the species scale.

We gave evidence that number of species has to be understood as entropy

$$N_{sp} = \mathcal{S}_{sp}$$

Can we take this analogy any further?

Can we define energy and temperature?

Is there a thermodynamical picture behind?

Species entropy and thermodynamics

[NC, Lüst, Montella '23]

Black hole thermodynamics

For the seminal work of Bekenstein and Hawking, we know that BHs can be described with the language of thermodynamics.

Consider Schwarzschild BH with radius R_{BH}. We have ($M_P = 1$)

$$M_{BH} = (R_{BH})^{d-3}$$
$$T_{BH} = (R_{BH})^{-1}$$
$$\mathcal{S}_{BH} = (R_{BH})^{d-2}$$

from which

$$\mathcal{S}_{BH} T_{BH}^{d-2} = 1$$

Temperature of species

From the previous relation $\mathcal{S}_{BH} T_{BH}^{d-2} = 1$, since $\mathcal{S}_{BH} \simeq N_{sp}$ it is natural to identify the **species temperature**

$$T_{sp} = \frac{1}{\mathcal{S}_{sp}^{\frac{1}{d-2}}} \equiv \Lambda_{sp}$$

- It is the temperature of a Schwarzschild BH with entropy $S_{sp} = N_{sp}$.
- It can be derived by tuning charges of non-extremal BHs in N=2 SUGRA, as those discussed in [NC, Dierigl, Gnecchi, Lüst, Scalisi '22].
- In a sense, it is a temperature of the moduli space of the EFT.

Energy of species

To find E_{sp}, let us proceed with a concrete example.

Consider a tower of N_{sp} species with step $\Delta E = \Lambda_{sp}/N_{sp}$.
The level k has energy $E_k = k\Delta E$ and the total energy of the tower is

$$E_{sp} = \sum_{k=1}^{N_{sp}} E_k \simeq \Lambda_{sp}^{3-d} = (\mathcal{S}_{sp})^{\frac{d-3}{d-2}}$$

- Similarly for a tower of string states, see [NC, Lüst, Montella '23]
- E_{sp} is the mass of a Schwarzschild BH with entropy $\mathcal{S}_{sp} = N_{sp}$
- T_{sp}, E_{sp} and $\mathcal{S}_{sp}$ obey the thermodynamic relation

$$\frac{1}{T} = \frac{\partial \mathcal{S}}{\partial E}$$

This is a non-trivial consistency check

- Given a tower of species, we identified its **entropy**, **temperature** and **energy**

- They obey standard thermodynamic relations

- We can formulate laws of **species thermodynamics**

- They allow for a thermodynamical analysis of the moduli space

The laws of species thermodynamics

- **Zero-th law**
  If two points in the moduli space have the same $\Lambda_{sp}(\phi)$, then they have the same $T_{sp}(\phi)$.
- **First law**
  Two neighboring stationary species towers are related by

$$\delta E_{sp} = T_{sp} \delta \mathcal{S}_{sp} + \dots$$

- **Second law**
  The species entropy does not increase when moving adiabatically towards the boundary of the moduli space

$$\delta \Lambda_{sp}(\phi) \leq 0, \qquad \delta \mathcal{S}_{sp}(\phi) \geq 0$$

- **Third law**
  It is impossible to reach the point $T_{sp} = 0$ with a finite sequence of steps.

Comments

- The 2nd law singles out a preferred direction over the moduli space, along which the number of species cannot decrease

$$\delta N_{sp} \geq 0$$

  For KK towers, this is towards large volume $\delta\mathcal{V} \geq 0$, while for string towers it is towards weak coupling $\delta g_s \leq 0$. Expected from **swampland conjectures** [Lee, Lerche, Weigand '19].

- The point $T_{sp} = 0$ is at infinite distance in the moduli space.

- If two towers of species coalesce, the final species scale is always less then the minimum of the initial ones

$$\Lambda_{sp_1+sp_2} \leq \min(\Lambda_{sp_1}, \Lambda_{sp_2})$$

Conclusions

- EFTs of gravity with N_{sp} species are valid at most at $\Lambda_{sp} = M_P / N_{sp}^{\frac{1}{d-2}}$
- In string theory, $N_{sp} = N_{sp}(\phi)$ and it has been proposed that this is related to the topological string free energy [Vafa, Van de Heisteeg, Wiesner, Wu '22]
- The proposal can be checked directly by studying black holes [NC, Lüst, Staudt '22]
- This strongly suggests that N_{sp} is an entropy
- One can formulate laws of species thermodynamics [NC, Lüst, Montella '23]

Future directions

- Understand moduli dependence of species scale with less than 8 supercharges, perhaps using [Martucci, Risso, Weigand '22]
- Understand better the role of the topological string and of the OSV conjecture
- Formulate species thermodynamics of Reissner-Nordstrom BHs
- How to interprete Q, pressure, ...?

Thank you!

Extra slides

A shortcut

One can arrive at the same result by using special geometry with coordinates (X^Λ, A) to rewrite the entropy as

$$\mathcal{S}_{BH} = \pi \left[e^{-K} + \frac{1}{6} c_{2i} \mathrm{Im} \frac{X^i}{X^0} \right]$$

Then, one can check that for the minimal charge configuration described above the two terms compete

$$e^{-K} \sim \frac{1}{6} c_{2i} \mathrm{Im} \frac{X^i}{X^0} \sim F_1$$

and thus

$$\mathcal{S}_{BH} = \pi \left[e^{-K} + \frac{1}{6} c_{2i} \mathrm{Im} \frac{X^i}{X^0} \right] \gtrsim F_1 \simeq \mathcal{S}_{BH,min}$$