String breaking in $N_{\mathrm{f}} = 2 + 1$ QCD

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- Introduction
- The static potential from lattice QCD
- String breaking with up, down and strange quarks
- Conclusions and Outlook

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Part I

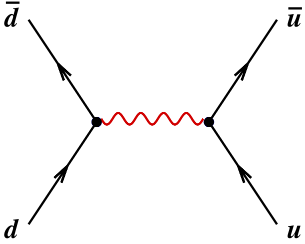
Introduction

Quantumchromodynamics (QCD)

QCD is the theory of strong interactions with the Lagrangian

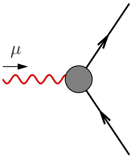
$$\mathcal{L}_{\mathrm{QCD}}(g_0, m_q) = -\frac{1}{2g_0^2}\mathrm{tr}\left\{F_{\mu\nu}F_{\mu\nu}\right\} + \sum_{q=u,d,s,c,b,t} \bar{q}\left(\gamma_\mu(\partial_\mu + A_\mu) + m_q\right) q$$

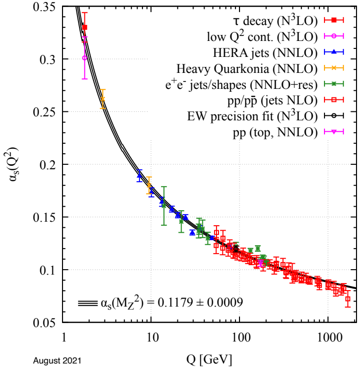
- bare parameters: gauge coupling g_0 and quark masses m_q
- Perturbation theory is an asymptotic expansion for small g_0



Asymptotic freedom

Renormalized coupling $\bar{g}$ and masses $\overline{m}$ depend on the renormalization scale μ and on the renormalization scheme

$\bar{g}(\mu) \quad =$ 



Asymptotic freedom for large μ ('t Hooft, 1972; Gross and Wilczek; Politzer, 1973):

$$\bar{g}^2(\mu) \overset{\mu\to\infty}{\sim} \frac{1}{2b_0 \log(\mu/\Lambda)}$$

[R.L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022)]

$\overline{\text{MS}}$ scheme, $\alpha_s = \bar{g}(\mu)^2/(4\pi)$

Confinement

Regimes of QCD

- Perturbation theory works at high energy $\gtrsim 2\,\mathrm{GeV}$ where α_s is small and quarks and gluons are (almost) free
- At lower energies, quarks and gluons are confined into hadrons $\Rightarrow$ we need a different tool to extract from $\mathcal{L}_{\mathrm{QCD}}$ the properties of hadrons: lattice formulation and computer simulations

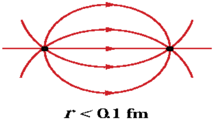
Probe of QCD at all scales

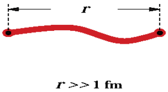
There is a quantity that can probe QCD in both regimes: it is the quark anti-quark static potential

Quark anti-quark static potential

Potential energy levels $V_n(r)$, $n = 0, 1, 2, \ldots$ of a quark-anti-quark static pair at distance r

Pure gauge theory (Yang-Mills, no fermions):





asymptotic freedom,
perturbation theory

confinement, flux tube
effective bosonic string theory

Short distance

Perturbation theory for $r \lesssim 0.1$ fm

$SU(N)$ + fermions: as $r \to 0$ perturbation theory can be applied (asymptotic freedom)

$$V(r) \overset{r \to 0}{\sim} -C_{\mathrm{F}} \frac{g_0^2}{4\pi r}, \quad C_{\mathrm{F}} = (N^2 - 1)/(2N)$$

Expansion of $V(r)$ in the $\overline{\mathrm{MS}}$ coupling is known to 3 loops [N. Brambilla, A. Pineda, J. Soto, A. Vairo, hep-ph/9907240, hep-ph/9903355; A. V. Smirnov, V. A. Smirnov and M. Steinhauser, 0911.4742; C. Anzai, Y. Kiyo and Y. Sumino, 0911.4335]

Large distance

$SU(N)$ Yang-Mills

As $r \to \infty$ the potential can be computed using an *effective bosonic string theory* [Y. Nambu, 1979; M. Lüscher, K. Symanzik, P. Weisz, 1980; M. Lüscher, 1981; M. Lüscher and P. Weisz, hep-th/0406205]: the string describes a flux tube joining the static sources and fluctuating in $d-2$ transverse directions

$$V(r) = \sigma r + \mu + \frac{\gamma}{r} + O(1/r^3)$$

σ is the string tension, μ a mass parameter; $\gamma = -\pi(d-2)/24$ depends only on the number of dimensions d

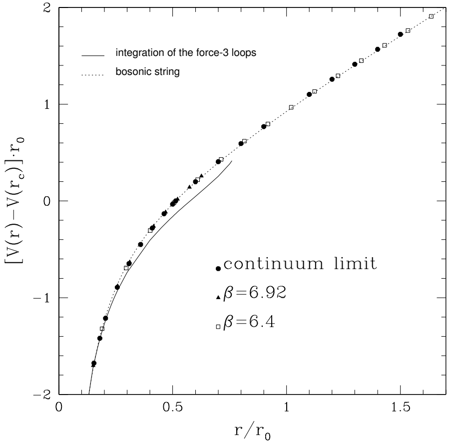
Broadening of the string: its width increases logarithmically in the distance r [M. Lüscher, G. Münster and P. Weisz, 1981]

Static force

$$F(r) = \frac{\mathrm{d}V}{\mathrm{d}r}$$

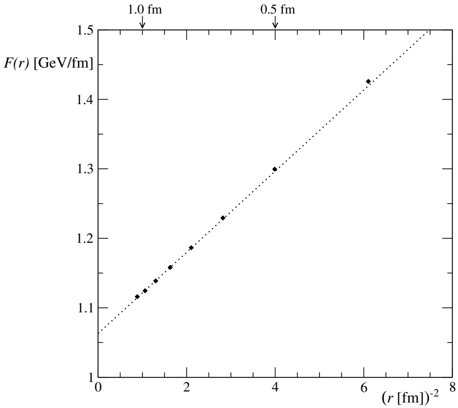
It is a renormalized quantity.

Static potential in $SU(3)$ Yang-Mills



[S. Necco and R. Sommer, `hep-lat/0108008`] continuum limit, $r_0 = 0.5$ fm [R. Sommer, `hep-lat/9310022`]

Static force in $SU(3)$ Yang-Mills



[M. Lüscher and P. Weisz, hep-lat/0207003] multi-level algorithm, $\beta = 6.0$.
Bosonic string:

$$F(r) = \sigma + \frac{\pi}{12r^2} + \mathrm{O}(1/r^4) \quad (r \to \infty)$$

For the ground state the string behavior already sets in for $r \geq 0.5 fm$

Large distance with fermions

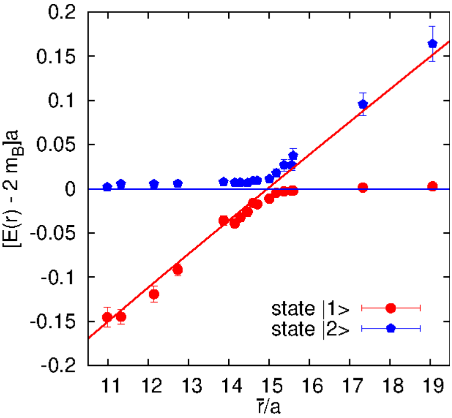
With dynamical (sea) quarks:

Around $r_c \approx 1.5\,\mathrm{fm}$ formation of two static-light mesons (string breaking)

String breaking

- String breaking describes the flattening of the potential $V_0(r)$ at large r
- *"The ground state potential $V_0(r)$ can therefore be called a static quark potential or a static meson potential"* [Sommer, Phys. Rept. 275(1) (1996)]
- Estimate $r_c \approx 1.5\,\mathrm{fm}$ from $V_0(r_c) = 2E_{\mathrm{stat-light}}$ [Alexandrou et al., Nucl.Phys. B414 (1994)]
- It has been observed as a mixing of "string-like" and two-meson operators [Drummond, 9805012; Philipsen and Wittig, 9807020; Knechtli and Sommer, 9807022, 0005021; Bali et al., 0505012, Bulava et al., 1902.04006]

Observation of string breaking in $N_\mathrm{f} = 2$ QCD



[SESAM, Bali, Neff, Düssel, Lippert, Schilling, Phys.Rev.D 71 (2005)]

$N_\mathrm{f} = 2$, 40×24^3, $m_\pi \approx 400\,$MeV,
$a \approx 0.083\,$fm
Minimal energy gap

A simple model of mixing [Knechtli, Günther, Peardon, Lattice Quantum Chromodynamics: Practical Essentials, Springer, 2017]

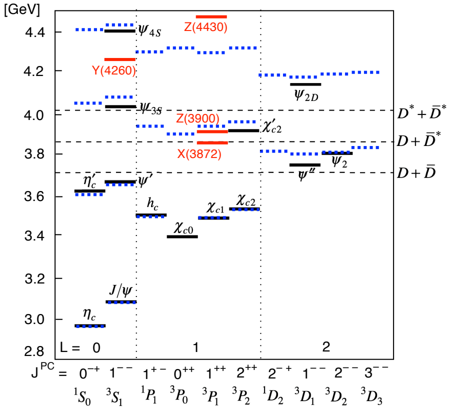
$$H(r) = \begin{pmatrix} \sigma r & g \\ g & 2E_\mathrm{stat-light} \end{pmatrix}$$

σr: linearly rising energy level;
$2E_\mathrm{stat-light}$: energy of a static-light meson pair; g: mixing.
In the string breaking region, where $\sigma r \approx 2E_\mathrm{stat-light}$ the energy levels are approximately

$$2(E_\mathrm{stat-light} \pm g)$$

“avoided level crossing” ($g \neq 0$)

Relation to quarkonium



charmonium, exotic XYZ states, quark model

[Hosaka et al., PTEP 2016 (2016) 6, 062C01]

Born-Oppenheimer approximation

- Cornell potential [Eichten et al., Phys.Rev.D 21 (1979)]

$$V_0(r) = -\frac{\kappa}{r} + \sigma r$$

- Schrödinger equation ($m_Q\,,\, m_Q v \gg \Lambda_{\mathrm{QCD}},\ v \ll 1$)

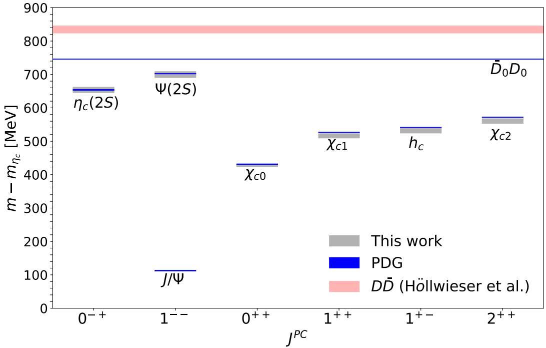
$$\left(-\frac{1}{2\mu}\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \frac{l(l+1)}{2\mu r^2} + V_0(r)\right) u(r) = E u(r)$$

  $\mu = m_Q/2$ from quark models [Godfrey and Isgur, Phys.Rev. D32 (1985); Capitani et al., Phys.Rev.D 99 (2019)]

  Quarkonium levels E

- Relativistic spin corrections to V_0 [Eichten and Feinberg, Phys.Rev.D 23 (1981)]

Charmonium spectrum



PhD thesis of J. A. Urrea-Niño, based on [Knechtli, Korzec, Peardon, Urrea-Niño, 2205.11564]

FOR 5269 https://confluence.desy.de/display/for5269

Wuppertal – DESY, Zeuthen – Dublin

We develop and apply new numerical techniques for spectroscopy computations of charmonium, glueballs and (hybrid) static potentials

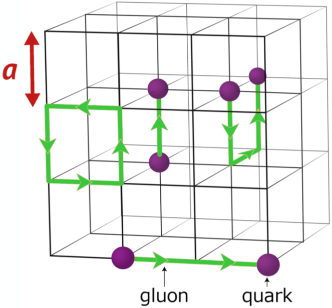
Part II

The static potential from lattice QCD

Lattice QCD setup

Numerical simulations of QCD using Monte Carlo methods

- well-established framework for non-perturbative QCD
- ab-initio calculations, action only input
- discretization of space-time,
- introduce lattice spacing a
- gluons, $SU(3)$ link variables
- quarks, covariant derivatives
- Dirac operator D, quark propagator D^{-1}
- discretized forms must reduce to continuum form in the limit $a \to 0$



Lattice QCD setup contd

Monte Carlo methods: statistical treatment of the theory

- create gluon configurations using QCD action
- average over configurations, error
- we need 100s to 1000s of (statistically independent) configurations
- observables: correlation functions in terms of “quark propagators”
- building block of hadronic measurements on the lattice
- solution of the Dirac equation $Dx_i = v_i$
  most intensive part of calculations
- very large (190Mx190M), but sparse matrix (most elements zero)
- highly optimized algorithms with good scaling behavior

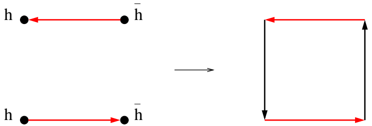
Distillation [Hadron Spectrum, Peardon et al, 0905.2160]

- quark field smearing with Laplacian eigenmodes $v_i[t]$
- we need inversions to get “quark perambulators” $v_i[t_1]^\dagger D^{-1}_{\alpha\beta} v_j[t_2]$
- stochastic representation of the perambulator [Morningstar et al, 1104.3870]

Wilson loops

Creation operator for a static quark at position $\vec{x}$ and a static anti-quark at $\vec{y}$ ($R = |\vec{r}| = |\vec{y} - \vec{x}|$) at $t = t_0$; annihilation operator at $t = t_1$, $T = |t_1 - t_0|$:

$$\begin{aligned} O(\vec{x}, \vec{y}) &= \overline{\psi}_h(\vec{x}) U_s(\vec{x}; \vec{y}) \gamma_5 \psi_{\bar{h}}(\vec{y}) \\ \bar{O}(\vec{y}, \vec{x}) &= -\overline{\psi}_{\bar{h}}(\vec{y}) U_s^\dagger(\vec{x}; \vec{y}) \gamma_5 \psi_h(\vec{x}) \end{aligned}$$



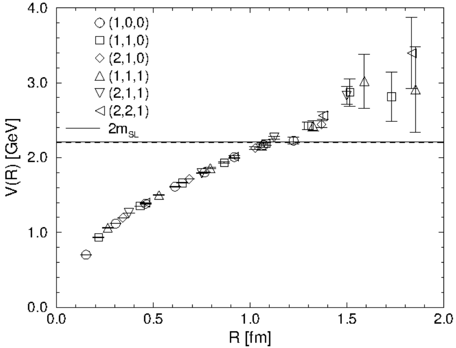
After integration over the heavy quarks ψ_h, $\psi_{\bar{h}}$ obtain a product of gauge links along a rectangle $\longrightarrow$ Wilson loop

$$\langle W(R, T) \rangle_{U, \psi, \overline{\psi}} \overset{N_t \to \infty}{\sim} \sum_n c_n c_n^* \mathrm{e}^{-V_n(r) T}$$

Static quark anti-quark potential

$$V_0(R) = -\lim_{T \to \infty} \partial_T \ln(\langle W(R, T) \rangle)$$

An overlap problem



$N_f = 2$, 32×16^3, $m_\pi \approx 460\,\text{MeV}$, $a \approx 0.15\,\text{fm}$

[Burkhalter for CP-PACS, Nucl.Phys.B Proc.Suppl. 73 (1999), LATTICE 98, Boulder]

Wilson loops have a tiny overlap with the broken string (need impractically large T). Mixing with static-light mesons is necessary to see string breaking.

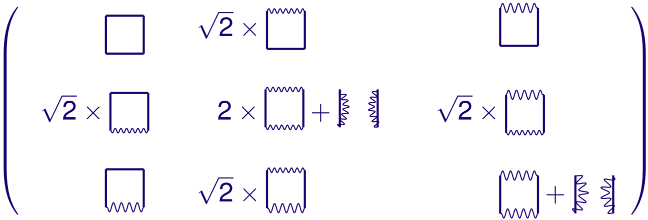
Part III

String breaking with up, down and strange quarks

Mixing matrix

N_f=2+1 QCD: degenerate up, down (light) quarks and a strange quark
static-light "B" and static strange "B_s" mesons

$$C(\mathbf{r},t) = \begin{pmatrix} \langle \mathcal{O}_W(t)\overline{\mathcal{O}}_W(0)\rangle & \langle \mathcal{O}_{B\bar{B}}(t)\overline{\mathcal{O}}_W(0)\rangle & \langle \mathcal{O}_{B_s\bar{B}_s}(t)\overline{\mathcal{O}}_W(0)\rangle \\ \langle \mathcal{O}_W(t)\overline{\mathcal{O}}_{B\bar{B}}(0)\rangle & \langle \mathcal{O}_{B\bar{B}}(t)\overline{\mathcal{O}}_{B\bar{B}}(0)\rangle & \langle \mathcal{O}_{B_s\bar{B}_s}(t)\overline{\mathcal{O}}_{B\bar{B}}(0)\rangle \\ \langle \mathcal{O}_W(t)\overline{\mathcal{O}}_{B_s\bar{B}_s}(0)\rangle & \langle \mathcal{O}_{B\bar{B}}(t)\overline{\mathcal{O}}_{B_s\bar{B}_s}(0)\rangle & \langle \mathcal{O}_{B_s\bar{B}_s}(t)\overline{\mathcal{O}}_{B_s\bar{B}_s}(0)\rangle \end{pmatrix}$$

$=$ 

- Two string-like operators
- Off-axis separations to increase the resolution in the string breaking region [Bresenham, 1965; Bolder et al., hep-lat/0005018]

CLS ensembles

$N_\mathrm{f} = 2+1$ ensembles from **Coordinated Lattice Simulations (CLS)**: CERN, DESY/NIC, Dublin, Berlin HU, Mainz, Madrid, Milan, Münster, Odense/CP^3-Origins, Regensburg, Roma-La Sapienza, Roma-Tor Vergata, Valencia, Wuppertal [M. Bruno et al., 1411.3982]

id	N_{conf}	N_{conf}^W	t_0/a^2	N_s	N_t	m_π[MeV]	m_K[MeV]	$m_\pi L$
N203	94	752	5.1433(74)	48	128	340	440	5.4
N200	104	1664	5.1590(76)	48	128	280	460	4.4
D200	209	1117	5.1802(78)	64	128	200	480	4.2

- lattice spacing $a = 0.06426(76)$ fm [M. Bruno, T. Korzec, S. Schaefer, 1608.08900]
- quark masses $m_\mathrm{u} = m_\mathrm{d} = m_l$ and m_s vary along $\sum_{f=\mathrm{u,d,s}} m_{\mathrm{bare},f}$=const.
- quark-mass parameter

$$\mu_l = \frac{3}{2}\frac{m_\pi^2}{\frac{1}{2}m_\pi^2 + m_K^2} \approx \frac{3}{2}\frac{2m_l}{2m_l + m_\mathrm{s}} \propto m_l$$

$N_\mathrm{f} = 3$ symmetric point: $\mu_l = 1$
physical point: $\mu_l = 0.1076$ ($m_\pi = 134.8$ MeV, $m_K = 494.2$ MeV isospin limit)

Extraction of the energy levels

Generalized Eigenvalue Problem (GEVP)

- For fixed inter-quark separation **r** solve the GEVP [Lüscher and Wolff, Nucl.Phys.B 339(1) (1990); Blossier et al., 0902.1265]

$$\begin{aligned} C(t)\, v_n(t,t_0) &= \lambda_n(t,t_0)\, C(t_0)\, v_n(t,t_0)\,, \quad n=0,1,\ldots,3\,, \quad t \geq t_0 \\ \lambda_n(t,t_0) &\simeq \exp\{-V_n(t-t_0)\} \end{aligned}$$

  We use $t_0 = 5$.

- We solve GEVP at time $t_d = 10$, project the correlator

$$\hat{C}_{ij} = (v_i(t_0,t_d), C(t) v_j(t_0,t_d))$$

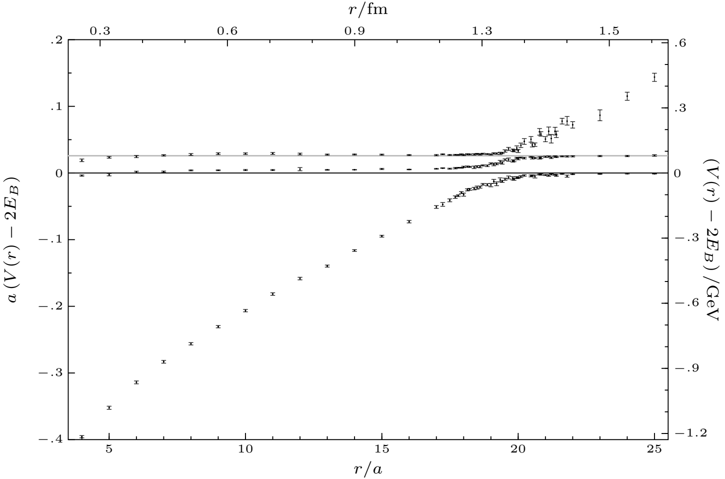
  and build the ratios

$$R_n(t) = \frac{\hat{C}_{nn}(t)}{C_B^2(t)}\,, \quad n = 0,1,2$$

  (C_B is the correlator of a single static-light meson, energy E_B)
  Correlated single-exponential fit yields $R_n = \mathrm{const} \times \exp[-t(V_n - 2E_B)]$

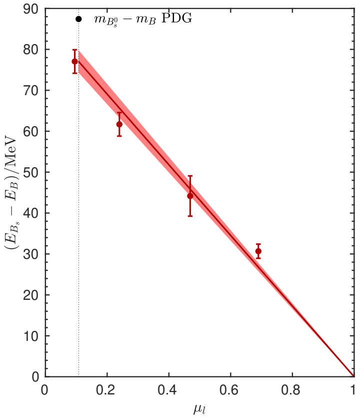
Potential levels

Lowest three levels of the static potential on N200 ($m_\pi = 280\,\text{MeV}$) with the thresholds twice the static-strange and twice the static-light meson masses



[Bulava, Hörz, Knechtli, Koch, Moir, Morningstar, Peardon, Phys.Lett.B 793 (2019)]

Static-light and static-strange mesons



At the physical point (preliminary [Knechtli, Koch, Peardon et al.])

$$E_{B_s} - E_B = 77.0(2.9)\ MeV \quad \text{(including the lattice spacing error)}$$

PDG: $m_{B_s^0} - m_B = 87\,\text{MeV}$; $m_{D_s^\pm} - m_{D^0} = 105\,\text{MeV}$

Model

Model Hamiltonian

$$H(r) \quad = \quad \begin{pmatrix} \hat{V}(r) & g_1 & g_2 \\ g_1 & \hat{E}_1 & 0 \\ g_2 & 0 & \hat{E}_2 \end{pmatrix}, \quad \hat{V}(r) = \hat{V}_0 + \sigma r$$

- *Correlated fit* to spectrum with 6 parameters: $a\hat{E}_1$, $a\hat{E}_2$, ag_1, ag_2, $a^2\sigma$, $a\hat{V}_0$
- $\hat{V}(r)$, $\hat{E}_1$, $\hat{E}_2$ are the asymptotic energy levels for $r \to \infty$ up to O(r^{-1})
- Notice: string tension σ is defined including the effects of *string breaking* (ground state potential is nowhere linear)

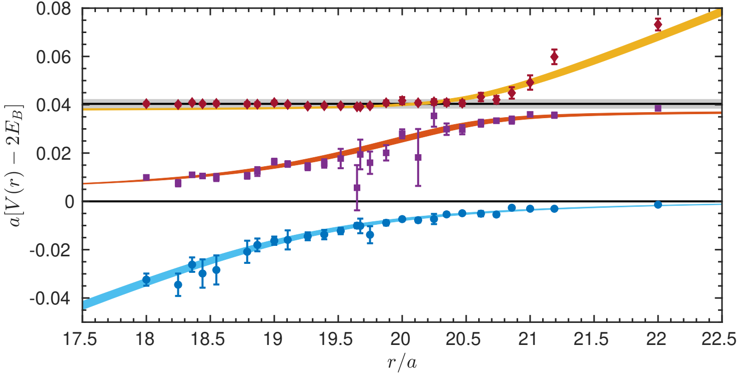
String breaking distances

Definition by minimum energy gap does not work (unlike for $N_\mathrm{f} = 2$)
Two string breaking distances r_c and r_{c_s} defined by the crossings

$$\hat{V}(r_c) = \hat{E}_1 \text{ (static-light)} \quad \text{and} \quad \hat{V}(r_{c_s}) = \hat{E}_2 \text{ (static-strange)}$$

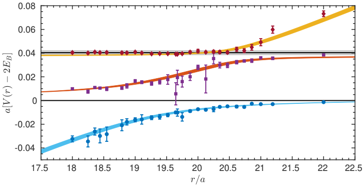
Model fits

D200 ($m_\pi = 200$ MeV)

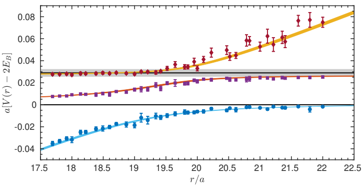


Model fits contd

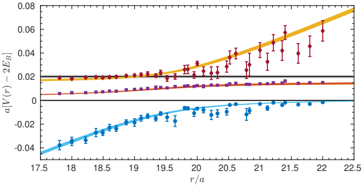
D200 ($m_\pi = 200$ MeV)



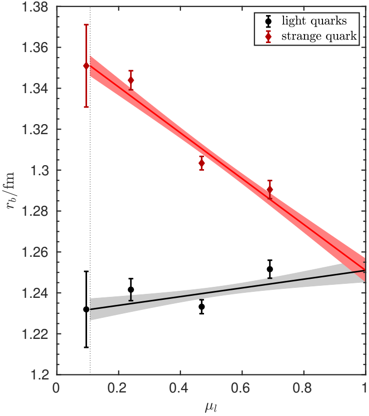
N200 ($m_\pi = 280$ MeV)



N203 ($m_\pi = 340$ MeV)



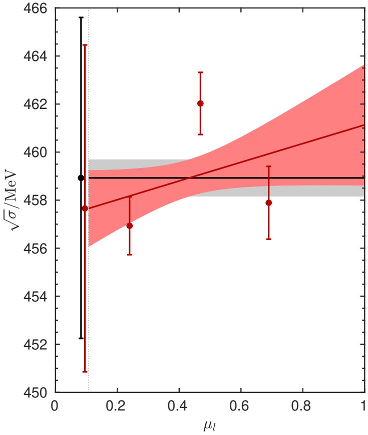
String breaking distances



Constrained extrapolation ($r_c = r_{c_s}$ at $\mu_l = 1$) linear in μ_l. Larger errors include the lattice spacing error. At the physical point (preliminary [Knechtli, Koch, Peardon et al.])

$$r_c = 1.232(19)\,\text{fm (light)}\,, \quad r_{c_s} = 1.351(20)\,\text{fm (strange)}$$

String tension

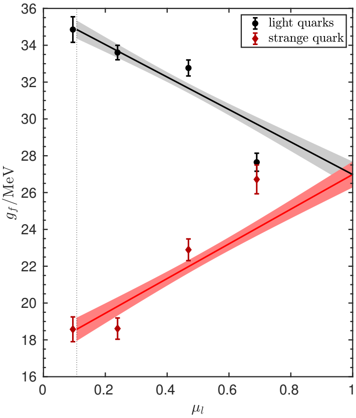


At the physical point (preliminary [Knechtli, Koch, Peardon et al.])

$$\sqrt{\sigma} = 458(7)\,\mathrm{MeV} \quad \text{linear extrapolation}$$

TUMQCD [Brambilla et al., 2206.03156], $N_\mathrm{f} = 2 + 1 + 1$: $\sqrt{\sigma} = 467(7)$ or $481(7)$ MeV

Mixing



Mixing per flavour $g_l = g_1/\sqrt{2}$ and $g_s = g_2$. Constrained extrapolation ($g_l = g_s$ at $\mu_l = 1$) linear in μ_l. At the physical point (preliminary [Knechtli, Koch, Peardon et al.])

$$g_l = 34.9(7)\,\mathrm{MeV}\ (\mathrm{light})\,, \quad g_s = 18.6(7)\,\mathrm{MeV}\ (\mathrm{strange})$$

Part IV

Conclusions and Outlook

Conclusions

- Computed three lowest levels of the static potential up to a separation $r = 1.6\,\mathrm{fm}$ in QCD with dynamical up, down and strange quarks
- Potential levels *in the region of string breaking* can be well fitted by a simple six parameter model $\longrightarrow$ robust definition of the string tension
- We have results for three sets of quark masses
- Extrapolations of the string breaking distances to the physical point

$$r_c = 1.232(19)\,\mathrm{fm}\ (\mathrm{light}) \quad r_{c_s} = 1.351(20)\,\mathrm{fm}\ (\mathrm{strange}) \quad \text{(preliminary)}$$

- Extrapolation of the string tension to the physical point

$$\sqrt{\sigma} = 458(7)\,\mathrm{MeV} \quad \text{(preliminary)}$$

- At the physical point the mixing coefficient *per* light quark is about twice as large as for the strange quark
- Our model of string breaking can be used as input to study quarkonia above threshold and heavy-light and heavy-strange coupled-channel meson scattering
- Outlook: hybrid potentials, Laplacian trial states [Höllwieser et al., 2212.08485]