



Primordial Gravitational Waves and Pulsar Timing Array Data

E.M., Morgante, Puchades Ibáñez, Ramberg, Ratzinger, Schenk, and Schwaller
JHEP **10** (2023), 171 [arXiv:2306.14856 [hep-ph]]

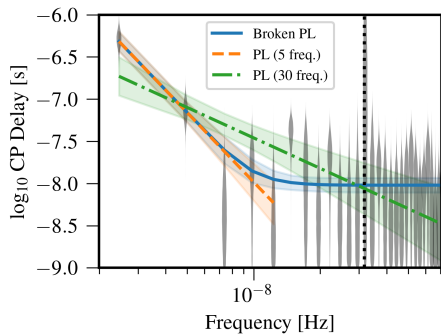
Eric Madge

DESY – November 1, 2023

Motivation

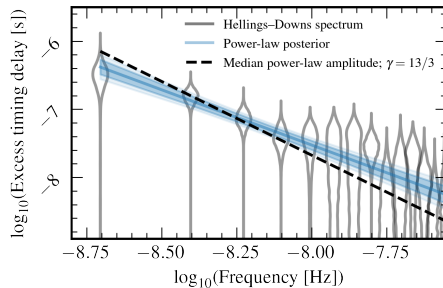
NANOGrav, 12.5 year dataset

[NANOGrav (APJL 2020)]



NANOGrav, 15 year dataset

[NANOGrav (APJL 2023)]



SGWB

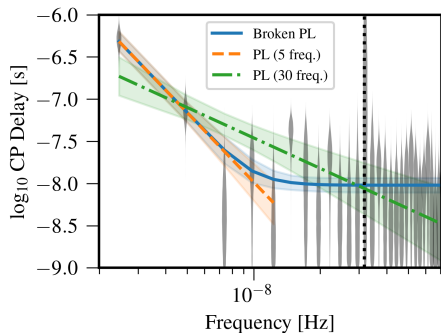
astrophysical?

cosmological?

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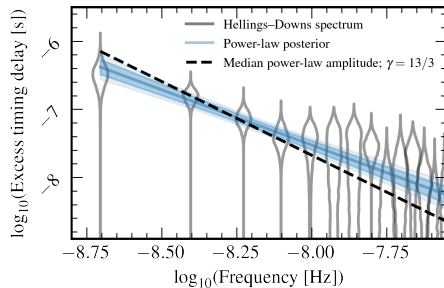
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NANOGrav, 15 year dataset

[NANOGrav (APJL 2023)]



SGWB

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cosmological?

here: ○ benchmark models
○ cosmological constraints

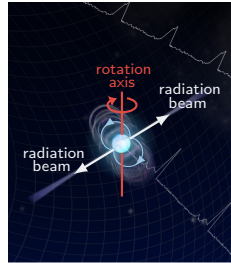
1. Pulsar Timing Arrays
2. Cosmological Phase Transitions
3. Domain Walls
4. Bosonic Instabilities
5. Conclusions

Pulsar Timing Arrays

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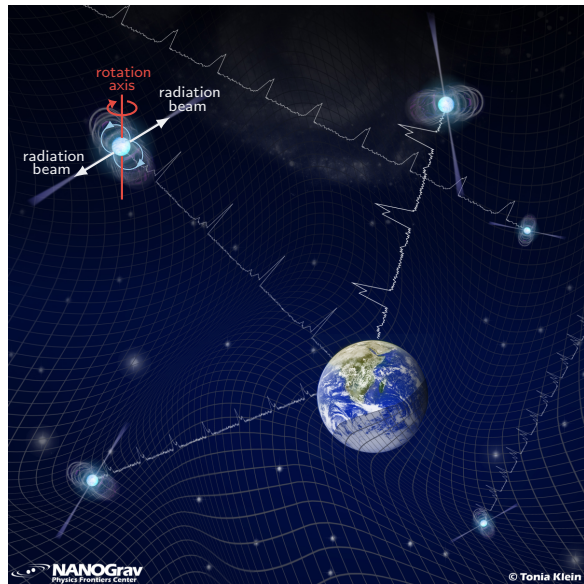
- **pulsar** = rapidly rotating neutron star
⇒ cosmic lighthouse



[Credit: T. Klein, NANOGrav; modified]

Pulsar Timing Arrays

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- **pulsar timing arrays (PTAs)**
GW detection via correlations in timing residuals ⇒ $f \sim \text{nHz}$

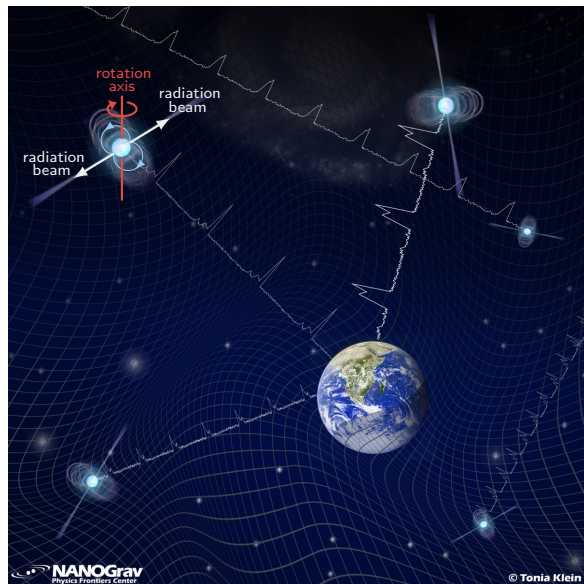


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Pulsar Timing Arrays

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GW detection via correlations in timing residuals ⇒ $f \sim \text{nHz}$
- **operating PTAs:**

collaboration	N_P	T_{obs}	ref.
EPTA	25	25 yrs	[A&A 2023]
NANOGrav	67	15 yrs	[APJL 2023]
PPTA	30	18 yrs	[PASA 2023]
InPTA	14	3.5 yrs	[PASA 2022]
CPTA	57	3.5 yrs	[in prep.]
IPTA	65	20 yrs	[MNRAS 2019]



[Credit: T. Klein, NANOGrav; modified]

PTA Signal

○ since Sept. 2020:

strong evidence for **common red-noise process**

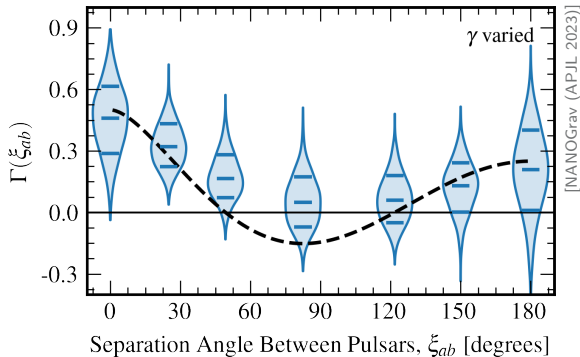
[NANOGrav (APJL 2020); PPTA (APJL 2021); EPTA (MNRAS 2021); IPTA (MNRAS 2022)]

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strong evidence for **Hellings-Downs correlations**

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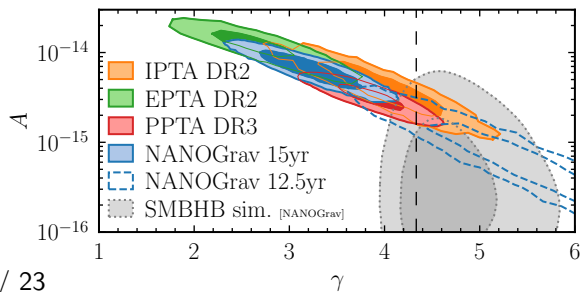
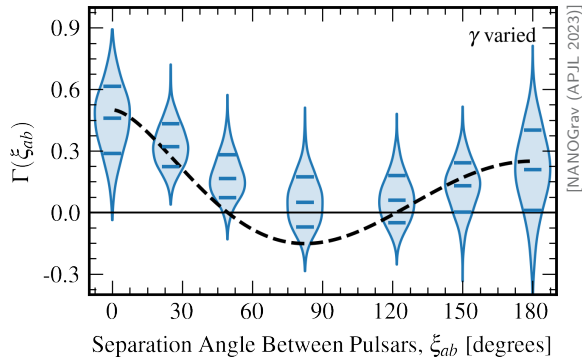
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⇒ **GW background**

- consistent with expected SGWB from **SMBHBs**:

$$h_c = A \left(\frac{f}{1 \text{ yr}^{-1}} \right)^{\frac{3-\gamma}{2}} \quad \text{with } \gamma = \frac{13}{3}$$



Primordial Sources of SGWBs

phase transitions, cosmic strings, domain walls, bosonic instabilities, inflation, ...

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○ amplitude: $\rho_{\text{GW}} \sim M_{\text{pl}}^2 \langle \dot{h}_{ij} \dot{h}_{ij} \rangle, \quad \square \bar{h}_{\mu\nu} \sim \frac{T_{\mu\nu}}{M_{\text{pl}}^2}$

$$\Rightarrow \Omega_{\text{GW}} \sim \left(\frac{a_*}{a_0}\right)^4 \frac{1}{\rho_c} \frac{\rho_{\text{source}}^2}{f_*^2 M_{\text{pl}}^2} \sim \left(\frac{a_*}{a_0}\right)^4 \left(\frac{H_0}{H_*}\right)^2 \left(\frac{H_*}{f_*}\right)^2 \Omega_{\text{source}}^2$$

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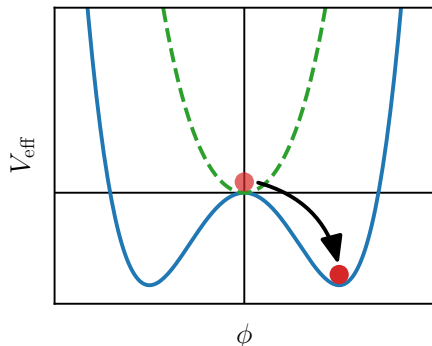
$$\Rightarrow \text{for PTAs: } T_* \sim \text{few MeV}, \quad \Omega_{\text{source}} \gtrsim 0.1$$

Cosmological Phase Transitions

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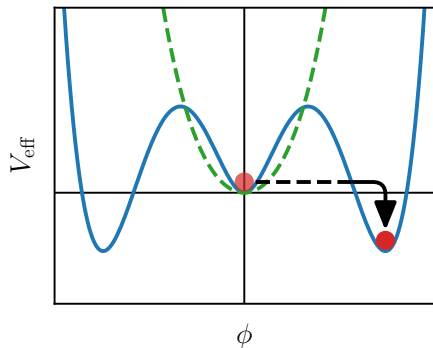
Cosmological Phase Transitions

- thermal corrections typically restore spontaneously broken symmetries at high temperatures
⇒ symmetry breaking phase transition
- can be **crossover** or first-order



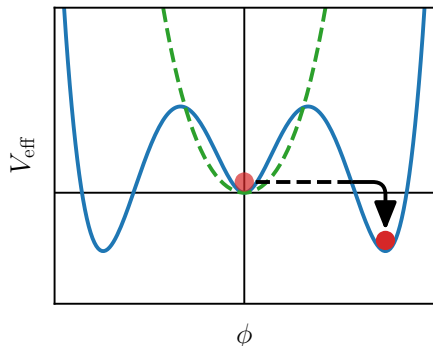
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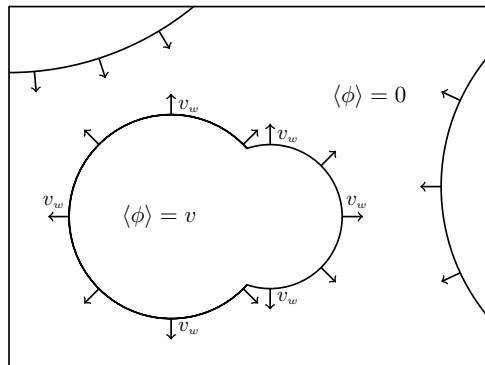
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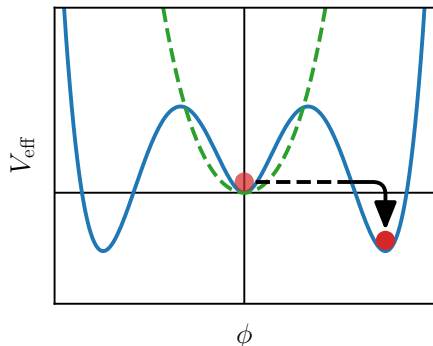
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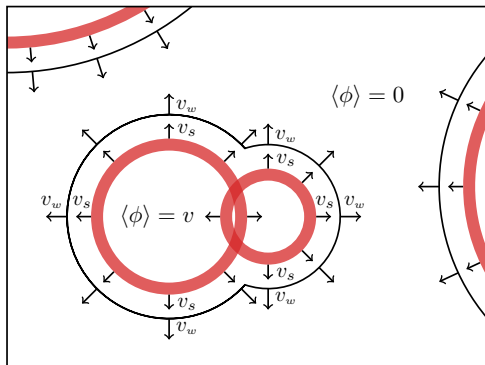
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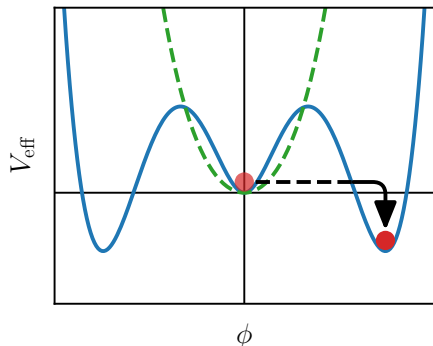
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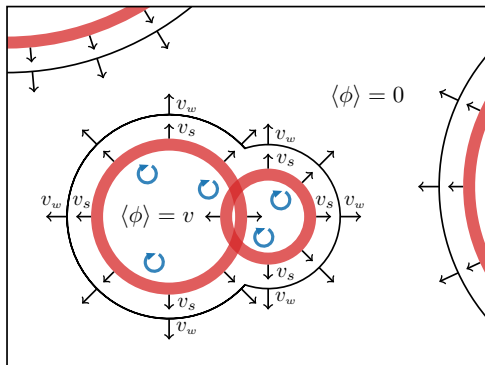
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GW production:

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2. sound waves collisions
3. turbulence and vortical motion



Gravitational Wave Spectrum

- peak frequency:

$$f_p \propto T_* \frac{1}{R_* H_*} \sim 10 \text{ nHz} \frac{T_*}{10 \text{ MeV}} \frac{\beta/H_*}{10}$$

$$\alpha \approx \frac{\Delta V}{\rho R}, \quad \beta = \frac{\dot{\Gamma}}{\Gamma} \approx \frac{3}{R_*}$$

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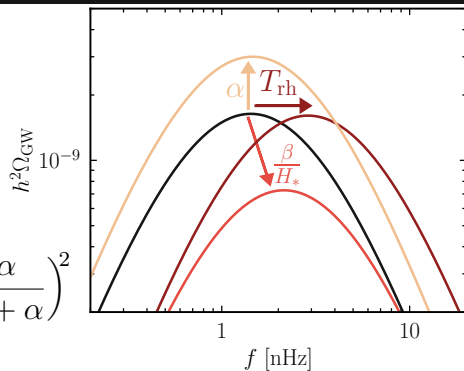
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- spectral shape: [from Lewicki, Vaskonen (EPJC 2023)]

$$\Omega_{\text{GW}} = \Omega_p \frac{(a + b)^c}{\left[b (f/f_p)^{-a/c} + a (f/f_p)^{b/c} \right]^c},$$

$$a = 2.41, \quad b = 2.42, \quad c = 4.08$$



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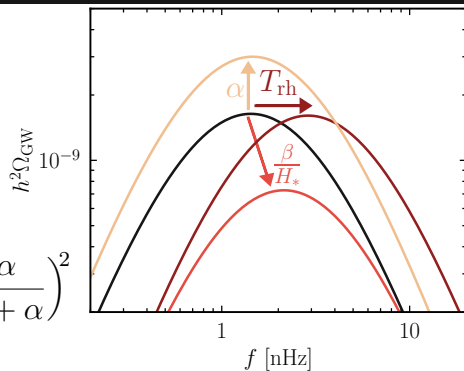
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Gravitational Wave Spectrum

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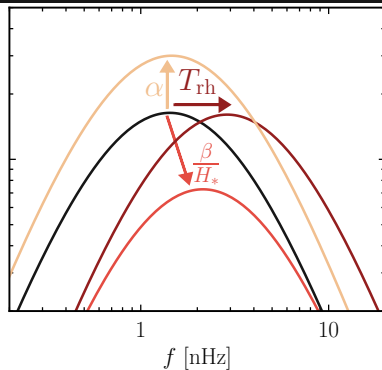
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\implies supercooled PTs \implies nearly conformal models



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Coleman-Weinberg Model

- classically scale-invariant gauged U(1) model

[Coleman, Weinberg (PRD '73)]

$$\mathcal{L} \supset -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + D_{\mu}\phi^{\dagger}D^{\mu}\phi - \frac{\lambda}{4}(\phi^{\dagger}\phi)^2$$

- 3 parameters: g, λ, M

$$M = \frac{\mu_R}{4\pi} e^{\gamma_E - \frac{1}{3}}$$



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
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 $m_A \propto g\langle\phi\rangle \propto M$

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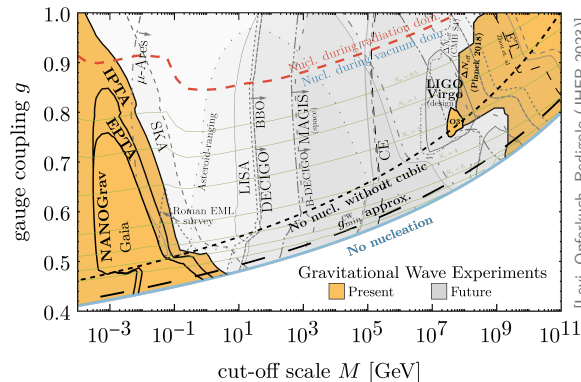
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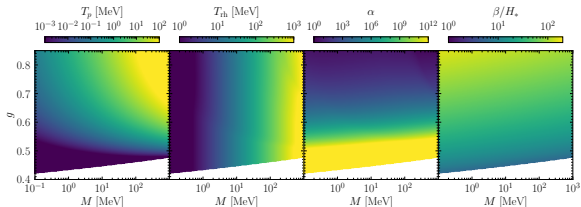
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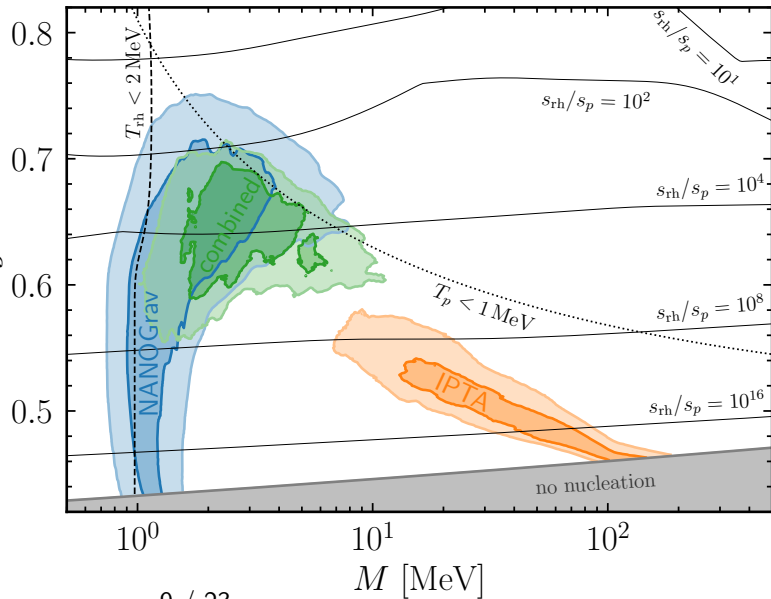
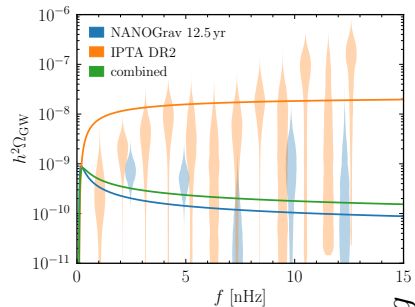
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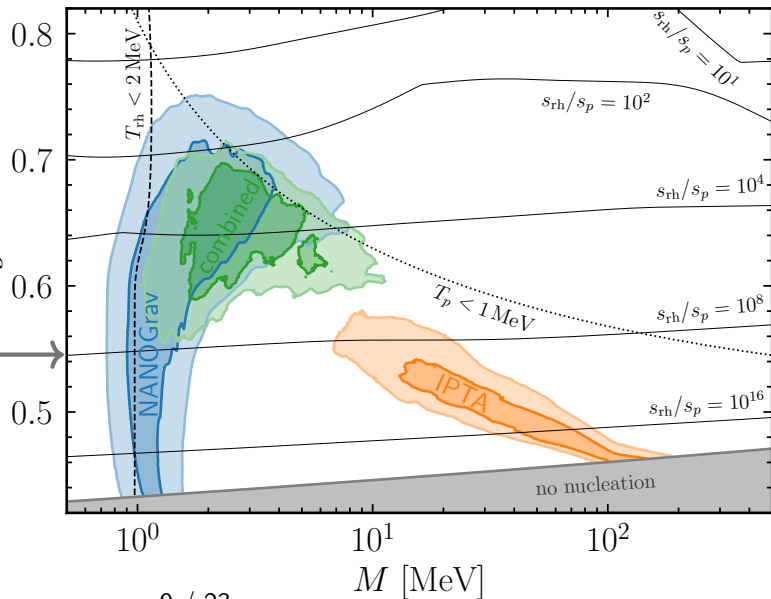
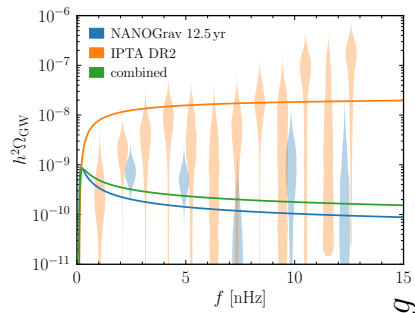
[Levi, Opferkuch, Redigolo (JHEP 2023)]



PTA fit - PhT in CW Model

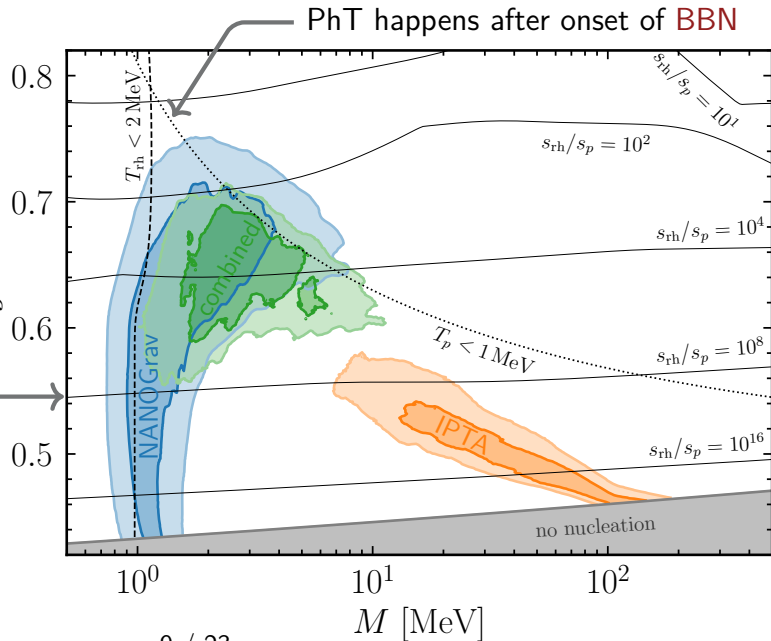
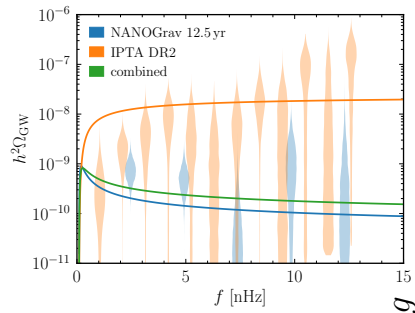


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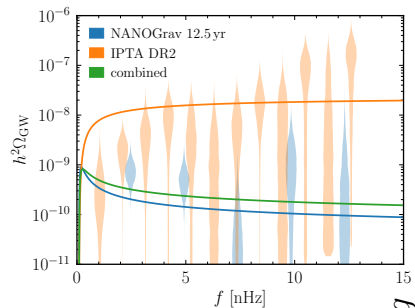
BAU: $\mathcal{O}(1)$ baryon asym. required before reheating (if $\frac{s_{\text{rh}}}{s_p} \sim 10^{10}$)

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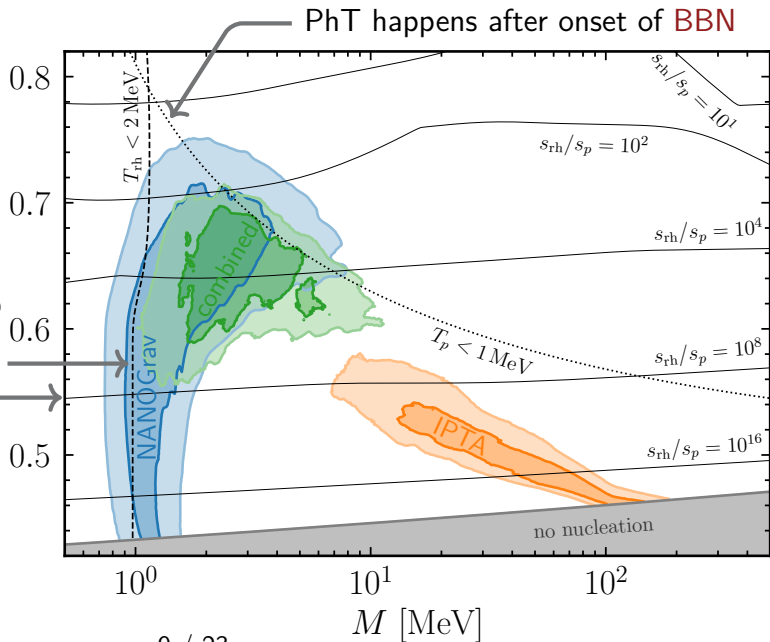
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ν -rethermalization

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Reheating

○ reheating: $\Delta V \longrightarrow \rho_R \implies$ requires $\Gamma_\phi \gtrsim H_{\text{rh}} \sim \frac{T_{\text{rh}}^2}{M_{\text{pl}}} \gtrsim 4 \times 10^{-20} \text{ MeV}$

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 - violates scale invariance
- direct coupling to e, γ : $\mathcal{L} \supset c_e \frac{|\phi|^2}{\Lambda^2} LH\bar{e} + c_\gamma \frac{|\phi|^2}{\Lambda^2} F_{\mu\nu}F^{\mu\nu}$ ✓
 - does not violate scale invariance
 - $\phi \rightarrow e^+e^-, \gamma\gamma$ searches e.g. at MESA

Domain Walls

1. Pulsar Timing Arrays
2. Cosmological Phase Transitions
3. **Domain Walls**
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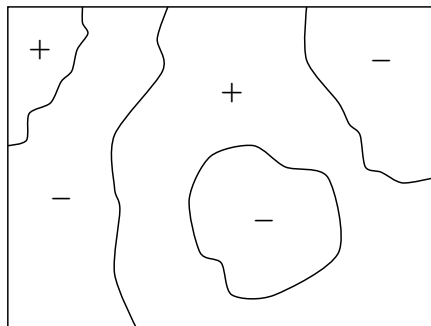
Domain Walls

- from spontaneously broken discrete symmetry (e.g. Z_2) after inflation
- DWs stretched to horizon size if friction negligible

⇒ **scaling regime:** ~ 1 DW/Hubble

$$\rho_{\text{DW}} \propto \frac{\sigma R^2}{R^3} \sim \sigma H$$

- **DW domination:** $\Omega_{\text{DW}} = \frac{\rho_{\text{DW}}}{3M_{\text{pl}}^2 H^2} \sim \frac{\sigma}{M_{\text{pl}} T^2} \Rightarrow T_{\text{dom}} \sim \sqrt{\frac{\sigma}{M_{\text{pl}}}}$



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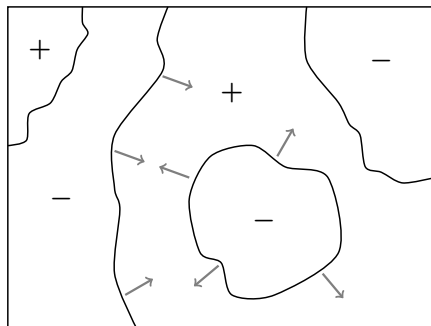
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- need to decay ⇒ add bias V_b

⇒ **DWs annihilate** when $V_b \sim \frac{\sigma}{R} \Rightarrow T_{\text{ann}} \sim 20 \text{ MeV} \sqrt{\frac{\text{TeV}^3}{\sigma} \frac{V_b}{\text{MeV}^4}}$



Domain Walls

- from spontaneously broken discrete symmetry (e.g. Z_2) after inflation
- DWs stretched to horizon size if friction negligible

⇒ **scaling regime**: ~ 1 DW/Hubble

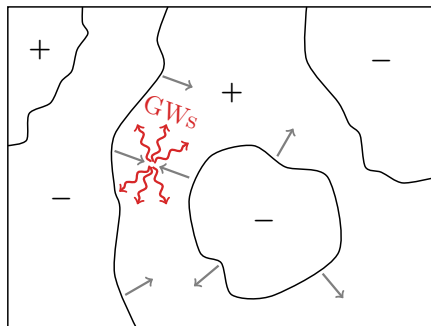
$$\rho_{\text{DW}} \propto \frac{\sigma R^2}{R^3} \sim \sigma H$$

- **DW domination**: $\Omega_{\text{DW}} = \frac{\rho_{\text{DW}}}{3M_{\text{pl}}^2 H^2} \sim \frac{\sigma}{M_{\text{pl}} T^2} \Rightarrow T_{\text{dom}} \sim \sqrt{\frac{\sigma}{M_{\text{pl}}}}$

- need to decay ⇒ add bias V_b

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- **GWs** dominantly produced in DW collisions around T_{ann}



Gravitational Wave Spectrum

GW production at T_{ann}

○ peak frequency:

$$f_p \sim \frac{a}{a_0} \frac{1}{R} \sim 1 \text{ nHz} \frac{T_{\text{ann}}}{10 \text{ MeV}}$$

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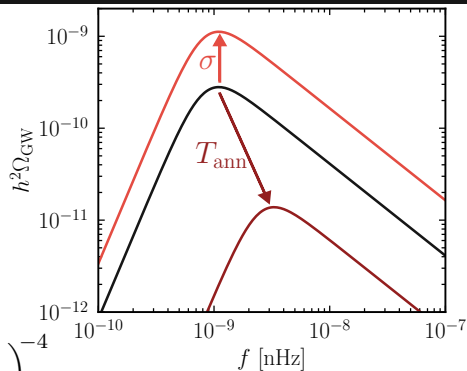
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$$\Omega_{\text{GW}} = \Omega_p \frac{4}{(f/f_p)^{-3} + 3 (f/f_p)}$$



[Ferreira et al. (JCAP 2023)
[Hitamatsu et al. (JCAP 2014)]

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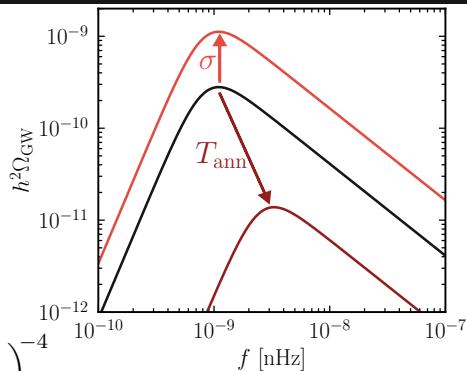
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- for PTAs: $T_{\text{ann}} \sim \text{few MeV}, \quad h^2 \Omega_{\text{DW}} \sim 0.1$



[Ferreira et al. (JCAP 2023)]
[Hitamatsu et al. (JCAP 2014)]

ALP Domain Walls

$$\mathcal{L} \supset \partial_\mu \phi^\dagger \partial^\mu \phi - \underbrace{\lambda \left(\phi^\dagger \phi - \frac{v^2}{2} \right)^2}_{\text{breaks U(1) spontaneously}} - \underbrace{m_a^2 f_a^2 \left[1 - \cos \frac{a}{f_a} \right]}_{\text{explicit breaking}}$$

w/ $\phi = \frac{v+\rho}{\sqrt{2}} e^{i\frac{a}{v}}$

U(1) \rightarrow Z_N
w/ $f_a = v/N$
 \Rightarrow DW stable if $N \geq 2$

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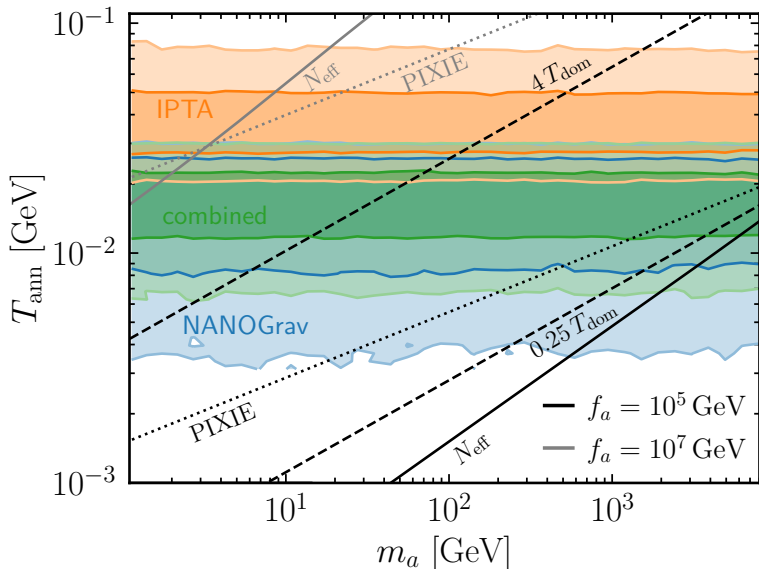
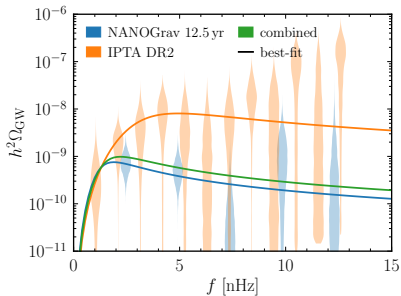
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○ additional breaking of $Z_N \implies V_b$

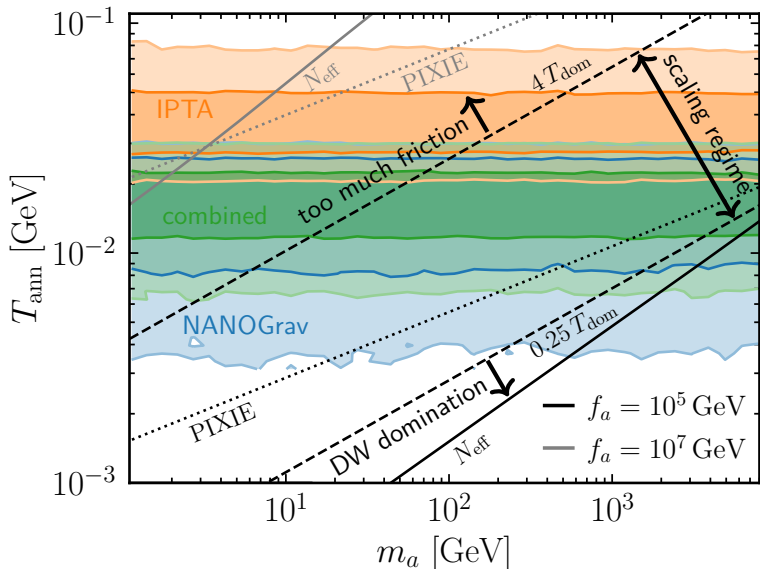
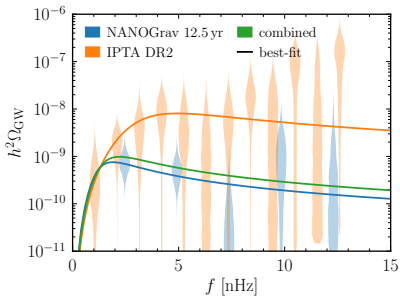
PTA Fit – ALP DWs

[see also Ferreira et al. (JCAP 2023)]



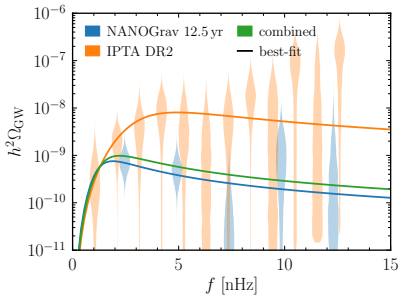
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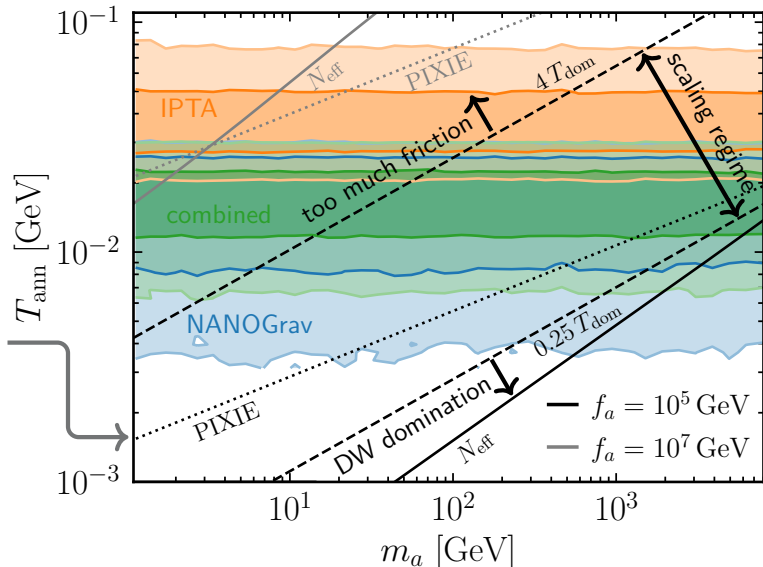
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μ -distortions in CMB

[Ramberg, Ratzinger, Schwaller (JCAP 2023)]

from acoustic waves in baryon-photon fluid induced via gravity



○ N axions:

$$V = \underbrace{\sum_{i=1}^N \lambda_i \left(\Phi_i^\dagger \Phi_i - \frac{f_i}{2} \right)^2}_{\text{breaks } U(1)^N \text{ spontaneously}} + \underbrace{\left[\sum_{i=1}^{N-1} \epsilon_i \Phi_i \Phi_{i+1}^3 + \text{h.c.} \right]}_{U(1)^{N-1} \text{ explicitly broken to discrete sym.}}$$

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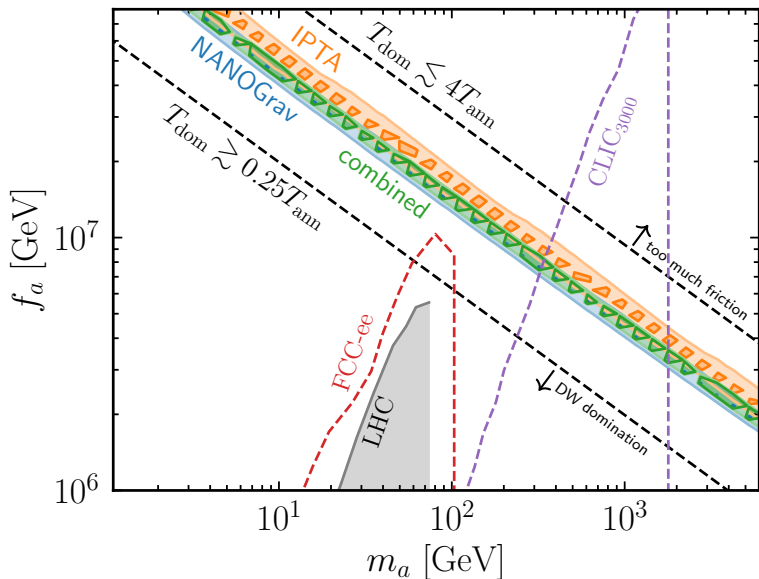
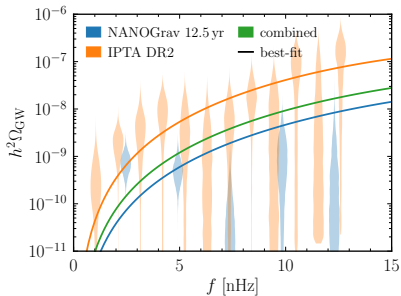
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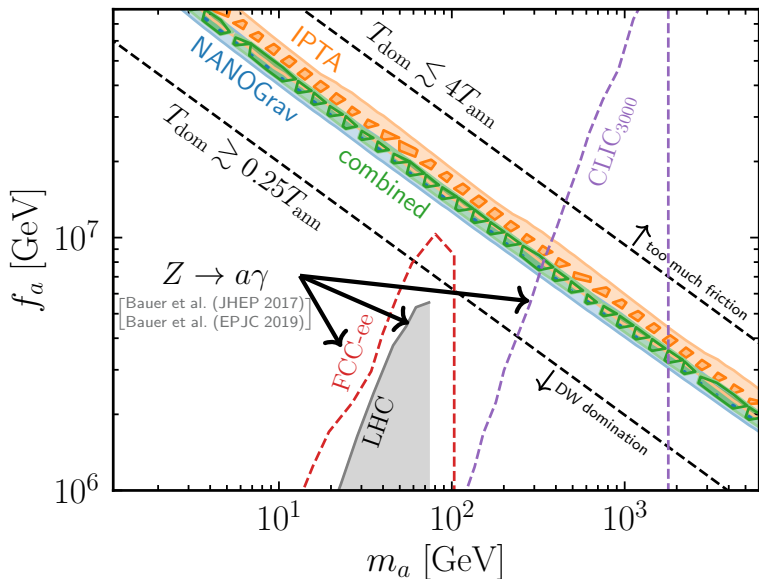
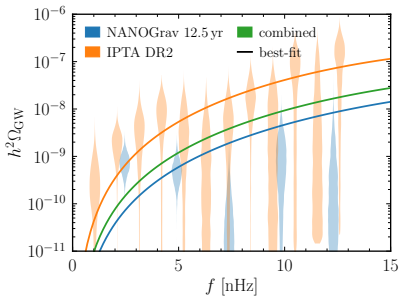
○ heavy axion decay to SM: $\Gamma_{a \rightarrow gg} \sim 10^{11} \text{ s}^{-1} \left(\frac{m_a}{10 \text{ GeV}} \right)^3 \left(\frac{10^7 \text{ GeV}}{f_a} \right)^2$

\Rightarrow decay before BBN, no N_{eff} constraints

PTA Fit – Aligned Axion DWs



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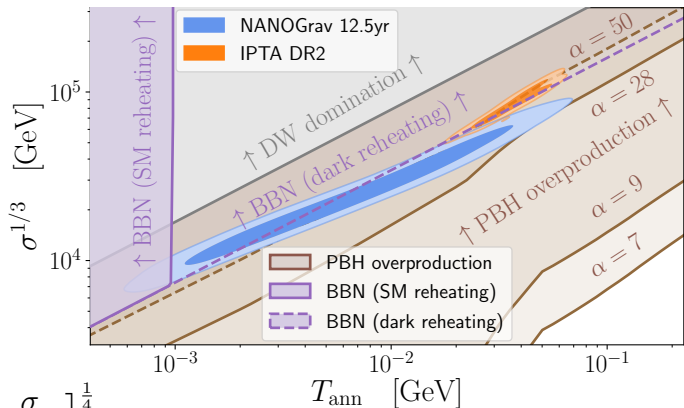
Primordial Black Hole Production

DWs can collapse to PBHs if

$$R < r_S = 2GM$$

$$\sim 2G \left(\frac{4\pi^2}{3} R^3 V_b + 4\pi R^2 \sigma \right)$$

$$\Rightarrow T_{\text{PBH}} \sim 100 \text{ MeV} \sqrt{\frac{T_{\text{ann}}}{10 \text{ MeV}}} \left[\frac{\sigma}{\text{TeV}^2} \right]^{\frac{1}{4}}$$



[Gouttenoire, Vitagliano, arXiv:2306.17841 [gr-qc]]

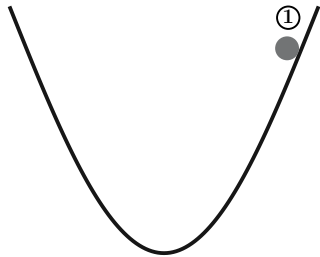
Bosonic Instabilities

1. Pulsar Timing Arrays
2. Cosmological Phase Transitions
3. Domain Walls
- 4. Bosonic Instabilities**
5. Conclusions

Misalignment Mechanism

$$\mathcal{L} \supset \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)$$

$$\Rightarrow \ddot{\phi} + 3H\dot{\phi} + m_a^2 \phi = 0$$

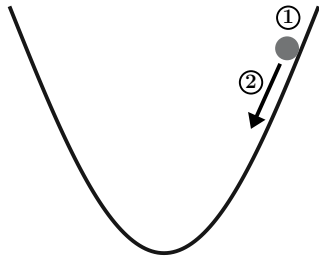


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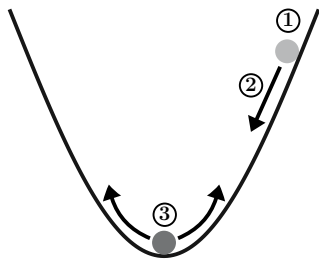
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Misalignment Mechanism

+

coupling to dark photon

$$\mathcal{L} \supset \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} - \frac{\alpha}{4} \frac{\phi}{f_a} X_{\mu\nu} \tilde{X}^{\mu\nu}$$

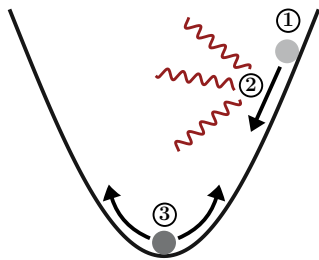
$$\implies \ddot{\phi} + 3H\dot{\phi} + m_a^2 \phi = -\frac{\alpha}{f_a} \langle X_{\mu\nu} \tilde{X}^{\mu\nu} \rangle$$

e.g. to deplete axion abundance

Agrawal, Marques-Tavares, Xue (JHEP, 2018)
Kitajima, Sekiguchi, Takahashi (PLB 2018)

or for dark-photon dark-matter

Dror, Harigaya, Narayan (PRD 2019);
Co, Pierce, Zhang, Zhao (PRD 2019);
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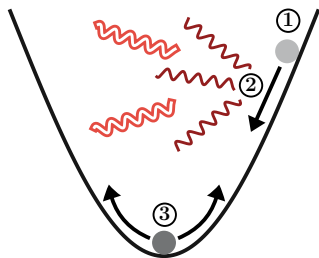
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\implies gravitational wave emission from dark photons

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dark photon EoM: $X_{\pm}''(\tau, k) + \omega_{\pm}^2(k)X_{\pm}(\tau, k) = 0$ $\omega_{\pm}^2(k) = \left(k^2 \mp k \frac{\alpha\phi'(\tau)}{f_a} \right)$

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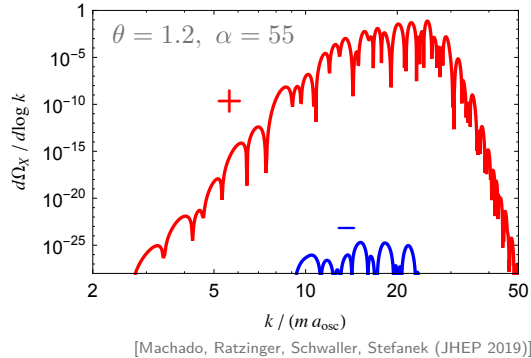
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dark photon spectrum:

- peaked around $\tilde{k} \approx a_{\text{osc}} m_a (\alpha \theta / 2)^{2/3}$
- first tachyonic helicity dominates

Gravitational Wave Spectrum

GWs generated at t_* around the time when the tachyonic band closes:

- peak frequency:

$$f_{\text{peak}} \sim 2 \frac{\tilde{k}_*}{a_0} \sim 4 \text{ nHz} \left(\frac{\alpha \theta}{100} \right)^{\frac{2}{3}} \left(\frac{m_a}{10^{-15} \text{ eV}} \right)^{\frac{1}{2}}$$

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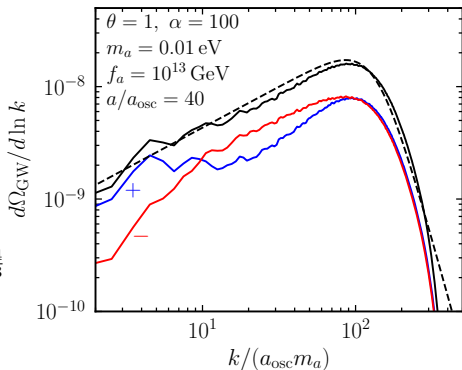
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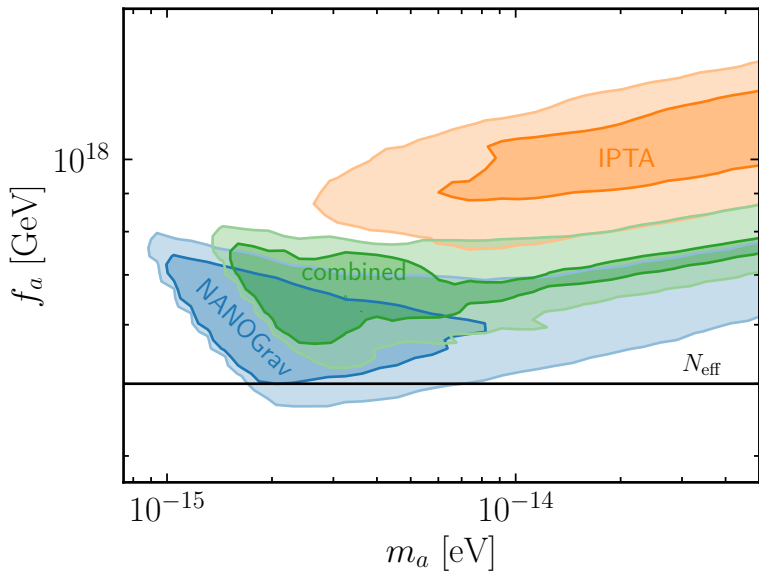
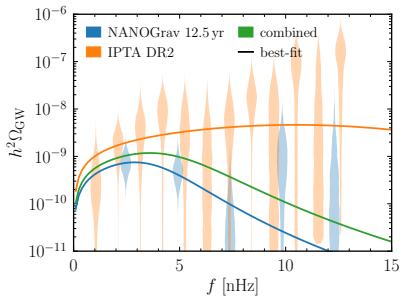
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- spectral shape from lattice: [Ratzinger, Schwaller, Stefanek (SciPost Phys. 2022)]

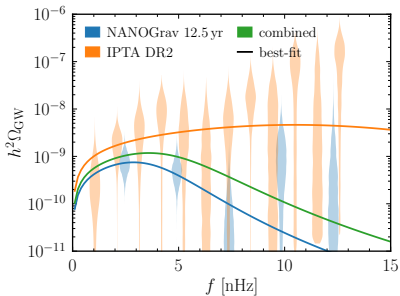
$$\Omega_{\text{GW}} = \Omega_{\text{GW}}^{\text{peak}} \mathcal{S}(f/f_{\text{peak}}), \quad \mathcal{S}(x) = x^{0.73} \left[\frac{1}{2} (1 + x^{4.2}) \right]^{\frac{-4.96 - 0.73}{4.2}}$$



PTA Fit – Audible Axions

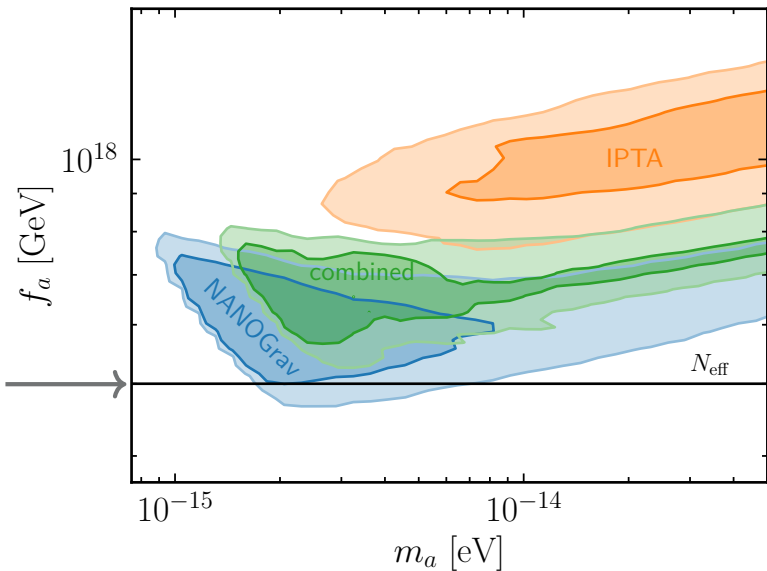


PTA Fit – Audible Axions



$$\Delta N_{\text{eff}} \sim 9.1 \left(\frac{\theta f_a}{M_{\text{pl}}} \right)^2$$

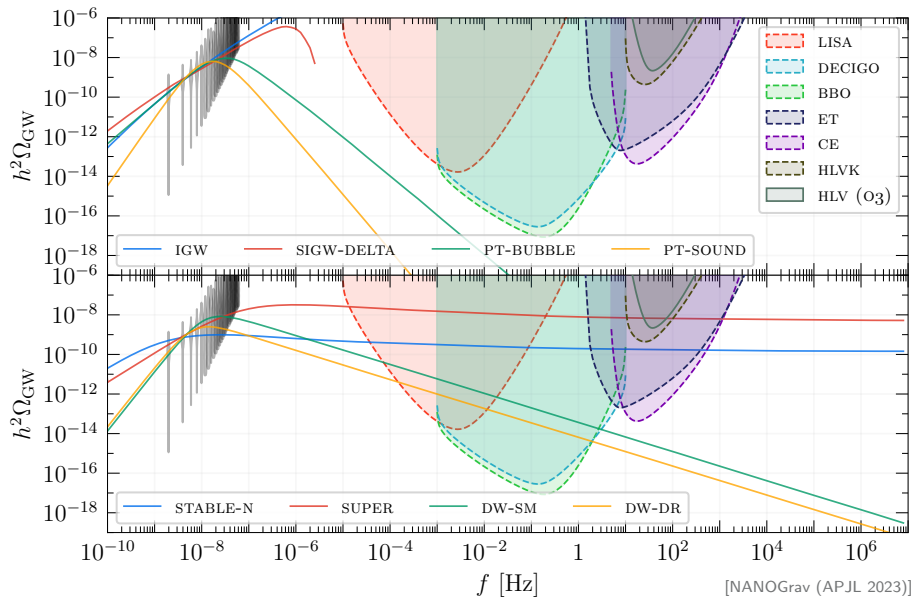
(also: $\Omega_a > \Omega_{\text{DM}}$)



Conclusions

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Fits to NANOGrav 15-year dataset



Conclusion

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Thank you for your attention!

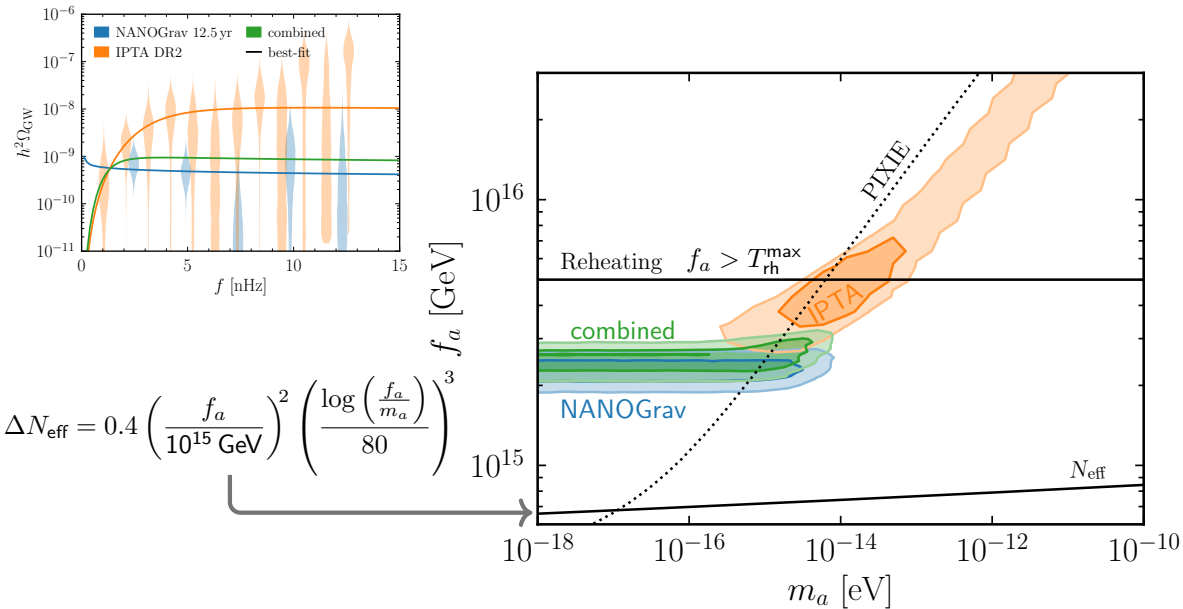


Primordial Gravitational Waves and Pulsar Timing Array Data

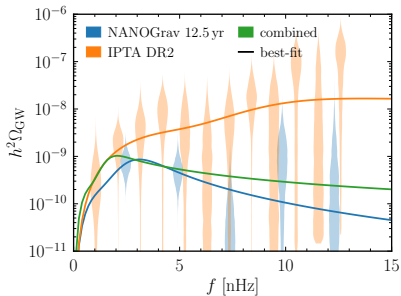
E.M., Morgante, Puchades Ibáñez, Ramberg, Ratzinger, Schenk, and Schwaller
JHEP **10** (2023), 171 [arXiv:2306.14856 [hep-ph]]

backup slides

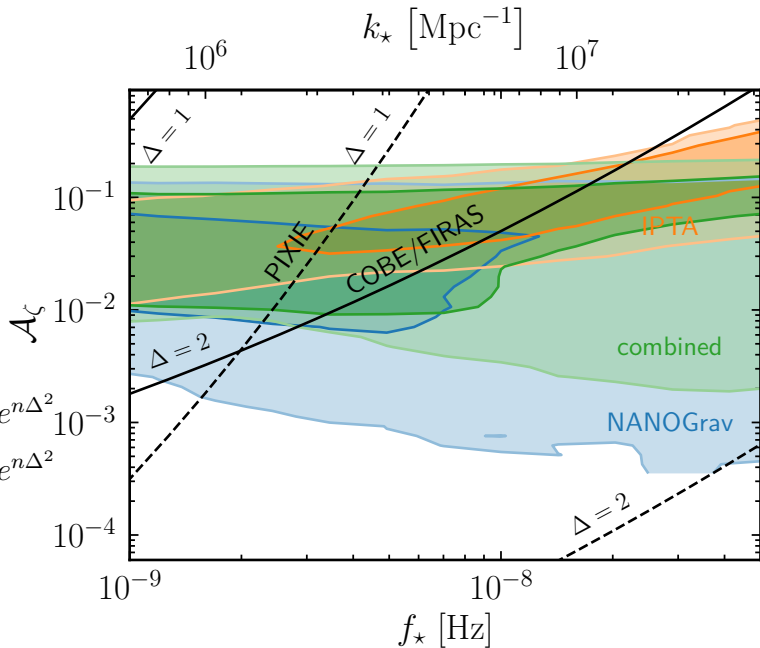
PTA Fit – Global ALP Strings



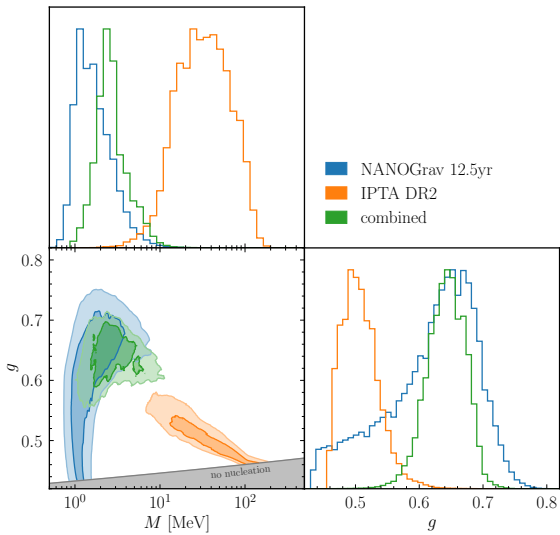
PTA Fit – Scalar-Induced GWs



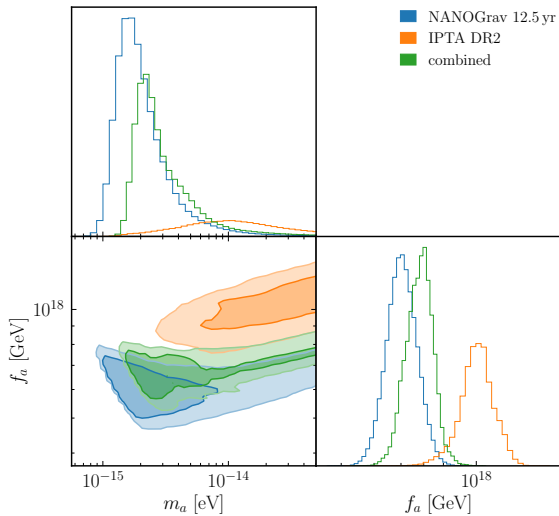
$$P_{\zeta}(k) = \begin{cases} \frac{A_{\zeta} e^{-\frac{\log^2(k/k_*)}{2\Delta^2}}}{\sqrt{2\pi} \Delta}, & \frac{k}{k_*} < e^{n\Delta^2} \\ \frac{A_{\zeta} e^{n\Delta^2}}{\sqrt{2\pi} \Delta} \left(\frac{k}{k_*}\right)^{-n}, & \frac{k}{k_*} > e^{n\Delta^2} \end{cases}$$



Triangle Plots I

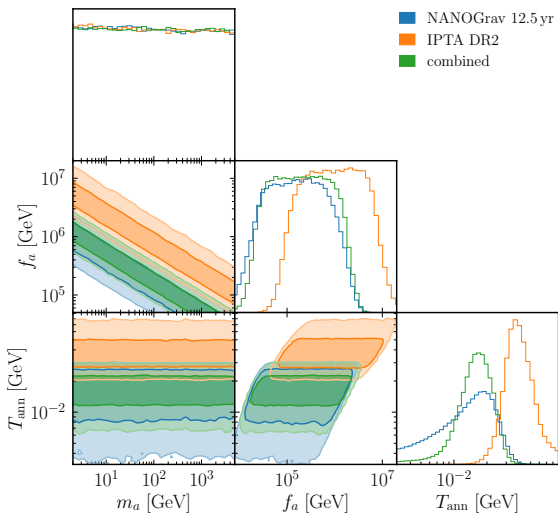


PhT in Coleman-Weinberg model

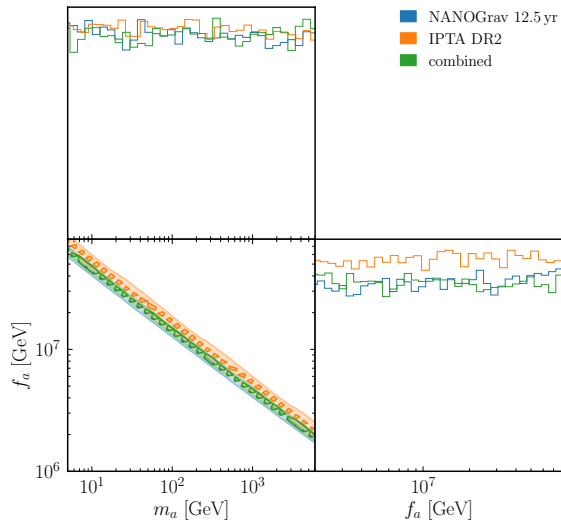


audible axions

Triangle Plots II

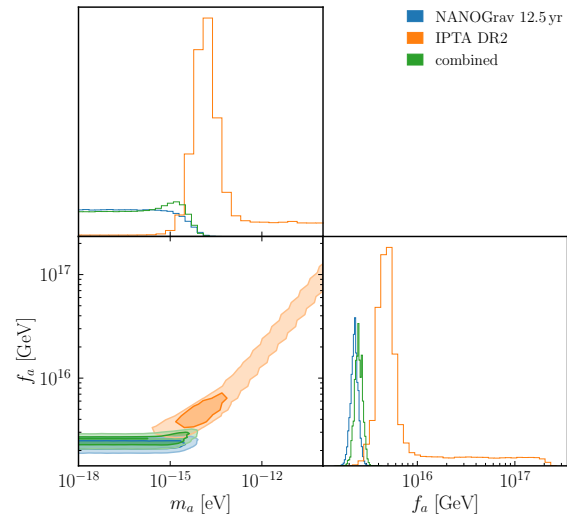


ALP domain walls

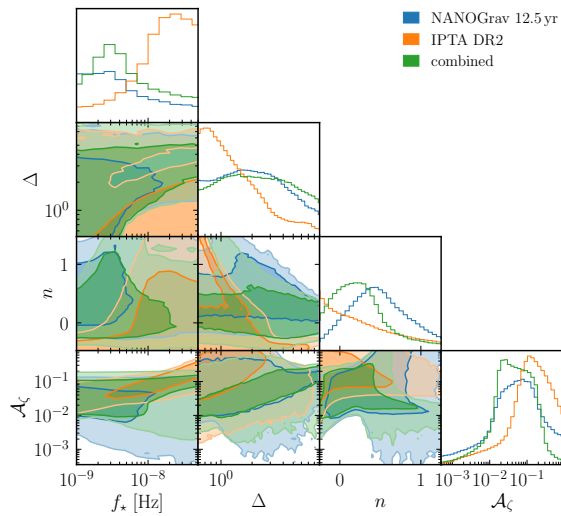


aligned axion domain walls

Triangle Plots III

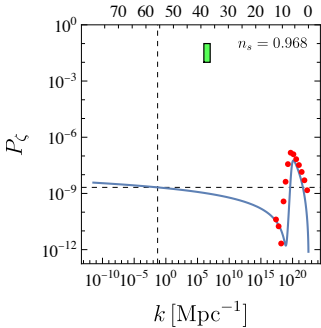
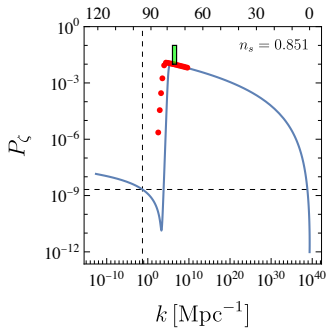
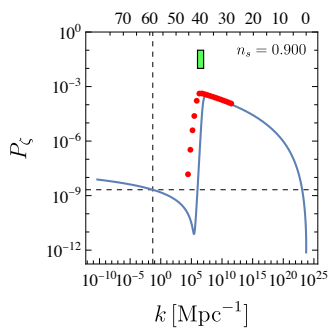
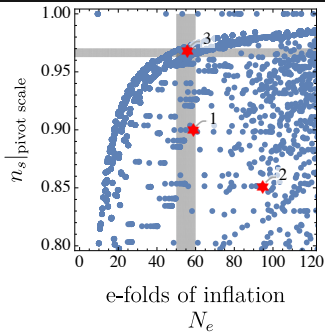
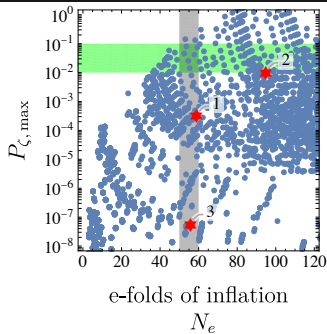
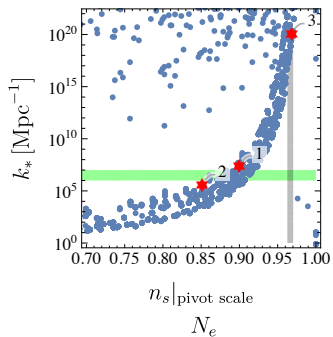


global ALP strings

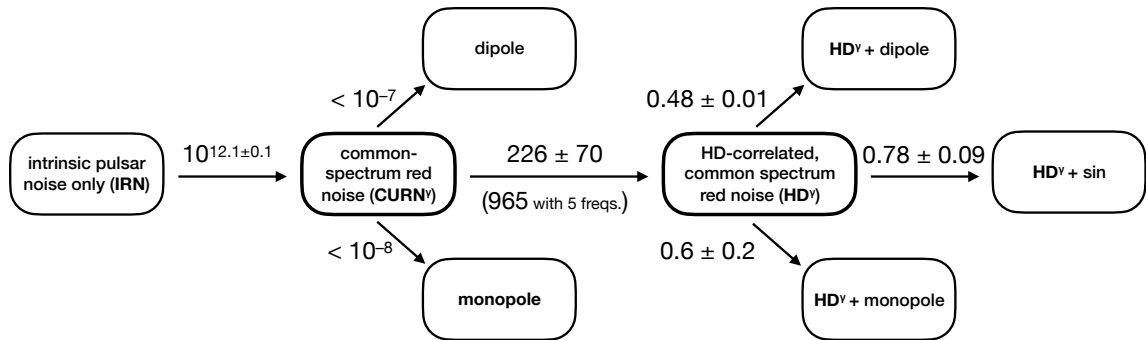


scalar-induced GWs

Single-field Inflation with Inflection Point



NANOGrav 15-year dataset: Bayesian Evidence



[NANOGrav (APJL 2023)]

NANOGrav 15-year dataset: Bayes Factors

