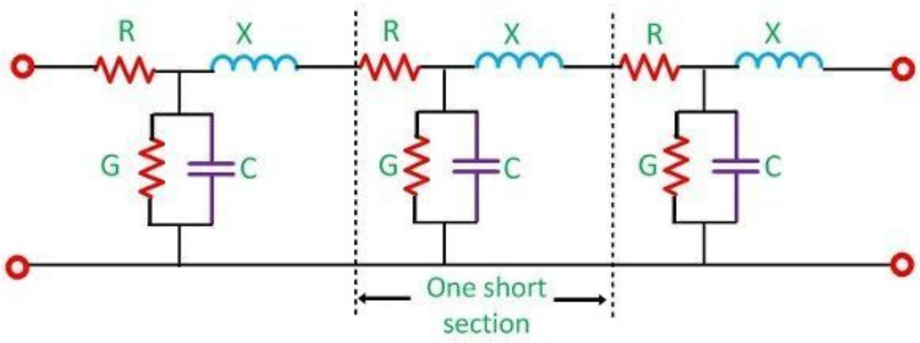
RF, SA, VNA, CB100 and more…

Analysis week Munich 2023

Topics (Part I):

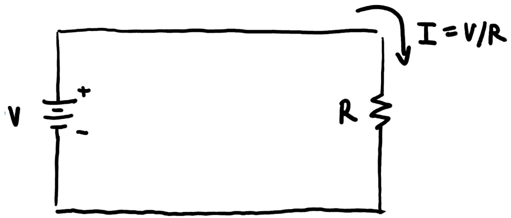
- Transmission Line Theory
  - Coaxial cables
  - Wave propagation in a transmission line
  - The lossless line
  - Terminated lossless transmission line
- Noise
  - Noise Power and Equivalent Noise Temperature
  - Y-Factor method (+amplifier noise)
  - Equivalent Noise Ratio (ENR)
  - LNA noise model

Transmission Line Theory



Transmission line Theory:

- Transmission lines may be a considerable fraction of the wavelength
- Voltages and current can vary in magnitude, phase and length

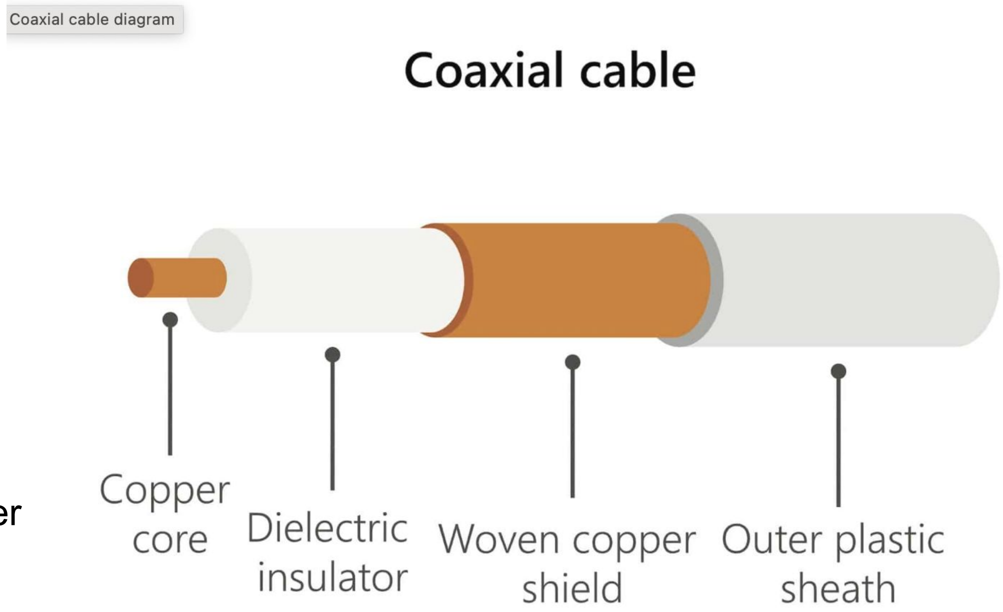


Circuit Theory:

- physical dimensions of the network are much smaller than the wavelength
- Voltages and currents do not vary

Coaxial Cables:

- Invented in 1858 (Transatlantic Telegraph communication)
- Used to carry high frequency electrical signals with low loss
- EM signal exists only between the inner and outer conductors:
- Protection of the signal from external EM interferance

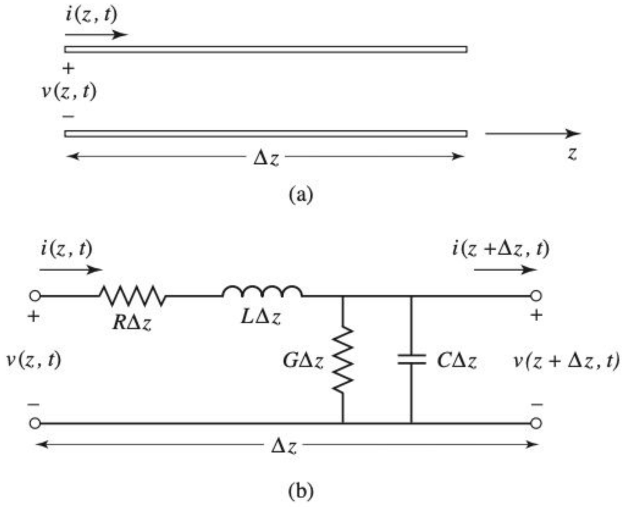


Telegrapher’s equations:

Telegrapher’s equations:

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t},$$

$$\frac{\partial i(z,t)}{\partial z} = -Gv(z,t) - C\frac{\partial v(z,t)}{\partial t}.$$



- R = Resistence per length
- G = Conductance per length

} Represent Loss

- L = Inductance per length
- C = Capacitance per length

Wave propagation in a Transmission line:

- Steady state Telegrapher's equations:

$$\frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0,$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0,$$

$$V(z) = V_o^+ e^{-\gamma z} + V_o^- e^{\gamma z},$$

$$I(z) = I_o^+ e^{-\gamma z} + I_o^- e^{\gamma z},$$

- Propagation constant:

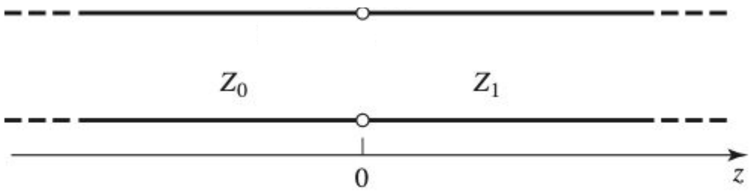
$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Characteristic Impedance and lossless line:

$$Z_0 = \frac{R + j\omega L}{\gamma} = \sqrt{\frac{R + j\omega L}{G + j\omega C}},$$

$$\frac{V_o^+}{I_o^+} = Z_0 = \frac{-V_o^-}{I_o^-}.$$

$$I(z) = \frac{V_o^+}{Z_0} e^{-\gamma z} - \frac{V_o^-}{Z_0} e^{\gamma z}$$



Lossless line:

$$\gamma = \alpha + j\beta = j\omega\sqrt{LC}$$

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z},$$

$$I(z) = \frac{V_o^+}{Z_0} e^{-j\beta z} - \frac{V_o^-}{Z_0} e^{j\beta z}.$$

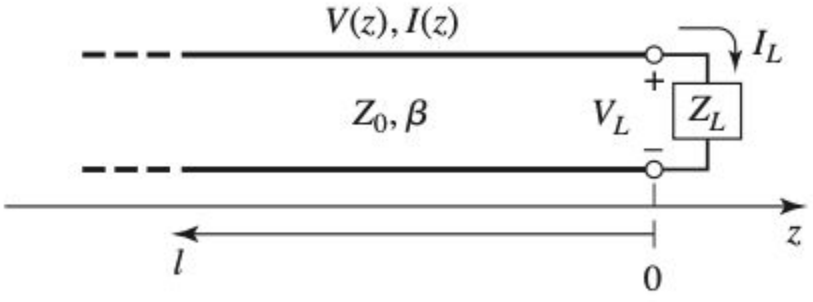
Terminated lossless line:

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}$$

$$I(z) = \frac{V_o^+}{Z_0} e^{-j\beta z} - \frac{V_o^-}{Z_0} e^{j\beta z}$$

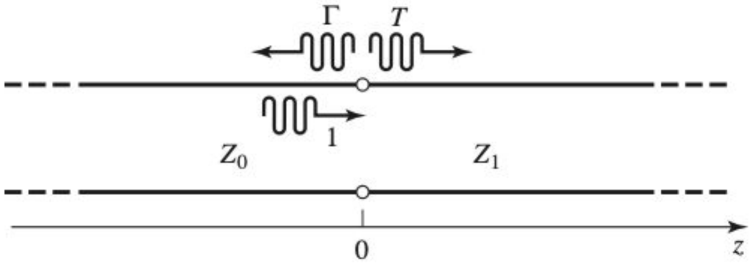
Load Impedance:

$$Z_L = \frac{V(0)}{I(0)} = \frac{V_o^+ + V_o^-}{V_o^+ - V_o^-} Z_0$$

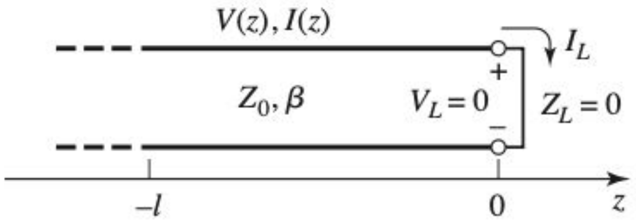


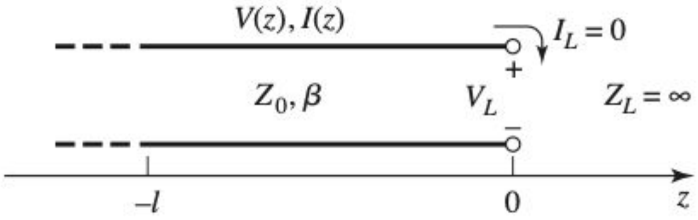
Voltage Reflection coefficient (Γ) :

$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

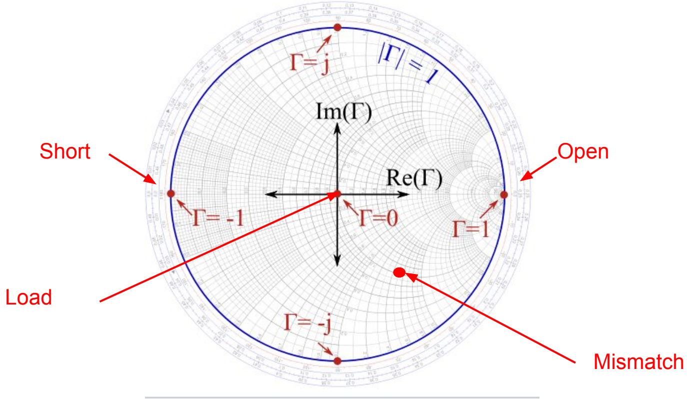


Short Open Load:



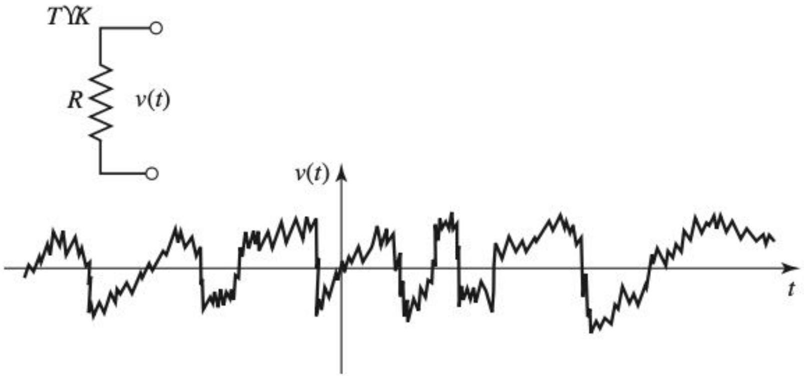


$$\Gamma = \frac{V_o^-}{V_o^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$



Noise

- Resistor at temperature T
- Random motions of electrons in the resistor :
  - Small random voltage fluctuations at the terminals
  - Average value = 0
  - RMS ≠ 0



$$V_n = \sqrt{\frac{4hf\ BR}{e^{hf/kT} - 1}},$$

→ Planck's black body Radiation law

Rayleigh-Jeans approximation:

$$V_n = \sqrt{\frac{4hf\ BR}{e^{hf/kT} - 1}},$$

hf ≪ kT

→

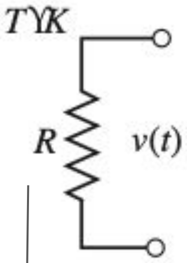
$$e^{hf/kt} - 1 \simeq \frac{hf}{kT}$$

$$V_n = \sqrt{4kTBR}.$$

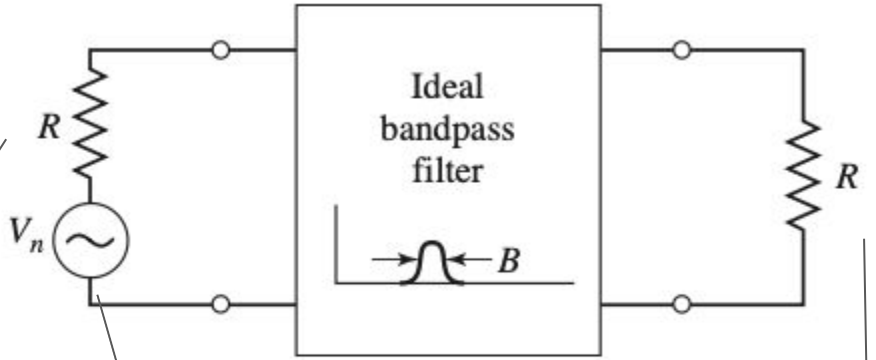
h = Planck's constant
k = Boltzmann's constant
T = Temperature (K)
B = Bandwidth of the system (Hz)
f = Center frequency of the bandwidth (Hz)
R = resistance (Ω)

Approximation valid for Microwave frequencies

Simulating Noisy Resistor:



Noisy Resistor



Noiseless Resistor

Voltage Generator:

$$V_n = \sqrt{4kTBR}.$$

Load Resistor

Power delivered to Load:

From the Rayleigh-Jeans approximation:

$$P_n = \left(\frac{V_n}{2R}\right)^2 R = \frac{V_n^2}{4R} = kTB,$$

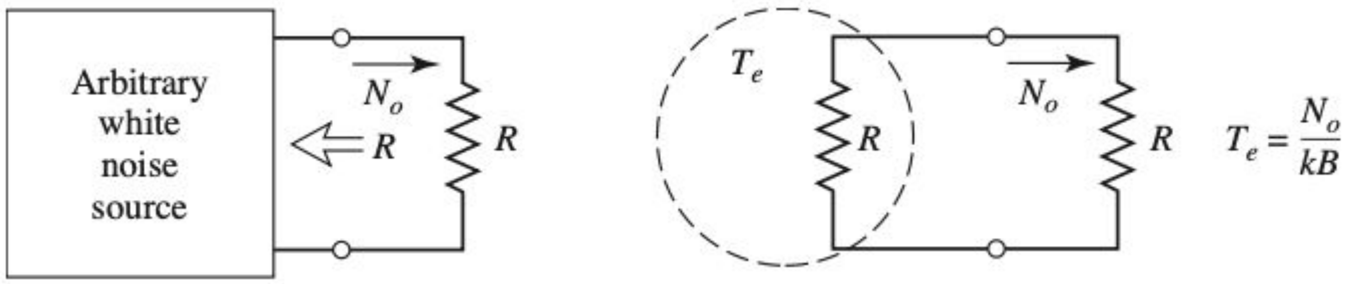
Power delivered to Load

More general if we use impedance:

$$P_{\text{avg}} = \frac{1}{2}\text{Re}\left\{V(z)I(z)^*\right\}$$

$$P_{\text{avg}} = \frac{1}{2}\frac{|V_o^+|^2}{Z_0}\left(1 - |\Gamma|^2\right),$$

Equivalent Noise Temperature:

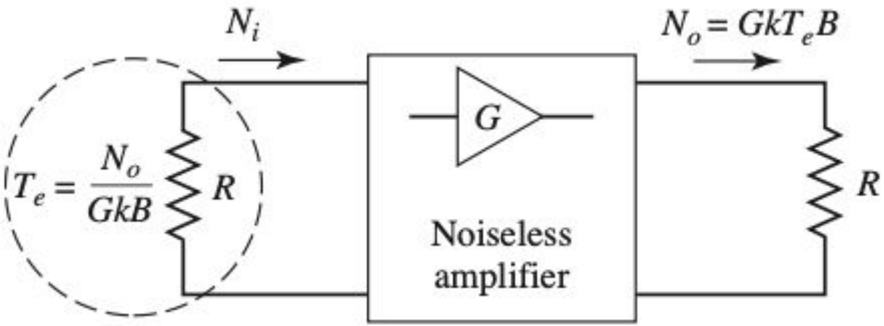


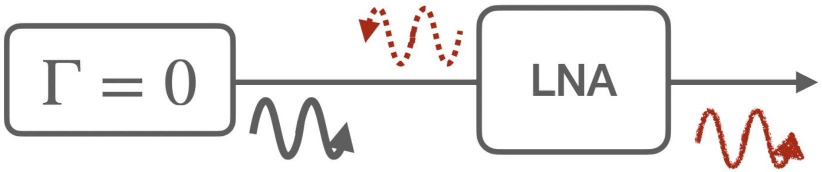
Any source of white noise can be modeled as an equivalent thermal noise source, and characterized with an Equivalent Noise Temperature (T_e)

$$T_e = \frac{N_o}{kB}$$

- N_o = Noise Power
- k = Boltzmann constant
- B = Bandwidth

Amplifiers:





Low Noise Amplifiers (LNA):

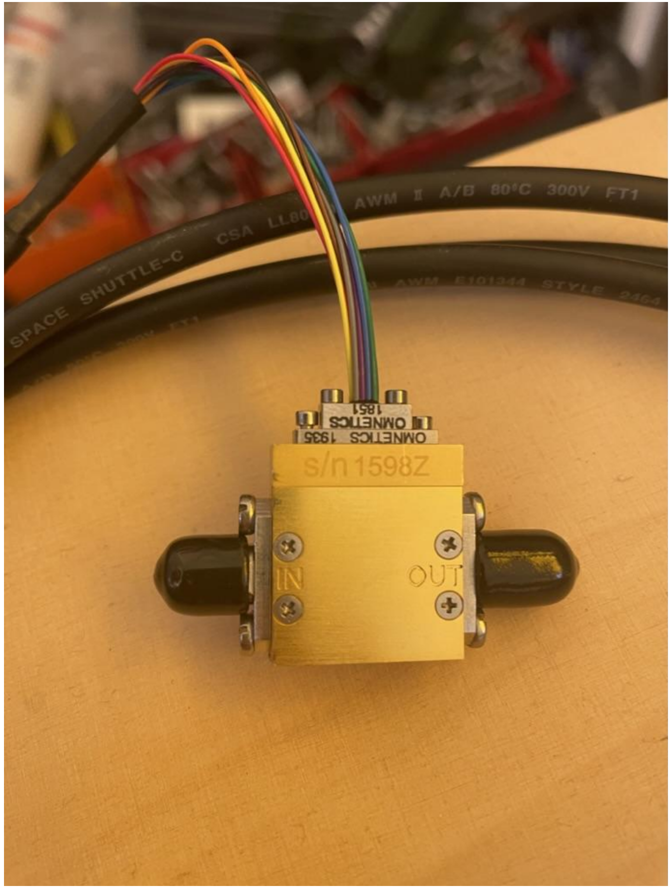
- The signal from the input is amplified by a factor G (gain)
- Our LNA ⟶ G=30dB (factor 1000)

$N_0 = GN_i$

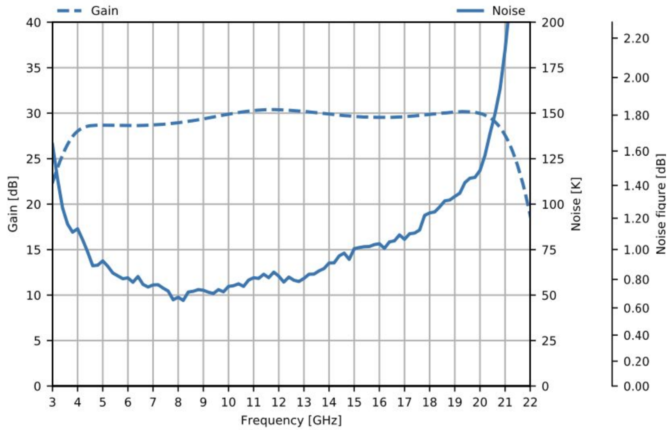
- Amplifiers are also Noisy!
- Contribute to a considerable amount of power in our case (150 K at room Temperature)

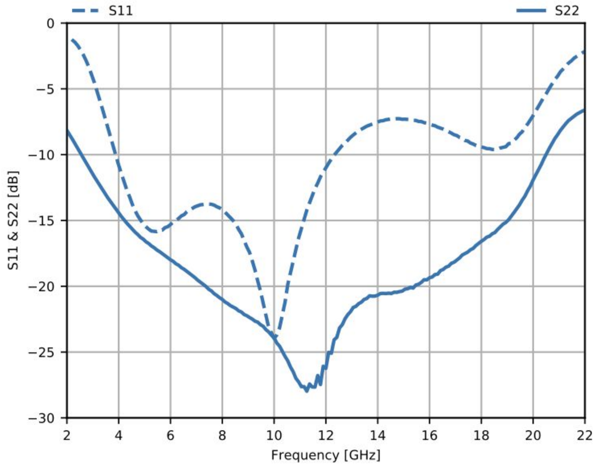
How do we calculate this?

Y-factor method!



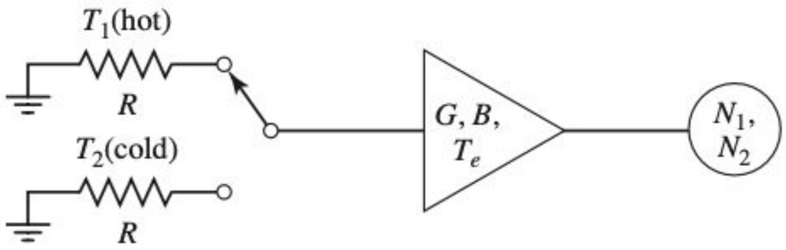
Measured data, Tamb=296 K





Noisy Amplifier (Y-factor method):

The simple case (Matched load)



Power measured at receiver:

$$N_1 = GkT_1B + GkT_eB,$$

$$N_2 = GkT_2B + GkT_eB,$$

Define Y-factor:

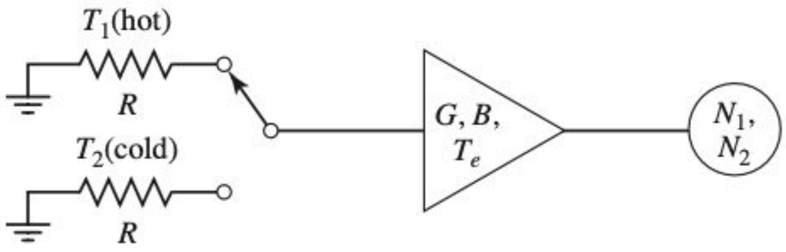
$$Y = \frac{N_1}{N_2} = \frac{T_1 + T_e}{T_2 + T_e} > 1,$$

$$T_e = \frac{T_1 - YT_2}{Y - 1}$$

LNA Noise Temperature

Noisy Amplifier (Y-factor method):

The not so simple case (Mismatched load)



Power measured at receiver:

$$N_1 = GkT_1B + GkT_eB,$$

$$N_2 = GkT_2B + GkT_eB,$$

"New" Y-factor:

LNA Noise Temperature
(mismatched load)

Excess Noise Ratio (ENR):

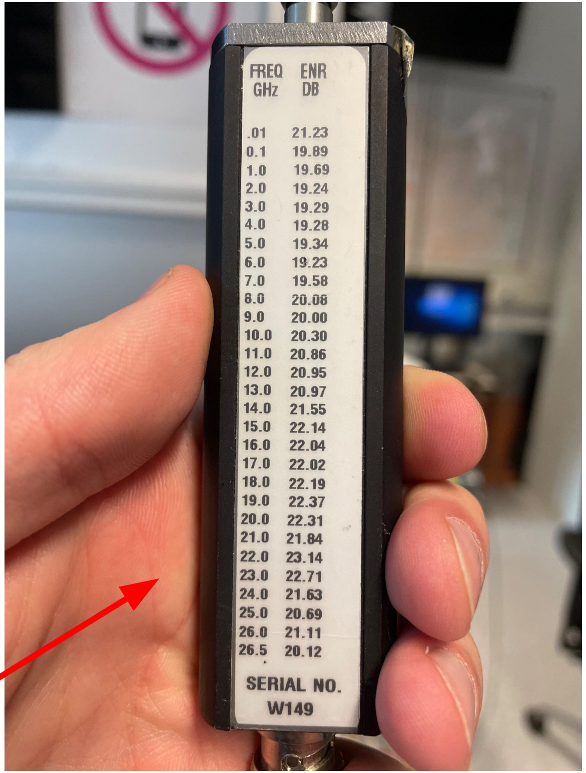
Two different ways of characterizing noise power:

1. Equivalent noise Temperature
2. Excess Noise Ratio

$$\text{ENR (dB)} = 10 \log \frac{N_g - N_o}{N_o} = 10 \log \frac{T_g - T_0}{T_0}$$

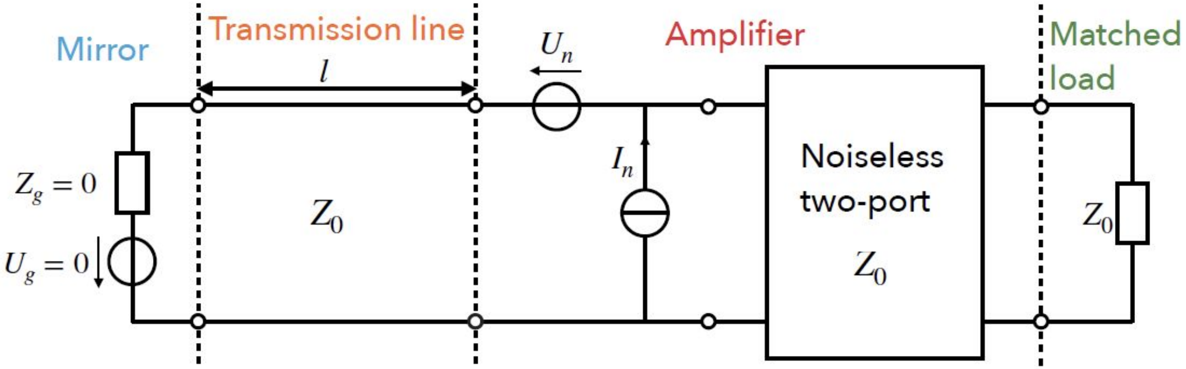
N_0, T_0 → Noise power/temperature of generator

N_g, T_g → Noise power/temperature of 290 K



Noise Diode

LNA Noise Model:



The model has three free parameters which can be obtained by measuring the noise signal from the standards: Short, Open and Load (SOL) + Mismatch (as crosscheck). For this we use the Spectrum analyser, SA.

The other parameters: Length of transmission line, Impedance, Gain, can be obtained through reflectivity measurements. For this we use the Vector Network Analyser, VNA.

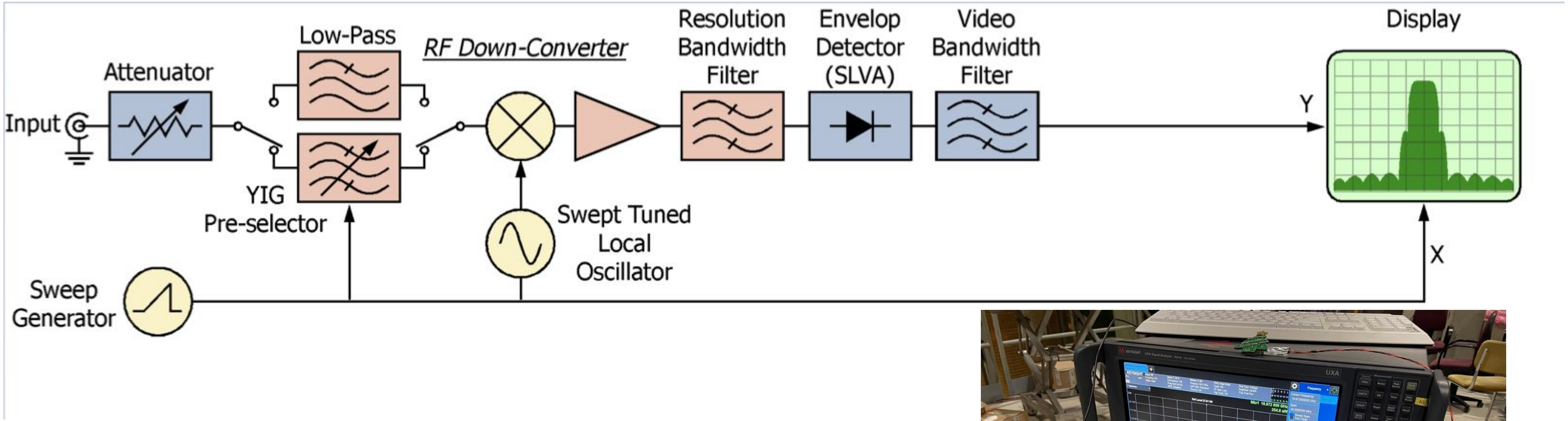
Break?

Topics (Part II):

- SA and VNA measurements
  - Spectrum Analyzer (SA)
  - Y-factor method
  - SA measurements
  - VNA measurements

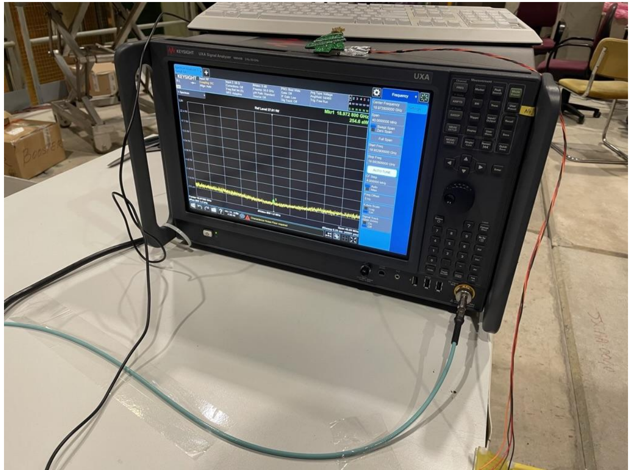
- CB100 setup and results
  - CB100 at Morpurgo
  - Different measurements
  - 2GHz and 40 MHz frequency range
  - How do we get to an exclusion limit?

The Spectrum Analyzer (SA):

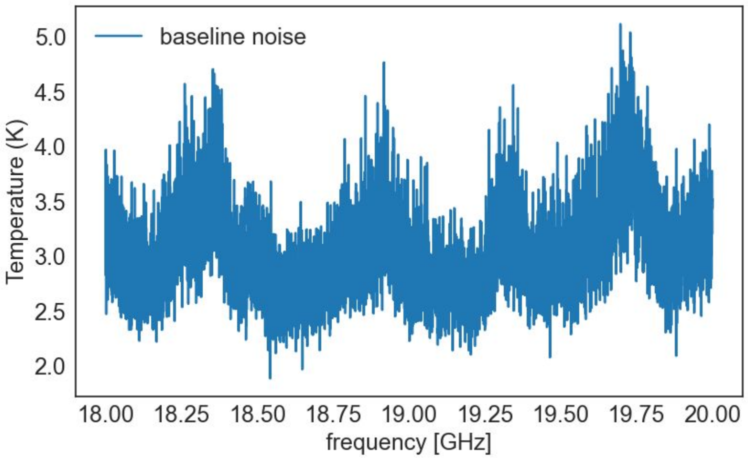


Parameters:

- Center/span
- Reference Level
- Resolution Bandwidth
- Video Bandwidth

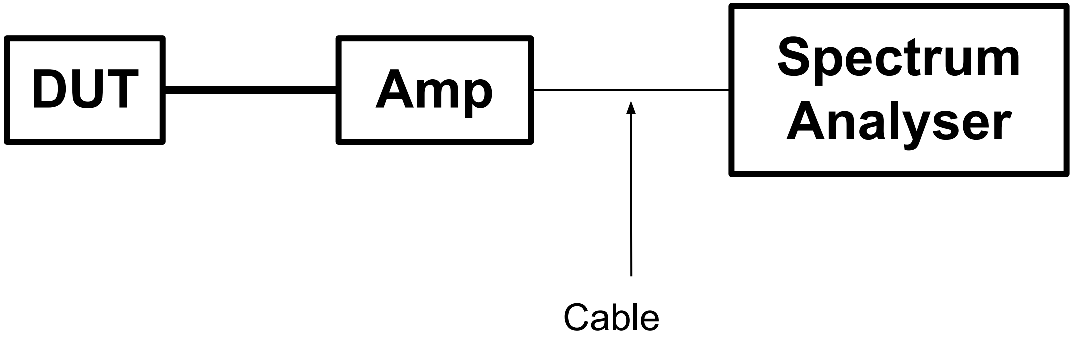


Baseline Measurement:



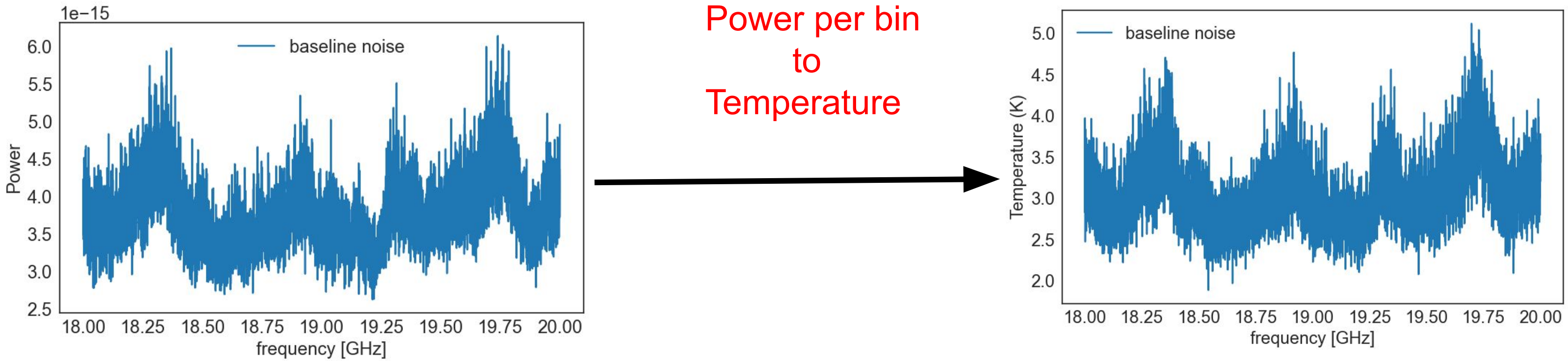
- The Spectrum analyser has some intrinsic noise (baseline)
- This is much higher than the noise of any device under test (DUT): Short, Open, Load, Booster

We need an LNA in order to amplify the signal and be able to see its spectrum

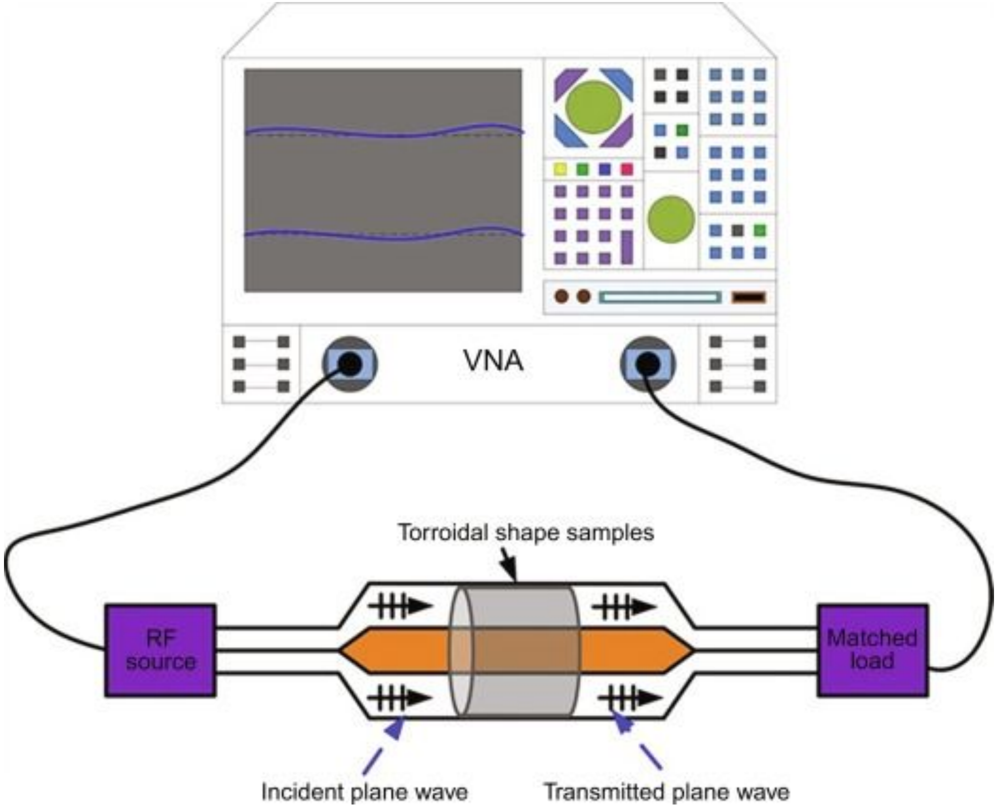


Power to Temperature: step by step

1. Measure Baseline (intrinsic Noise of SA)
2. Measure noise with Diode (on and off)
3. Y-factor to obtain Temperature of LNA + Power to Noise conversion
4. One to one conversion between Power per bin to Temperature



The Vector Network Analyzer (VNA):



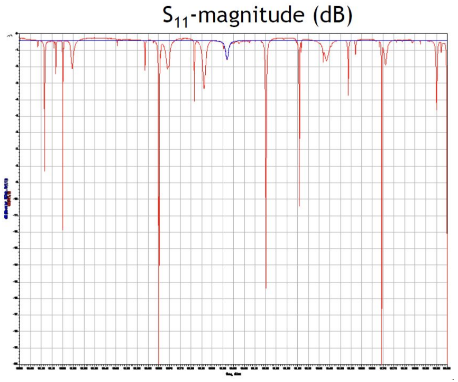
Computes reflectivity and transmissivity through the DUT:

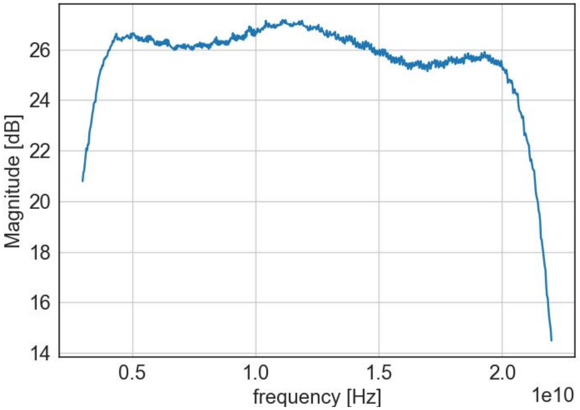
- s-parameters
- reflectivity (Γ)
- smith chart

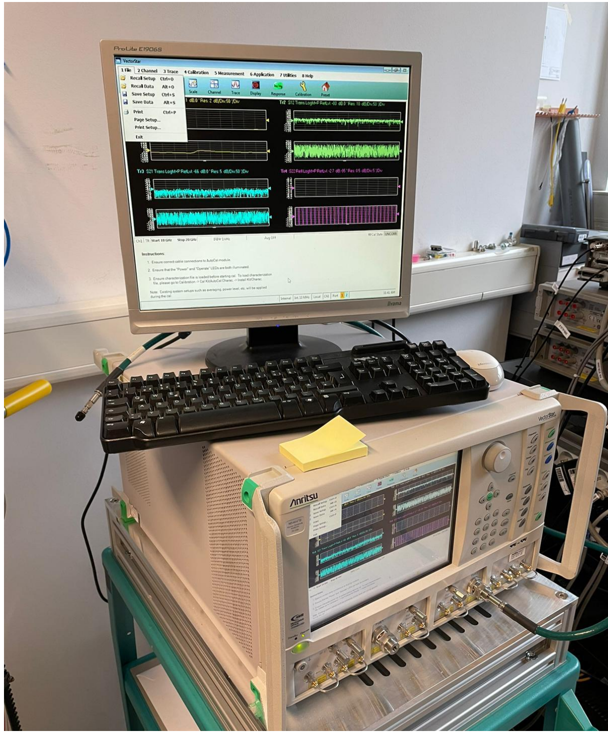
Allows us to obtain certain parameters for our model:

- Gain of LNA
- electrical length of DUT

The Vector Network Analyzer (VNA):

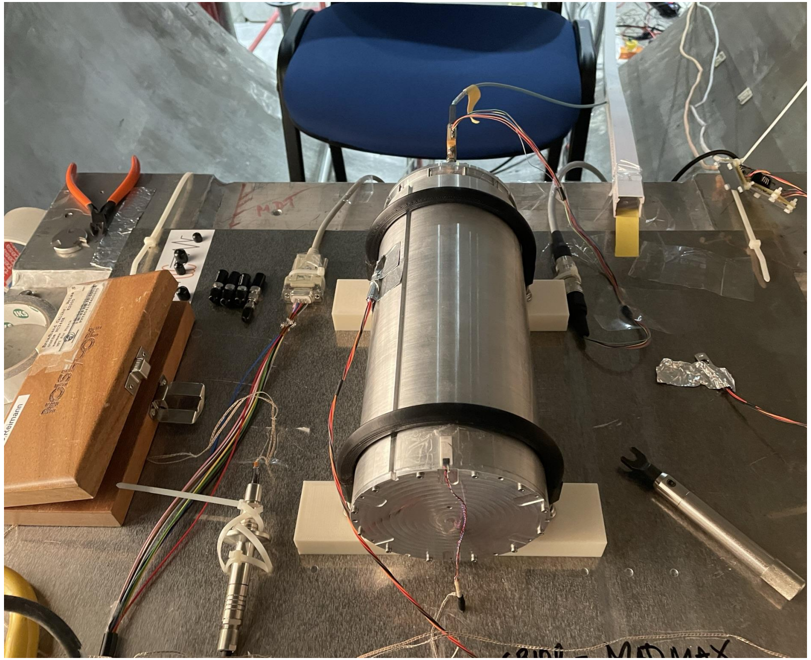


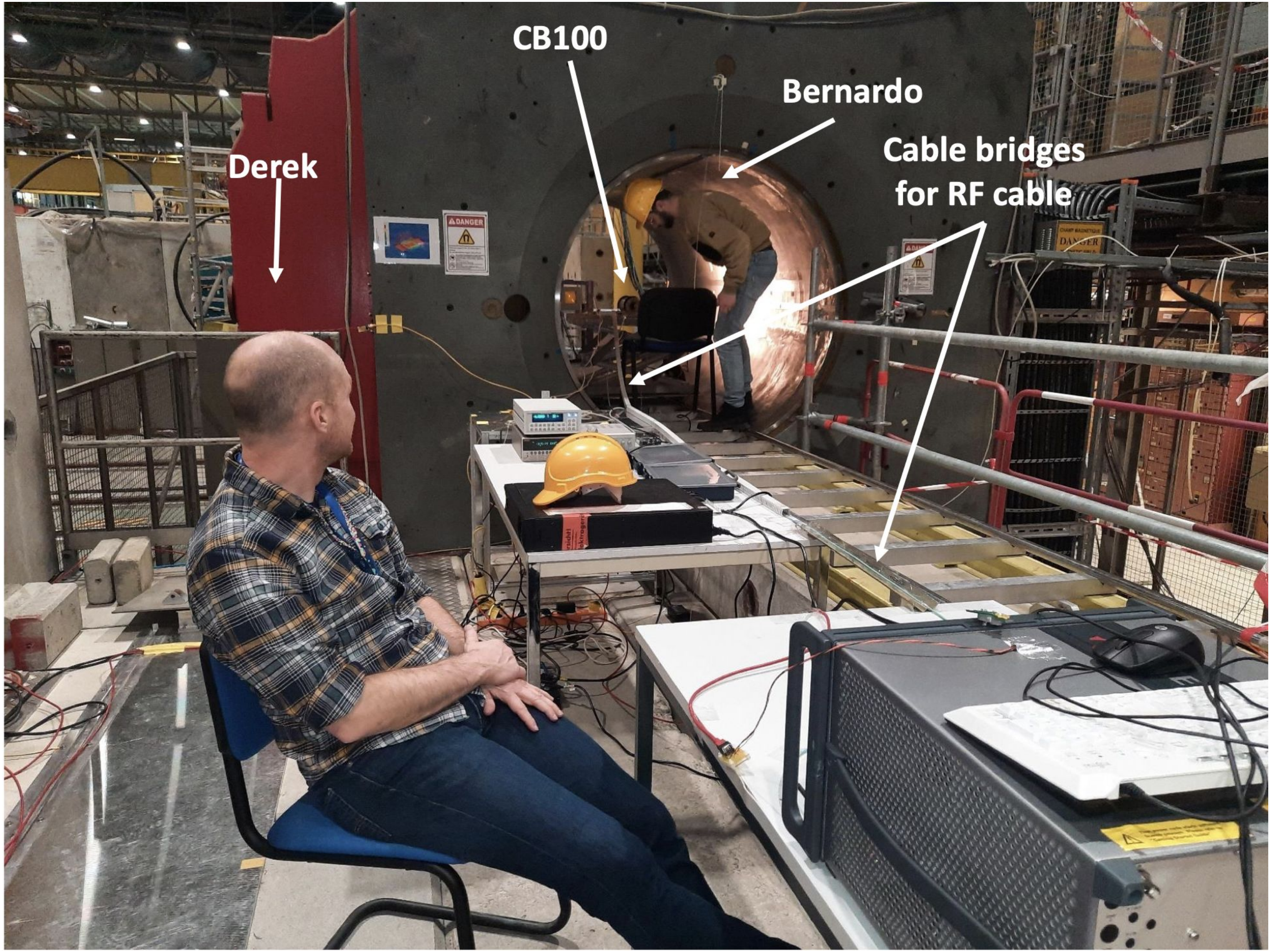


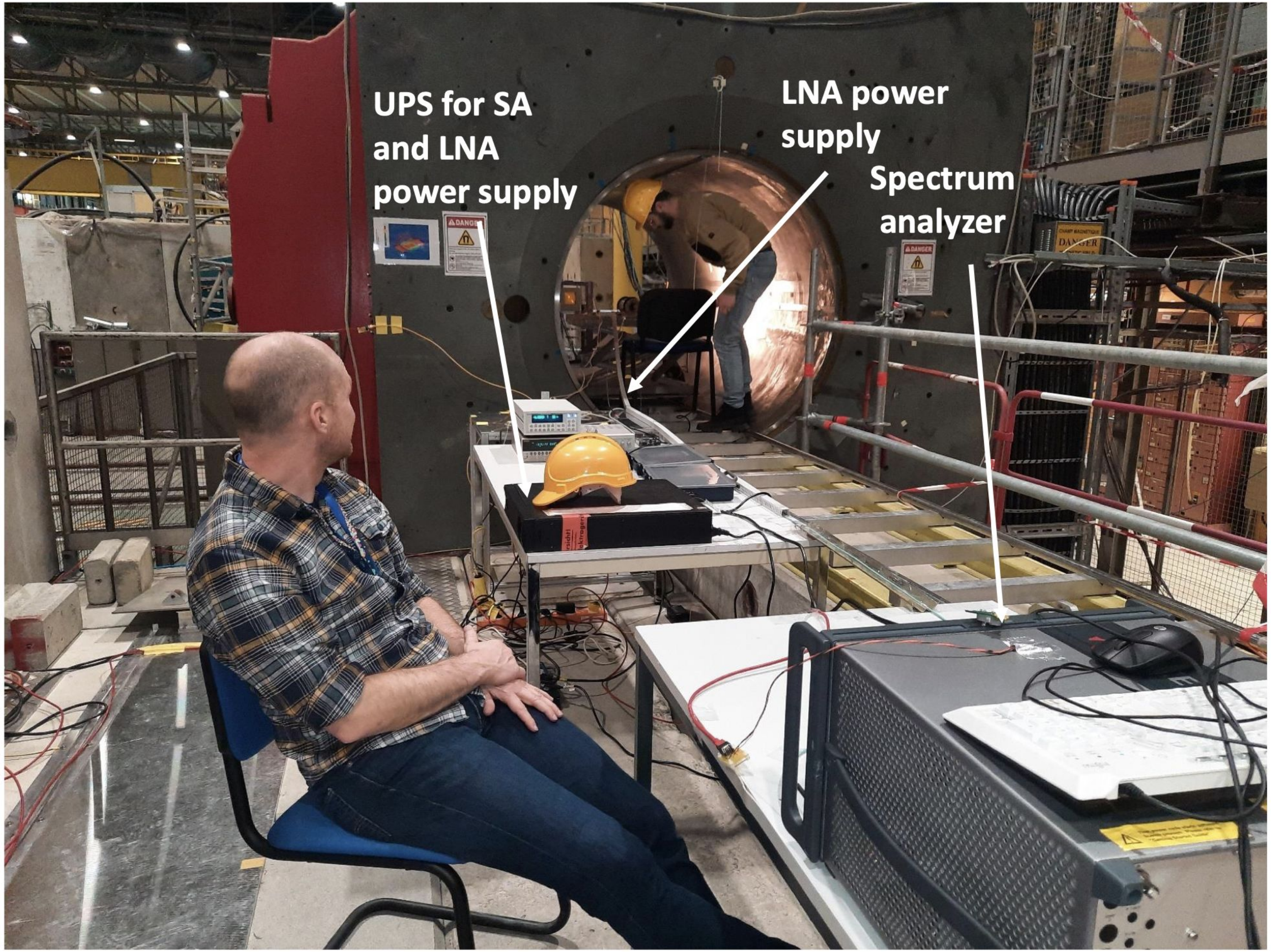


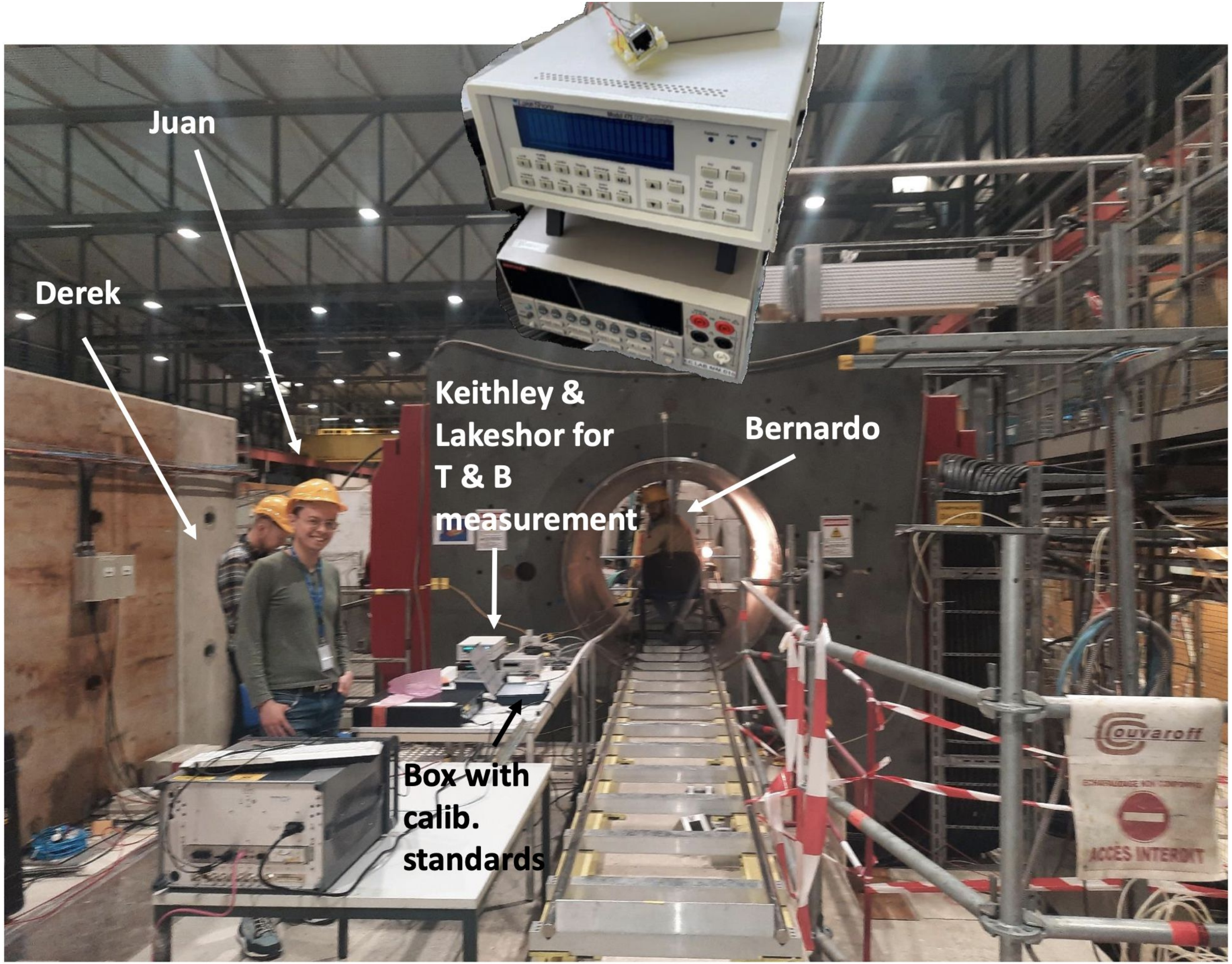
- Booster reflectivity measurement
- LNA: gain (S_{21}), reflectivity (S_{11}), impedance,length
- Standards (SOLM)

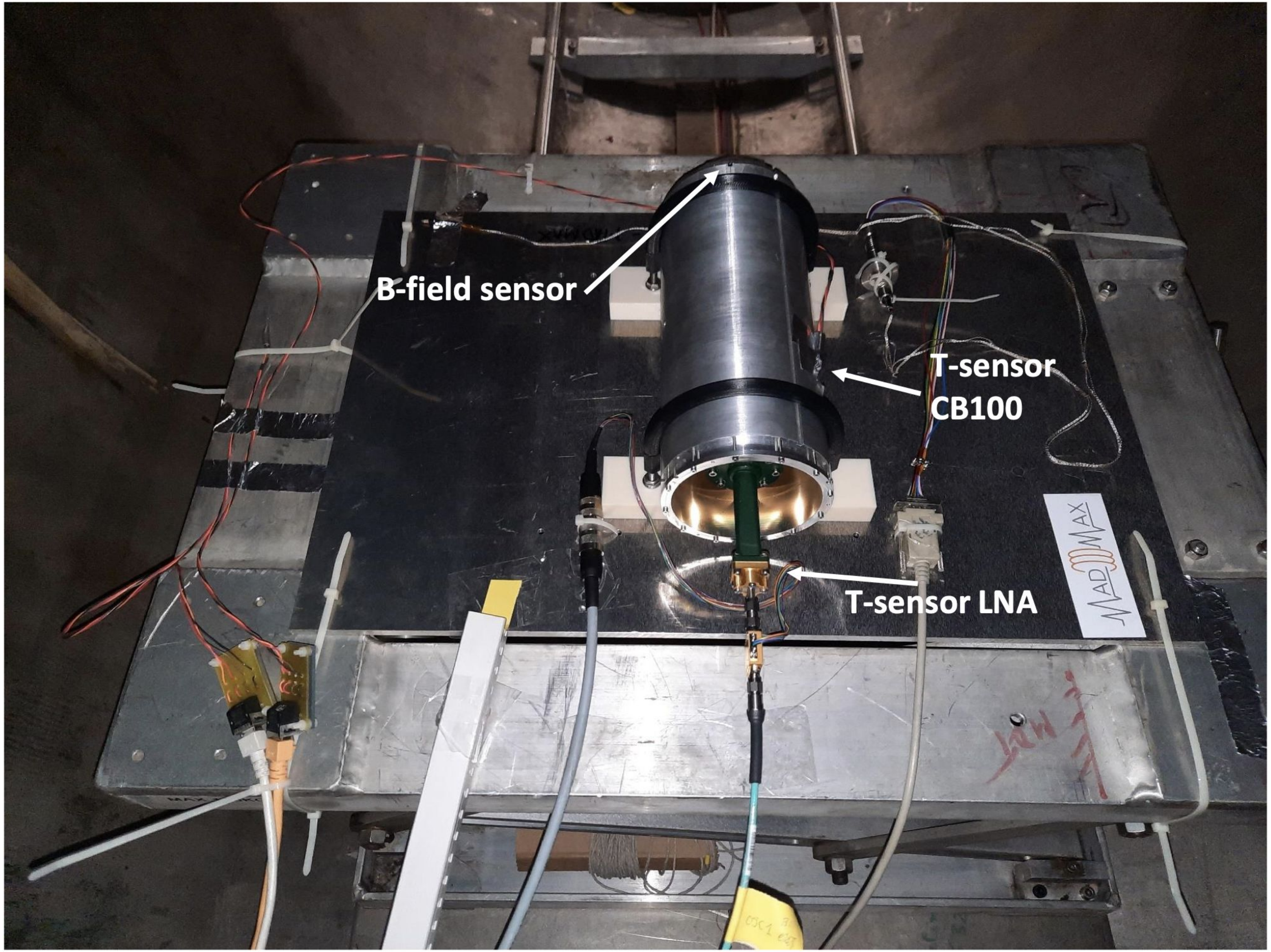
CB100 Setup and Measurements




CB100
Bernardo
Cable bridges
for RF cable
Derek
DANGER

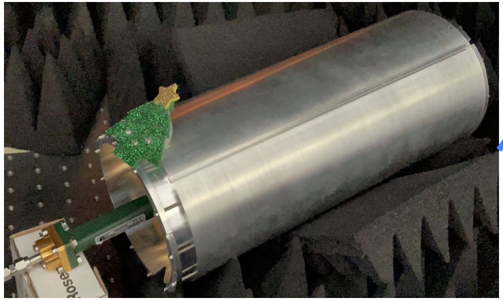

UPS for SA and LNA power supply
LNA power supply
Spectrum analyzer


Juan
Derek
Keithley &
Lakeshor for
T & B
measurement
Bernardo
Box with
calib.
standards
Couvaroff
ACCES INTERDIT

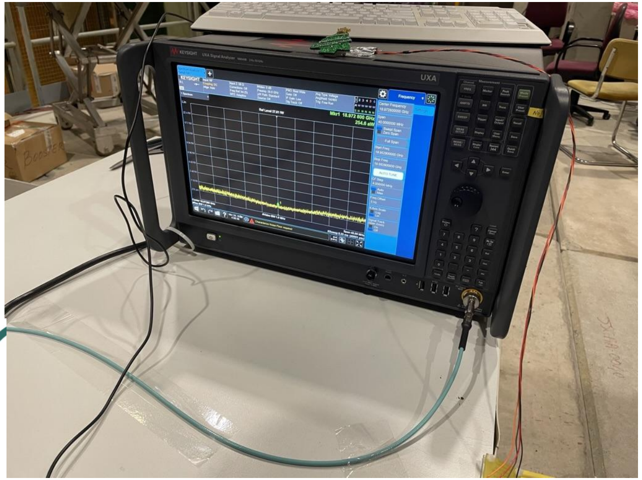

B-field sensor
T-sensor
CB100
T-sensor LNA
MADMAX

Setup of CB100:

Standards
+
Booster

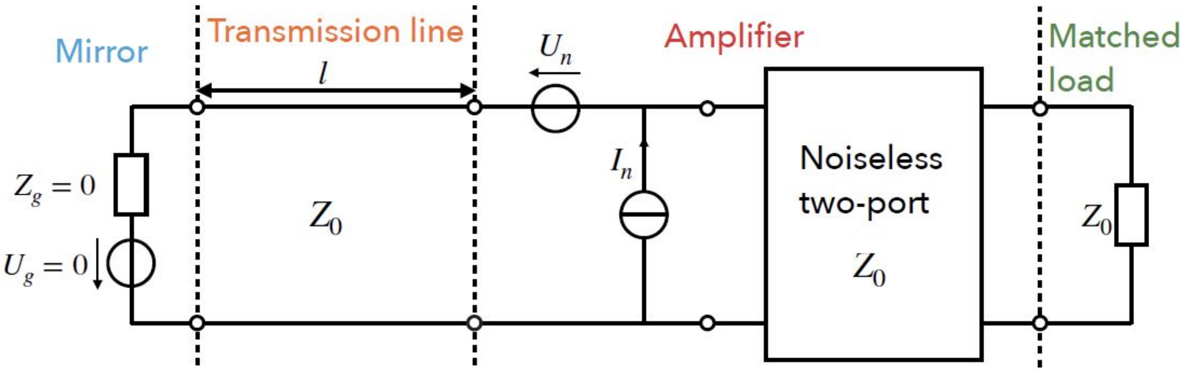


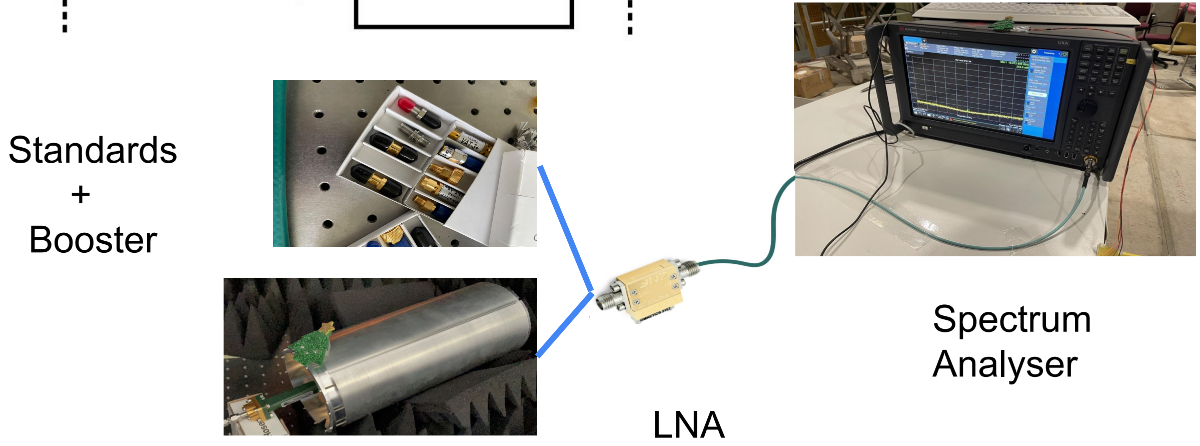
LNA



Spectrum Analyser

What do we need to measure?





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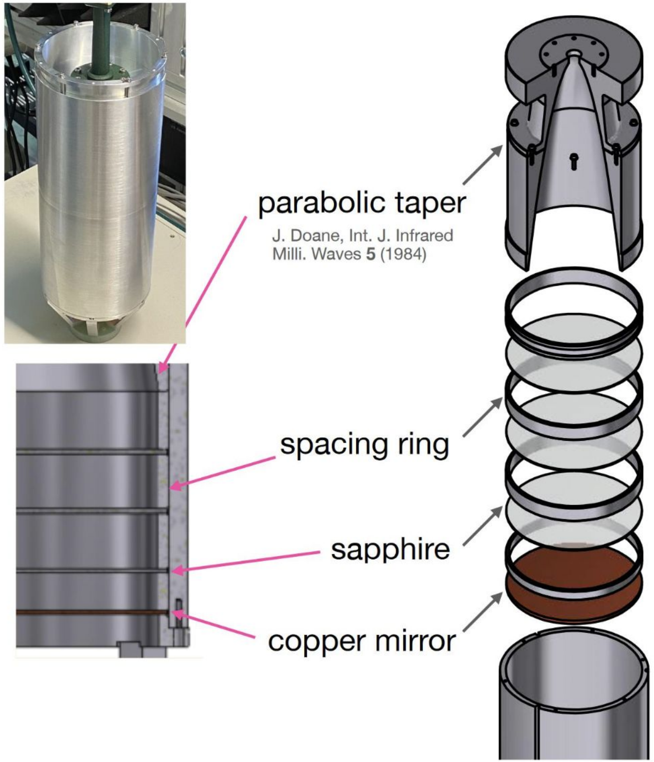
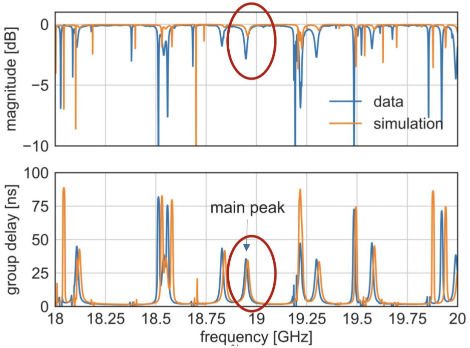
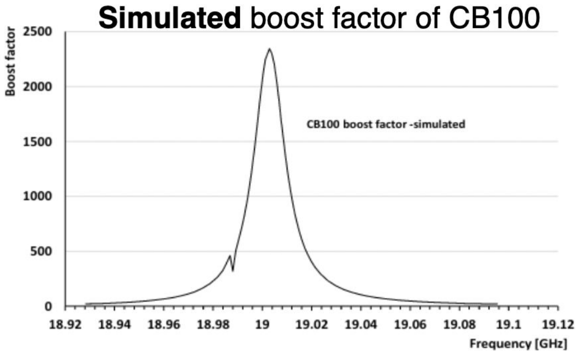
7 measurements in total!

CB100 Simulations and Measurements:



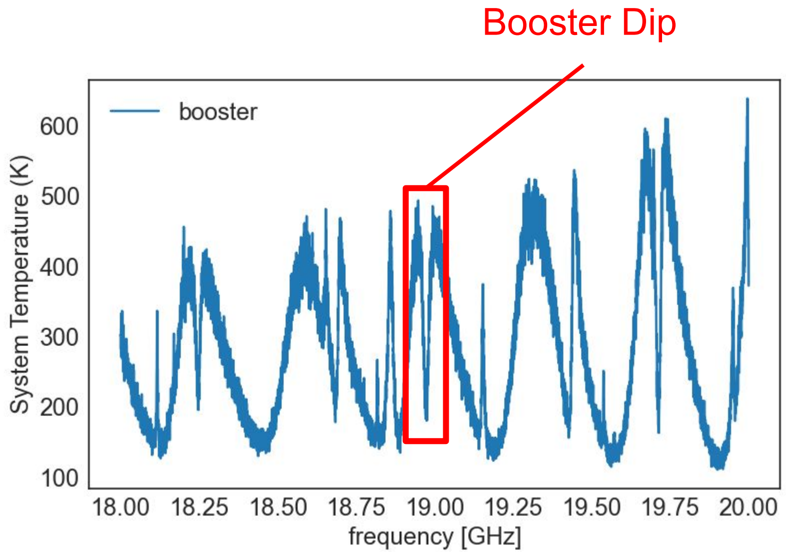
- 100 mm Sapphire Discs (ε = 9.4)
- Discs in fixed Position
- Optimized for Boost factor β^2~ 2000??
- 40 MHz bandwidth

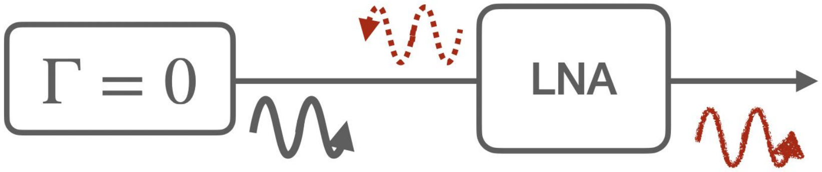


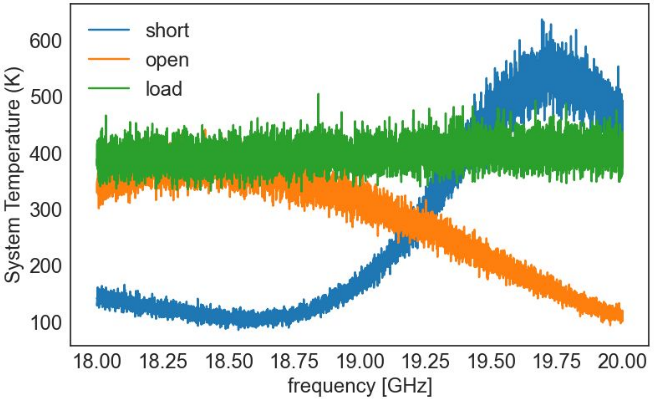


2 GHz Measurements:

- Gives a better perspective off frequency dependence of system
- Comparison with other measurements allows to see if something changed in booster

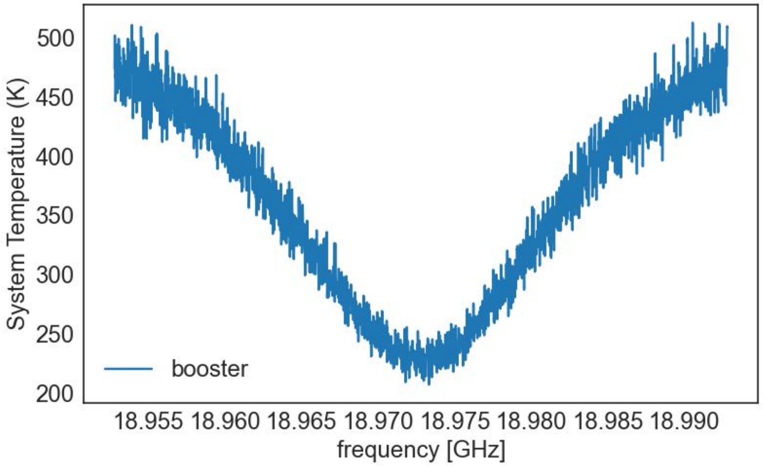


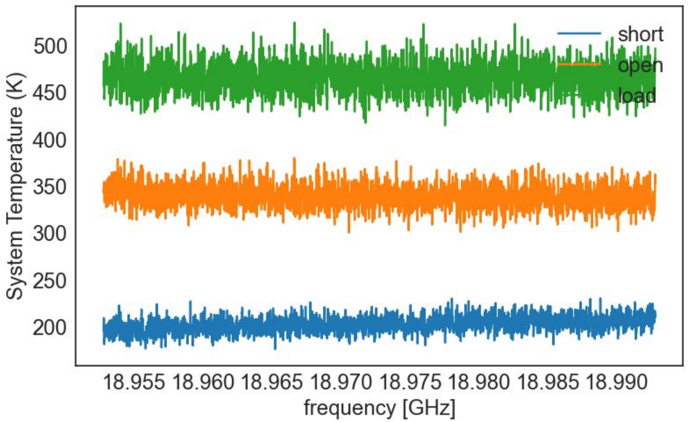




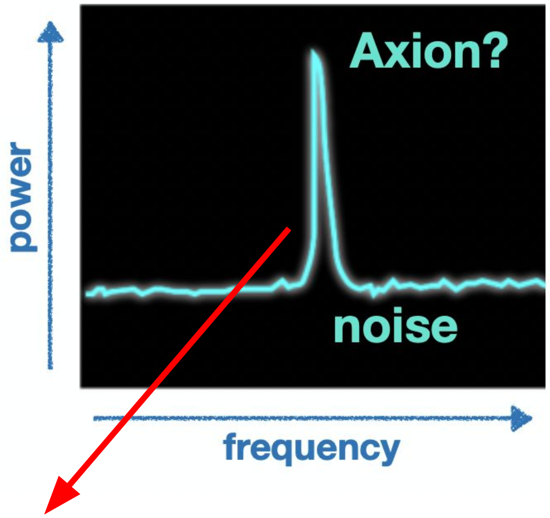
40 MHz Measurements:

- Long term measurement of 40MHz
- Center frequency ~ 18,97 GHz
- Booster dip in the middle of 40 MHz window

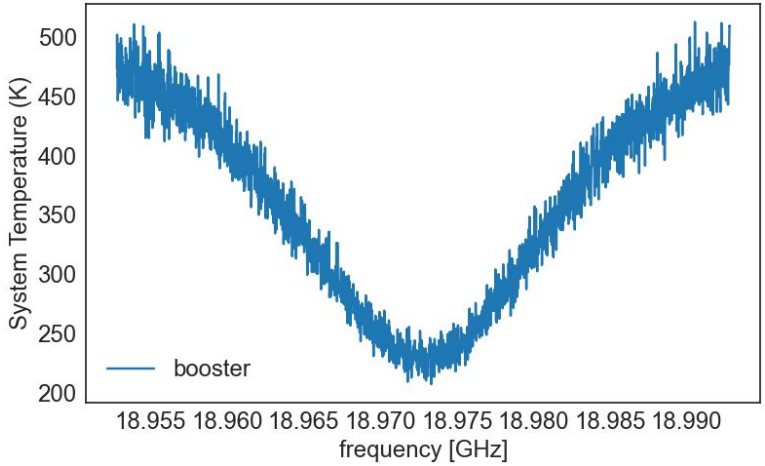


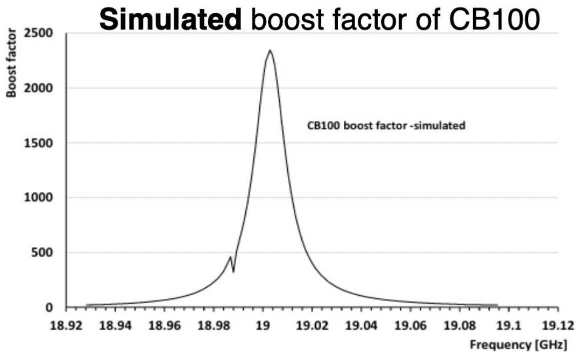


What does an Axion look like?



$\Delta\nu$ ≃ 20 kHz





Sensitivity of CB100 2023?

$$C_{a\gamma} = 234.3 \left(\frac{1m^2}{A}\right)^{1/2} \left(\frac{1.6T}{B_e}\right) \left(\frac{300MeV}{\rho_a}\right)^{1/2} \left(\frac{2200}{\beta^2}\right)^{1/2} \left(\frac{T_{sys}}{410}\right)^{1/2} \left(\frac{\Delta\nu}{20kHz}\right)^{1/4} \left(\frac{4days}{\Delta t}\right)^{1/4} \left(\frac{SNR}{5}\right)^{1/2}$$

How to do this?